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Earth's core turbulence and anisotropic diffusion

Several anisotropy rotating magnetoconvection models

Main equations and parameters

Comparison of different anisotropies' effects

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- Anisotropies, in particular the strong ones, allow the preference (or/and existence) of some modes, what is not possible in the isotropic case
- Usually the anisotropies with greater diffusivity in vertical direction facilitate convection, but the anisotropy in magnetic diffusivity influences only some case of convection
- The case with only thermal and magnetic diffusivity anisotropic, is really unique and this highlights the importance of these two turbulent diffusivities; however, some surprising behaviour with greater diffusivity in vertical direction suggests that also anisotropy in viscosity is important

anisotropy = anisotropic diffusion, anisotropic diffusive coefficients, anisotropic diffusivities

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There is strong belief that the Earth's Core is in turbulent state

The Earth's core is driven into motion by buoyancy forces so strong that the flow and field are turbulent, fluctuating on every length and time scale, as it is accepted by the most of geophysicists. Traditional approach to turbulence (see, e.g., Krause and Rädler, 1980) splits each variables into mean and fluctuating ones, e.g., $\mathbf{u} = \overline{\mathbf{u}} + \mathbf{u}'$, $\mathbf{B} = \overline{\mathbf{B}} + \mathbf{B}'$, thus the mean IE is:

$$\frac{\partial \overline{\mathbf{B}}}{\partial t} = \eta_0 \nabla^2 \overline{\mathbf{B}} + \boldsymbol{\nabla} \times (\overline{\mathbf{u}} \times \overline{\mathbf{B}}) + \boldsymbol{\nabla} \times \boldsymbol{\mathcal{E}}, \quad \boldsymbol{\mathcal{E}} = \overline{\mathbf{u}' \times \mathbf{B}'}$$

within suitable approximations and conditions we can say that $\boldsymbol{\nabla} \times \boldsymbol{\mathcal{E}} = -\boldsymbol{\nabla} \times (\beta \boldsymbol{\nabla} \times \overline{\mathbf{B}})$, the well known "$\beta$-effect" In general, due to turbulence the diffusive coefficients are anisotropic; for instance the buoyancy has a preferred direction $\Rightarrow$ local turbulence may be significantly anisotropic with respect to gravity direction

$$\boldsymbol{\nabla} \times \boldsymbol{\mathcal{E}} = -\boldsymbol{\nabla} \times (\beta \boldsymbol{\nabla} \times \overline{\mathbf{B}}) \rightarrow \boldsymbol{\nabla} \times \boldsymbol{\mathcal{E}} = -\boldsymbol{\nabla} \times (\boldsymbol{\beta} \boldsymbol{\nabla} \times \overline{\mathbf{B}}) = \boldsymbol{\nabla} \cdot (\boldsymbol{\beta} \boldsymbol{\nabla} \overline{\mathbf{B}})$$

where $\boldsymbol{\beta}$ is a tensor quantity, thus, in general we should speak about an "anisotropic beta-effect" or anisotropic magnetic diffusivity, anisotropic $\boldsymbol{\eta}$ tensor ($\boldsymbol{\eta} = \boldsymbol{\eta}_0 + \boldsymbol{\beta}$), which parameterizes mean electromotive force, $\boldsymbol{\mathcal{E}}$.
Analogous considerations can be made (see, e.g., Fearn and Roberts, 2007) for Navier-Stokes and heat equations

$$\rho \left[\frac{\partial \overline{\mathbf{u}}}{\partial t} + \overline{\mathbf{u}} \cdot \boldsymbol{\nabla} \overline{\mathbf{u}} + 2\Omega \hat{\mathbf{k}} \times \overline{\mathbf{u}} \right] = \boldsymbol{\nabla} \overline{p} + \nu \nabla^2 \overline{\mathbf{u}} + (1/\mu_0) \overline{\mathbf{B}} \cdot (\boldsymbol{\nabla} \overline{\mathbf{B}}) + \boldsymbol{\nabla} \cdot \overline{\mathbf{u}'\mathbf{u}' - \mathbf{B}'\mathbf{B}'}$$

where, within some approximations $\boldsymbol{\nabla} \cdot \overline{\mathbf{u}'\mathbf{u}' - \mathbf{B}'\mathbf{B}'} = \boldsymbol{\nabla} \cdot (\boldsymbol{\nu}_T \boldsymbol{\nabla} \overline{\mathbf{u}})$, tensorial turbulent viscosity, $\boldsymbol{\nu}_T$, parameterizes Reynolds and Maxwell stresses related to $\overline{\mathbf{u}'\mathbf{u}' - \mathbf{B}'\mathbf{B}'}$.

$$\frac{\partial \overline{\Theta}}{\partial t} + \overline{\mathbf{u}} \cdot \boldsymbol{\nabla} \overline{\Theta} = \kappa \nabla^2 \overline{\Theta} - \boldsymbol{\nabla} \cdot \overline{\mathbf{u}'\Theta'}$$

where, in some cases $-\boldsymbol{\nabla} \cdot \overline{\mathbf{u}'\Theta'} = \boldsymbol{\nabla} \cdot (\boldsymbol{\kappa}_T \boldsymbol{\nabla} \overline{\Theta})$, $\boldsymbol{\kappa}_T$ is the parameterized tensorial turbulent thermal diffusivity.

