Experimental and numerical study of the resonant feature of internal gravity waves in the case of 'dead water' phenomenon

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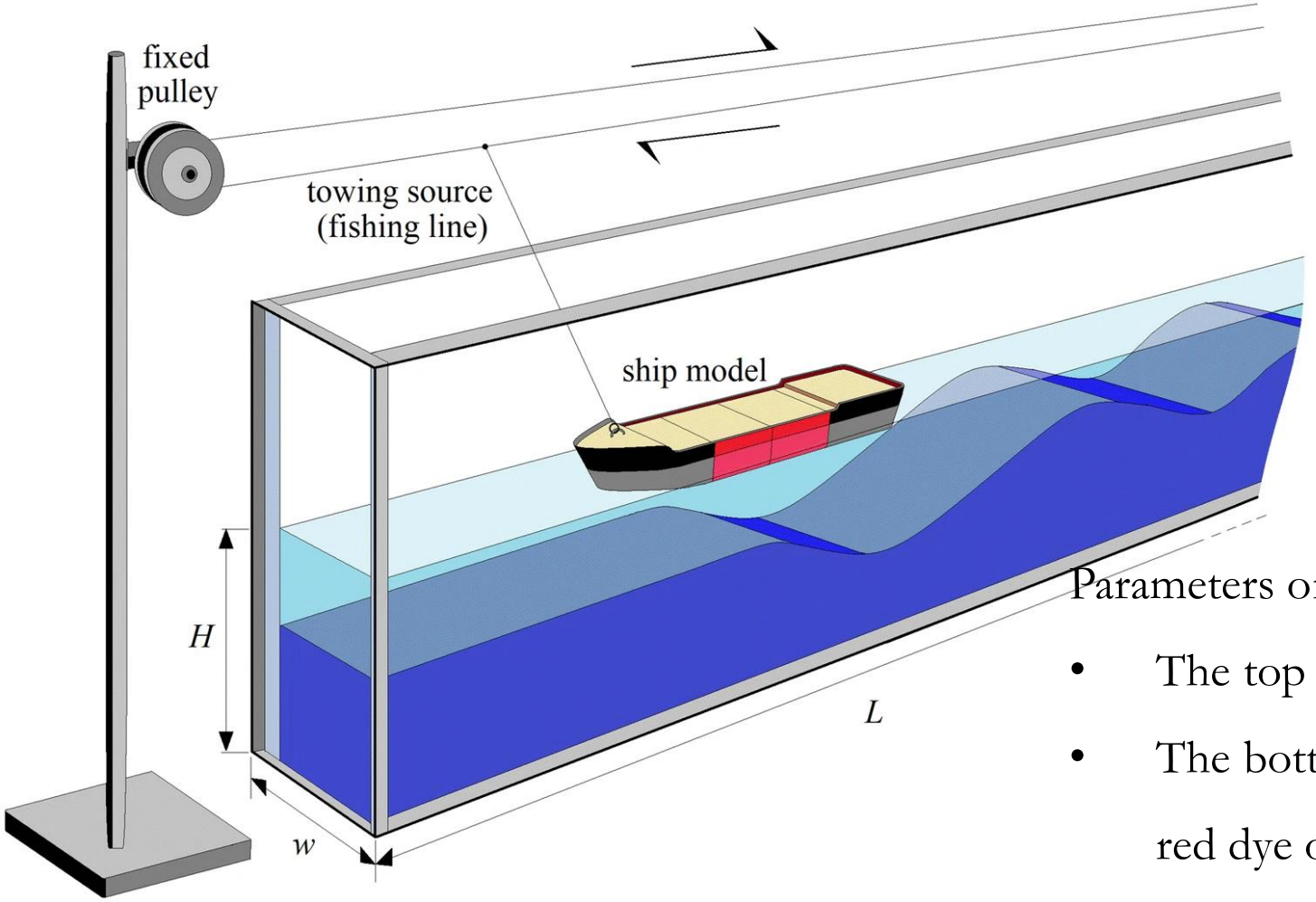
Motivation

- The “dead water” phenomenon has been known to a Norwegian fishermen for centuries.
- The ships were hard to maneuver and/or slowed down almost to a standstill, in highly stratified water.
- Simple experiment, difficult to understand.

Georg Anton Rasmussen, Norwegian Fjord Landscape with a Ship

The experimental setup



L = 239 cm
w = 8.8 cm
H = 12 cm

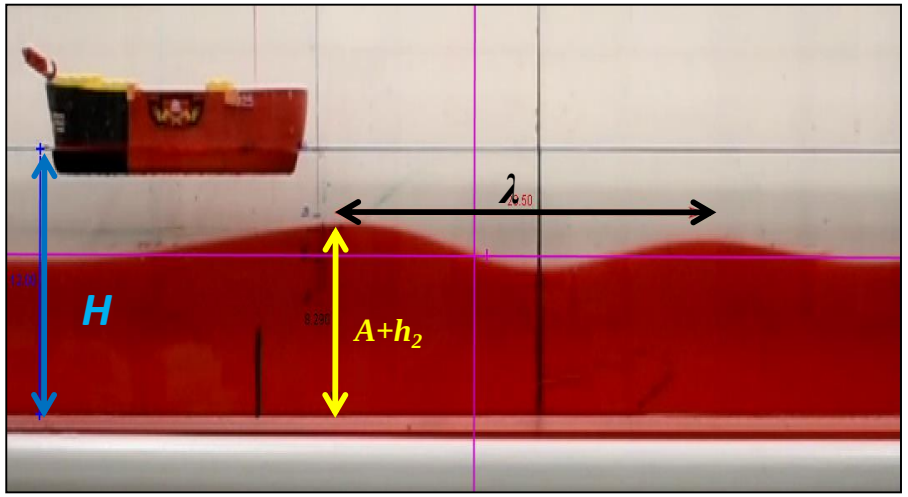
Parameters of the experiments:

