Mapping 3-D mantle electrical conductivity using Swarm, Cryosat-2 and ground observatory data

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Existing global 3-D models of mantle conductivity

- Models differ much
- Lateral variations of conductivity are (unrealistically?) large
Some details on obtaining the models

• Variations of magnetic field from global net of geomagnetic observatories are used

• Work in frequency domain

• Variations are in period range 2 days – 100 days; “mid” mantle (400 – 1600 km) is probed

• Assumption on the source – magnetospheric ring current, described near the Earth via first zonal harmonic

• This assumption allows researchers to exploit local C-response concept (Banks, 1969) to obtain 3-D conductivity models

Challenges

• Spatial irregularity of observatories precludes obtaining trustworthy 3-D conductivity distributions beneath the regions with the scarcity of observations (in particular, beneath the oceans)

• The ring current source is not so simple; ignoring it’s more complex spatial structure might lead to artefacts in the recovered 3-D models
Potential solution: complement observatory data with satellite data

Significantly improves spatial coverage with the data. This allows: a) to specify the source more accurately; b) to obtain information about mantle structures beneath oceans.
Work with satellite data (in frequency domain)

• Standard concept based on local C-responses does not work since satellites move in space

• Matrix Q-responses* overcome the above problem

\[
t^l_k(\omega) = \sum_{n=1}^{N} \sum_{m=-n}^{n} Q_{kn}^{lm}(\omega)\varepsilon_n^m(\omega)
\]

\[
\delta\mathbf{B}(\mathbf{r}, \omega) = -\text{grad} \left\{ a \sum_{n=1}^{N} \sum_{m=-n}^{n} \varepsilon_n^m(\omega) \left( \frac{r}{a} \right)^n Y_n^m(\vartheta, \varphi) + \sum_{k=1}^{K} \sum_{l=-k}^{k} t^l_k(\omega) \left( \frac{a}{r} \right)^{k+1} Y_k^l(\vartheta, \varphi) \right\}
\]

\(\delta\mathbf{B}\) - field due to magnetospheric ring current

\(\varepsilon_n^m\) - external coefficients (due to the source)

\(t^l_k\) - induced coefficients (due to 3-D EM induction)

\(Q_{kn}^{lm}\) - matrix Q-responses; note that for 1-D Earth: \(t_n^m(\omega) = Q_n(\omega)\varepsilon_n^m(\omega)\)

General scheme to estimate matrix Q-responses

- Subtract from the data core, lithospheric and ionospheric fields
- Take (residual) data only from mid latitudes
- Specify $N$ and $K$ (maximum degrees in SH expansion of external and induced contributions)
- Estimate time series of external and induced coefficients (by least squares)
- Estimate matrix Q-responses

\[
I_k^l(\omega) = \sum_{n=1}^{N} \sum_{m=-n}^{n} Q_{kn}^{lm}(\omega) \varepsilon_n^m(\omega)
\]

\[
\delta \mathbf{B}(\mathbf{r}, \omega) = -\text{grad} \left\{ a \sum_{n=1}^{N} \sum_{m=-n}^{n} \varepsilon_n^m(\omega) \left( \frac{r}{a} \right)^n Y_n^m(\theta, \varphi) + \sum_{k=1}^{K} \sum_{l=-k}^{k} I_k^l(\omega) \left( \frac{a}{r} \right)^{k+1} Y_{k}^{l}(\theta, \varphi) \right\}
\]
Actual implementation

- Data are 5 years of Swarm (A/B/C) + Cryosat-2 + observatory measurements
- Subtract from the data the CM* core, lithospheric and ionospheric fields
- Take (residual) data from $\pm 55^0$ geomagnetic latitudes
- Specify $N = 2$ and $K = 3$
- Estimate time series of external and induced coefficients with cadence of 6 hours
- Estimate matrix Q-responses at 16 periods between 2 and 31 days

$$t_k^l(\omega) = \sum_{n=1}^{2} \sum_{m=-n}^{n} Q_{kn}^{lm}(\omega) \epsilon_n^m(\omega)$$

$$\delta \mathbf{B}(\mathbf{r},\omega) = -\nabla \left\{ a \sum_{n=1}^{2} \sum_{m=-n}^{n} \epsilon_n^m(\omega) \left( \frac{r}{a} \right)^n Y_n^m(\theta,\varphi) + \sum_{k=1}^{3} \sum_{l=-k}^{k} t_k^l(\omega) \left( \frac{a}{r} \right)^{k+1} Y_k^l(\theta,\varphi) \right\}$$