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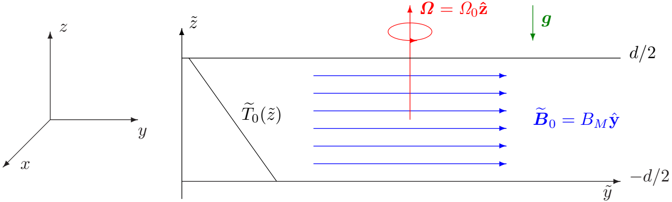
Geometry of the models

Turbulence can cause anisotropy, therefore it is worth to introduce anisotropy in the rotating magnetoconvection models; this was done for the first time in the model by Šoltis and Brestenský (2010), which considered only viscosity and thermal diffusivity anisotropic, and recently it was advanced by Filippi et al. (2019), by deeming also the magnetic diffusivity as anisotropic.
These models deals about Rotating Magnetoconvection in horizontal plane layer rotating about vertical axis and permeated by homogeneous horizontal magnetic field (Roberts and Jones, 2000), influenced by anisotropic diffusivities.



Model of rotating magnetoconvection with homogeneous horizontal basic magnetic field in the infinite horizontal unstable stratified fluid layer with temperature profile, $T_0(\tilde{z})$.

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Different anisotropic rotating magnetoconvection models studied

k	isotropic	anisotropic	name	reference
i	ν, κ, η	–	isotropy	Roberts and Jones (2000)
h	ν, η	κ	pure-h anisotropy	Donald and Roberts (2004)
m	ν, κ	η	pure-m anisotropy	FB
p	η	ν, κ	partial-p anisotropy	Šoltis and Brestenský (2010)
q	ν	κ, η	partial-q anisotropy	FB
f	–	ν, κ, η	full anisotropy	Filippi et al. (2019)

Table: Different anisotropy cases, $k = i$, h, m, p, q and f, with related isotropic or/and anisotropic diffusivities, ν, κ and η. Cases i, p and f have been already studied and published, cases m and q are new (FB) and case h is not yet published.

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Anisotropic diffusive coefficients in SA and BM anisotropy (partial and full)

SA and BM anisotropic diffusions modelling

Partial anisotropy model by Šoltis and Brestenský (2010) with $\boldsymbol{\nu}$, $\boldsymbol{\kappa}$ anisotropic tensors and $\boldsymbol{\eta}_0$ isotropic tensor, where

$$\boldsymbol{\nu} = \begin{pmatrix} \nu_{xx} & 0 & 0 \\ 0 & \nu_{yy} & 0 \\ 0 & 0 & \nu_{zz} \end{pmatrix}, \quad \boldsymbol{\kappa} = \begin{pmatrix} \kappa_{xx} & 0 & 0 \\ 0 & \kappa_{yy} & 0 \\ 0 & 0 & \kappa_{zz} \end{pmatrix}, \quad \boldsymbol{\eta}_0 = \begin{pmatrix} \eta_0 & 0 & 0 \\ 0 & \eta_0 & 0 \\ 0 & 0 & \eta_0 \end{pmatrix}.$$

Magnetic diffusion also due to turbulence

inspired a full anisotropy model (Filippi et al., 2019) with anisotropic $\boldsymbol{\nu}$, $\boldsymbol{\kappa}$, $\boldsymbol{\eta}$, where $\eta_{xx} = \eta_0 + \beta_{xx}$, $\eta_{yy} = \eta_0 + \beta_{yy}$, $\eta_{zz} = \eta_0 + \beta_{zz}$.

Later, we will speak about another kind of anisotropy, heat transport anisotropy, a partial anisotropy with $\boldsymbol{\nu}$ also isotropic.

SA, Stratification anisotropy:

$\nu_{xx} = \nu_{yy} \neq \nu_{zz}$, $\kappa_{xx} = \kappa_{yy} \neq \kappa_{zz}$, $\eta_{xx} = \eta_{yy} = \eta_{zz} = \eta_0$ and $\eta_{xx} = \eta_{yy} \neq \eta_{zz}$.

Gravity or/and the Archimedean buoyancy force are dominant

BM by Braginsky and Meytlis (1990):

$\nu_{xx} < \nu_{yy} = \nu_{zz}$, $\kappa_{xx} < \kappa_{yy} = \kappa_{zz}$, $\eta_{xx} = \eta_{yy} = \eta_{zz} = \eta_0$ and $\eta_{xx} < \eta_{yy} = \eta_{zz}$.