- The top layer of thickness h_1 (pure tap water)
- The bottom layer thickness h_2 (water + salt + red dye of density ρ_2)
- five small ships of width of *5.9* cm and different lengths *(l = 9.8; 16.2; 22.6; 29; 35.4 cm)*,
- Towing speed U

Methods

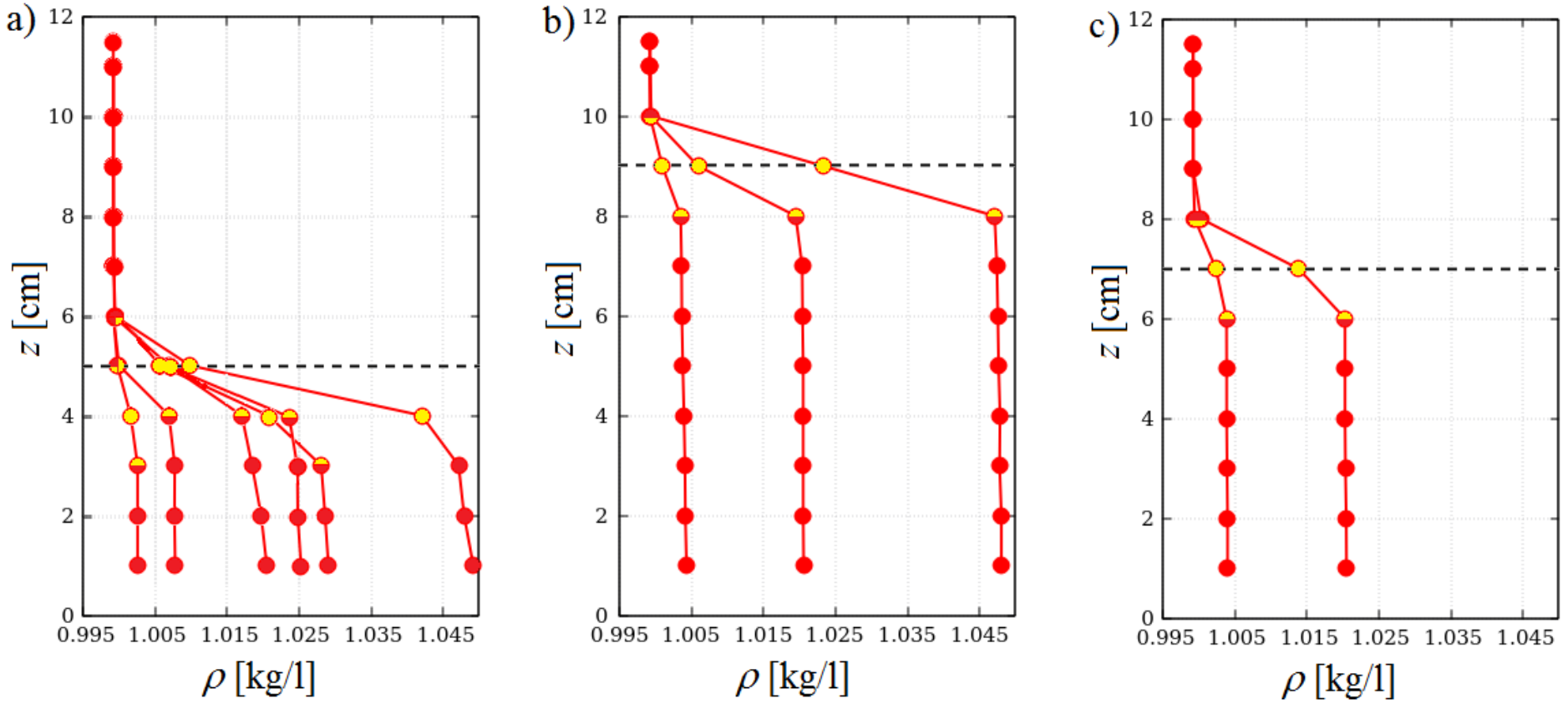
Measured quantities:

- Amplitude A
- Wavelength λ (wave number $k = 2\pi/\lambda$)

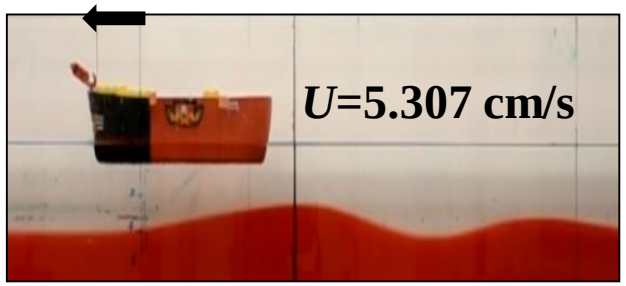


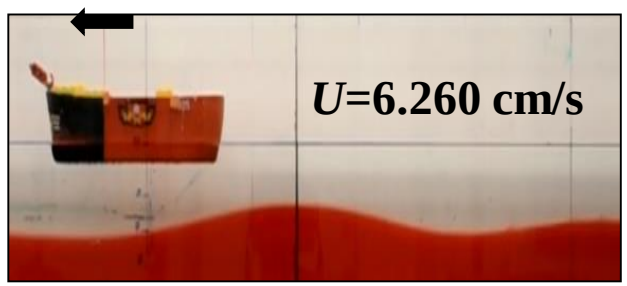
the shape of the inner layer boundary tracking by processing videos using *Tracker*, an open-source correlation-based feature tracking software .

