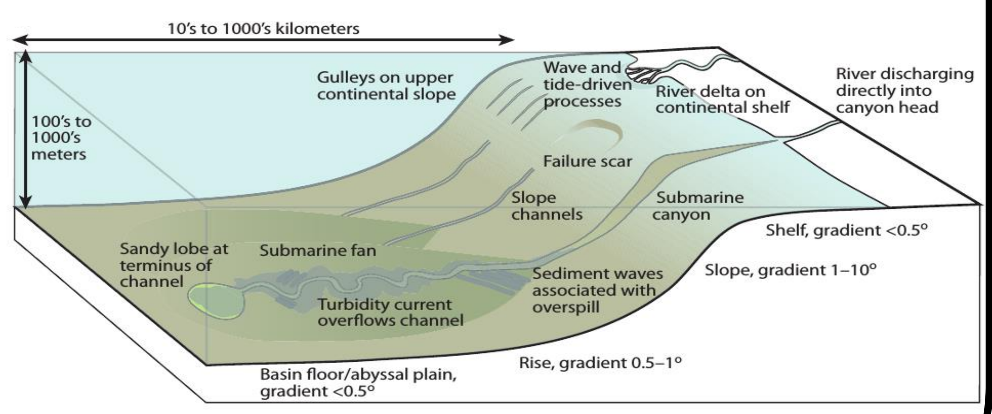
# An efficient cellular flow model for cohesive particle flocculation in turbulence

Kunpeng Zhao, Bernhard Vowinckel, Tom Hsu, Thomas Köllner, Bofeng Bai, Eckart Meiburg
Department of Mechanical Engineering, University of California, Santa Barbara, CA, USA
State Key Laboratory of Multiphase Flow in Power Engineering, Xi'an Jiaotong University, Xi'an 710049, China
Leichtweiß-Institut für Wasserbau, Technische Universität Braunschweig, 38106 Braunschweig, Germany
Center for Applied Coastal Research, Department of Civil & Environmental Engineering,
University of Delaware, Newark, DE 19716, USA

## Motivation and goals

- Manage estuaries and benthic habitats
- Underwater landslides
- Carbon cycle modeling
- Deep sea hydrocarbon exploration

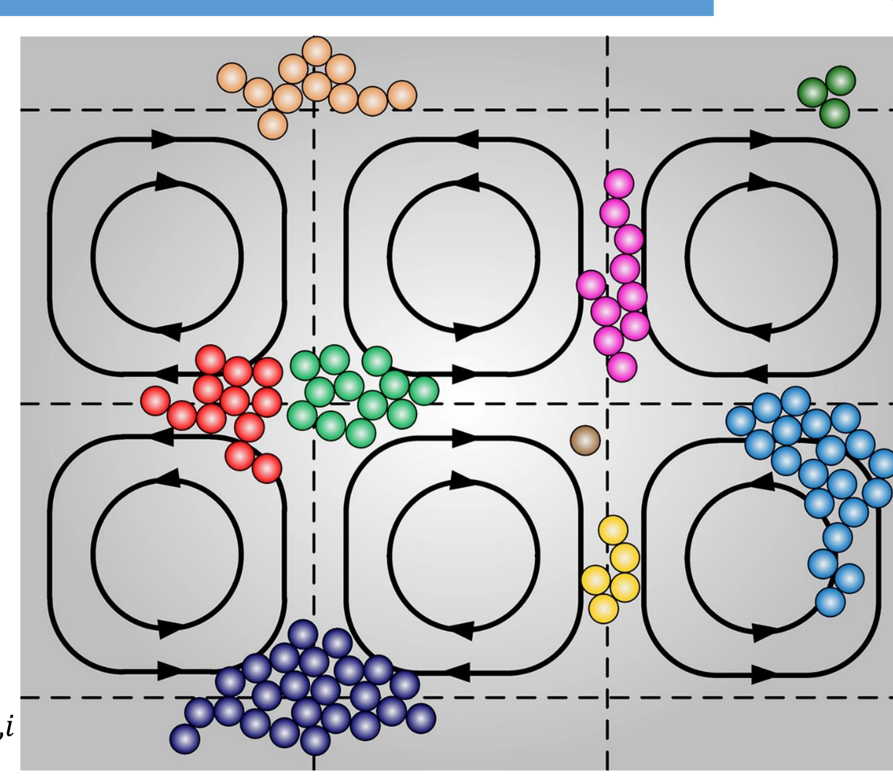
} Modelled by Population Balance Equations