Rotation and magnetic field are dominant

There is “horizontal isotropy” in SA, but not in BM

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Heat transport anisotropy

Donald and Roberts (2004) developed a dynamo model which introduces anisotropy only in the thermal diffusivity. Inspired by them we began to study also another case of anisotropy in rotating magnetoconvection, we call it Heat transport anisotropy.

$$\boldsymbol{\nu} = \begin{pmatrix} \nu_0 & 0 & 0 \\ 0 & \nu_0 & 0 \\ 0 & 0 & \nu_0 \end{pmatrix}, \quad \boldsymbol{\kappa} = \begin{pmatrix} \kappa_{xx} & 0 & 0 \\ 0 & \kappa_{yy} & 0 \\ 0 & 0 & \kappa_{zz} \end{pmatrix}, \quad \boldsymbol{\eta}_0 = \begin{pmatrix} \eta_0 & 0 & 0 \\ 0 & \eta_0 & 0 \\ 0 & 0 & \eta_0 \end{pmatrix}.$$

This new simplified model is developed for wider chance to compare the effects of more anisotropic models.

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Dimensionless governing equations and parameters 1/2

Now, let us move to magnetoconvection problems, for simplicity we use for the fluctuating variables the symbols, $\tilde{\mathbf{u}}$, $\tilde{\mathbf{b}}$, $\tilde{\theta}$ instead of $\mathbf{u}'$, $\mathbf{B}'$, Θ' used when we spoke about turbulence and anisotropy.

In the most general anisotropy model we get the following main dimensionless equations, after using standard procedures

$$R_o \partial_t \mathbf{u} + \hat{\mathbf{z}} \times \mathbf{u} = -\boldsymbol{\nabla} p + \Lambda_z (\boldsymbol{\nabla} \times \mathbf{b}) \times \hat{\mathbf{y}} + R\vartheta \hat{\mathbf{z}} + E_z \nabla_\nu^2 \mathbf{u},$$

$$\partial_t \mathbf{b} = \boldsymbol{\nabla} \times (\mathbf{u} \times \hat{\mathbf{y}}) + \nabla_\eta^2 \mathbf{b},$$

$$q_z^{-1} \partial_t \vartheta = \hat{\mathbf{z}} \cdot \mathbf{u} + \nabla_\kappa^2 \vartheta,$$

$$\nabla \cdot \mathbf{u} = 0, \qquad \nabla \cdot \mathbf{b} = 0,$$

where $\tilde{z} = dz$, $\tilde{\mathbf{b}} = B_M \mathbf{b}$ and

$$\tilde{\mathbf{u}} = U\mathbf{u} = \frac{\eta_{zz}}{d}\mathbf{u}, \quad \tilde{t} = \frac{d}{U}t = \frac{d^2}{\eta_{zz}}t, \quad \tilde{p} = 2\Omega_0 \eta_{zz} \rho_0 p, \quad \tilde{\vartheta} = \frac{\eta_{zz} \Delta T}{\kappa_{zz}} \vartheta$$

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Dimensionless governing equations and parameters 2/2

$$R_o = \frac{\eta_{zz}}{2\Omega_0 d^2},\ \Lambda_z = \frac{B_M^2}{2\Omega_0 \rho_0 \mu_0 \eta_{zz}},\ E_z = \frac{\nu_{zz}}{2\Omega_0 d^2},\ R = \frac{\alpha_T g \Delta T d}{2\Omega_0 \kappa_{zz}},\ q_z = \frac{\kappa_{zz}}{\eta_{zz}},$$

$$\alpha_\nu = \frac{\nu_{xx}}{\nu_{zz}}, \quad \alpha_\kappa = \frac{\kappa_{xx}}{\kappa_{zz}}, \quad \alpha_\eta = \frac{\eta_{xx}}{\eta_{zz}}$$

SA anisotropy:

$$E_z \nabla_\nu^2 \mathbf{u} = E_z[(1-\alpha_\nu)\partial_{zz} + \alpha_\nu \nabla^2]\mathbf{u}, \quad \nabla_\kappa^2 \vartheta = [(1-\alpha_\kappa)\partial_{zz} + \alpha_\kappa \nabla^2]\vartheta,$$

$$\nabla_\eta^2 \mathbf{b} = [(1-\alpha_\eta)\partial_{zz} + \alpha_\eta \nabla^2]\mathbf{b}$$

BM anisotropy:

$$E_z \nabla_\nu^2 \mathbf{u} = E_z[(\alpha_\nu - 1)\partial_{xx} + \nabla^2]\mathbf{u}, \quad \nabla_\kappa^2 \vartheta = [(\alpha_\kappa - 1)\partial_{xx} + \nabla^2]\vartheta,$$

$$\nabla_\eta^2 \mathbf{b} = [(\alpha_\eta - 1)\partial_{xx} + \nabla^2]\mathbf{b}$$

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Method of solution in the most general case

A solution can be found by some methods developed, e.g., in Chandrasekhar (1961):

$$\nabla \cdot \mathbf{v} = \nabla \cdot \mathbf{B} = 0 \Rightarrow \mathbf{u} = a^{-2}[\nabla \times (\nabla \times w\hat{\mathbf{z}}) + \nabla \times \omega\hat{\mathbf{z}}] \quad \text{and} \quad \mathbf{b} = a^{-2}[\nabla \times (\nabla \times b\hat{\mathbf{z}}) + \nabla \times j\hat{\mathbf{z}}]$$

