Non-linear resonant instability of short surface waves as the first stage "bag-breakup" process at the air-sea interface at high winds

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Introduction

The recent experimental study [1], [2] identify **"bag breakup"** fragmentation as the **dominant mechanism** by which spume droplets are generated at hurricane wind speeds. These droplets can **significantly** affect the exchanging processes in the air-ocean boundary layer. In order to estimate spray-mediated heat, momentum and mass fluxes we need not only reliable experimental data, but a **theoretical model** of this process.

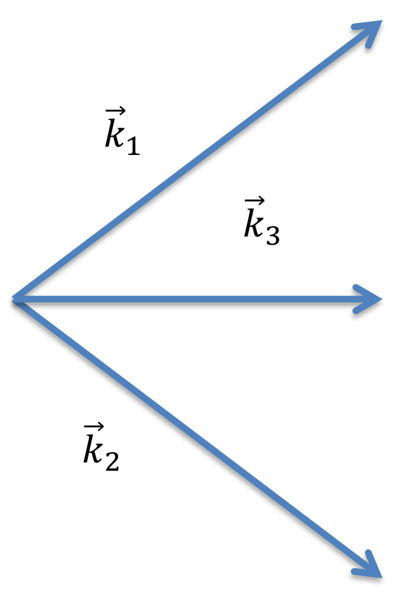
The "bag-breakup" fragmentation is a strongly **non-linear** process, and we focus only on its first stage which includes the small-scale elevation of the water surface.

Our model of the **bag's initiation** is based on a weak nonlinear interaction of a **longitudinal surface wave** and **two oblique waves** propagating at equal and opposite angles to the flow as it was done in [3], [4].

Model

Let's consider the piecewise velocity profile as

$$\bar{u}(z) = \begin{cases} z & -1 \le z \le 0 \\ -1 & -\infty \le z \le -1 \end{cases},$$



which was along OX axis. All quantities are made dimensionless relative to the stream velocity $\boldsymbol{V}$, density $\boldsymbol{\rho}$ of the fluid and the thickness $\boldsymbol{\delta}$ of the turbulent boundary layer.

We assume that the interaction occurs between three waves: two oblique, which propagate at the same angle to the wind flow and one longitudinal wave.

Assuming the fluid incompressible and inviscid, the components of the velocity perturbation should satisfy the non-linear equations of motion:

$$\partial_t v_x + u_0 \partial_x v_x + v_z u_0' + \frac{1}{\rho}\partial_x p = -(\vec{v}, \nabla) v_x$$

$$\partial_t v_z + u_0 \partial_x v_z + \frac{1}{\rho}\partial_z p = -(\vec{v}, \nabla) v_z$$

with non-linear boundary conditions:

1) Dynamic boundary conditions (taking into account capillary effect and gravity)

$$-p\Big|_{z=\eta_1} + (G\eta + \Sigma\Delta_\perp \eta) = 0$$

$$p\Big|_{z=-1+\eta_2} - p\Big|_{z=-1-\eta_2} = 0$$

2) Kinematic boundary conditions at $z_0 = -0; -1 \pm 0$

$$\partial_t \eta + (\vec{v}_\perp\Big|_{z=z_0+\eta}, \nabla_\perp)\eta = v_z\Big|_{z=z_0+\eta}$$

3) Decay at infinity

$$v, v' \to 0 (z \to -\infty)$$

First order

We defined velocity perturbation, using the perturbation steam function

$$v_{j_x} = \partial_z \Psi_j \quad v_{j_z} = -\partial_{x_j}\Psi_j = -ik_j\Psi_j$$

We seek a uniform expansion by using the method of multiple scales in the form

$$\psi_j(\vec{r}, t) = \varepsilon\psi_j^{(1)}(\vec{r}, t, \varepsilon t, \dots) + \varepsilon^2\psi_j^{(2)}(\vec{r}, t, \varepsilon t, \dots) + \cdots$$

