Optimal processing for seismic noise correlations

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Summary

- A wide spectrum of processing schemes is commonly applied during the calculation of seismic noise correlations.
- Many processing schemes are nonlinear, thus breaking linear physics of seismic wave propagation.
- This naturally raises the question: To what extent are the resulting noise correlations physically meaningful quantities?
- Here we demonstrate that many processing methods introduce an unphysical component into noise correlations.
- Profound consequences: Processed correlations cannot be entirely explained by any combination of Earth structure and noise sources, and that inversion results may thus be polluted.
- The positive component of our work: A new class of processing schemes that are optimal in the sense of
  (1) completely avoiding the unphysical component, while
  (2) closely approximating the desirable effects of conventional processing schemes.

Summary

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- Many processing schemes are nonlinear, thus breaking linear physics of seismic wave propagation.
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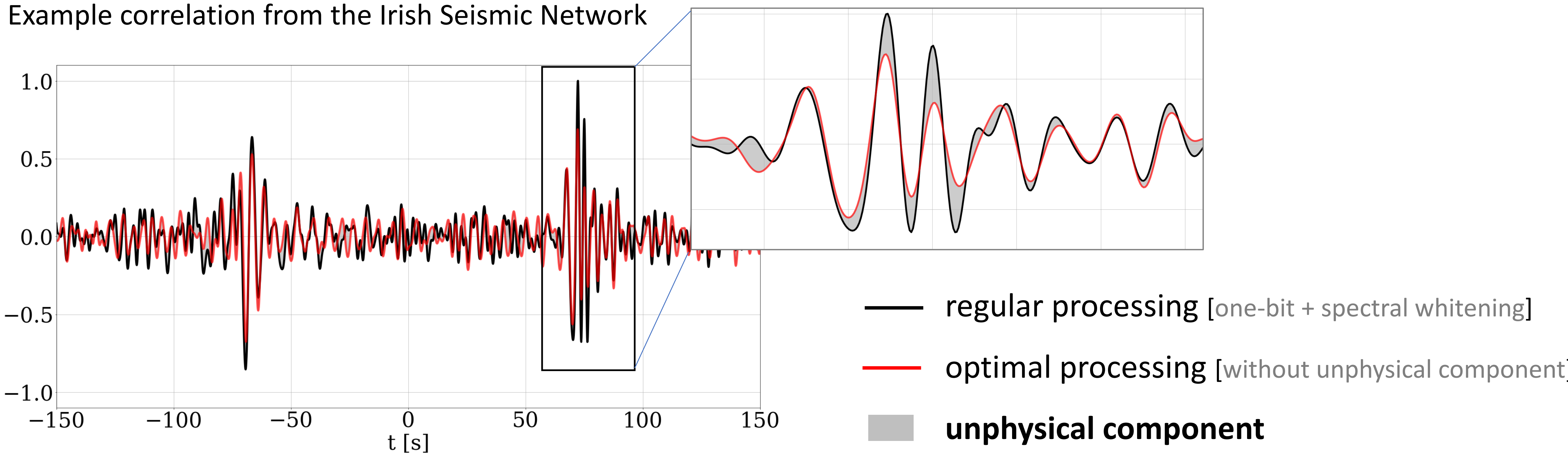
- Here we demonstrate that many processing methods introduce an unphysical component into noise correlations.
- Profound consequences: Processed correlations cannot be entirely explained by any combination of Earth structure and noise sources, and that inversion results may thus be polluted.
- The positive component of our work: A new class of processing schemes that are optimal in the sense of
  (1) completely avoiding the unphysical component, while
  (2) closely approximating the desirable effects of conventional processing schemes.

Example correlation from the Irish Seismic Network



Processing in noise interferometry:

- suppress high-amplitude transients
- compensate for heterogeneous noise sources
- enhance certain phases
- accelerate convergence towards ... ?
- ...

Much of this processing is nonlinear:

- spectral whitening
- one-bit normalisation
- rms clipping
- phase-weighted stacking
- ...

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Explanations: A wide spectrum of processing schemes is commonly applied during the calculation of seismic noise correlations. This is intended to suppress large-amplitude transient and monochromatic signals, to accelerate convergence of the correlation process, or to modify raw correlations into more plausible approximations of inter-station Green's functions.

Processing in noise interferometry:

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- compensate for heterogeneous noise sources
- enhance certain phases
- accelerate convergence towards ... ?
- ...

Much of this processing is nonlinear:

- spectral whitening
- one-bit normalisation
- rms clipping
- phase-weighted stacking
- ...

Processing transforms the wavefield:

$$u(\boldsymbol{x}, t) \rightarrow \hat{u}(\boldsymbol{x}, t)$$

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Explanations: Generally speaking, the act of processing transforms the original seismic wavefield u into its processed version û.

Processing in noise interferometry:

- suppress high-amplitude transients
- compensate for heterogeneous noise sources
- enhance certain phases
- accelerate convergence towards ... ?
- ...

Much of this processing is nonlinear:

- spectral whitening
- one-bit normalisation
- rms clipping
- phase-weighted stacking
- ...

Processing transforms the wavefield:

$$u(\boldsymbol{x},t) \rightarrow \hat{u}(\boldsymbol{x},t)$$

$u(\boldsymbol{x},t)$ satisfies the wave equation:

$$\partial_t^2 u(\boldsymbol{x},t) = c^{-2}\Delta u(\boldsymbol{x},t)$$

$\hat{u}(\boldsymbol{x},t)$ does not:

$$\partial_t^2 \hat{u}(\boldsymbol{x},t) \neq c^{-2}\Delta \hat{u}(\boldsymbol{x},t)$$

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Explanations: The original wavefield, u, satisfies a wave equation, such as the one written above. However, when processing is nonlinear, the processed wavefield û does not satisfy a wave equation. It is not a physical wavefield.

Processing in noise interferometry:

- suppress high-amplitude transients
- compensate for heterogeneous noise sources
- enhance certain phases
- accelerate convergence towards ... ?
- ...

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$$\partial_t^2 \hat{u}(\boldsymbol{x},t) \neq c^{-2}\Delta \hat{u}(\boldsymbol{x},t)$$

Yet we pretend it does!

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Explanations: Yet, in most of ambient noise interferometry, we silently pretend that the processed wavefield û does actually satisfy a wave equation. Obviously, there is a contradiction!

1.

How physically meaningful are processed correlations?

2.

Can unphysical components be reduced?

while still having a useful processing

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Explanations: This contradiction naturally raises these two questions. In what follows, we will answer these questions, first by developing some theory, and then by showing some real-world examples.