All perturbations (w, ω, b, j and ϑ) have a form like
$f(x, y, z, t) = \Re e[F(z) \exp(ilx + imy) \exp(\lambda t)]$ with $a = \sqrt{l^2 + m^2}$, $\lambda = i\sigma \in \mathbb{C}$,
$F(z) = W(z)$, $\Omega(z)$, $B(z)$, $J(z)$, and $\Theta(z)$
If we take some curls of momentum, induction and heat equations, introduce inverted magnetic Prandtl number $p = \eta_{zz}/\nu_{zz} = R_o/E_z$ and we consider the simplest boundary conditions at $z = \pm 1/2$, i.e.:

– stress free boundaries: $W = D^2W = D\Omega = 0$
– perfectly thermally conducting boundaries: $\Theta = 0$
– perfectly electrically conducting boundaries: $B = DJ = 0$.

"dispersion relation"

$$Ra^2 \frac{q_z(K_\eta^2 + \lambda)}{(q_z K_\kappa^2 + \lambda)} = \frac{K^2 N_f^2 + \pi^2 \left(K_\eta^2 + \lambda\right)^2}{N_f} \quad (1)$$

where N_f

$$N_f = (K_\eta^2 + \lambda)(K_\nu^2 + p\lambda)E_z + \Lambda_z m^2, \quad (2)$$

$K^2 = \pi^2 + a^2$,
$K_\kappa^2 = \pi^2 + \alpha_\kappa a^2$, $K_\nu^2 = \pi^2 + \alpha_\nu a^2$, $K_\eta^2 = \pi^2 + \alpha_\eta a^2$ in SA
$K_\kappa^2 = \pi^2 + \alpha_\kappa l^2 + m^2$, $K_\nu^2 = \pi^2 + \alpha_\nu l^2 + m^2$, $K_\eta^2 = \pi^2 + \alpha_\eta l^2 + m^2$ in BM

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Critical Rayleigh numbers in stationary SA (partial, full) anisotropy

From now on, for simplicity we assume that in partial and full anisotropy:

$$\alpha_\kappa = \alpha_\nu = \alpha, \qquad \alpha_\kappa = \alpha_\nu = \alpha_\eta = \alpha$$

$\lambda = 0,\ 1_\alpha = (K_\alpha/K)^2$

$$R_p^s = \frac{\pi(\pi^2 + a^2)^{1/2} K_\alpha^2}{a^2}\left(C_p + \frac{1}{C_p}\right),\ R^s = \frac{\pi(\pi^2+a^2)^{3/2}}{a^2}\left(C_\alpha + \frac{1_\alpha^2}{C_\alpha}\right).$$

where $C_p = \frac{E_z K^2 K_\alpha^2 + \Lambda m^2}{\pi K}$ and $C_\alpha = \frac{E_z K_\alpha^4 + \Lambda m^2}{\pi K}$
in SA $K_\alpha^2 = \pi^2 + \alpha a^2$ and

$$C_p = \frac{E_z(\pi^2+a^2)(\pi^2+\alpha a^2) + \Lambda m^2}{\pi(\pi^2+a^2)^{1/2}},\ C_\alpha = \frac{E_z(\pi^2+\alpha a^2)^2 + \Lambda m^2}{\pi(\pi^2+a^2)^{1/2}} \qquad (3)$$

$$\frac{\partial R_p^s}{\partial C_p} = 0,\ \frac{\partial R_p^s}{\partial a^2} = 0 \qquad \text{and} \qquad \frac{\partial R^s}{\partial C_\alpha} = 0,\ \frac{\partial R^s}{\partial a^2} = 0$$

we have critical C_p, a^2 and C_α, a^2 in case of partial and full anisotropy respectively thus

$$C_{pc} = 1,\ a_c^2 = \pi^2 2_\alpha = \pi^2 \frac{1+\sqrt{1+8\alpha}}{2\alpha} \quad \text{and} \quad C_{\alpha c} = 1_\alpha, a_c^2 = \pi^2 2_\alpha \qquad (4)$$

by the (3) and (4) it is possible to obtain m_c and l_c

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Critical Rayleigh numbers in stationary SA m-case anisotropy

$\alpha_\nu = \alpha_\kappa = 1, \ \alpha_\eta = \alpha$

$$R^s_m = \frac{\pi}{a^2} K^2 K_\alpha \left(\frac{C_m}{1_\alpha} + C_m^{-1} \right)$$

where

$$C_m = \frac{K_\alpha^2 K^2 E + \Lambda_z m^2}{\pi K_\alpha}$$

$$\frac{\partial R^s_m}{\partial C_m} = 0 \qquad \text{and} \qquad \frac{\partial R^s_m}{\partial a^2} = 0$$

$$C_{mc} = \sqrt{1_\alpha} \qquad \text{and} \qquad a_c^2 = 2\pi^2$$

a_c in pure-m anisotropy is identical to the isotropic one

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Critical Rayleigh numbers in stationary heat transport and partial-q SA anisotropy