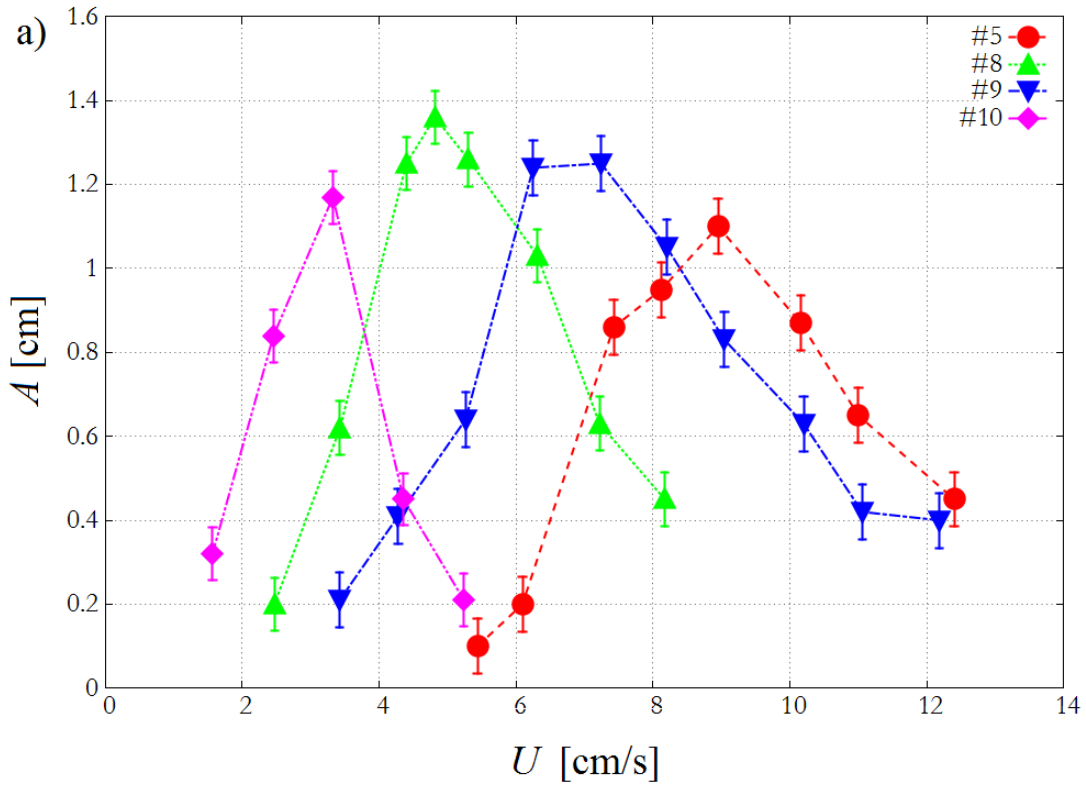
Density profiles from the experiments: 3 layer thickness configurations



resonance curves: ship velocity vs. amplitude

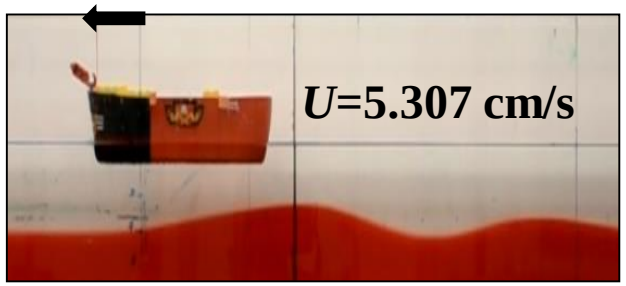


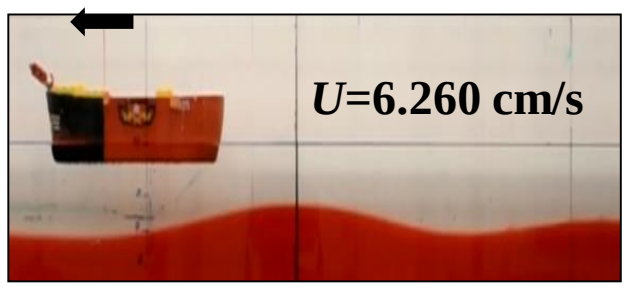




The critical velocity (U*) for the maximum amplitude is highly dependent on the stratification.