Theory

processing → unphysical wavefield

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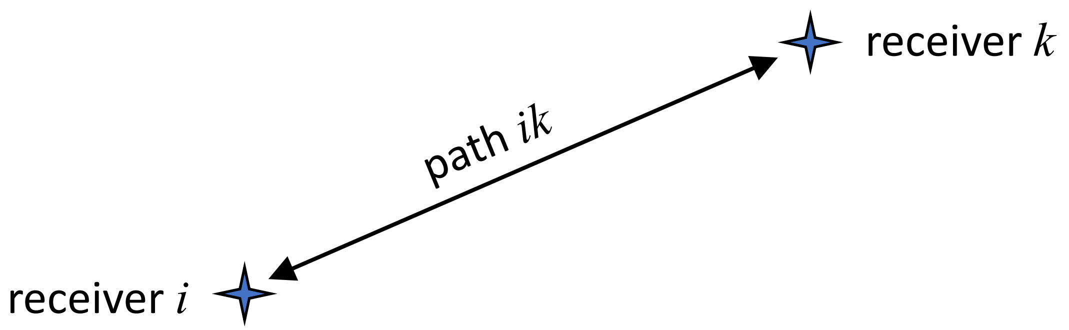
Explanations: So, let us start with a theory that describes how to compute noise correlations from processed seismic recordings. This theory has two components:

Processing operator

[Fichtner et al. (2017)]

P_{path}^{source}

processing operator



Explanations: The first component involves the processing operator, i.e., the operator that takes a raw correlation into a processed correlation. The action of the processing operator **P** depends on (1) the path connecting a pair of receivers, and (2) the sources acting during the time interval for which we have recordings.

Processing operator

[Fichtner et al. (2017)]

$$P_{path}^{source} = g_{path}\, f^{source} + e_{path}^{source}$$

processing operator

path corrector

source corrector

factorisation residual

factor analysis of P

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Explanations: The processing operator **P** can be decomposed using a standard factor analysis. One factor, **g**, depends only on the path between receivers. We call this the *path corrector*, for reasons that will become clear later. The second factor, the *source corrector* **f** only depends on the sources of the wavefield. Finally, there is a *factorisation residual* that depends on both the path and the sources.

Processing operator

[Fichtner et al. (2017)]

$$P_{path}^{source} = g_{path}\, f^{source} + e_{path}^{source}$$

processing operator — path corrector — source corrector — factorisation residual

Inter-station correlations

[Woodard (1997), Tromp et al. (2010), Hanasoge (2013), ...]

$$C_{path} = \int G_{path} \left[G_{path}^{*} \; S \; \right] d\boldsymbol{x}$$

correlation — Green's functions — source psd

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Explanations: The second part of the theory concerns the computation of an inter-station correlation. In the absence of any processing, a correlation can be written – without any approximations – as an integral over a suitable Green's function of the medium (or, more precisely, a numerical model of the medium) and the power-spectral density of the wavefield sources, e.g., noise sources. This result has been known for more than 20 years!

Processing operator

[Fichtner et al. (2017)]

$$P_{path}^{source} = g_{path} \, f^{source} + e_{path}^{source}$$

processing operator — path corrector — source corrector — factorisation residual

Inter-station correlations

[Woodard (1997), Tromp et al. (2010), Hanasoge (2013), …]

$$C_{path} = \int G_{path} \left[G_{path}^{*} \; S \; \right] dx$$

processing

$$\hat{C}_{path} = \int G_{path}^{eff} \left[G_{path}^{*} \; S^{eff} \; \right] dx \; \ldots$$

processed correlation — effective Green‘s function ($\leftarrow g_{path}$) — effective source ($\leftarrow f^{source}$)

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Explanations: When processing is applied, this equation changes. The raw correlation turns into a processed correlation, and the original Green‘s function becomes an effective Green‘s function. Also, the original source power-spectral density becomes an effective one.

processing → effective medium + effective source

Should be taken into account when solving an inverse problem.

$$\hat{C}_{path} = \int G_{path}^{eff} \left[G_{path}^{*} S^{eff} \right] d\boldsymbol{x} \ldots$$

processed correlation

effective Green's function

effective source

↑ g_{path}

↑ f^{source}

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Explanations: Hence, in a nutshell, processing makes us see a correlation wavefield that propagates through an effective medium, and that is excited by an effective source.

$$\hat{C}_{path} = \int G_{path}^{eff} \left[G_{path}^{*} S^{eff} \right] d\boldsymbol{x} \ldots$$

$$+ \int G_{path} \left[G_{path}^{*} S_{path}^{eff} \right] d\boldsymbol{x}$$

Effective **source** that depends on the **path**.

Different receiver pairs see **different sources**.