From now on, for simplicity we assume that in heat transport and partial-q anisotropy:

$$\alpha_\kappa = \alpha, \alpha_\nu = \alpha_\eta = 1, \qquad \alpha_\nu = 1, \alpha_\kappa = \alpha_\eta = \alpha$$

$\lambda = 0,\ 1_\alpha = (K_\alpha/K)^2$

$$R_h^s = 1_\alpha \frac{\pi(\pi^2+a^2)^{3/2}}{a^2}\left(C + \frac{1}{C}\right),\ R_q^s = \frac{\pi K_\alpha^3}{a^2}\left(\frac{C_q}{1_\alpha} + C_q^{-1}\right).$$

in SA $K_\alpha^2 = \pi^2 + \alpha a^2$ and

$$C = \frac{EK^4 + \Lambda m^2}{\pi K},\ C_q = \frac{K_\alpha^2 K^2 E + \Lambda_z m^2}{\pi K_\alpha}$$

$$\frac{\partial R_h^s}{\partial C} = 0,\ \frac{\partial R_h^s}{\partial a^2} = 0 \quad \text{and} \quad \frac{\partial R_q^s}{\partial C_q} = 0,\ \frac{\partial R_q^s}{\partial a^2} = 0$$

$$C_c = 1,\ a_c^2 = \pi^2 2_\alpha \quad \text{and} \quad C_{qc} = \sqrt{1_\alpha},\ a_c^2 = \pi^2 2_\alpha$$

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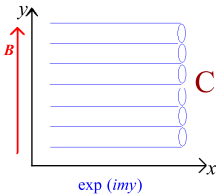
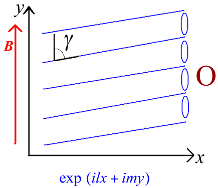
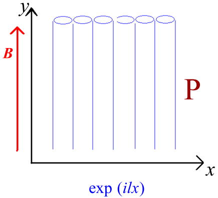
The orientation of the rolls of convection

Roberts and Jones (2000)







Cross rolls (stationary convection SC mode)

Oblique roll (stationary convection SO mode)

Parallel rolls (stationary convection P mode)

$\gamma = \arctan\left(\frac{m}{l}\right)$ is the angle between the axis of rolls and the magnetic field $\boldsymbol{B}$.

$l = 0 \Rightarrow \gamma = \pi/2$ rolls $\perp$ $\boldsymbol{B}$, $m = 0 \Rightarrow \gamma = 0$ rolls $\parallel$ $\boldsymbol{B}$

graphical results about partial and full Sa, So and BM anisotropy

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Comparison of all anisotropic rotating magnetoconvection models studied

Several models of anisotropic magnetoconvection studied: isotropy (i), heat transport (h), partial-p (p), partial-q (q) and full (f) anisotropies

$$i - \text{case} \Rightarrow N_i = (K^2+\lambda)(K^2+p\lambda)E + \Lambda m^2, \quad R\frac{q_z(K^2+\lambda)}{(q_z K^2+\lambda)}a^2 = \frac{K^2 N_i^2 + \pi^2\left(K^2+\lambda\right)^2}{N_i}$$

$$h - \text{case} \Rightarrow N_h = (K^2+\lambda)(K^2+p\lambda)E + \Lambda m^2, \quad R\frac{q_z(K^2+\lambda)}{(q_z K_\alpha^2+\lambda)}a^2 = \frac{K^2 N_h^2 + \pi^2\left(K^2+\lambda\right)^2}{N_h}$$

$$m - \text{case} \Rightarrow N_m = (K_\alpha^2+\lambda)(K^2+p\lambda)E + \Lambda_z m^2, \quad R\frac{q_z(K_\alpha^2+\lambda)}{(q_z K^2+\lambda)}a^2 = \frac{K^2 N_m^2 + \pi^2\left(K^2+\lambda\right)^2}{N_m}$$

$$p - \text{case} \Rightarrow N_p = (K^2+\lambda)(K_\alpha^2+p\lambda)E_z + \Lambda m^2, \quad R\frac{q_z(K^2+\lambda)}{(q_z K_\alpha^2+\lambda)}a^2 = \frac{K^2 N_p^2 + \pi^2\left(K^2+\lambda\right)^2}{N_p}$$

$$q - \text{case} \Rightarrow N_q = (K_\alpha^2+\lambda)(K^2+p\lambda)E + \Lambda_z m^2, \quad R\frac{q_z(K_\alpha^2+\lambda)}{(q_z K_\alpha^2+\lambda)}a^2 = \frac{K^2 N_q^2 + \pi^2\left(K_\alpha^2+\lambda\right)^2}{N_q}$$

$$f - \text{case} \Rightarrow N_f = (K_\alpha^2+\lambda)(K_\alpha^2+p\lambda)E_z + \Lambda_z m^2, \quad R\frac{q_z(K_\alpha^2+\lambda)}{(q_z K_\alpha^2+\lambda)}a^2 = \frac{K^2 N_f^2 + \pi^2\left(K_\alpha^2+\lambda\right)^2}{N_f}$$

some regime diagrams about h, p and f anisotropy, m, q and f anisotropy and all cases

