Sensing Ionospheric Turbulence Using GNSS

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Motivation

• Turbulence in the troposphere has been studied in the Kolmogorov sense

• $2/3 \cdot 5/3$ power law well verified in time and space by VLBI/GNSS/SAR (other)...

• Turbulence in the ionosphere not so studied (author impression) in Kolmogorov sense

• This study aims to check whether signal coming from GNSS is compatible with Kolmogorov turbulence theory
Kolmogorov Turbulence Theory Prediction

• Kolmogorov turbulence theory predict refractive index that varies with a power law of 2/3

\[ \chi(d) = C \times d^{2/3} \]

• A radio wave passing through a turbulent medium should have a phase structure function (variogram) that varies with a power law of 5/3

\[ D(d) = \langle (\phi(x) - \phi(x + d))^2 \rangle = C \times d^{5/3} \]

• If we assume that the variation in time are caused by spatial pattern moving in space (frozen flow) the same should be observed in time

\[ D(t) = \langle (\phi(x) - \phi(x + t))^2 \rangle = C \times t^{5/3} \]
Treuhaft Lanyi Model (I)

- Describe the variation of the signal of two electromagnetic ways passing through a turbulent medium.

- Assume a planar geometry and a layer with homogeneous isotropic turbulence.

- Gives the variogram of the signal given the distance of the receiver, and the azimuth and elevation of the satellite.

- Key element is the thickness of the turbulent layer.
Treuhaft Lanyi Model (II)

- Two power laws emerge from the model (in between smooth transition):
  - $5/3$ when distance from station is much lower than the layer thickness
  - $2/3$ when station distance is much larger than layer thickness
- Verified for the troposphere (VLBI, SAR, GNSS) observation
- Ionosphere is also a stratified medium so might be relevant

![Diagram of Treuhaft Lanyi model](image)

LOFAR Study

- Recently (Mevius et al 2016) structure function for the ionospheric delay have been evaluated using the LOw-Frequency Radio interferometer ARray (LOFAR)

- They found power law of $\sim 1.9$ greater than 1.66 (5/3) (Kolmogorov 3D range)

Sun Influence

• Sun is the main driver of ionospheric variation (daily variation)

• This should have an impact on the signal variogram

• It would make sense to remove such daily variation from the signal and test whether the residual has a turbulent behaviour
TEC Caused Delay on GNSS Signals (I)

• The GNSS signal is delayed by the Total Electron Component proportionally with the inverse of the frequency of the signal.

• GNSS satellites emit ranging signals on multiple frequencies this allows to make measurement of the integral of the TEC along the ray-path.

• At the main frequency L1 (GPS, GALILEO) 1 TEC -> 16 cm of delay.

• Tracking of the phase of the signal can be done up to mm accuracy (~ 0.02 TEC) but with an unknown number of cycles.

• Differencing signal from same satellite/receiver and different frequencies all the geometric part can be removed (geometry free combination)
TEC Caused Delay on GNSS Signals (II)

\[
\varphi_1 - \varphi_2 = \left( \frac{1}{f_1} - \frac{1}{f_2} \right) \cdot \int TEC + N_1 \lambda_1 - N_2 \lambda_2 + \beta^r_1 - \beta^r_1 + \beta^s_1 - \beta^s_1
\]

- Phase difference
- Terms depending on TEC through frequency
- Unknown number of cycle, if signal tracking is not lost constant in time
- Electronic bias of the receiver (mostly constant in time) same for two satellite seen from the same receiver
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- Differencing in time all the terms non depending from ionsphere disappears
Daily Variation

- We need a model to remove daily variation
- IONEX model form IGS centers is a good candidate:
  - single layer model
  - 3D grid with resolution of 2.5° in latitude, 5° in longitude and 1h in time
Geostationary Beidou (I)

- Beidou Satellite System has 5 geostationary satellites.
- Their signal pass through the same ionosphere (almost).
- It is a good chance to study the variability in time.
- 3 receivers studied along the equator (data from IGS network)
Geostationary Beidou (II)

- 1.9 power law for the raw signal (same as LOFAR)

Structure function of the signal exhibiting a 1.9 power law (dashed line)
Geostationary Beidou (III)

- Once the daily signal is removed the structure function exhibit a power law close to 5/3
- No clear sign of 2/3 power law

Structure function of the signal with 5/3 power law superimposed (dashed line)
Does the Variability Depend on Sun Flux?