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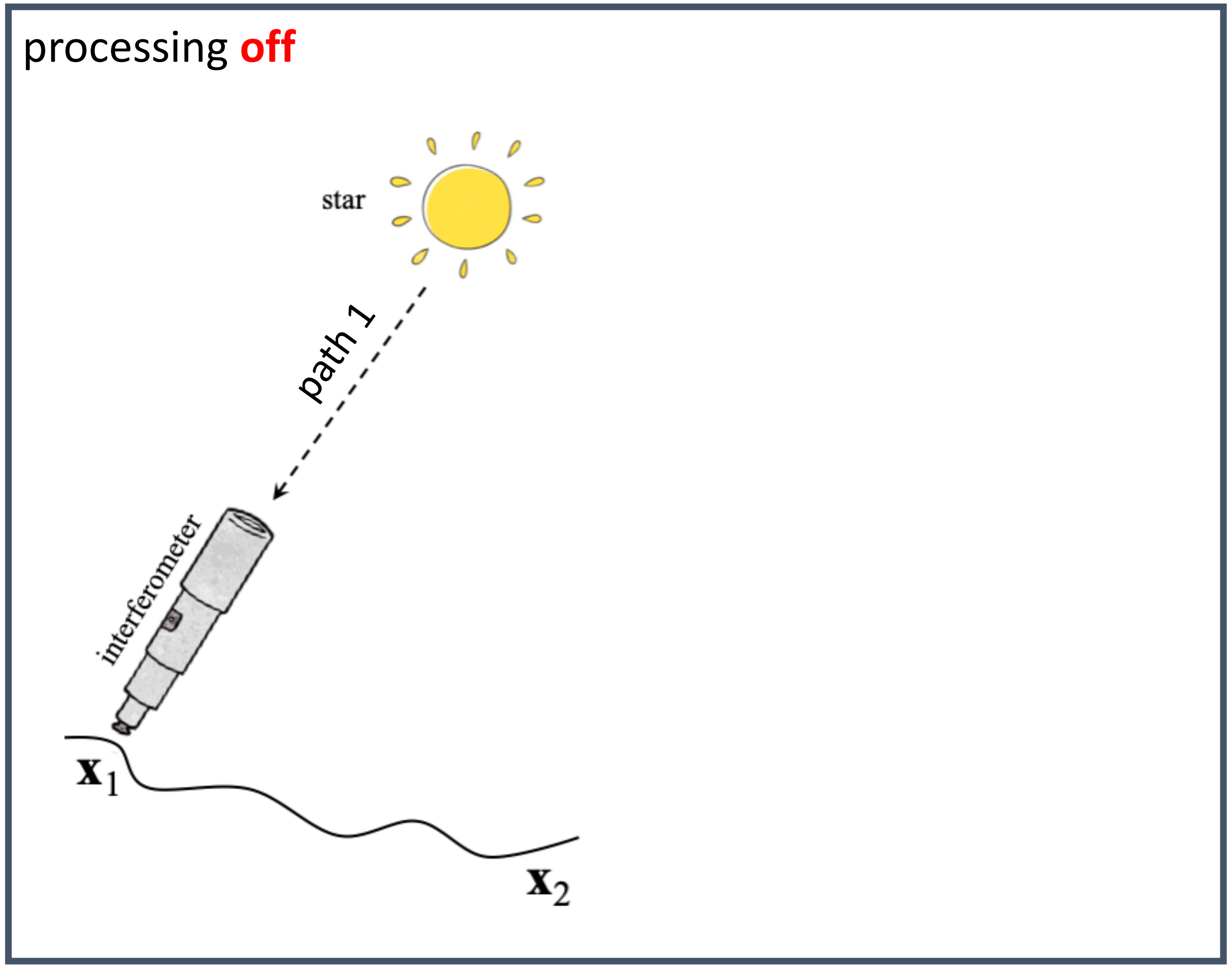
Explanations: But this is not everything. There is a nasty additional term with an effective wavefield source that depends on the inter-station path. Hence, different receiver pairs actually see different sources. The following slide illustrates that this is physically absurd:

Illustration



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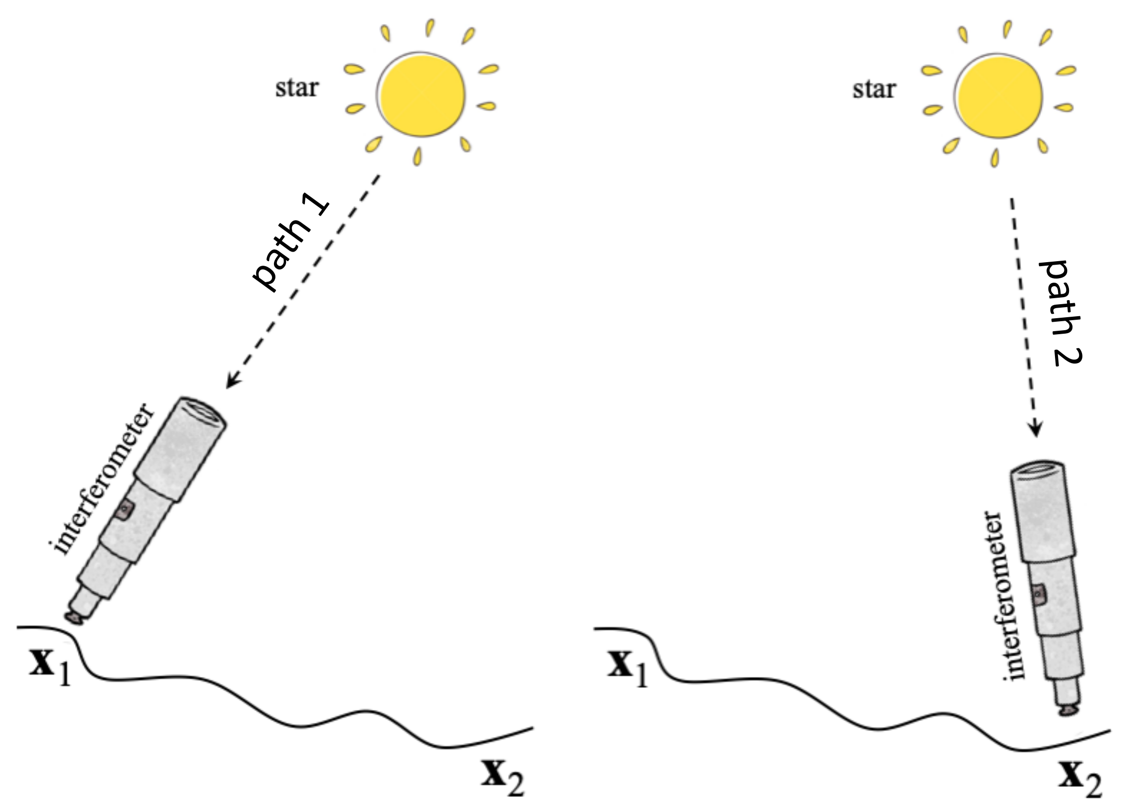
Explanations: Assume processing is turned off. We observe a star (source of an electromagnetic wavefield) using a interferometer at position $\mathbf{x}_1$.