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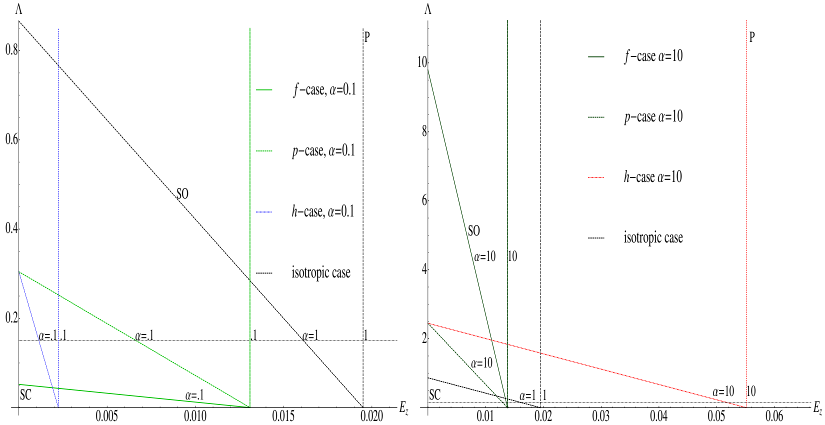
ΛE_z Regime diagrams in SA anisotropy (h, p, f with $\alpha = 0.1, 10$)



The ΛE_z regime diagrams for steady convection in SA anisotropy in h, p and f cases compared with isotropy. In the left figure atmospheric anisotropy (Sa), in the right figure oceanic anisotropy (So).

...back to comparison

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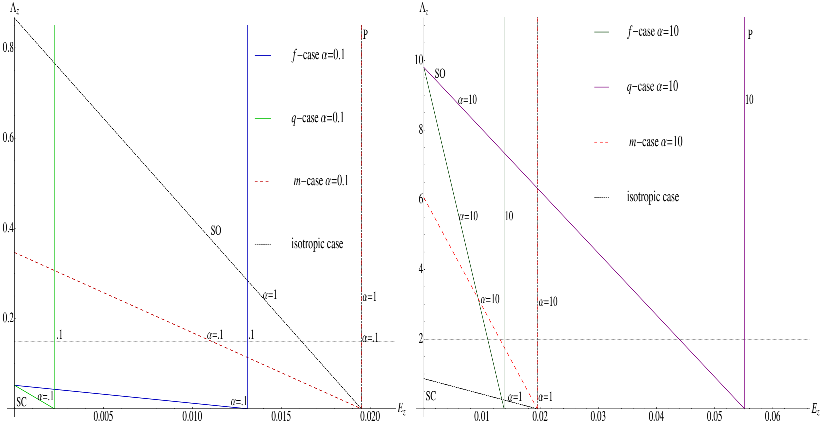

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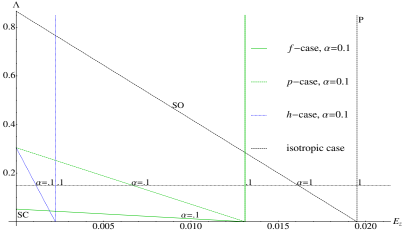
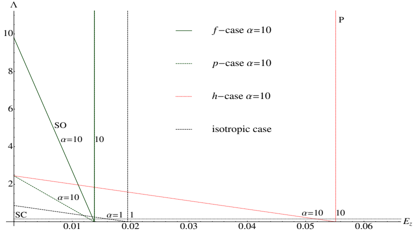
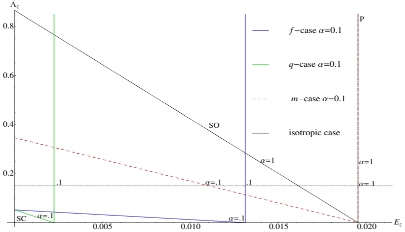
Some ΛE_z Regime diagrams in SA anisotropy with $\alpha = 0.1, 10$



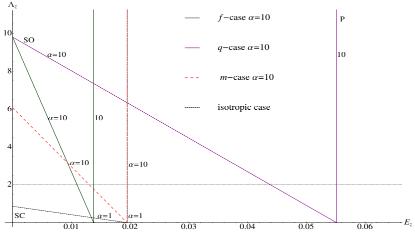
h, p and f, α=0.1



h, p and f, α=10



m, q and f, α=0.1



m, q and f, α=10

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Bibliography

Braginsky, S.I. and Meytlis, V.P., Local turbulence in the Earth's core. *Geophys. Astrophys. Fluid Dyn.*, 1990, **55**, 71–87.

Chandrasekhar, S., *Hydrodynamic and Hydromagnetic Stability*, 1961 (Oxford: Clarendon Press).

Donald, J.T. and Roberts P.H., The effect of anisotropic heat transport in the earth's core on the geodynamo. *Geophys. Astrophys. Fluid Dyn.*, 2004, **98**, 367–384.

Fearn, D., and Roberts, P.H., The Geodynamo, in *Mathematical Aspects of Natural Dynamos*, Edited by Dormy, E., Soward, A.M., Taylor and Francis Group, Boca Raton, 2007.