Variance at 240 second distance. The variance is computed each half hour. The sinus of sun elevation on the piercing point is superimposed.

- C01
- C03
- C04

Light dependence but not very clear.
Special Case: Scintillation

- Scintillation was detected on receiver 3 data
- Interesting to study its variogram
- Also close to 5/3 power law
Spatial variogram: Double Difference Analysis

- Evaluate the variogram in space would be interesting.
- Ambiguity terms and electronic bias prevent this analysis directly.
- Ambiguity terms can be fixed (and thus eliminated) in post processing.
- Electronic bias can be removed through double differencing of geometry free observables ($gf$).

\[ \Delta Vgf = (gf_{r1}^{s1} - gf_{r2}^{s1}) - (gf_{r1}^{s2} - gf_{r2}^{s2}) \]

- Treuhaft Lanyi model is suitable only for single difference, it has to be extended to double difference (DD).
The Treuhaft-Lanyi model has been extended to Double difference DD, the variogram can be written as:

\[
V(d, \theta_1, \theta_2, \phi_1, \phi_2, C_0) = C_0 \left( \frac{1}{\sin(\theta_1)^2} \int_{h_1}^{h_2} \int_{h_1}^{h_2} D(r_1^1 - r_1^2)^{2/3} - D(r_1^1 - r_1^1)^{2/3} \, dz \right) \]

\[
+ \frac{1}{\sin(\theta_2)^2} \int_{h_1}^{h_2} \int_{h_1}^{h_2} D(r_2^2 - r_2^2)^{3/3} - D(r_2^2 - r_2^2)^{3/3} \, dz \right) \]

\[
+ \frac{1}{\sin(\theta_1) \cdot \sin(\theta_2)} \int_{h_1}^{h_2} \int_{h_1}^{h_2} 2D(r_1^1 - r_1^2)^{2/3} - D(r_1^1 + r_1^2)^{2/3} \, dz \cdot dz' \]

where \( d \) is the distance between the receiver

\( \theta_1, \theta_2 \): are the two elevations for the satellites

\( \phi_1, \phi_2 \): are the two elevations for the satellites

\( C_0 \): is the turbulence constant

\( r_s^s \): is the raypath from receiver \( r \) to satellite \( s \) along which the integration occurs

\( D() \): is the euclidean distance operator

The integrals can be evaluated numerically

When using satellite using same elevation for both satellite the variogram saturates as in figure on the right.
Case Study: Japan

• Japan has a dense network of homogeneous GNSS receiver (better for GPS ambiguity resolution) with precise GPS pseudoranging code (needed to solve ambiguity in dual frequency case)

• Analysis on GPS L1 L2 frequencies.

• 120 station processed in network mode.
Spatial Structure Function (I)

Geometry of the two satellites

Light agreement of model and data.

<table>
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<th>El 1 [°]</th>
<th>El 2 [°]</th>
<th>Az 1 [°]</th>
<th>Az 2 [°]</th>
<th>Layer height [km]</th>
<th>Layer thickness[km]</th>
<th>C0</th>
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<td>45.95</td>
<td>350</td>
<td>100</td>
<td>5.50E-09</td>
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Spatial Structure Function (II)

Light agreement of model and data.

<table>
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<th>EI 1 [°]</th>
<th>EI 2 [°]</th>
<th>Az 1 [°]</th>
<th>Az 2 [°]</th>
<th>Layer height [km]</th>
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<tbody>
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<td>-2.62</td>
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Conclusion

- The temporal and spatial variability of ionospheric delay on GNSS has been studied looking for Kolmogorov-like turbulent behaviour

- Raw signal temporal structure function match the power law found in LOFAR studies

- A daily variation + turbulent signal model has been proposed

- Structure function in time computed using geostationary satellite show a power law close to the one from kolmogorov turbulence once the daily signal is removed

- Treuhaft Lanyi model has been extended to double difference to study the spatial structure function from GNSS

- Experimental data computed from 1 day on dense GNSS Japanese network shows light agreement with the model
Future work

• Need for denser data in space -> PPP processing with ambiguity resolution

• Calibrate Ionospheric bias of the receiver -> from double difference processing back to single difference processing

• The model has to be validated in more places on the earth

• Account for inomougenity in the vertical profile of tutbulence