Illustration

processing **off**



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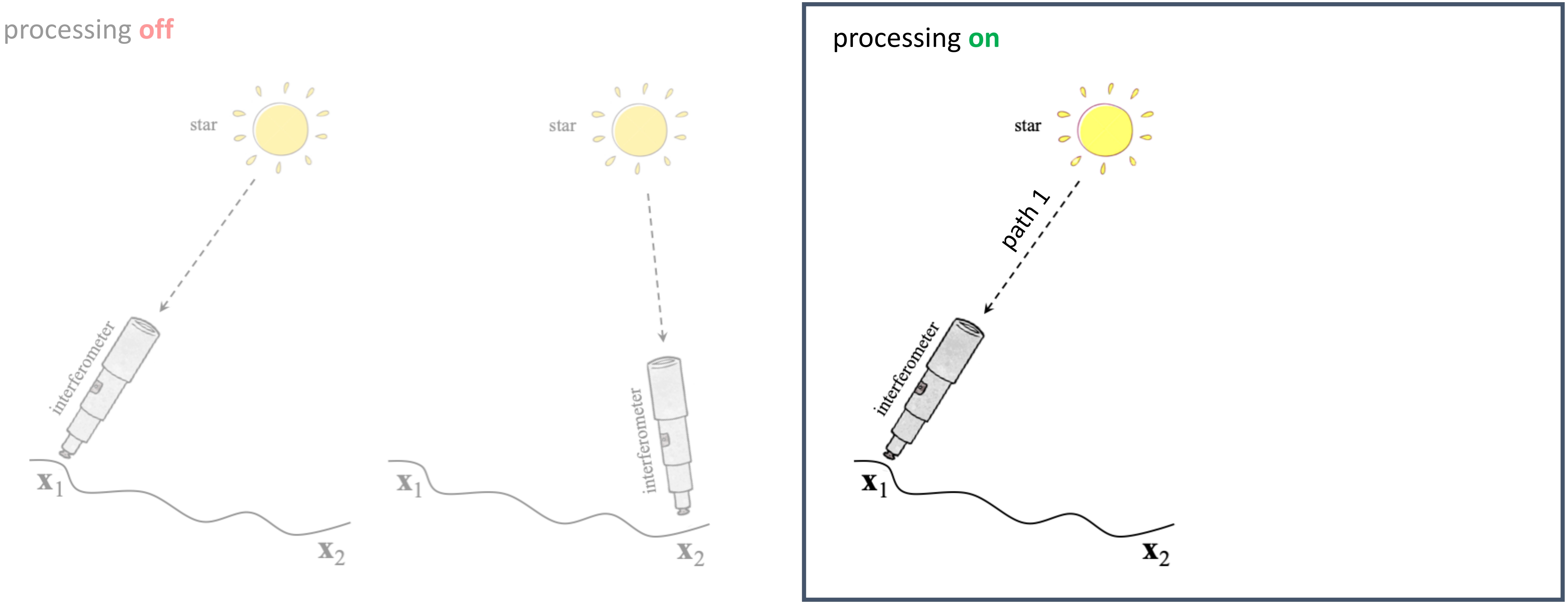
Explanations: If we move the interferometer to a different position $\mathbf{x}_2$, we of course see the same source, i.e., the same star. This is how it should be.

Illustration



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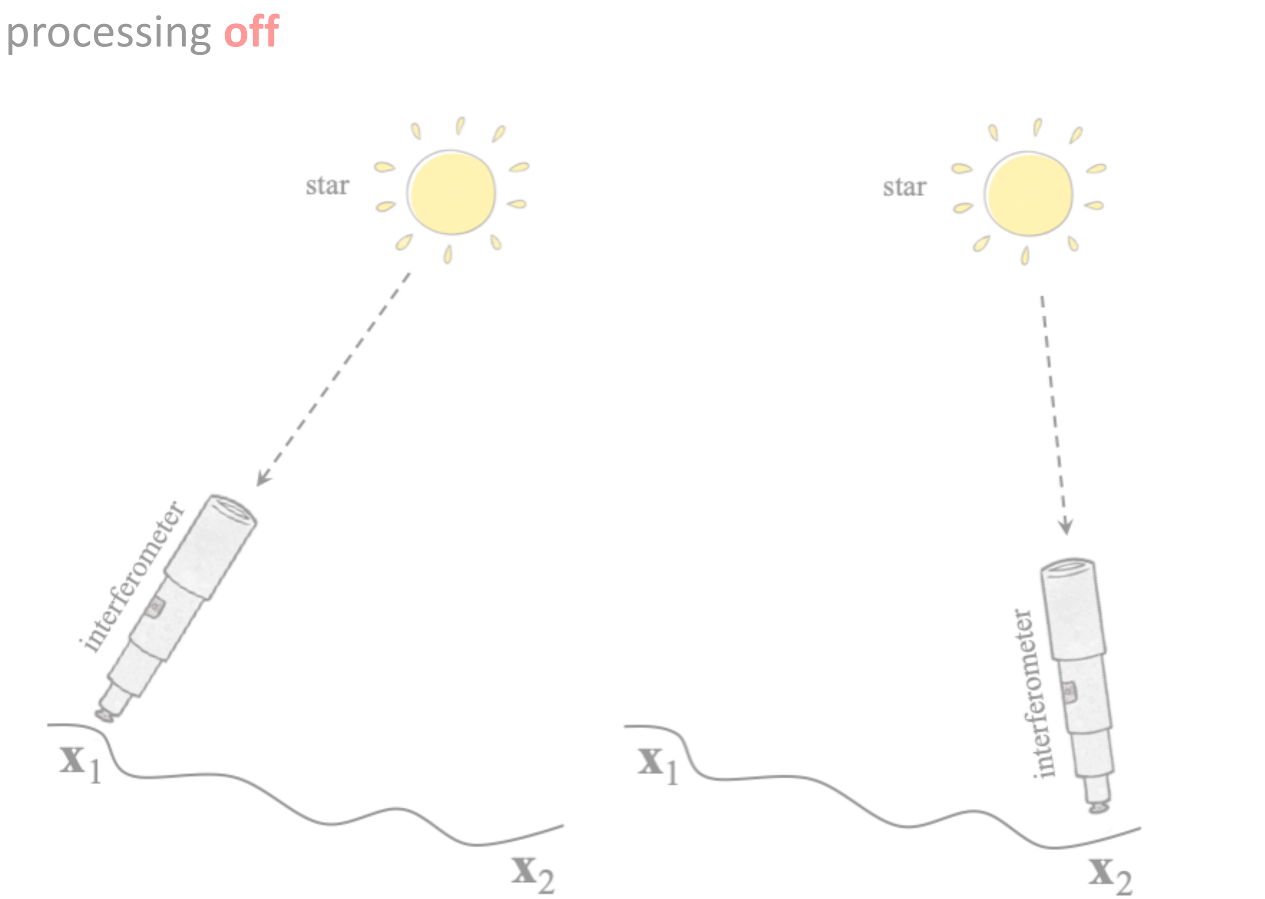
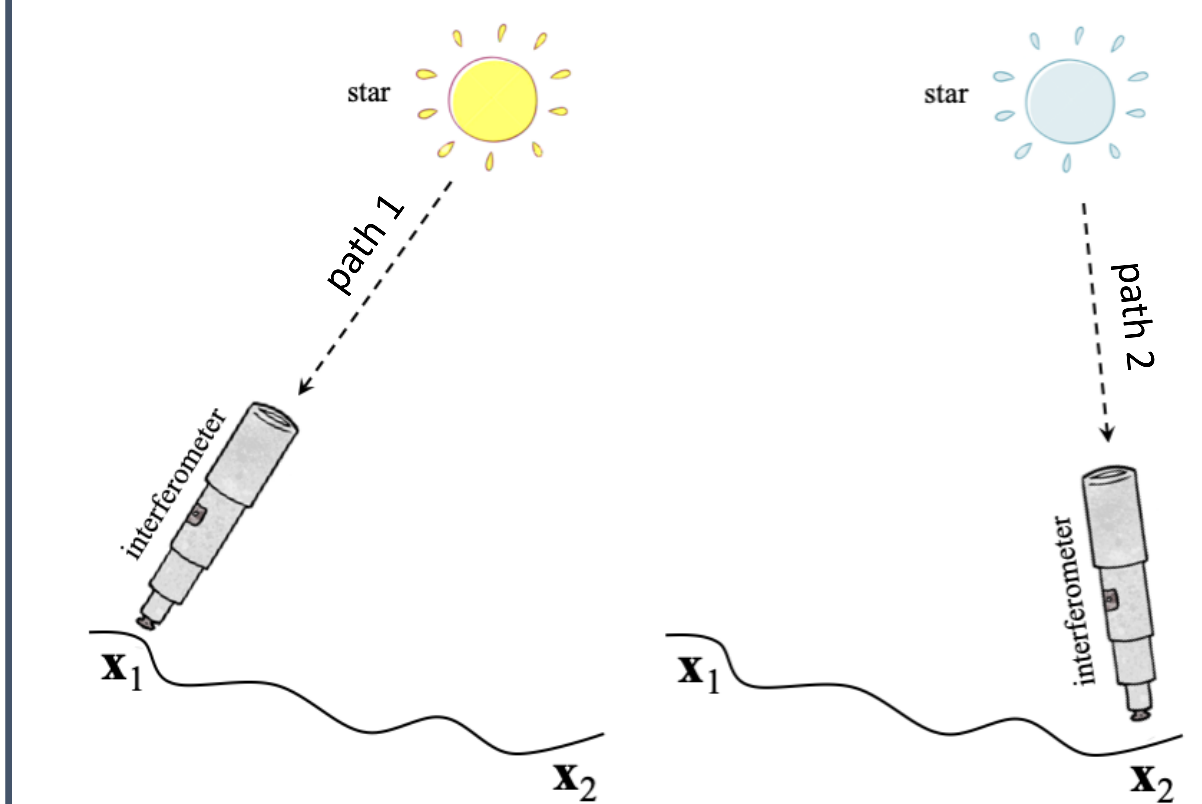
Explanations: Now we turn on the processing. By design, the processing makes the star appear a bit brighter. This is nice and good, but ...