[Meiburg & Kneller, 2010]

## Computational model

- One-way coupled point particles in Taylor-Green vortices
  [Zhao et al., 2020]
- 2D, steady, periodic
- Particle motion

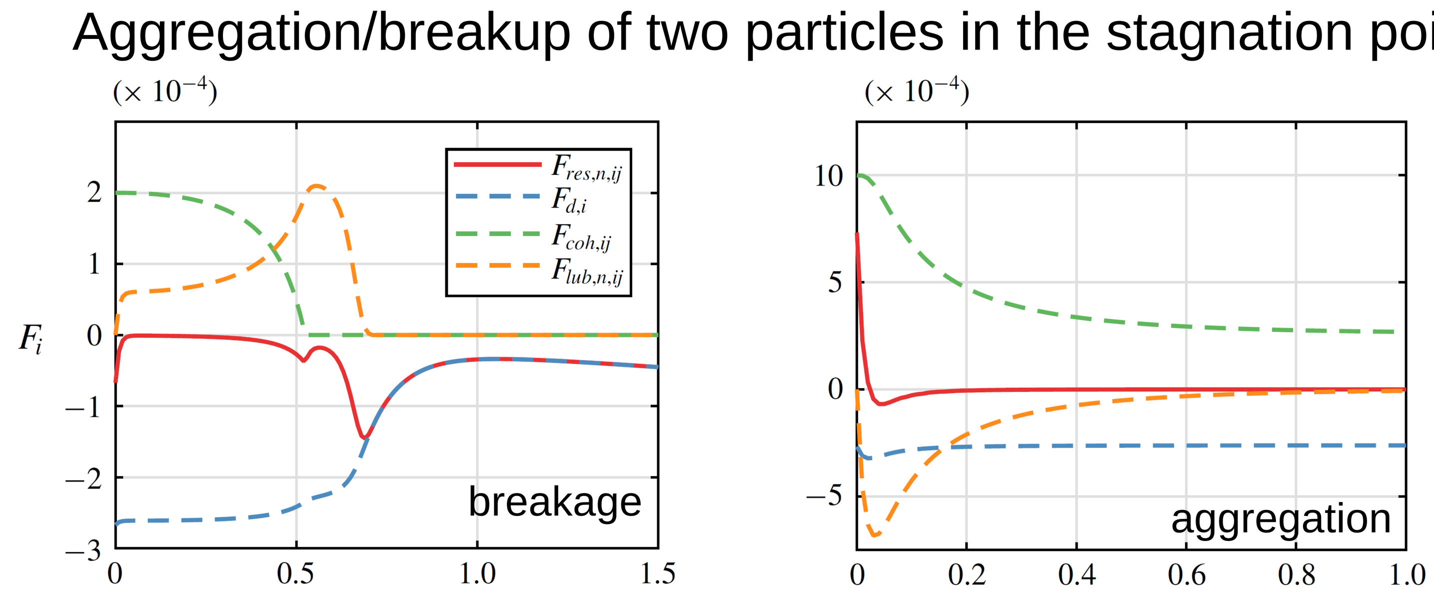


$$m_p \frac{d\mathbf{u}_{p,i}}{dt} = \mathbf{F}_{d,i} + \mathbf{F}_{g,i} + \mathbf{F}_{c,i} \qquad I_p \frac{d\boldsymbol{\omega}_{p,i}}{dt} = \mathbf{T}_{c,i}$$

