Does lateral mixing really take place along neutral surfaces in the double-diffusive regions of the ocean?

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EGU online, 04 May 2020
Evidence for isopycnal control of ocean water masses

Antarctic Intermediate Water (AIW) seems to follow isopycnal surfaces (defined here in terms of $\sigma_1$)

Idea dating back to Iselin, Montgomery etc...
Consequence for mixing and stirring of tracers

\[ \overline{v'C'} = -\mathbf{K} \nabla \overline{C} \]

Redi (1982) key assumption: Symmetric part of mixing tensor \( \mathbf{K} \) is diagonalized by the isopycnal and diapycnal directions

\[ \frac{1}{2} (\mathbf{K} + \mathbf{K}^T) \nabla_i \overline{C} = K_i \nabla_i \overline{C} \]

Isopycnal mixing coefficient

\[ \frac{1}{2} (\mathbf{K} + \mathbf{K}^T) \nabla_d \overline{C} = K_d \nabla_d \overline{C} \]

Turbulent diapycnal mixing coefficient
Consequence for mixing and stirring of tracers

\[ \overline{\nu'C'} = -\mathbf{K} \nabla \overline{C} \]

Redi (1982) key assumption: Symmetric part of mixing tensor \( \mathbf{K} \) is diagonalized by the isopycnal and diapycnal directions

\[ \frac{K_i}{K_d} = O(10^7) \]

Essential to get the isopycnal directions right to avoid spurious diapycnal mixing!!!
Numerical evidence for isopycnal mixing
Abernathey, Ferreira and Klocker, 2013

MITgcm, Linear equation of state, function of T only, no salinity

Mixing directions, obtained by diagonalizing the mixing tensor (black)

\[
\begin{bmatrix}
\overline{v'c_1} & \overline{v'c_2} & \ldots & \overline{v'c_6} \\
\overline{w'c_1} & \overline{w'c_2} & \ldots & \overline{w'c_6}
\end{bmatrix}
= -\begin{bmatrix}
K_{yy} & K_{yz} \\
K_{zy} & K_{zz}
\end{bmatrix}
\begin{bmatrix}
\partial \overline{c_1}/\partial y & \partial \overline{c_2}/\partial y & \ldots & \partial \overline{c_6}/\partial y \\
\partial \overline{c_1}/\partial z & \partial \overline{c_2}/\partial z & \ldots & \partial \overline{c_6}/\partial z
\end{bmatrix}
\]

Isopycnal directions (white)
McDougall (1987): Potential density surfaces are very sensitive to choice of reference pressure and may exhibit considerable differences.

Cause: Thermobaricity

(Pressure dependence of thermal expansion coefficient)
McDougall (1987): Potential density surfaces are very sensitive to choice of reference pressure and may exhibit considerable differences.

Cause: Thermobaricity

Solution: Neutral Density?

Nova Scotia

Straits of Gibraltar
Neutral density: What is it?

General problem: Density-spiciness ($\gamma$, $\xi$) re-mapping of $(S, \theta)$ space

$$\rho = \rho(S, \theta, p) = \hat{\rho}(\gamma, \xi, p)$$
Neutral density: What is it?

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\[
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Neutral Density \(\gamma^N \approx \gamma^N(S, \theta)\) is only density-like variable satisfying

\[
\rho \approx \hat{\rho}(\gamma^N, p) \quad \frac{\partial \hat{\rho}}{\partial \xi} \approx 0
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$\gamma^N$ makes seawater look like a single component fluid with all attendant benefits: materially conserved PV, perfect predictor of vertical stability and shear, zero buoyancy surface, geostrophic streamfunction, etc...
Looking for observational evidence for superiority of neutral density over potential density (Pingree 1972)

Spread in \((\theta, S)\) properties reduced on potential density surfaces referenced to reference pressures close to actual pressures

Question: Are neutral surfaces minimizing spread in \((\theta, S)\) properties?
McDougall (1987) analyzed the problem and discussed many examples where spread in $(\theta, S)$ properties is less on neutral surfaces than on potential density surfaces.

Isopycnal gradients can be as much as 4 times larger on potential density surfaces than on neutral surfaces.
IF ONLY LIFE WAS THAT SIMPLE…
\[
\frac{\nabla_{\sigma_4} \theta}{\nabla_h \theta}
\]

\[
\frac{\nabla_{\sigma_4} S}{\nabla_h S}
\]

\[\gamma^N = 27.9366\]

27W, 30N, 1750 m

Isopycnal gradients (using \(\sigma_4\)) of \(\theta\) and \(S\) can be reduced by as much as a factor of 2 relative to isoneutral gradients about 2000 dbar deeper than actual pressure.
\[ \theta \text{ on } \gamma^N = 27.9366 \]

\[ \theta \text{ on } \sigma_4 = 45.5302 \]
Density surfaces and Turner angle

- Differences between density variables exacerbated in salt finger ($45 < T_u < 90$)
- Density differences minimal in doubly-stable regions ($-45 < T_u < 45$)
Theory: generalization of McDougall (1987a,b)

\[
\frac{\nabla_{\gamma} \theta}{\nabla_{n} \theta} = \frac{c(R_\rho - 1)}{R_\rho - c} \quad \frac{\nabla_{\gamma} S}{\nabla_{n} S} = \frac{R_\rho - 1}{R_\rho - c}
\]

\[
R_\rho = \frac{\alpha \theta_z}{\beta S_z}
\]

\[
c = \frac{\gamma_s}{\gamma_\theta} (S, \theta) \frac{\rho_\theta}{\rho_s} (S, \theta, \rho)
\]

Doubly stable:
\(\gamma^N\) better than \(\sigma(S, \theta, \rho_r)\) for S but not for \(\theta\) or vice versa

Doubly diffusive:
There is always a \(\sigma(S, \theta, \rho_r)\) variable better than \(\gamma^N\)
Explanation for van Sebille et al. (2011) finding that $\sigma_2$ outperforms $\gamma_N$ for tracing Labrador Sea Water to Abaco Line in Western Boundary Current?

Van Sebille et al. (2011)
Summary

- Neutral density is optimal for making seawater look like a simple component fluid with all attendant benefits. Optimality for predicting vertical stability, vertical shear, conservative PV, etc...
  
  => Justify the construction of globally defined density variable maximizing neutrality as discussed by Trevor McDougall and Geoff Stanley in next 2 talks
Summary

• Neutral density is optimal for making seawater look like a simple component fluid with all attendant benefits. Optimality for predicting vertical stability, vertical shear, conservative PV, etc...

• However, neutral density is not optimal for the following properties that are more relevant to stirring and mixing:
  • Minimising the spread in \((\theta, S)\) properties
  • Minimising the energy cost of adiabatic and isohaline parcel exchange (material surfaces exist for which energy cost is negative (Tailleux, 2016))

• Implications: Assumption that neutral rotated diffusion tensor is diagonal questionable

• Double-diffusive region: testbed for mixing parameterisations
Data/Ocean State Estimate misfit for T at 300 m

Forget et al. (2015)

Sensitivity of Stratification to
(a) Mesoscale eddy
(b) Isopycnal Mixing
(c) Diapycnal Mixing
"It does not matter how beautiful your theory or how smart you are. If it does not agree with experiments, it’s wrong."

Richard P. Feynman
I would rather have questions that can't be answered than answers that can't be questioned.

— Richard P. Feynman —