Anisotropic turbulent diffusivities and rotating magnetoconvection problems

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EGU General Assembly 2020

Outline

Earth’s core turbulence and anisotropic diffusion

Several anisotropy rotating magnetoconvection models

Main equations and parameters

Comparison of different anisotropies’ effects

Outline  Conclusion  Anisotropic diffusion  Several models  Main equations  Compared anisotropies
Anisotropies, in particular the strong ones, allow the preference (or/and existence) of some modes, what is not possible in the isotropic case.

Usually the anisotropies with greater diffusivity in vertical direction facilitate convection, but the anisotropy in magnetic diffusivity influences only some case of convection.

The case with only thermal and magnetic diffusivity anisotropic, is really unique and this highlights the importance of these two turbulent diffusivities; however, some surprising behaviour with greater diffusivity in vertical direction suggests that also anisotropy in viscosity is important.

anisotropy = anisotropic diffusion, anisotropic diffusive coefficients, anisotropic diffusivities
Anisotropic turbulent diffusivities and rotating magnetoconvection problems

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There is strong belief that the Earth’s Core is in turbulent state

The Earth’s core is driven into motion by buoyancy forces so strong that the flow and field are turbulent, fluctuating on every length and time scale, as it is accepted by the most of geophysicists. Traditional approach to turbulence (see, e.g., Krause and Rädler, 1980) splits each variables into mean and fluctuating ones, e.g., \( u = \bar{u} + u' \), \( B = \bar{B} + B' \), thus the mean IE is:

\[
\frac{\partial \bar{B}}{\partial t} = \eta_0 \nabla^2 \bar{B} + \nabla \times (\bar{u} \times \bar{B}) + \nabla \times \mathcal{E}, \quad \mathcal{E} = u' \times B'
\]

within suitable approximations and conditions we can say that \( \nabla \times \mathcal{E} = -\nabla \times (\beta \nabla \times \bar{B}) \), the well known ”\( \beta \)-effect” In general, due to turbulence the diffusive coefficients are anisotropic; for instance the buoyancy has a preferred direction \( \Rightarrow \) local turbulence may be significantly anisotropic with respect to gravity direction

\[
\nabla \times \mathcal{E} = -\nabla \times (\beta \nabla \times \bar{B}) \rightarrow \nabla \times \mathcal{E} = -\nabla \times (\beta \nabla \times \bar{B}) = \nabla \cdot (\beta \nabla \bar{B})
\]

where \( \beta \) is a tensor quantity, thus, in general we should speak about an ”anisotropic beta-effect” or anisotropic magnetic diffusivity, anisotropic \( \eta \) tensor (\( \eta = \eta_0 + \beta \)), which parameterizes mean electromotive force, \( \mathcal{E} \).

Analogous considerations can be made (see, e.g., Fearn and Roberts, 2007) for Navier-Stokes and heat equations

\[
\rho \left[ \frac{\partial \bar{u}}{\partial t} + \bar{u} \cdot \nabla \bar{u} + 2\Omega \hat{k} \times \bar{u} \right] = \nabla \bar{p} + \nu \nabla^2 \bar{u} + (1/\mu_0) \bar{B} \cdot (\nabla \bar{B}) + \nabla \cdot (\nu T \nabla \bar{u})
\]

where, within some approximations \( \nabla \cdot (\nu T \nabla \bar{u}) = \nabla \cdot (\nu_T \nabla \bar{u}) \), tensorial turbulent viscosity, \( \nu_T \), parameterizes Reynolds and Maxwell stresses related to \( \bar{u}' \bar{u}' - \bar{B}' \bar{B}' \).

\[
\frac{\partial \bar{\Theta}}{\partial t} + \bar{u} \cdot \nabla \bar{\Theta} = \kappa \nabla^2 \bar{\Theta} - \nabla \cdot (\nu_T \nabla \bar{\Theta})
\]

where, in some cases \( -\nabla \cdot (\nu_T \nabla \bar{\Theta}) = \nabla \cdot (\kappa_T \nabla \bar{\Theta}) \), \( \kappa_T \) is the parameterized tensorial turbulent thermal diffusivity.
Turbulence can cause anisotropy, therefore it is worth to introduce anisotropy in the rotating magnetoconvection models; this was done for the first time in the model by Šoltis and Brestenský (2010), which considered only viscosity and thermal diffusivity anisotropic, and recently it was advanced by Filippi et al. (2019), by deeming also the magnetic diffusivity as anisotropic.