Illustration





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Explanations: ... when we move the interferometer to position $\mathbf{x}_2$, we suddenly see a different star. Apparently, the source of the wavefield looks different now, because processing makes a wavefield source that depends in the path along which we observe the wavefield.

Illustration

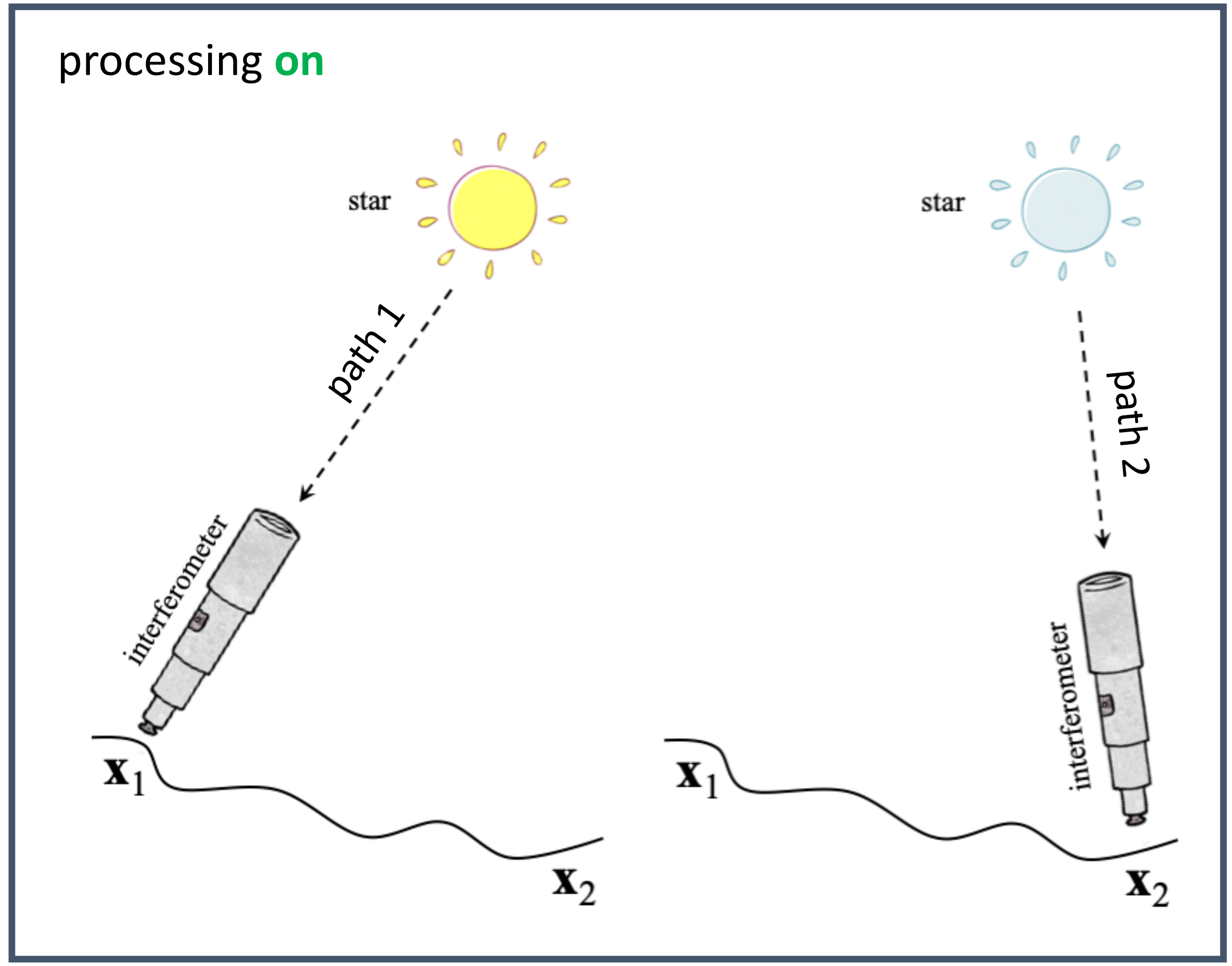
Moving observer‘s inference:

Atmosphere must have changed!

