Rapid Global Finite-Frequency Ambient Noise Source Inversion

Jonas Igel¹ (jonas.igel@erdw.ethz.ch), Laura Ermert², Andreas Fichtner¹ ¹Institute of Geophysics, ETH Zurich; ²Department of Earth and Planetary Sciences, Harvard University

1. Introduction

Full waveform ambient noise tomography methods (e.g. Sager et al., 2017) are able to circumvent some of the main assumptions made in ambient noise seismology (e.g. that the cross-correlation is equal to the Green's function) if the heterogeneous noise source distribution is known. Efficient forward modelling of global cross-correlations $C(x_1, x_2)$ in the microseismic frequency range (currently up to 0.2 Hz) is now possible since we have optimised the cross-correlation modelling by implementing:

- Green's functions *G* from pre-computed wavefields (e.g. AxiSEM, Nissen-Meyer et al. 2014)
- Power spectral density of noise distribution S on spatially variable grids

We use a gradient-based iterative method, employing finite-frequency sensitivity kernels to **rapidly** invert for the power spectral density of the noise source distribution of the secondary microseisms. Since the mechanism behind the secondary microseisms is well understood (e.g. Ardhuin et al., 2011), data inversions can be seen as a new observable of the ocean state and could improve full waveform ambient noise tomography methods.

2. Forward Model and Gradient

A spatially variable grid enables us to reduce the number of grid points by a factor of 5 and still have a high spatial resolution in our area of interest. Combined with pre-computed Green's function databases, we can forward model global cross-correlations and sensitivity kernels within a few seconds (e.g. Ermert et al., 2017):

 $C(x_1, x_2) = \int G(x_1, \xi) [G^*(x_2, \xi) S(\xi)] d\xi$



Figure 2: Synthetic source sensitivity kernel with corresponding cross-correlation and measurement windows



The sensitivity kernel K for each station pair is a simple multiplication of a measurement-dependent adjoint source f(w) and the Green's functions in the frequency domain.

We measure the logarithmic energy ratio between the causal and acausal branch of the cross-correlations (see Figure 2). This is mostly not affected by unknown 3D Earth structures (Sager et al., 2018).

3. Inversion framework

Our inversion framework is based on the gradient-based iterative method employing finite-frequency sensitivity kernels (e.g. Ermert et al., 2017). Usually a homogeneous noise distribution in the ocean is used as the initial model.



Figure 1: Homogeneous and spatially variable grid adapted to data

$K(\xi,\omega) = 2f(\omega) \cdot (G^*(x_1,\xi,\omega) \cdot G(x_2,\xi,\omega))$

4. Synthetic Inversions

As a first application, we aim to resolve artifical noise sources in the North Atlantic. We perform several synthetic inversions for different station locations in North America and Europe. Synthetic data is computed for a noise distribution based on the significant wave height of a WaveWatch III model as seen in Figure 3 (left).

- starting model is a homogeneous distribution in the ocean
- cross-correlations filtered between 0.1 and 0.2 Hz
- observed cross-correlations selected based on clear surface wave arrival in causal/acausal window (see Figure 1) → 6400 cross-correlations for 179 stations
- 10 iterations within 100 minutes on 600 cores

Several inversions for different station locations and inversion parameters are performed. More stations do not necessarily lead to a better final model, as redundant station pairs can be omitted without information loss. The dominant noise sources are visible after two iterations and reasonably well resolved after 10 iterations

5. Data Inversions

As a real data application we invert for observed cross-correlations for three consecutive days from the 3rd Oct 2019. The data is automatically downloaded, processed, and cross-correlated using the Python package ObsPy (Krischer et al., 2015) and processing toolkit ants (github.com/lermert/ants 2). We use the same approach as for the synthetic inversions with a homogeneous distribution in the ocean as starting model. The spatiotemporal variations in the source distribution of the secondary microseism are visible, with some inversion artefacts in areas where we do not expect source locations, e.g. on land. 05/10/2019 07/10/2019 06/10/2019 ral 0.2 Density



Figure 4: Inversion model after 10 iterations for three consecutive days using real data. The PSD is set to 0 where the station sensitivity is below 1%. The spatiotemporal variations of ambient noise sources are visible.



Figure 3: Model after 10 iterations (right, PSD set to 0 below 1% station sensitivity) of a synthetic inversion (right) for a noise source distribution based on the significant wave height of a WaveWatch III model (left). The station locations (A) are chosen based on data availability and a subset is selected by picking a minimum distance between the stations to remove clusters.

References

Ermert, L., Sager, K., Afanasiev, M., Boehm, C. and Fichtner, A., 2017. Ambient Seismic Source Inversion in a Heterogeneous Earth: Theory and Application to the Earth's Hum. Journal of Geophysical Research: Solid Earth, 122(11), pp.9184-9207. Ardhuin, F., Stutzmann, E., Schimmel, M. and Mangeney, A., 2011. Ocean wave sources of seismic noise. Journal of Geophysical Research: Oceans, 116(C9). Krischer, L., Megies, T., Barsch, R., Beyreuther, M., Lecocq, T., Caudron, C. & Wassermann, J. (2015) ObsPy: A bridge for seismology into the scientific Python ecosystem. Compu-tational Science and Discovery, 8, 0–17, IOP Publishing. doi:10.1088/1749-4699/8/1/014003 Nissen-Meyer, T., Driel, M.V., Stähler, S., Hosseini, K., Hempel, S., Auer, L., Colombi, A. and Fournier, A., 2014. AxiSEM: broadband 3-D seismic wavefields in axisymmetric media. Solid Earth, (1), pp.425-445. Sager, K., Ermert, L., Boehm, C. and Fichtner, A., 2017. Towards full waveform ambient noise inversion. Geophysical Journal International, 212(1), pp.566-590. Sager, K., Boehm, C., Ermert, L., Krischer, L., & Fichtner, A. (2018). Sensitivity of Seismic Noise Correlation Functions to Global Noise Sources. Journal of Geophysical Research: Solid Earth, 123(8), 6911–6921. https://doi.org/10.1029/2018JB016042