These models deals about Rotating Magnetoconvection in horizontal plane layer rotating about vertical axis and permeated by homogeneous horizontal magnetic field (Roberts and Jones, 2000), influenced by anisotropic diffusivities.
**Different anisotropic rotating magnetoconvection models studied**

<table>
<thead>
<tr>
<th>$k$</th>
<th>isotropic</th>
<th>anisotropic</th>
<th>name</th>
<th>reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i$</td>
<td>$\nu, \kappa, \eta$</td>
<td>$-$</td>
<td>isotropy</td>
<td>Roberts and Jones (2000)</td>
</tr>
<tr>
<td>$h$</td>
<td>$\nu, \eta$</td>
<td>$\kappa$</td>
<td>pure-$h$ anisotropy</td>
<td>Donald and Roberts (2004)</td>
</tr>
<tr>
<td>$m$</td>
<td>$\nu, \kappa$</td>
<td>$\eta$</td>
<td>pure-$m$ anisotropy</td>
<td>FB</td>
</tr>
<tr>
<td>$p$</td>
<td>$\eta$</td>
<td>$\nu, \kappa$</td>
<td>partial-$p$ anisotropy</td>
<td>Šoltis and Brestenský (2010)</td>
</tr>
<tr>
<td>$q$</td>
<td>$\nu$</td>
<td>$\kappa, \eta$</td>
<td>partial-$q$ anisotropy</td>
<td>FB</td>
</tr>
<tr>
<td>$f$</td>
<td>$-$</td>
<td>$\nu, \kappa, \eta$</td>
<td>full anisotropy</td>
<td>Filippi et al. (2019)</td>
</tr>
</tbody>
</table>

**Table:** Different anisotropy cases, $k = i, h, m, p, q$ and $f$, with related isotropic or/and anisotropic diffusivities, $\nu, \kappa$ and $\eta$. Cases $i, p$ and $f$ have been already studied and published, cases $m$ and $q$ are new (FB) and case $h$ is not yet published.
Partial anisotropy model by Šoltis and Brestenský (2010) with $\nu$, $\kappa$ anisotropic tensors and $\eta_0$ isotropic tensor, where

$$\nu = \begin{pmatrix} \nu_{xx} & 0 & 0 \\ 0 & \nu_{yy} & 0 \\ 0 & 0 & \nu_{zz} \end{pmatrix}, \quad \kappa = \begin{pmatrix} \kappa_{xx} & 0 & 0 \\ 0 & \kappa_{yy} & 0 \\ 0 & 0 & \kappa_{zz} \end{pmatrix}, \quad \eta_0 = \begin{pmatrix} \eta_0 & 0 & 0 \\ 0 & \eta_0 & 0 \\ 0 & 0 & \eta_0 \end{pmatrix}.$$ 

Magnetic diffusion also due to turbulence inspired a full anisotropy model (Filippi et al., 2019) with anisotropic $\nu$, $\kappa$, $\eta$, where

$$\eta_{xx} = \eta_0 + \beta_{xx}, \quad \eta_{yy} = \eta_0 + \beta_{yy}, \quad \eta_{zz} = \eta_0 + \beta_{zz}.$$

Later, we will speak about another kind of anisotropy, heat transport anisotropy, a partial anisotropy with $\nu$ also isotropic.

SA, Stratification anisotropy:

$$\nu_{xx} = \nu_{yy} \neq \nu_{zz}, \quad \kappa_{xx} = \kappa_{yy} \neq \kappa_{zz}, \quad \eta_{xx} = \eta_{yy} = \eta_{zz} = \eta_0 \quad \text{and} \quad \eta_{xx} = \eta_{yy} \neq \eta_{zz}.$$ 

Gravity or/and the Archimedean buoyancy force are dominant.

BM by Braginsky and Meytlis (1990):

$$\nu_{xx} < \nu_{yy} = \nu_{zz}, \quad \kappa_{xx} < \kappa_{yy} = \kappa_{zz}, \quad \eta_{xx} = \eta_{yy} = \eta_{zz} = \eta_0 \quad \text{and} \quad \eta_{xx} < \eta_{yy} = \eta_{zz}.$$ 

Rotation and magnetic field are dominant.

There is “horizontal isotropy” in SA, but not in BM.
Donald and Roberts (2004) developed a dynamo model which introduces anisotropy only in the thermal diffusivity. Inspired by them we began to study also another case of anisotropy in rotating magnetoconvection, we call it Heat transport anisotropy.

\[ \nu = \begin{pmatrix} \nu_0 & 0 & 0 \\ 0 & \nu_0 & 0 \\ 0 & 0 & \nu_0 \end{pmatrix}, \quad \kappa = \begin{pmatrix} \kappa_{xx} & 0 & 0 \\ 0 & \kappa_{yy} & 0 \\ 0 & 0 & \kappa_{zz} \end{pmatrix}, \quad \eta_0 = \begin{pmatrix} \eta_0 & 0 & 0 \\ 0 & \eta_0 & 0 \\ 0 & 0 & \eta_0 \end{pmatrix}. \]

This new simplified model is developed for wider chance to compare the effects of more anisotropic models.
Dimensionless governing equations and parameters 1/2

Now, let us move to magnetoconvection problems, for simplicity we use for the fluctuating variables the symbols, $\tilde{u}$, $\tilde{b}$, $\tilde{\theta}$ instead of $u'$, $B'$, $\Theta'$ used when we spoke about turbulence and anisotropy.