But in reality:

Just a processing artefact!

Moving observer paradox.



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Explanations: The observer with the processing interferometer must therefore come to the incorrect conclusion that the atmosphere along path 1 is different from the atmosphere along path 2. In seismological terms: We would see a different Earth depending on the station-station path that we use for the observation.

Optimal processing

Avoiding the unphysical contribution

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Explanations: Is there a way of avoiding this obviously unphysical effect? Indeed, there is! The processing schemes that do not produce unphysical effects are called *optimal processing*.

Regular processing:

includes potentially unphysical effects

$$P_{path}^{source} = g_{path} f^{source} + \boxed{e_{path}^{source}}$$

factorisation residual

sole responsible for the unphysical component

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Explanations: To see how optimal processing works, we recall the factorisation of the processing operator that we saw a couple of slides before. It turns out that the factorisation residual is the only responsible for the unphysical effects.

Regular processing:

includes potentially unphysical effects

$$P_{path}^{source} = g_{path}\, f^{source} + e_{path}^{source}$$

Optimal processing:

no unphysical effects

$$\Pi_{path}^{source} = g_{path}\, f^{source}$$

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Explanations: So, we can design new processing schemes by simply omitting this term. The procedure now is as follows: (1) Apply the regular processing you like. (2) Based on this, compute the factorisation. (3) Omit the factorisation residual, and just apply **f** and **g** as processing. **This ensures that you have a processing scheme that is as close as possible to the one you designed originally, while totally avoiding any unphysical effects.**

Example

Estimating the unphysical contribution

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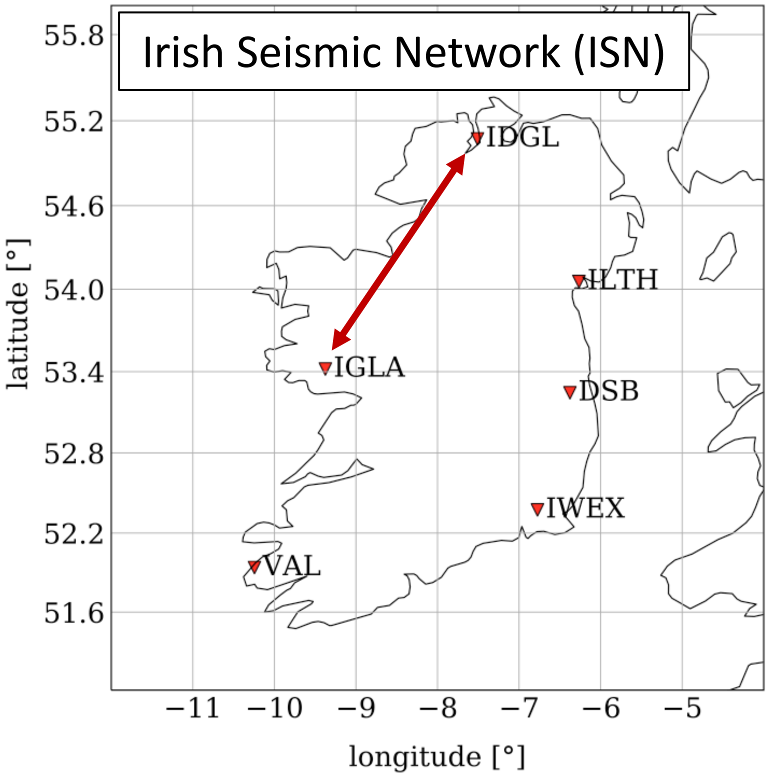
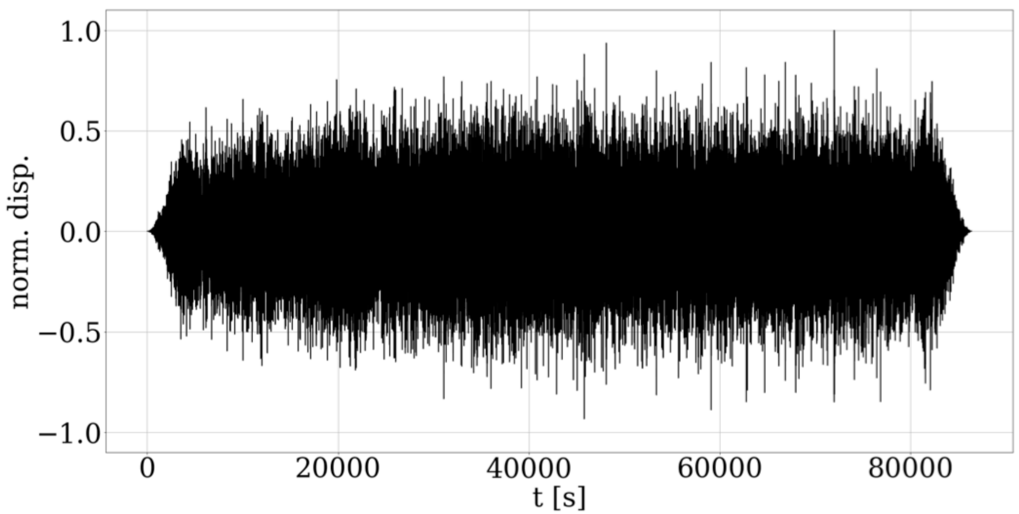
Explanations: This was somewhat dry theory. Now let us look at a real-world example.



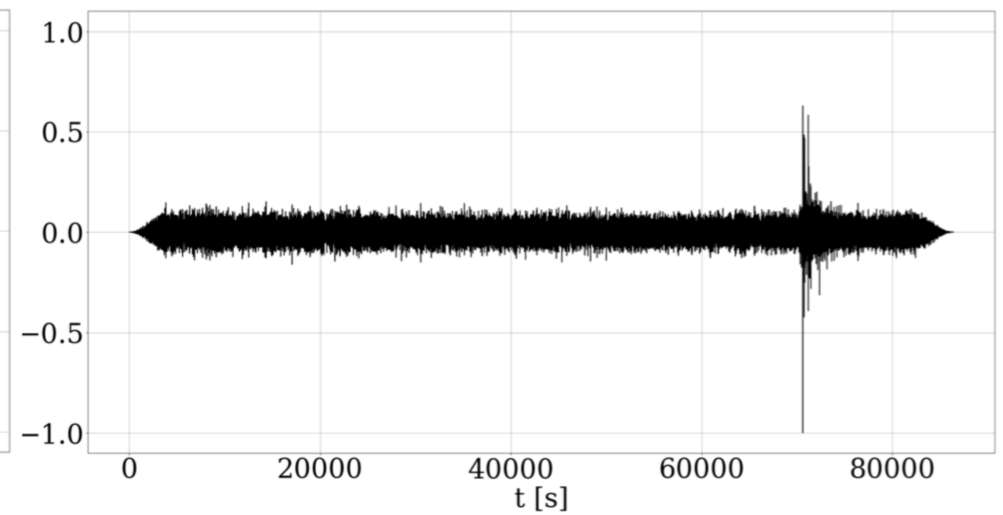
Data

- Irish Seismic Network stations
- 31 days of continuous data from January 2019
- **Processing**: one-bit normalisation + spectral whitening