*Sabaka T., L. Tøffner-Clausen, N. Olsen, and C. C. Finlay. 2018. "A comprehensive model of Earth’s magnetic field determined from 4 years of Swarm satellite observations." Earth, Planets and Space, 70 (1).
As a result we estimate $N(N+2) \times K(K+2) = 8 \times 15$ elements of Q-matrices at 16 periods ($N = 2; K = 3$).

$$
\begin{align*}
\begin{pmatrix}
\ell^{-1}_1(\omega_j) \\
\ell^0_1(\omega_j) \\
\vdots \\
\ell^3_3(\omega_j)
\end{pmatrix} &=
\begin{pmatrix}
Q^{-11}_1(\omega_j) & Q_{11}^{-10}(\omega_j) & \cdots & Q_{12}^{-12}(\omega_j) \\
Q_{11}^{01}(\omega_j) & Q_{11}^{00}(\omega_j) & \cdots & Q_{12}^{02}(\omega_j) \\
\vdots & \vdots & & \vdots \\
Q_{31}^{31}(\omega_j) & Q_{31}^{30}(\omega_j) & \cdots & Q_{32}^{32}(\omega_j)
\end{pmatrix}
\begin{pmatrix}
\varepsilon^{-1}_1(\omega_j) \\
\varepsilon^0_1(\omega_j) \\
\vdots \\
\varepsilon^2_2(\omega_j)
\end{pmatrix}
\end{align*}
$$

$j = 1, 2, \ldots, 16$
The best resolved are diagonal elements
Multiple squared coherencies (MSC) is a measure that estimates the extent to which the output (time series of induced coefficients of a given degree $k$ and order $l$) can be predicted from the input (time series of external coefficients up to degree 2) using a linear model.

The closer MSC to 1 the better determination/separation of external and induced coefficients.
Multiple squared coherencies (MSC) for induced coefficients up to $k = 2$ and up to $l = 2$ for different data sets

$$i^l_k(\omega) = \sum_{n=1}^{2} \sum_{m=-n}^{n} Q_{kn}^{lm}(\omega) c_n^m(\omega)$$

$k \leq 2$, $l \leq 2$

Note different scale for $i^0_1(\omega)$
More on these plots

• The smallest MSC is for the scenario when only observatory data are used to separate external and induced coefficients.

• The largest MSC is for the scenario when both observatory and satellite data are used.

• If observatory data are not included, adding Cryosat to Swarm data improves MSC.

• If observatory data are included, adding Cryosat to Swarm and observatory data does not improve MSC.

Exploiting satellite data significantly improves separation of non P10 external and induced coefficients.
Inverting matrix Q-responses in terms of 3-D mantle conductivity

- Matrix Q-responses are estimated using Swarm + Cryosat + observatory data

- Responses are inverted using quasi-Newton optimization algorithm applied to penalty function which consists of misfit and regularization terms (note that the inverse problem is highly non-linear)

- Conductivity is recovered in layers: 410-520, 520-670, 670-900 and 900-1150 km

- Conductivity in these layers is parameterized by spherical harmonics up to degree 3

- Forward modellings are performed on lateral grid of $5^\circ \times 5^\circ$

- Outside target depth range 410-1150 km, conductivity model is fixed to 1-D distribution similar to Grayver et al (2017)*; it also includes thin layer of known, laterally-varying, conductance. This layer approximates nonuniform oceans and continents

Results of inversion

- Recovered anomalies are concentrated in/around Pacific Ocean

- Inversion results are rather robust with respect to different inversion set ups (parameterization, regularization, number of Q-matrix elements involved into inversion)

- The model is of low-resolution, since with existing data only low-degree and order external and induced coefficients can be determined, and thus low-degree and order Q-matrices can be estimated and inverted

Two comments:

a) in considered period range (2 – 30 days) 670 – 900 and 900 – 1150 km layers should be better resolved

b) since non-polar data is not involved in the analysis, results in polar regions should be taken with caution
Summary and outlook

- Exploiting satellite data significantly improves separation of non-P10 external and induced coefficients.

- If observatory data are not included into analysis, adding Cryosat to Swarm data improves separation.

- The presented satellite-based 3-D conductivity model is of very low (ocean scale) resolution and is confined to the depths 400 – 1150 km.

- More (well-calibrated) multi-platform (Grace, Iridium, ...) data would potentially improve separation of external and induced contributions and give an opportunity to resolve higher-degree and order terms, and thus to refine satellite-based 3-D models.

- Longer measurements by Swarm will allow for probing deeper structures (> 1150 km).

- Natural way forward is to build/compile global multi-resolution 3-D conductivity model for the whole depth column (0 – 1500 km) using multi-source, multi-data and multi-response approach.