$\mathbf{F}_{d,i}$… Hydrodynamic forces
$\mathbf{F}_{g,i}$… Buoyancy
$\mathbf{F}_{c,i}$… Particle interaction (contact, lubrication, cohesion)
$\mathbf{T}_{c,i}$… Torque due to contact and lubrication

[Biegert et al., 2017]
[Vowinckel et al., 2019a,b]

### Aggregation/breakup of two particles in the stagnation point




→ Smooth forcing throughout the flocculation process

**Acknowledgments:**

E.M. gratefully acknowledges support through NSF grants CBET-1803380 and OCE-1924655, as well as by the Army Research Office through grant W911NF-18-1-0379. T.-J.H. received support through NSF grant OCE-1924532. K.Z. is supported by the China Scholarship Council, as well as by the China National Fund for Distinguished Young Scientists through grant 51425603. B.V. gratefully acknowledges support through German Research Foundation (DFG) grant VO2413/2-1. Computational resources for this work used the Extreme Science and Engineering Discovery Environment (XSEDE), which is supported by NSF grant TG-CTS150053.

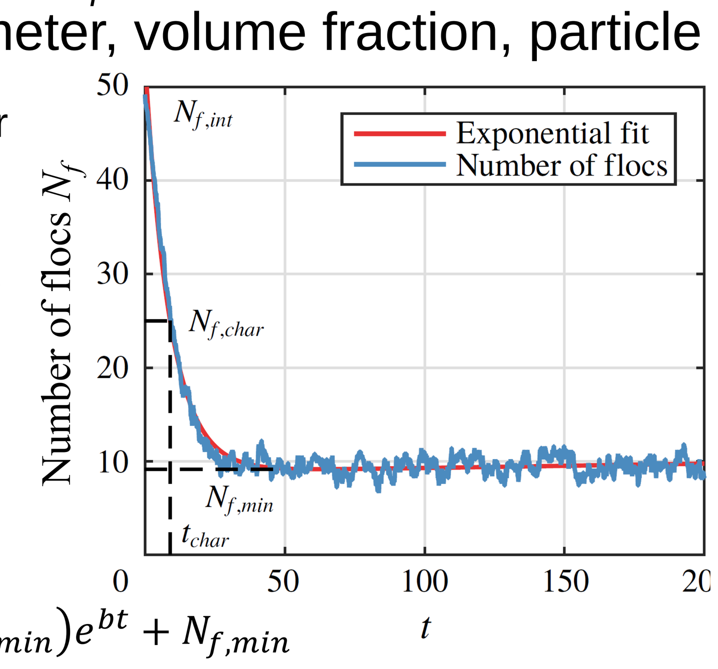
**References**:

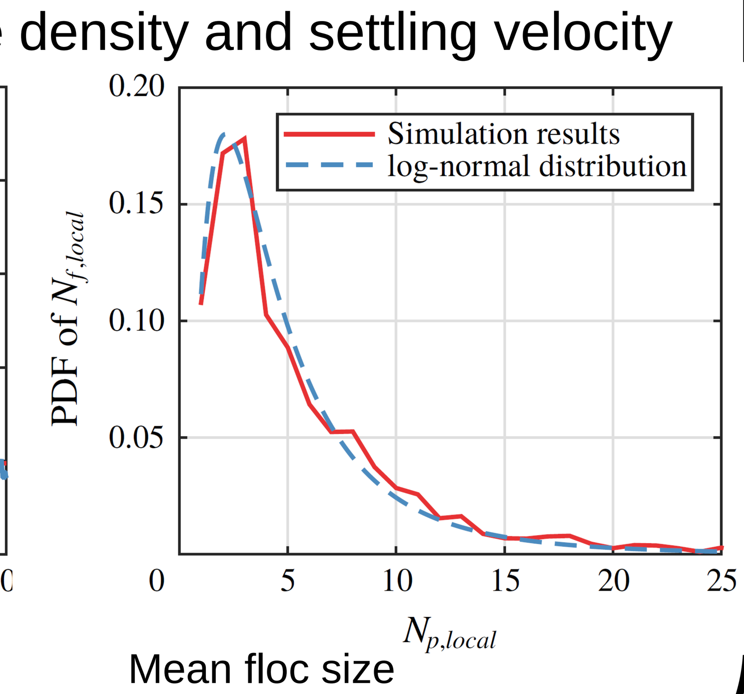
Biegert, E., Vowinckel, B., & Meiburg, E. (2017). *J. Comp. Phys.*, *340*, 105-127.
Kuprenas, R., Tran, D., & Strom, K. (2018). *J. Geophys. Res.: Oceans*, *123*(9), 6736-6752.
Tran, D., Kuprenas, R., & Strom, K. (2018). *Cont. Shelf Res.*, *158*, 1-14.
Meiburg, E., & Kneller, B. (2010). *Annu. Rev. Fluid Mech.*, *42*, 135-156.
Vowinckel, B., Biegert, E., Luzzatto-Fegiz, P., & Meiburg, E. (2019a). *Phys. Rev. Fluids*, *4*(7), 074305.
Vowinckel, B., Withers, J., Luzzatto-Fegiz, P., & Meiburg, E. (2019b). *J. Fluid Mech.*, *858*, 5-44.
Winterwerp, J. C. (1998). *J. Hydraul. Res.*, *36*(3), 309-326.
Zhao, K., Vowinckel, B., Hsu, T. J., Köllner, T., Bai, B., & Meiburg, E. (2020). *J. Fluid Mech*, R3, *889*.

## Flocculation statistics

300 runs with $N_p = 50$ particles to vary cohesiveness, Stokes number, particle diameter, volume fraction, particle density and settling velocity

→Aggregation over time
→Track $N_{p,local}$ number particles in a floc
→log-normal floc-size distribution at steady state




Exponential fit

$$N_f = (N_{f,int} - N_{f,min})e^{bt} + N_{f,min}$$

Mean floc size

$$\bar{N}_{p,local} = \frac{1}{(N_{f,int}/N_p - N_{f,min}/N_p)e^{bt} + N_{f,min}/N_p} \Rightarrow \bar{D}_f = (\bar{N}_{p,local})^{1/n_f} D_p$$

## Population balance

Fitting simulation results to find $N_{f,min}$ and $b$

$$\frac{N_p}{N_{f,min}} = 8.5a_1 St^{0.65} Co^{0.58} D_p^{-2.9} \phi^{0.39} \rho_s^{-0.49} (W+1)^{-0.38}$$