6. Conclusions and Outlook

Using a gradient-based iterative method with pre-computed wavefields and spatially variable grids has several advantages:

- Absence of approximations
- Efficient forward modelling of crosscorrelations and sensitivity kernels
- Insensitivity to unknown Earth structures → Rapid global noise source inversions

Synthetic inversions show promising results with a strong dependency on the station locations. Implementing an **optimal design** approach to select the station pairs could further improve the resolution. Automating the data acquisition and cross-correlation process could lead to publicly available daily ambient seismic noise source maps. These could help improve methods in full waveform ambient noise tomography and near real-time subsurface monitoring.

> © Authors. All rights reserved.

1. Introduction

Full waveform ambient noise tomography methods (e.g. Sager et al., 2017) are able to circumvent some of the main assumptions made in ambient noise seismology (e.g. that the cross-correlation is equal to the Green's function) if the heterogeneous noise source distribution is known. **Efficient forward modelling of global cross-correlations** $C(x_1, x_2)$ in the microseismic frequency range (currently up to 0.2 Hz) is now possible since we have optimised the cross-correlation modelling by implementing:

- Green's functions G from pre-computed wavefields
 - (e.g. AxiSEM, Nissen-Meyer et al. 2014)
- Power spectral density of noise distribution S on spatially variable grids

We use a gradient-based iterative method, employing finite-frequency sensitivity kernels to **rapidly invert for the power spectral density of the noise source distribution** of the secondary microseisms. Since the mechanism behind the secondary microseisms is well understood (e.g. Ardhuin et al., 2011), data inversions can be seen as a **new observable of the ocean state** and could **improve full waveform ambient noise tomography methods**.

2. Forward Model and Gradient

A spatially variable grid enables us to reduce the number of grid points by a factor of 5 and still have a high spatial resolution in our area of interest. Combined with pre-computed Green's function databases, we can **forward model global cross-correlations and sensitivity kernels within a few seconds** (e.g. Ermert et al., 2017):

$$C(x_1, x_2) = \int G(x_1, \xi) [G^*(x_2, \xi) S(\xi)] d\xi$$



Figure 2: Synthetic source sensitivity kernel with corresponding cross-correlation and measurement windows



Figure 1: Homogeneous and spatially variable grid adapted to data

The sensitivity kernel K for each station pair is a simple multiplication of a measurement-dependent adjoint source f(w) and the Green's functions in the frequency domain.

 $K(\xi,\omega) = 2f(\omega) \cdot (G^*(x_1,\xi,\omega) \cdot G(x_2,\xi,\omega))$

We measure the logarithmic energy ratio between the causal and acausal branch of the cross-correlations (see Figure 2). This is mostly not affected by unknown 3D Earth structures (Sager et al., 2018).

3. Inversion framework

Our inversion framework is based on the gradient-based iterative method employing finite-frequency sensitivity kernels (e.g. Ermert et al., 2017). Usually a homogeneous noise distribution in the ocean is used as the initial model.



4. Synthetic Inversions

As a first application, we aim to resolve artifical noise sources in the North Atlantic. We perform several synthetic inversions for different station locations in North America and Europe. Synthetic data is computed for a noise distribution based on the significant wave height of a WaveWatch III model as seen in Figure 3 (left).

- starting model is a homogeneous distribution in the ocean
- cross-correlations filtered between 0.1 and 0.2 Hz
- observed cross-correlations selected based on clear surface wave arrival in causal/acausal window (see Figure 1)
- \rightarrow 6400 cross-correlations for 179 stations
- 10 iterations within 100 minutes on 600 cores

Several inversions for different station locations and inversion parameters are performed. **More stations do not necessarily lead to a better final model**, as redundant station pairs can be omitted without information loss. The dominant noise sources are visible after two iterations and reasonably well resolved after 10 iterations



Figure 3: Model after 10 iterations (right, PSD set to 0 below 1% station sensitivity) of a synthetic inversion (right) for a noise source distribution based on the significant wave height of a WaveWatch III model (left). The station locations (\blacktriangle) are chosen based on data availability and a subset is selected by picking a minimum distance between the stations to remove clusters.

5. Data Inversions

As a real data application we invert for observed cross-correlations for three consecutive days from the 3rd Oct 2019. The data is **automatically downloaded**, **processed**, **and cross-correlated** using the Python package ObsPy (Krischer et al., 2015) and processing toolkit ants (github.com/lermert/ants_2). We use the same approach as for the synthetic inversions with a homogeneous distribution in the ocean as starting model. The spatiotemporal variations in the source distribution of the secondary microseism are visible, with some inversion artefacts in areas where we do not expect source locations, e.g. on land.



Figure 4: Inversion model after 10 iterations for three consecutive days using real data. The PSD is set to 0 where the station sensitivity is below 1%. The spatiotemporal variations of ambient noise sources are visible.

6. Conclusions and Outlook

Using a gradient-based iterative method with pre-computed wavefields and spatially variable grids has several advantages:

- Absence of approximations
- Efficient forward modelling of crosscorrelations and sensitivity kernels
- Insensitivity to unknown Earth structures
- → Rapid global noise source inversions

Synthetic inversions show promising results with a strong dependency on the station locations. Implementing an **optimal design** approach to select the station pairs could further improve the resolution. Automating the data acquisition and cross-correlation process could lead to publicly available **daily ambient seismic noise source maps**. These could help **improve methods in full waveform ambient noise tomography and near real-time subsurface monitoring**.