First order of the method of multiple scales gives us that all waves propagate independently, and we obtain a homogeneous system of algebraic equations for each wave in the following form

$$-i\omega_j\eta_j^{(1)} - \Phi_{1-j}^{(1)}e^{-k_j} - \Phi_{1+j}^{(1)}e^{k_j} = 0$$

$$-i(\omega_j + k_j\cos\alpha_j)\eta_{w_j}^{(1)} - \Phi_{1-j}^{(1)} - \Phi_{1+j}^{(1)} = 0$$

$$-i(\omega_j + k_j\cos\alpha_j)\eta_{w_j}^{(1)} - \Phi_{2+j}^{(1)} = 0$$

$$\Phi_{1-j}^{(1)}(-\omega_j + \cos\alpha_j)e^{-k_j} + \Phi_{1+j}^{(1)}(\omega_j + \cos\alpha_j)e^{k_j} - (G + \Sigma k_j^2)k_j\eta_j^{(1)} = 0$$

$$-\Phi_{2+j}^{(1)}(\omega_j + k_j\cos\alpha_j) + \Phi_{1w-j}^{(1)}(-\omega_j + (1-k_j)\cos\alpha_j) + \Phi_{1w+j}^{(1)}(\omega_j + (1+k_j)\cos\alpha_j) = 0$$

where $j = 1,2,3$.

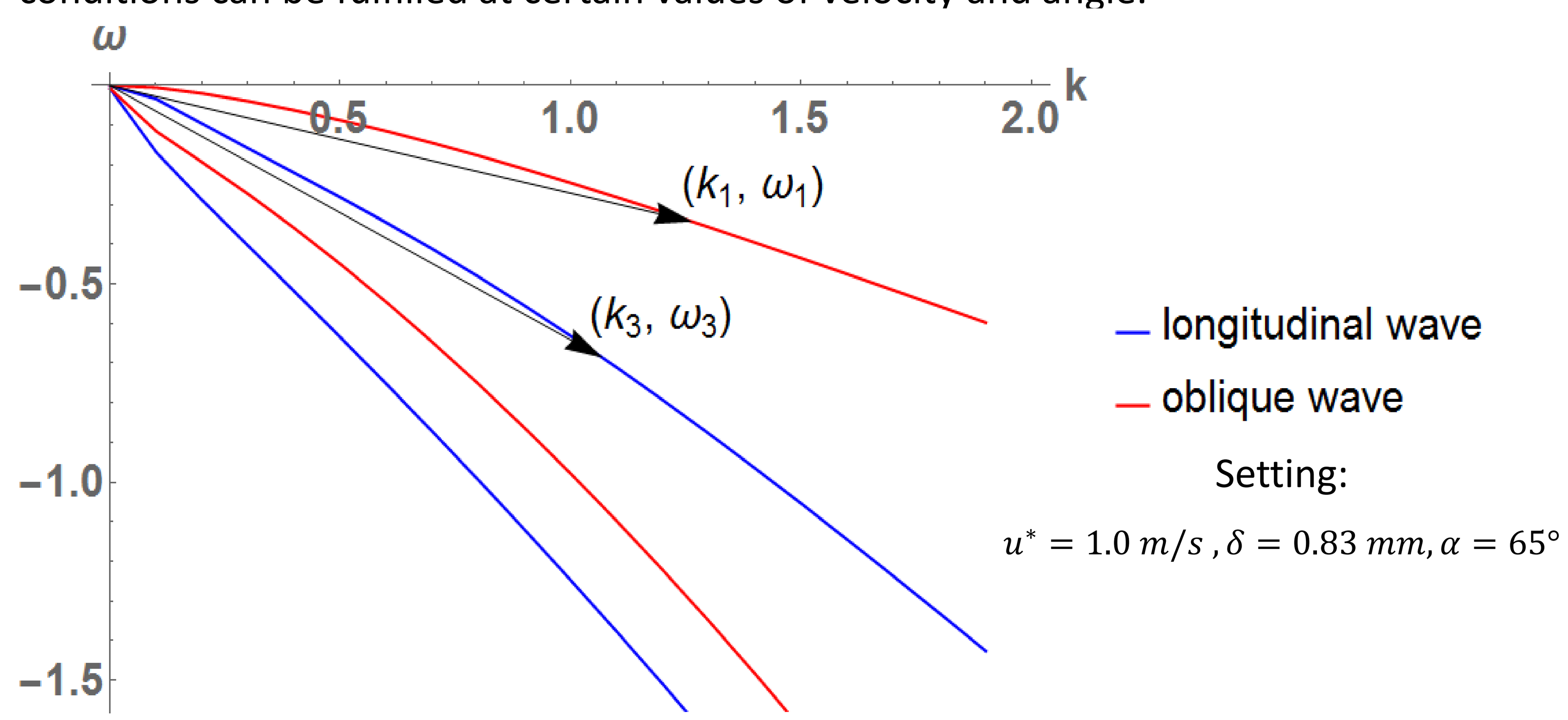
It can be rewritten as $L(\omega_j, k_j, \alpha_j)\boldsymbol{X}_j^{(1)} = 0$