station DSB – 1 January 2019



station DSB – 5 January 2019



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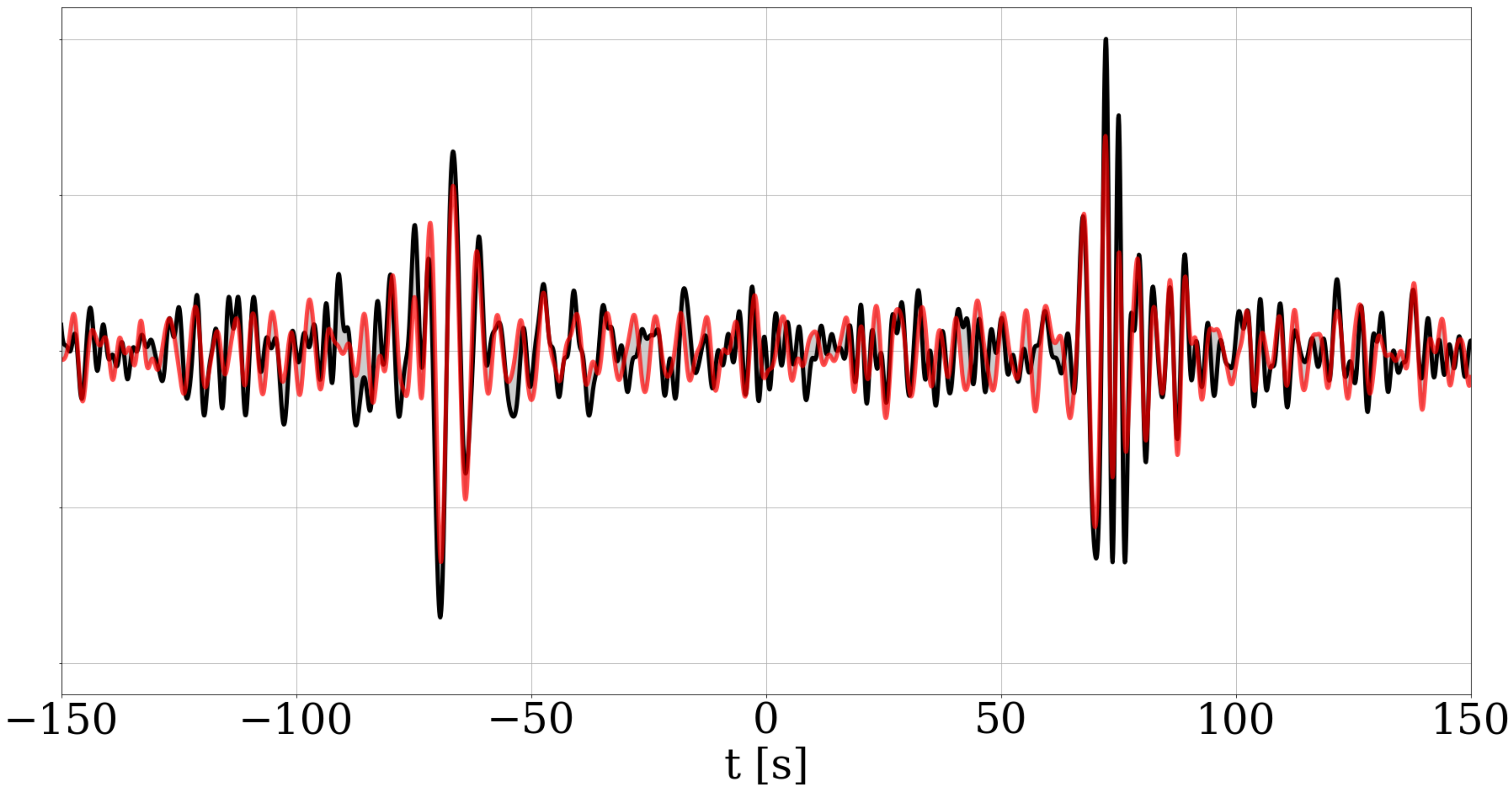
Explanations: The example is from the Irish Seismic network. In total, we used 31 days of continuous recordings from January 2019. The data contain both microseismic noise and a few earthquake signals. Hence, the combination of one-bit normalisation and spectral whitening is reasonable.

Time-domain correlation



regular processing [one-bit + spectral whitening]

optimal processing [without unphysical component]

unphysical component [only explainable by path-dependent source]