resonance curves: ship velocity vs. amplitude

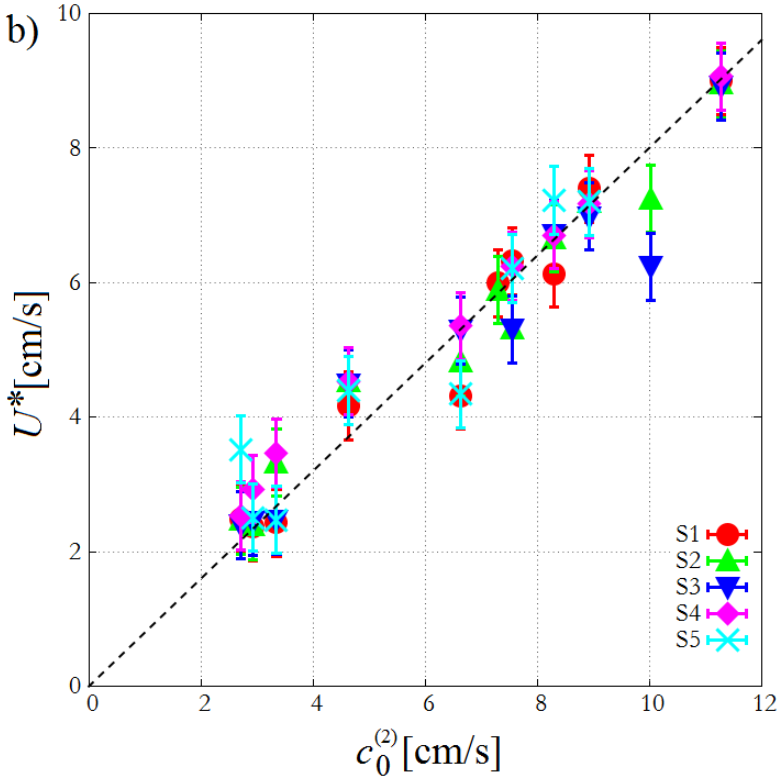




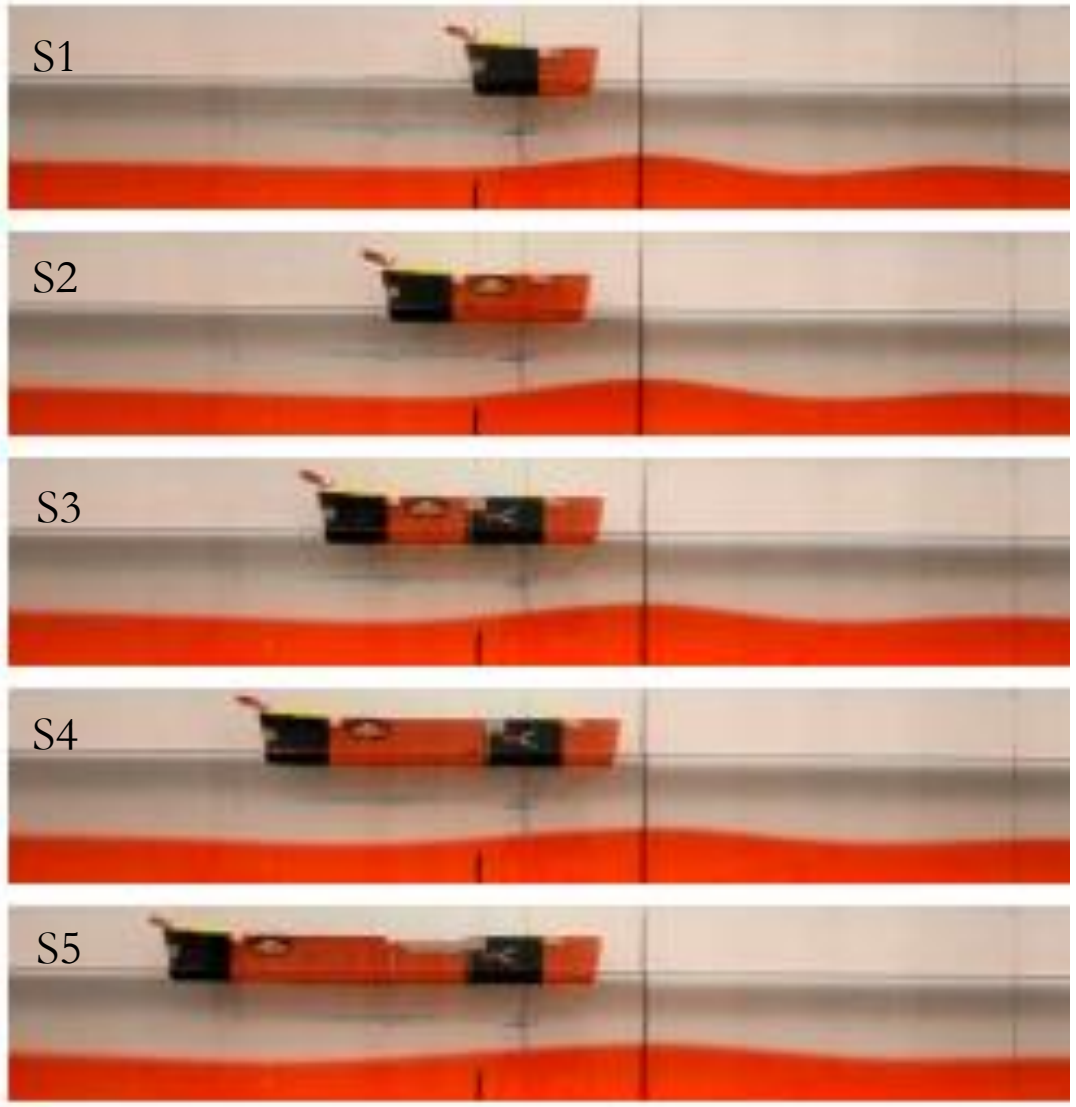
The ship velocity scaled by the two-layer longwave velocity ($c_0^{(2)}$).