For a nontrivial solutions, the determinants of the coefficient matrices must be zero; that is $\det L(\omega_j, k_j, \alpha_j) = \mathcal{D}(\omega_j, k_j, \alpha_j) = 0$ – dispersion relation for each of the waves.

$$\mathcal{D}(\omega_j, k_j, \alpha_j) = \omega_j^2 \frac{\cos\alpha_j \cosh k_j - (\omega_j + k_j\cos\alpha_j)e^{k_j}}{\cos\alpha_j \sinh k_j - (\omega_j + k_j\cos\alpha_j)e^{k_j}} + \omega_j\cos\alpha_j - (G + \Sigma k_j^2)k_j = 0$$

Dispersion relations

We assumed that the amplitudes of oblique waves are the same. For effective interaction of these waves, the synchronism conditions $k_3 = 2k_1\cos\alpha$, $Re\,\omega_3(k_3) = Re\,2\omega_1(k_1)$ must be satisfied. Analysis of the dispersion relations shows that these conditions can be fulfilled at certain values of velocity and angle.



Second order

To derive the three-wave interaction system, it is necessary to take into account second order terms, but the term $\psi_j^{(2)}$, unlike $\psi_j^{(1)}$, contains not only an irrotational part, but a rotational one. As a result, the stream function can be written in the following way

$$\psi_j(\vec{r}, t) = \varepsilon A_j(\varepsilon t)\phi_j^{(1)}(z)\exp[ik_jx_j - i\omega_jt] + \varepsilon^2(\phi_j^{(2)}(z) + i\frac{W_j(z)}{k_j})\exp[ik_jx_j - i\omega_jt]$$

Considering the nonlinearity of the boundary conditions, we get an inhomogeneous system of equations for each of the waves in the form: $L(\omega_j, k_j, \alpha_j)\boldsymbol{X}_j^{(2)} = \boldsymbol{b}_{w_j} + \partial_\tau A_j(\tau)\boldsymbol{b}_{\tau_j} + \boldsymbol{b}_{nl_j}$

Their homogeneous parts will have a nontrivial solution if their right-hand sides are orthogonal to the solution of the adjoint homogeneous problem.

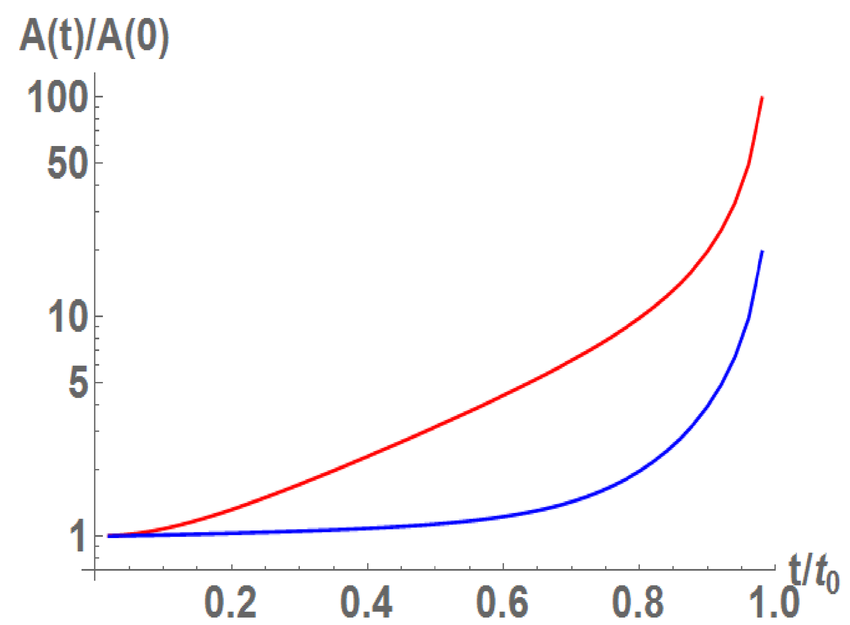
It could be shown that $\left(\boldsymbol{b}_{\tau_j}, \boldsymbol{u}_j\right) \propto \frac{\partial\mathcal{D}(\omega_j, k_j, \alpha_j)}{\partial\omega_j}$ and $\left(\boldsymbol{b}_{w_j}, \boldsymbol{u}_j\right) = \int_{-1}^{0}(W_j'' - W_j)\phi_j^{(1)}(z)\,dz$