Filippi, E., Brestenský, J., Šoltis, T., Effects of anisotropic diffusion on onset of rotating magnetoconvection in plane layer; stationary modes, *Geophys. Astrophys. Fluid Dyn.*, 2019, **113**, 80–106.

Krause, F. and Rädler, K.H., *Mean-Field Magnetohydrodynamics and Dynamo Theory*, Akademie-Verlag, Berlin, 1980.

Roberts P.H. and Jones, C. A., The onset of magnetoconvection at large Prandtl number in a rotating layer I. Finite magnetic diffusion. *Geophys. Astrophys. Fluid Dyn.*, 2000, **92**, 289–325.

Šoltis, T. and Brestenský, J., Rotating magnetoconvection with anisotropic diffusivities in the Earth's core, *Phys. Earth Planet. Inter.*, **178**, 27–38, 2010.

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Anisotropic diffusive coefficients in SA and BM anisotropy (partial and full)

Assumption 1: following anisotropy in the viscosity and thermal diffusivity holds

$$\boldsymbol{\nu} = \begin{pmatrix} \nu_{xx} & 0 & 0 \\ 0 & \nu_{yy} & 0 \\ 0 & 0 & \nu_{zz} \end{pmatrix}, \qquad \boldsymbol{\kappa} = \begin{pmatrix} \kappa_{xx} & 0 & 0 \\ 0 & \kappa_{yy} & 0 \\ 0 & 0 & \kappa_{zz} \end{pmatrix}.$$

Assumption 2: same diffusion in two directions, but a different diffusion in the third one. This is called *partial anisotropy* for reasons which will be clearer later. However, in a more general case we need to deal also with the anisotropy in the magnetic diffusion also due to turbulence

$$\eta_{xx} = \eta_0 + \beta_{xx}, \; \eta_{yy} = \eta_0 + \beta_{yy}, \; \eta_{zz} = \eta_0 + \beta_{zz}$$

We call this situation *full anisotropy* (Filippi et al., 2019) considering magnetic diffusivity also anisotropic

SA, Stratification anisotropy:

$\nu_{xx} = \nu_{yy} \neq \nu_{zz}, \; \kappa_{xx} = \kappa_{yy} \neq \kappa_{zz}, \eta_{xx} = \eta_{yy} = \eta_{zz} = \eta_0$ and $\eta_{xx} = \eta_{yy} \neq \eta_{zz}$.

Gravity or/and the Archimedean buoyancy force lead the dynamics of turbulent eddies

BM by Braginsky and Meytlis (1990):

$\nu_{xx} < \nu_{yy} = \nu_{zz}, \; \kappa_{xx} < \kappa_{yy} = \kappa_{zz}, \; \eta_{xx} = \eta_{yy} = \eta_{zz} = \eta_0$ and $\eta_{xx} < \eta_{yy} = \eta_{zz}$.

Rotation and magnetic field lead the dynamics of turbulent eddies

There is “horizontal isotropy” in SA, but not in BM

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Method of solution in the most general case

We look for a solution by using some methods developed, e.g., in Chandrasekhar (1961): $\mathbf{u}$ and $\mathbf{B}$ are divergenceless, therefore

$$\mathbf{u} = a^{-2}[\boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times w\hat{\mathbf{z}}) + \boldsymbol{\nabla} \times \omega\hat{\mathbf{z}}] \quad \text{and} \quad \mathbf{b} = a^{-2}[\boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times b\hat{\mathbf{z}}) + \boldsymbol{\nabla} \times j\hat{\mathbf{z}}]$$

All perturbations $(w, \omega, b, j$ and $\vartheta)$ have a form

$$f(x, y, z, t) = \Re e[F(z) \exp(ilx + imy) \exp(\lambda t)]$$

$a = \sqrt{l^2 + m^2}$, $\lambda = i\sigma \in \mathbb{C}$, $F(z) = W(z)$, $\Omega(z)$, $B(z)$, $J(z)$, and $\Theta(z)$

$$[E_z\mathcal{D}_\nu - R_o\lambda]\Omega + \mathrm{D}W + \mathrm{i}m\Lambda_z J = 0, \tag{1}$$

$$(\mathrm{D}^2 - a^2)[E_z\mathcal{D}_\nu - R_o\lambda]W - \mathrm{D}\Omega$$
$$+\mathrm{i}m\Lambda_z(\mathrm{D}^2 - a^2)B = a^2 R\Theta, \tag{2}$$

$$(\mathcal{D}_\eta - \lambda)J + \mathrm{i}m\Omega = 0, \tag{3}$$
$$(\mathcal{D}_\eta - \lambda)B + \mathrm{i}mW = 0, \tag{4}$$
$$(\mathcal{D}_\kappa - \zeta\lambda)\Theta + W = 0, \tag{5}$$

where $\zeta = q_z^{-1}$, $D = d/dz$ and $\mathcal{D}_\kappa$, $\mathcal{D}_\nu$, $\mathcal{D}_\eta$ are equal to $D^2 - \alpha_\kappa l^2 - m^2$, $D^2 - \alpha_\nu l^2 - m^2$, $D^2 - \alpha_\eta l^2 - m^2$ and $D^2 - \alpha_\kappa a^2$, $D^2 - \alpha_\nu a^2$, $D^2 - \alpha_\eta a^2$ for BM and SA types of anisotropies, respectively.