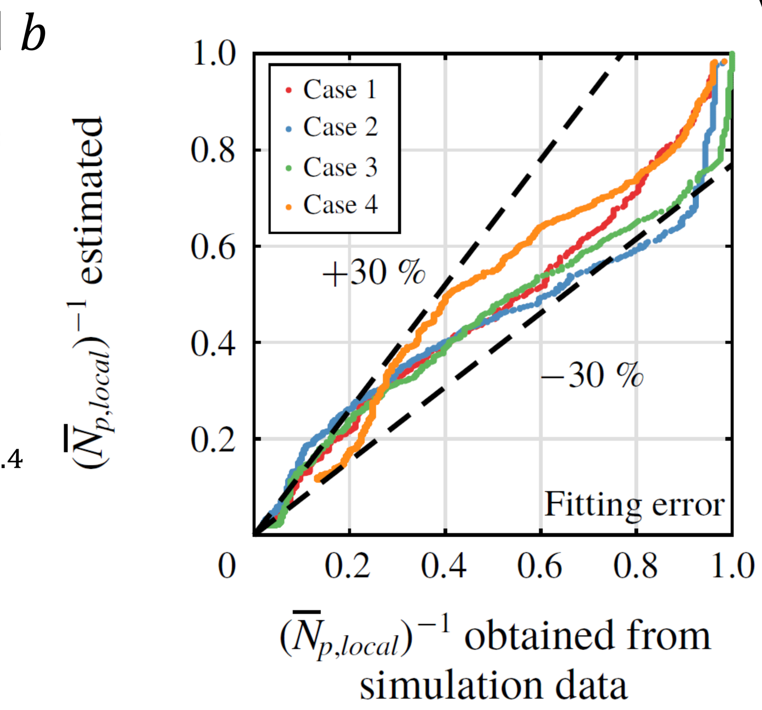
and

$$b = -0.7a_2 St^{0.36} Co^{-0.017} D_p^{-0.36} \phi^{0.75} \rho_s^{-0.11} (W+1)^{-1.4}$$

for $St \le 0.7$

$$b = -0.3a_2 St^{0.38} Co^{-0.0022} D_p^{-0.61} \phi^{0.67} \rho_s^{-0.033} (W+1)^{-1.4}$$

for $St > 0.7$




Yields good prediction within ±30% bound for $a_1 = a_2 = 1$

Application to experimental data:

- Recalibrate $a_1 = 500$ and $a_2 = 35$
- Comparison to models by Winterwerp (1998)

$$\frac{d\bar{D}_f}{dt} = \frac{k_A' D_p^{n_f-3}}{n_f \rho_p} Gc\bar{D}_f^{4-n_f} - \frac{k_B'}{n_f}\left(\frac{\mu_f}{F_y}\right)^q \left(\frac{\bar{D}_f - D_p}{D_p}\right)^p G^q \bar{D}_f^{2q+1}$$

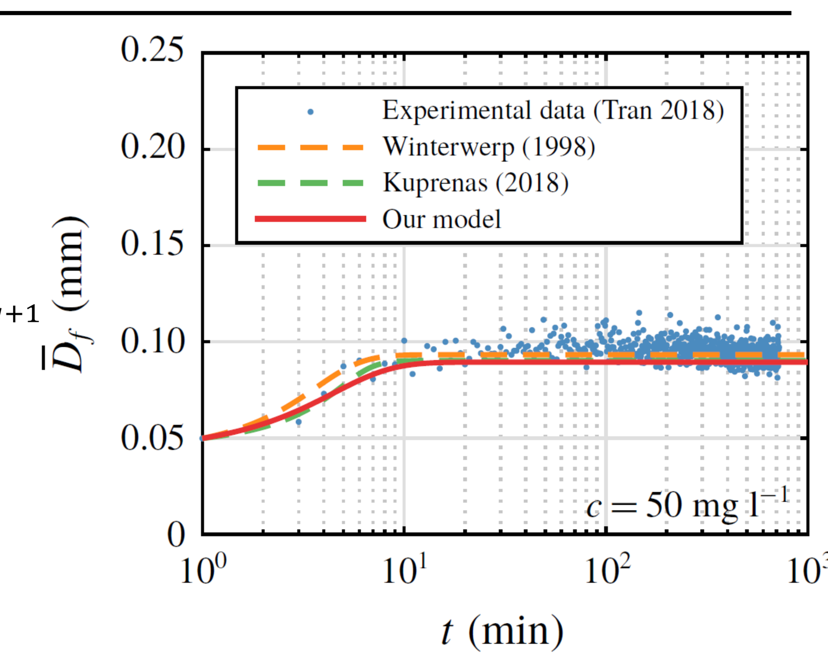
$q = 0.5$

with recalibrated $k_A' = 1.35$; $k_B' = 1.29 \times 10^{-5}$

and modified model by Kuprenas et al. (2018)