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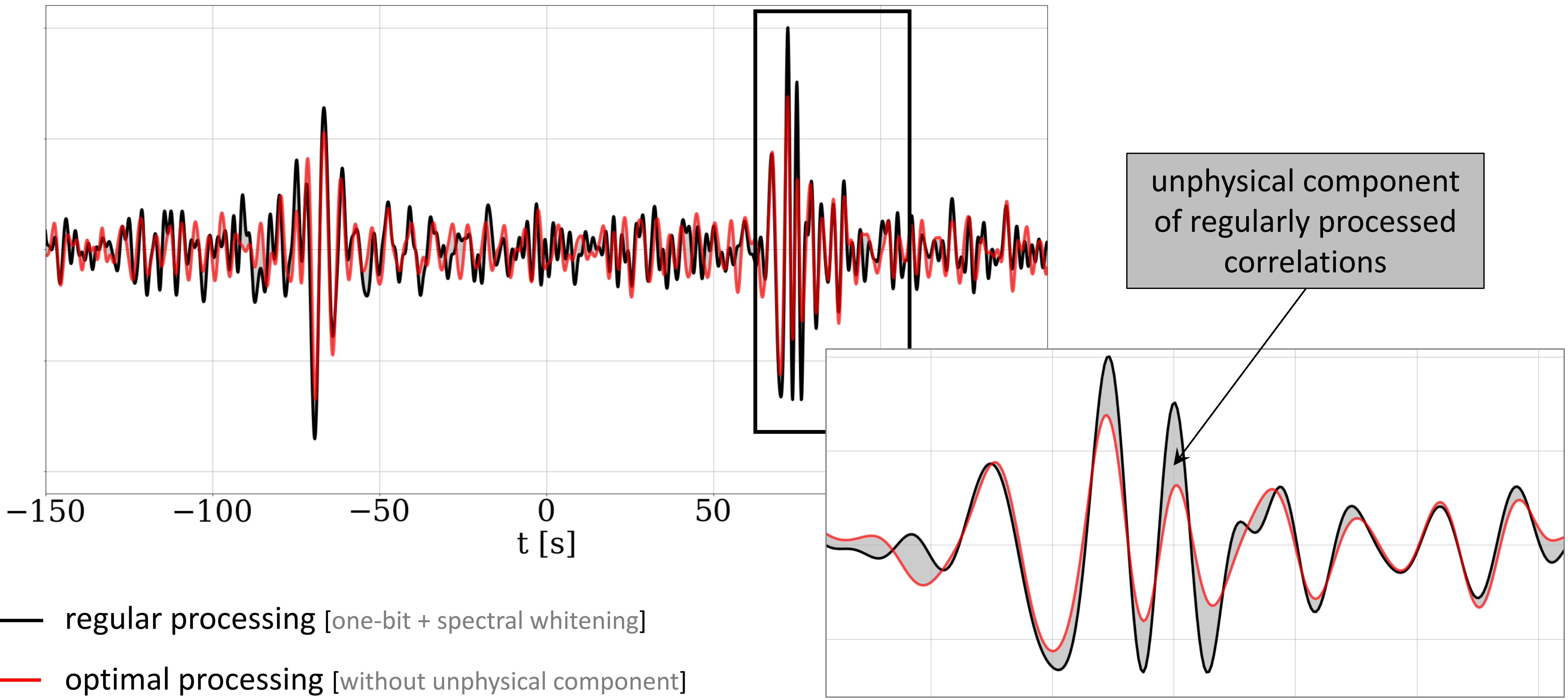
Explanations: Here you see an example of an inter-station correlation where we applied the regular processing (black) and the optimal processing (red). The difference between the two, shaded in grey, is the unphysical component of the correlation wavefield, i.e, that part of the correlation that can only be explained by having an unphysical noise source that depends on the station-station path.

Time-domain correlation



regular processing [one-bit + spectral whitening]

optimal processing [without unphysical component]

unphysical component [only explainable by path-dependent source]

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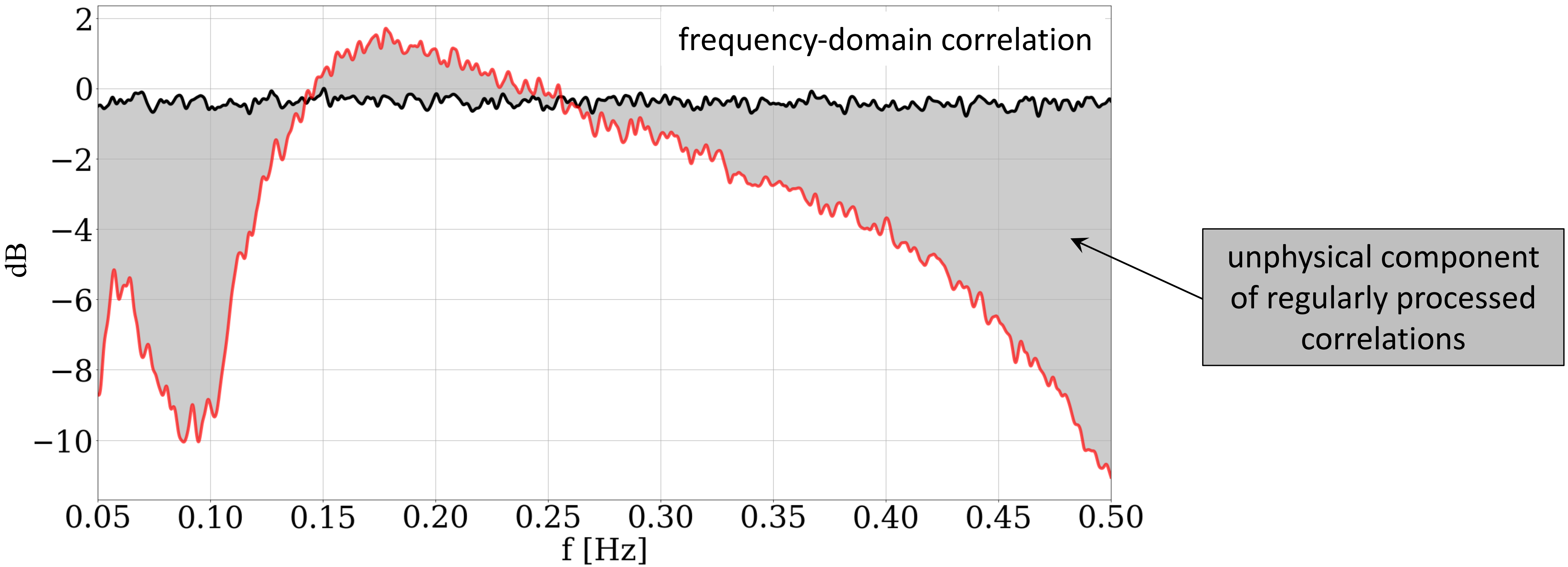
Explanations: These differences seem small at first, but when we zoom in, we discover that the unphysical component can actually be quite substantial. It affects both phase and amplitude, but the former mostly less than the latter.

Frequency-domain correlation



regular processing [one-bit + spectral whitening]

optimal processing [without unphysical component]

unphysical component [only explainable by path-dependent source]

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Explanations: In the frequency domain, this becomes much more pronounced. In fact, at some frequencies, the unphysical contribution introduced by nonlinear processing can be quite large.

Conclusions

1. Processing may introduce unphysical components into noise correlations.
   - Take the form of path-specific noise sources [moving observer paradox].
   - Can be misinterpreted as variations in medium properties [Earth structure].

2. The unphysical component can be large.
   - Affects both amplitude and phase [former more than the latter].
   - Must be considered when trying to exploit details of noise correlations.

3. Optimal processing.
   - Processing scheme that is closest to the original one ...
   - ... and completely avoids the unphysical component.

some elements of the theory: Fichtner et al., 2017. Generalised Interferometry I.
happy to share research code: andreas.fichtner@erdw.ethz.ch

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Explanations: These are the most important conclusions of our work so far. Some elements of the theory can be found in Fichtner et al., 2017 (Generalised Interferometry I.). We are happy to share research code: andreas.fichtner@erdw.ethz.ch.