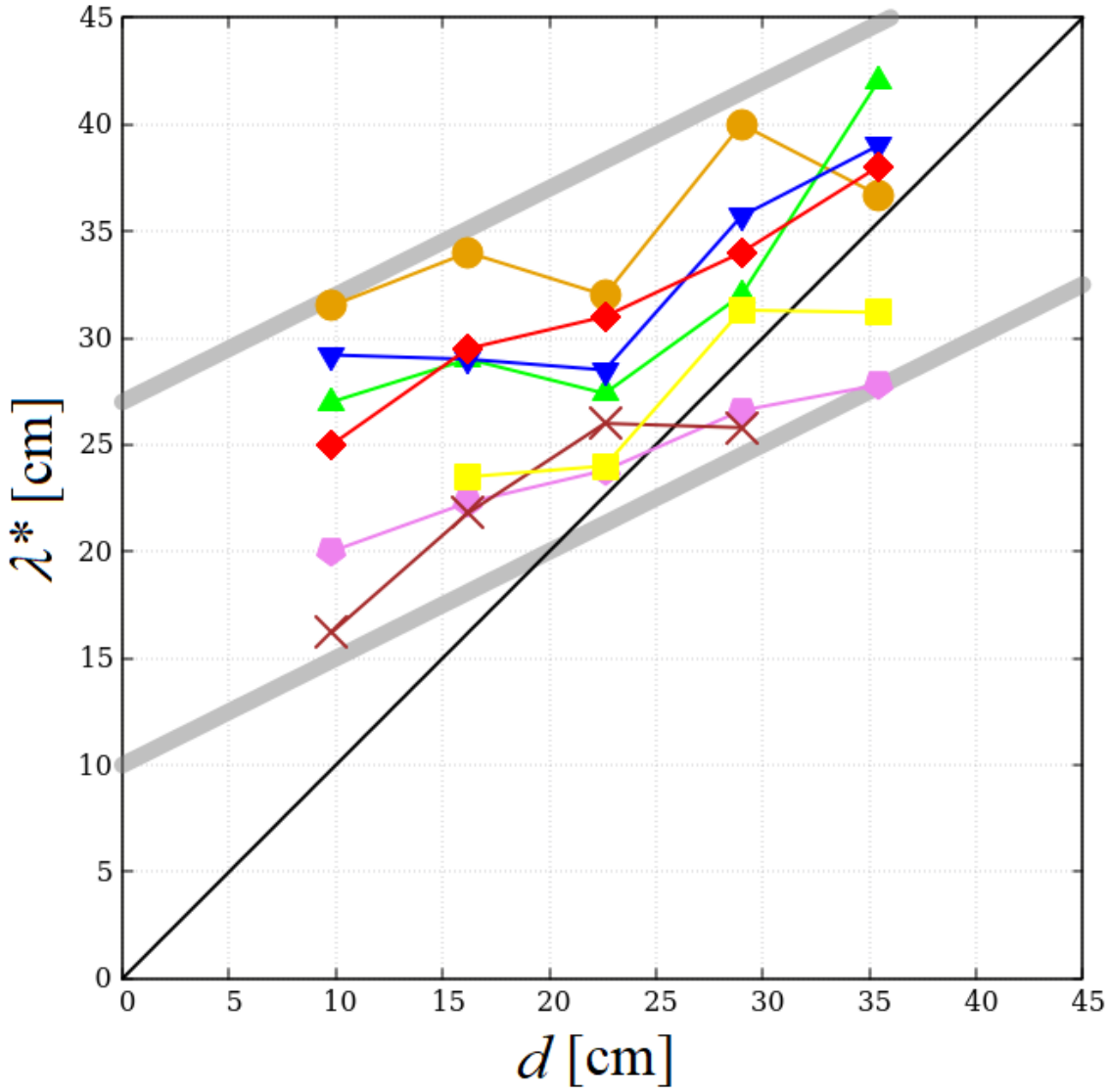
$$c_0^{(2)} = \sqrt{g\frac{\rho_2 - \rho_1}{\rho_1}\frac{H_1 H_2}{H}}$$

$$U^* \approx 0.8\, C_0^{(2)}$$



Critical wavelength λ * the maximum amplitude dependence on the ship of length d



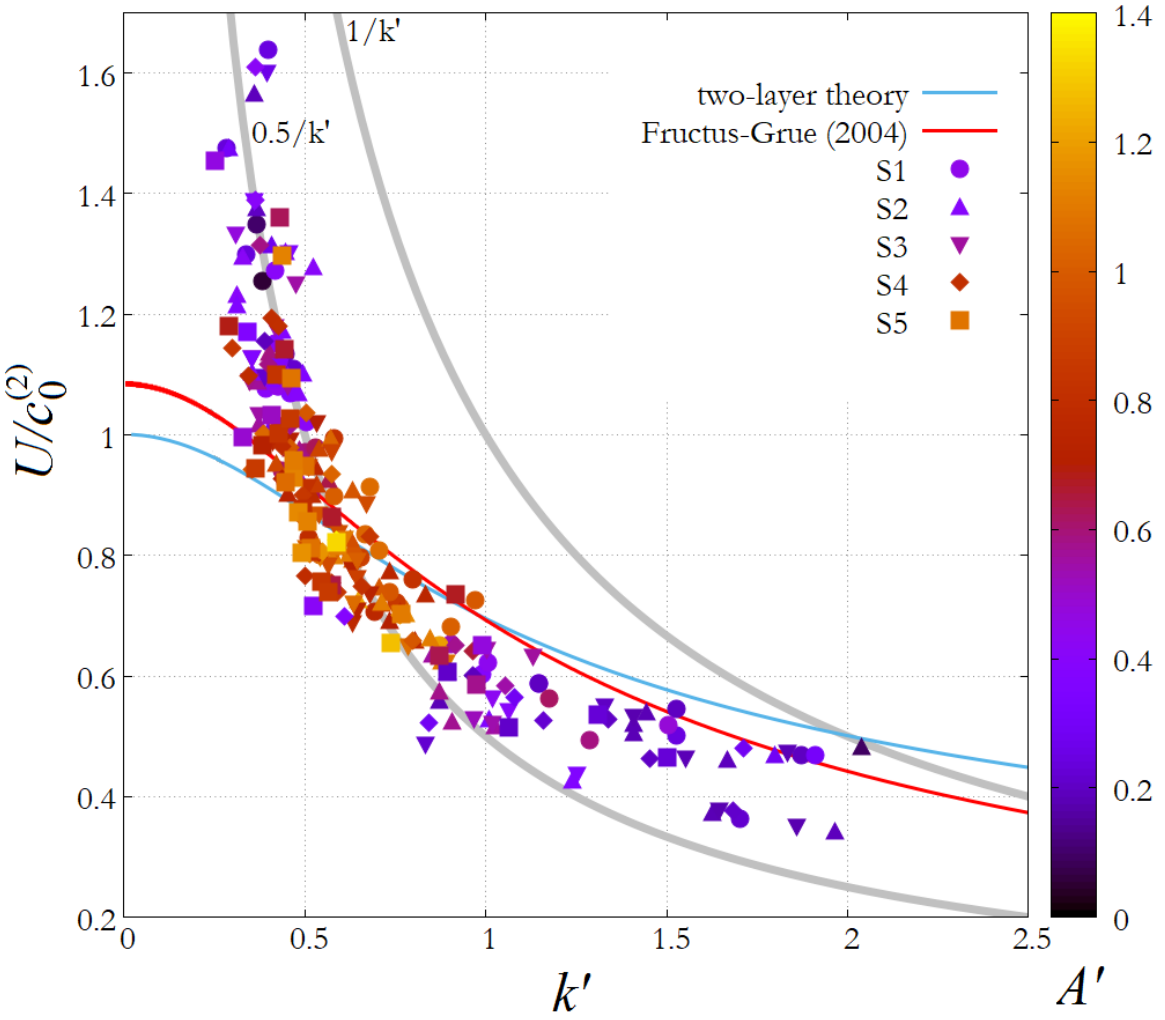


(non-dimensional) velocity-wavenumber

Two and three-layer approximations based on the linear theory.