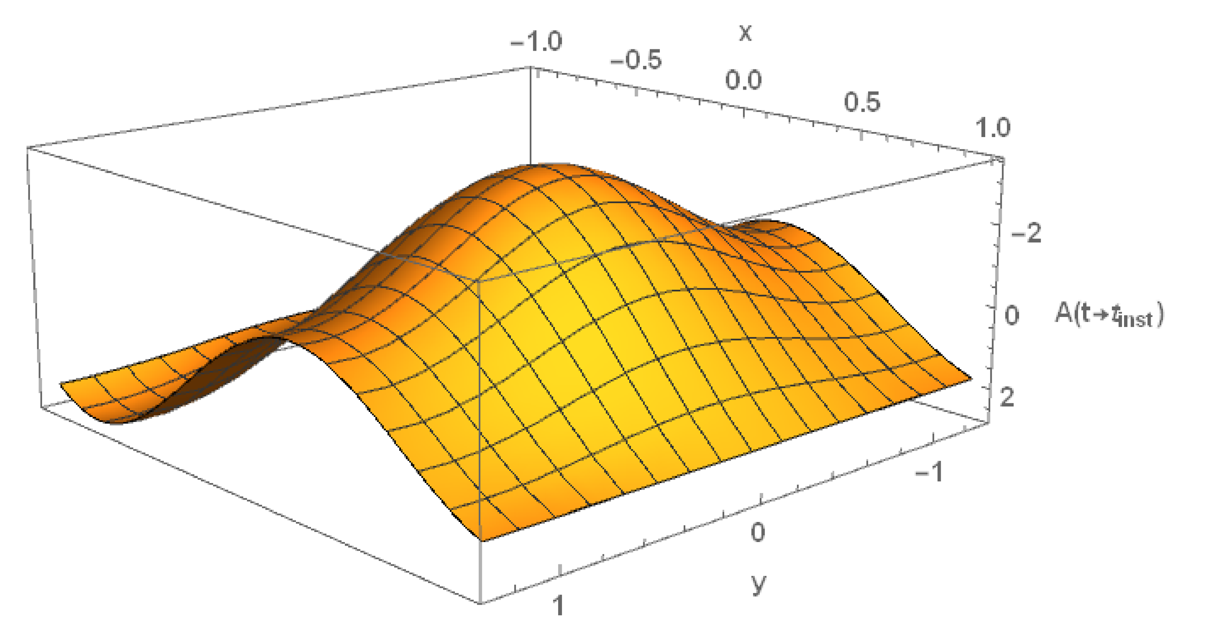
The irrotational term $W_j(z)$ is determined from the equations of motion of the second order in the inviscid limit. However, in this case, the integral will have a high-order pole in the critical layer at $z = z_0$ $(-1 \le z_0 \le 0)$. The solution of the viscous problem for a purely shear flow is described in detail in [4] and all mathematical calculations for present study almost coincide with [4]. The evaluation of the last term $\left(\boldsymbol{b}_{w_j}, \boldsymbol{u}_j\right)$ is quite straightforward. That resulted in the equations describing the modulation of amplitudes A_1, A_3 as follow:

$$\frac{dA_1}{dt} = \Lambda_1 A_3 A_1^*$$

$$\frac{dA_3}{dt} = \Lambda_3 A_1^2$$

Numerical simulation of the receiving system demonstrates explosive growth within finite time t_{inst}.





The dynamics of the simulated structure is similar to the initial stage of the "bag-breakup" phenomenon. To compare the experimental data and our model, we define the transverse scale of the structure formed by three surface waves as $L_\perp = \frac{2\pi\delta}{k\sin\alpha} = \frac{4\pi\delta}{k_3\tan\alpha}$, where we come back to dimensional variables.

Numerical calculations showed the transverse scale of the most unstable triads $L_\perp$ and its time of growth t_{inst} have following dependencies on the friction velocity

$$L_\perp \propto u_*^{-1.23} \quad t_{inst} \propto u_*^{-1.89}$$

As it was reported in [1] the experimental data give us the following dependencies for size of bags and their lifetime

$$< R_1 > \propto u_*^{-1} \quad < \tau > \propto u_*^{-2}$$

Outlook

In the future research, we will continue develop this model and will focus on more thorough comparison with the "bag-breakup" instability.

References

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