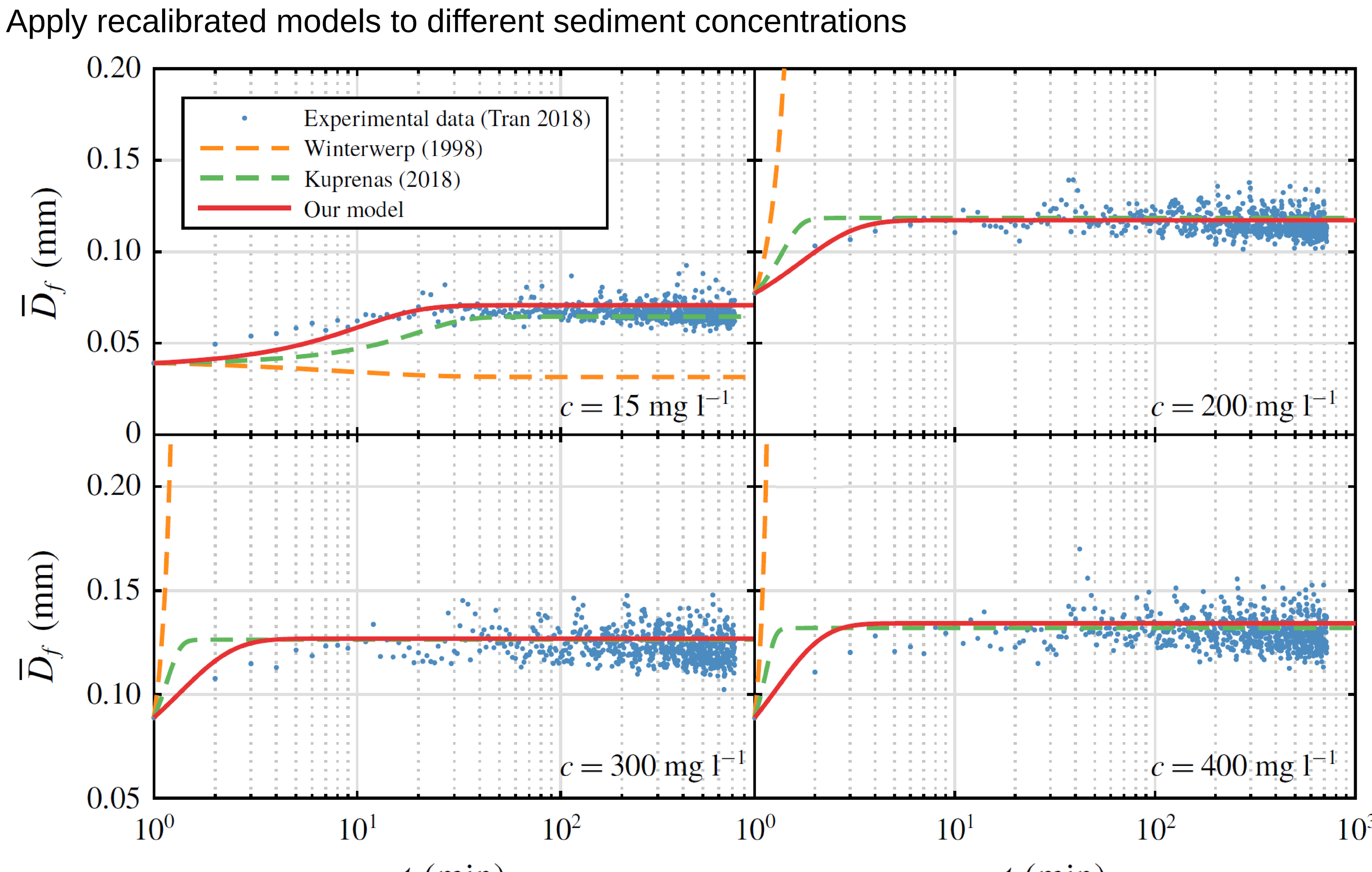
$q = 0.5 + 1.5\bar{D}_f/\eta$

with recalibrated $k_A' = 0.45$; $k_B' = 1.16 \times 10^{-6}$




Validation:

Apply recalibrated models to different sediment concentrations




Excellent agreement with experimental data, especially during transient stages