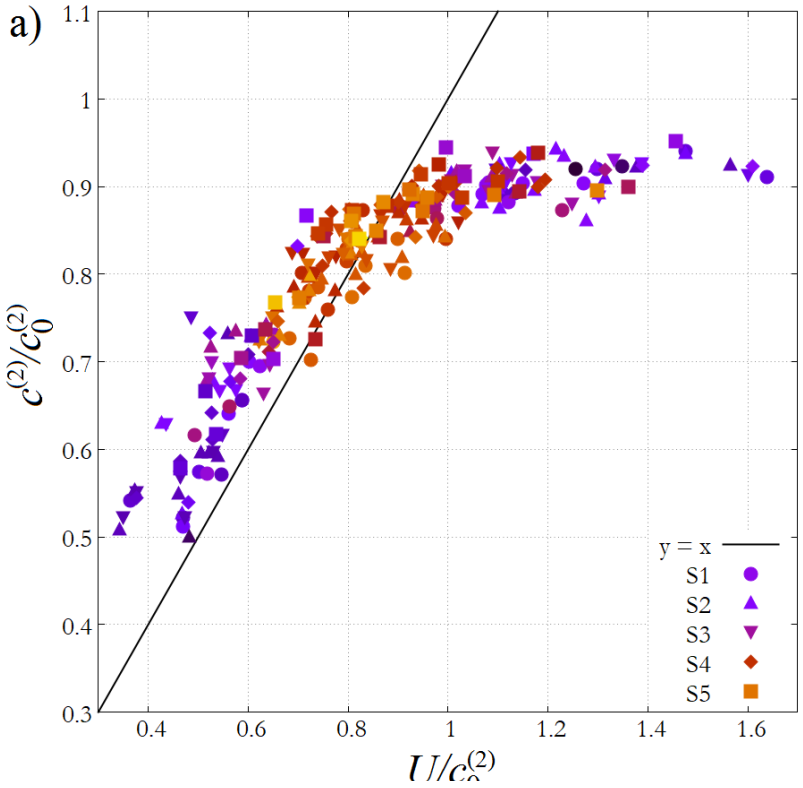
The scaled wavenumber k' by the vertical scale:
$H_r = (h_1\, h_2)/\ (h_1 + h_2)$

At lower wavenumbers (higher velocities; U>U*) the points follow a hyperbolae (the frequency is constant in each stratification).



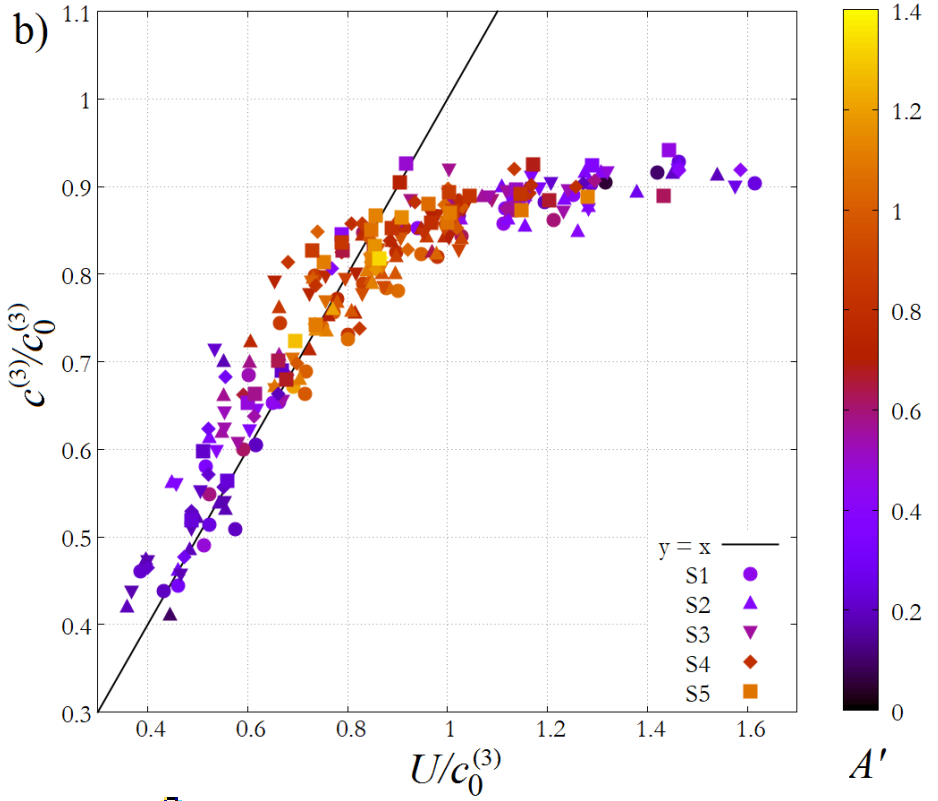
“Free” wave propagation in the speed range U <U *

Two-layer approximation



$$c^{(2)} = \frac{\omega}{k} = \sqrt{\frac{g}{k}\frac{\rho_2 - \rho_1}{\rho_1 \coth(H_1^{(2)}k) + \rho_2 \coth(H_2^{(2)}k)}}$$

Three-layer approximation (Fructus & Grue, 2004)