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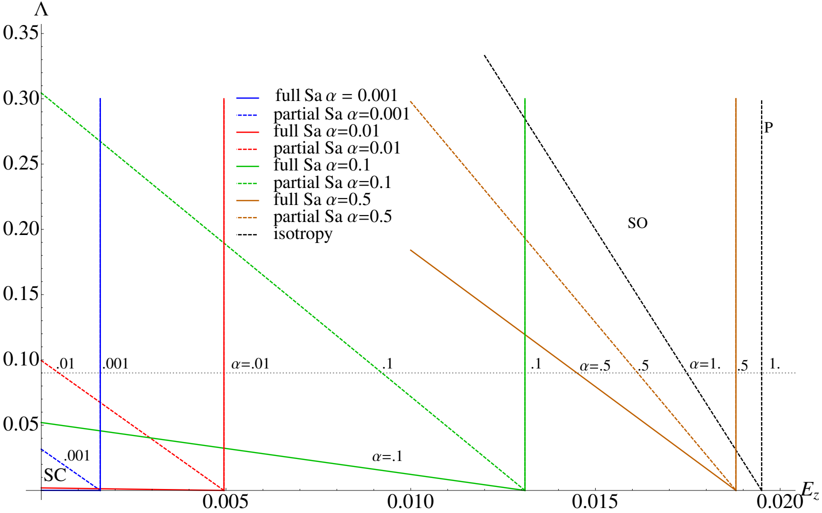
Regime diagrams in Sa anisotropy, $\alpha < 1$

ΛE_z diagrams for several cases of Sa anisotropy compared with isotropic case.



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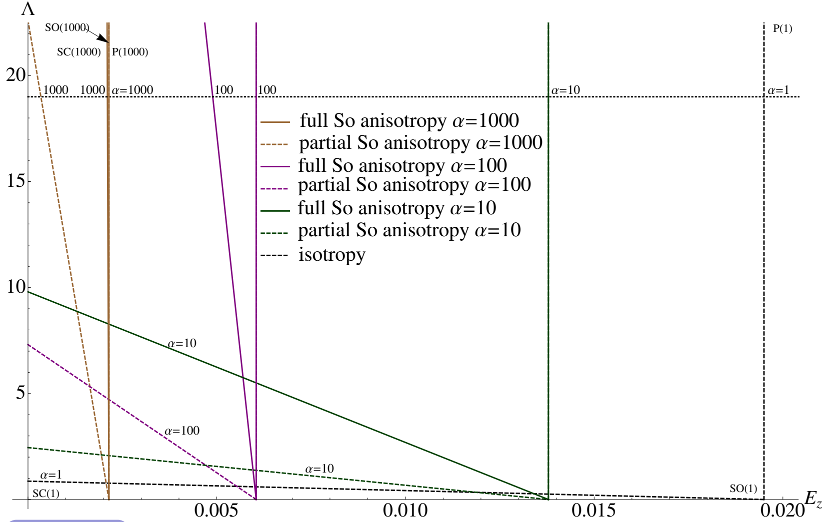
Regime diagrams in So anisotropy, $\alpha > 1$

ΛE_z diagrams for several cases of So anisotropy compared with isotropic case.



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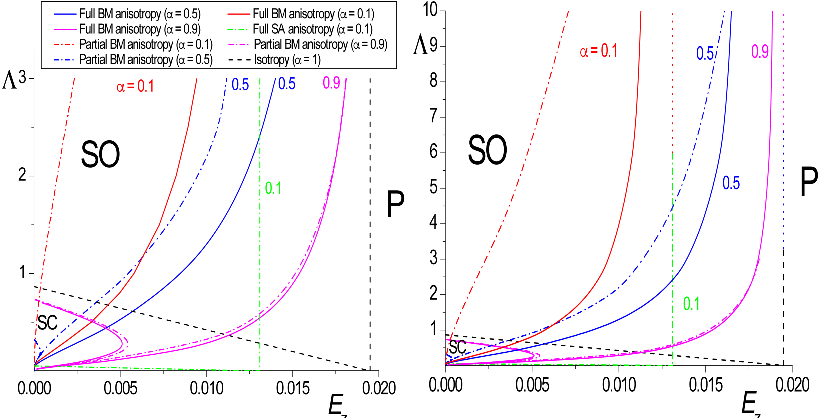
ΛE_z-Regime diagrams in BM anisotropy



The regime diagrams for steady convection in various cases of partial and full BM anisotropy in two different Λ-axis scales. For comparison cases of isotropy, $\alpha = 1$, and of strong Sa, $\alpha = 0.1$, are added; there are three asymptotes at $\Lambda \to \infty$, represented by the dotted vertical lines, for six SO/P lines for $\alpha =$ 0.1, 0.5, 0.9.

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