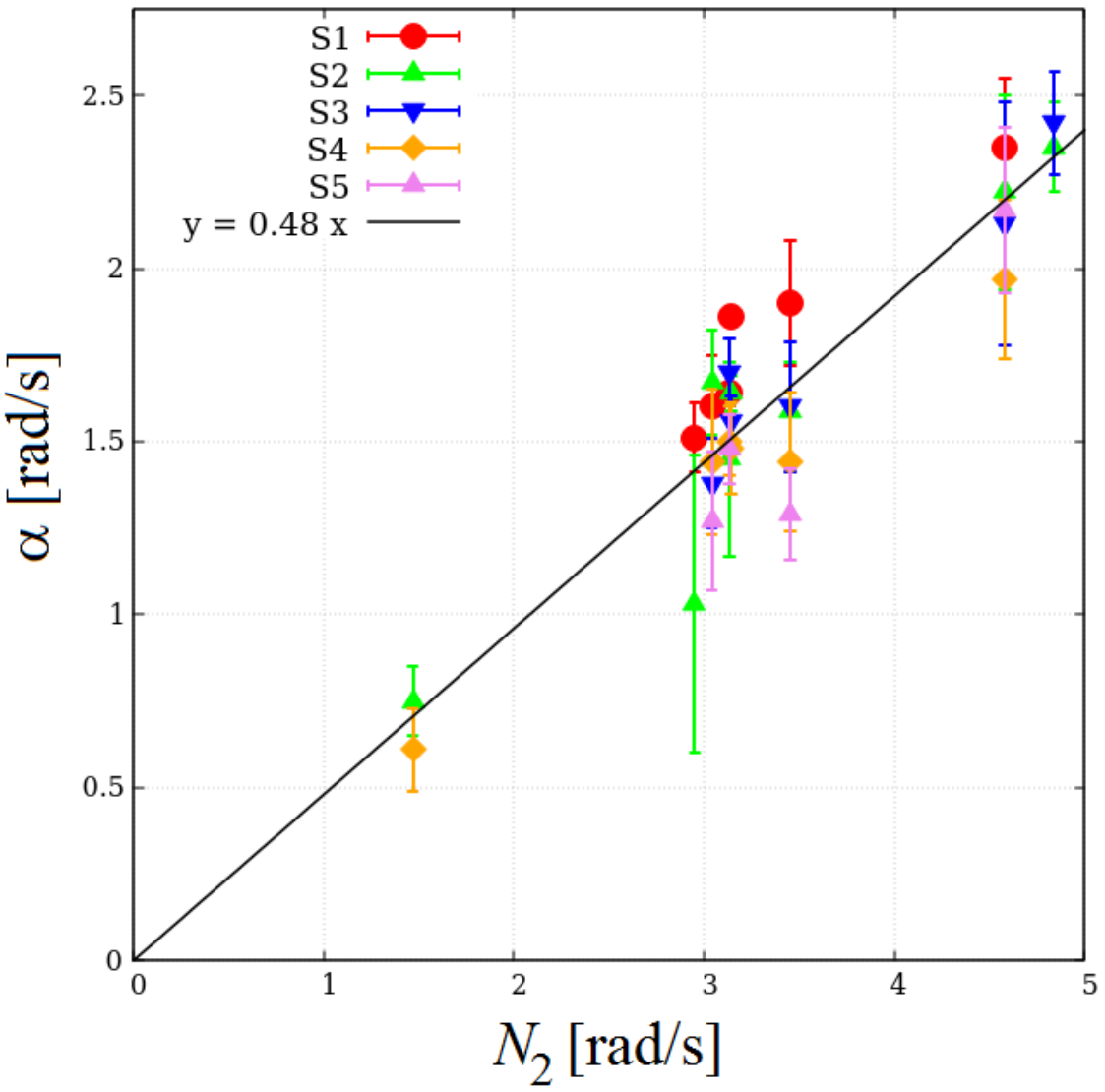


$$K_2^2 - T_1T_2 - T_1T_3 - T_2T_3 = 0$$

$$N_j \equiv \sqrt{-\frac{g}{\rho_0}\frac{\mathrm{d}\rho}{\mathrm{d}z}\Big|_j} \qquad T_j = K_j \cot(K_j h'_j) \qquad K_j = \sqrt{N_j^2/(C'')^2 - k^2}$$

numerical solution!

In the velocity range U <U, the (linear) three-layer approximation performs well.
U> U *, both models fail: there is “different physics”.



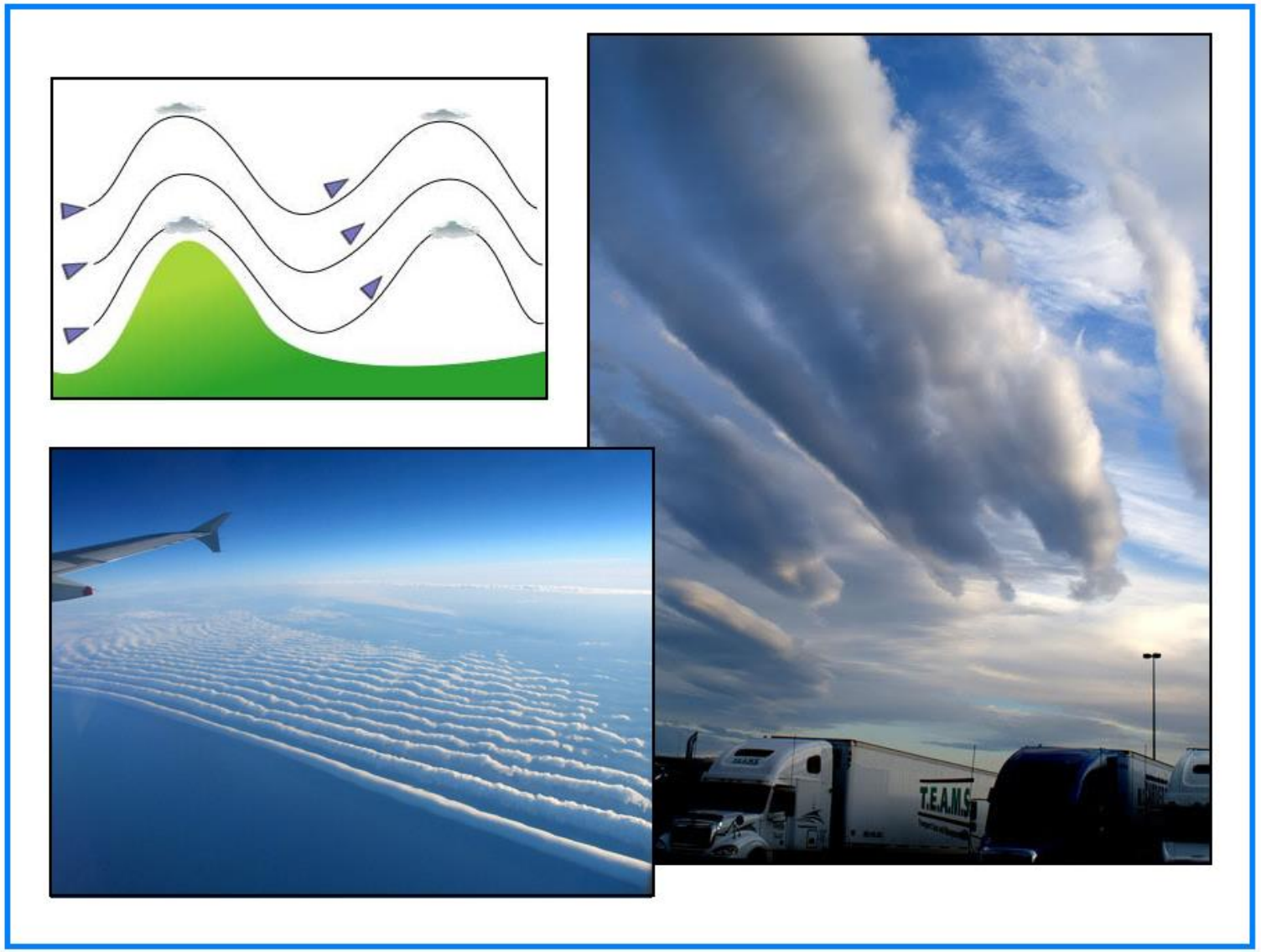
In the range U> U* there is a very strong correlation between the Brunt - Väisälä frequency of the middle layer and the observed α frequency.

$$N = \left(-\frac{g}{\rho_0} \frac{d\rho_0}{dz} \right)^{1/2}$$

Lee waves

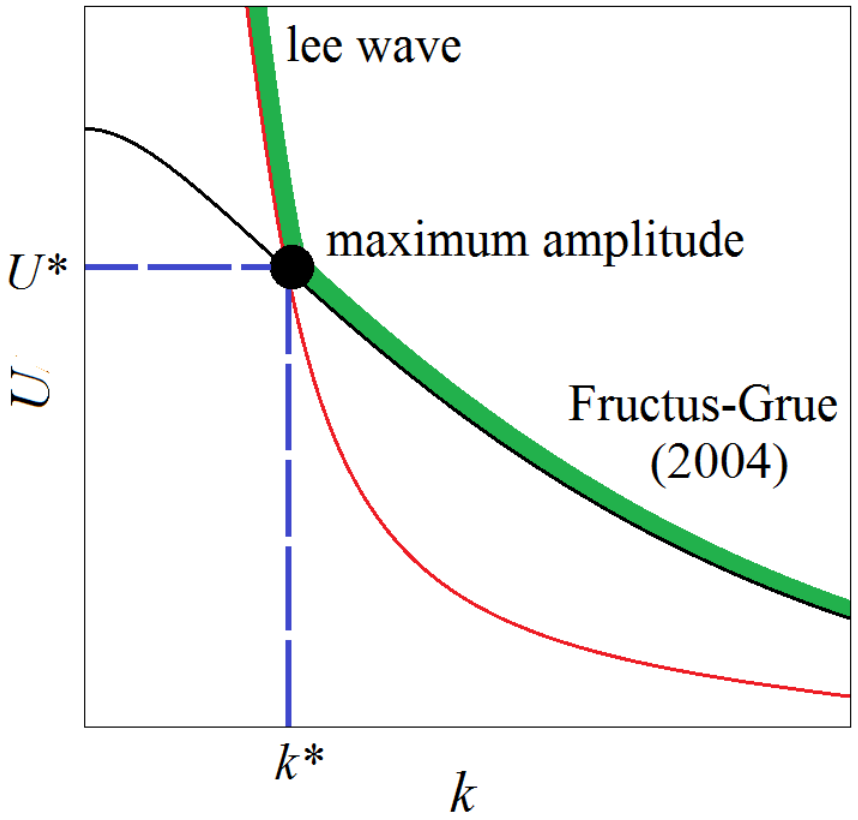
Lee waves characterized by a constant frequency in the continues stratified system.

$$U = Ncos(\varphi)/k_{lee}$$



interpretation

the appearance of the maximum amplitude (in the intersection point marked by black filled circle) by two different types of internal wave types ("free" and "lee") caused by constructive interference.



The observed $U(k)$ domain is the green curve.

Summary

- We experimentally (and numerically) modeled the phenomenon of the "dead water".
- Internal wave generation is a resonance phenomenon, different behaviors for sub- and supercritical wavelengths.
- It's been shown that constructive interference between two types of wave propagation results is this amplification.
- λ^* are increasing at increasing ship lengths.
- Although the amplitudes are large but the linear theory performs fairly well.

- For more details:

K. Medjdoub, I.M. Jánosi, M. Vincze: *Laboratory investigations on the resonant feature of „dead water" phenomenon.* (2020)

Article: https://link.springer.com/content/pdf/10.1007/s00348-019-2830-2.pdf

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