In the most general anisotropy model we get the following main dimensionless equations, after using standard procedures

$$R_o \partial_t u + \hat{z} \times u = -\nabla p + \Lambda_z (\nabla \times b) \times \hat{y} + R\vartheta \hat{z} + E_z \nabla^2_{\nu} u,$$

$$\partial_t b = \nabla \times (u \times \hat{y}) + \nabla^2_{\eta} b,$$

$$q_z^{-1} \partial_t \vartheta = \hat{z} \cdot u + \nabla^2_{\kappa} \vartheta,$$

$$\nabla \cdot u = 0, \quad \nabla \cdot b = 0,$$

where $\tilde{z} = dz$, $\tilde{b} = B_M b$ and

$$\tilde{u} = U u = \frac{\eta_{zz}}{d} u, \quad \tilde{t} = \frac{d}{U} t = \frac{d^2}{\eta_{zz}} t, \quad \tilde{p} = 2\Omega_0 \eta_{zz} \rho_0 p, \quad \tilde{\vartheta} = \frac{\eta_{zz} \Delta T}{\kappa_{zz}} \vartheta.$$
Dimensionless governing equations and parameters 2/2

\[ R_o = \frac{\eta_{zz}}{2\Omega_0 d^2}, \quad \Lambda_z = \frac{B_M^2}{2\Omega_0 \rho_0 \mu_0 \eta_{zz}}, \quad E_z = \frac{\nu_{zz}}{2\Omega_0 d^2}, \quad R = \frac{\alpha_T g \Delta T d}{2\Omega_0 \kappa_{zz}}, \quad q_z = \frac{\kappa_{zz}}{\eta_{zz}}, \]

\[ \alpha_\nu = \frac{\nu_{xx}}{\nu_{zz}}, \quad \alpha_\kappa = \frac{\kappa_{xx}}{\kappa_{zz}}, \quad \alpha_\eta = \frac{\eta_{xx}}{\eta_{zz}} \]

**SA anisotropy:**

\[ E_z \nabla^2_\nu u = E_z [(1 - \alpha_\nu) \partial_{zz} + \alpha_\nu \nabla^2] u, \quad \nabla^2_\kappa \vartheta = [(1 - \alpha_\kappa) \partial_{zz} + \alpha_\kappa \nabla^2] \vartheta, \]

\[ \nabla^2_\eta b = [(1 - \alpha_\eta) \partial_{zz} + \alpha_\eta \nabla^2] b \]

**BM anisotropy:**

\[ E_z \nabla^2_\nu u = E_z [(\alpha_\nu - 1) \partial_{xx} + \nabla^2] u, \quad \nabla^2_\kappa \vartheta = [(\alpha_\kappa - 1) \partial_{xx} + \nabla^2] \vartheta, \]

\[ \nabla^2_\eta b = [(\alpha_\eta - 1) \partial_{xx} + \nabla^2] b \]
Method of solution in the most general case

A solution can be found by some methods developed, e.g., in Chandrasekhar (1961):

\[ \nabla \cdot \mathbf{v} = \nabla \cdot \mathbf{B} = 0 \Rightarrow \mathbf{u} = a^{-2}[\nabla \times (\nabla \times \mathbf{w}\hat{z}) + \nabla \times \mathbf{\omega}\hat{z}] \quad \text{and} \quad \mathbf{b} = a^{-2}[\nabla \times (\nabla \times \mathbf{b}\hat{z}) + \nabla \times \mathbf{j}\hat{z}] \]

All perturbations \((w, \omega, b, j\) and \(\vartheta)\) have a form like

\[ f(x, y, z, t) = \Re[F(z) \exp(\imath lx + \imath my) \exp(\lambda t)] \quad \text{with} \quad a = \sqrt{l^2 + m^2}, \quad \lambda = \imath \sigma \in \mathbb{C}, \]

\[ F(z) = W(z), \ \Omega(z), \ \mathbf{B}(z), \ \mathbf{J}(z), \ \text{and} \ \Theta(z) \]

If we take some curls of momentum, induction and heat equations, introduce inverted magnetic Prandtl number \(p = \eta_{zz}/\nu_{zz} = R_o/E_z\) and we consider the simplest boundary conditions at \(z = \pm 1/2\), i.e.:

- stress free boundaries: \(W = D^2W = D\Omega = 0\)
- perfectly thermally conducting boundaries: \(\Theta = 0\)
- perfectly electrically conducting boundaries: \(B = DJ = 0\).

"dispersion relation"

\[ Ra^2 \frac{q_z(K^2_{\eta} + \lambda)}{(q_z K^2_{\kappa})^2 + \lambda^2} = \frac{K^2 N_f^2 + \pi^2 (K^2_{\eta} + \lambda)^2}{N_f} \]

where \(N_f\)

\[ N_f = (K^2_{\eta} + \lambda)(K^2_{\nu} + p\lambda)E_z + \Lambda_z m^2, \]

\[ K^2_{\eta} = \pi^2 + a^2, \quad K^2_{\kappa} = \pi^2 + \alpha_\kappa a^2, \quad K^2_{\nu} = \pi^2 + \alpha_\nu a^2, \quad K^2_{\eta} = \pi^2 + \alpha_\eta a^2 \quad \text{in SA} \]

\[ K^2_{\kappa} = \pi^2 + \alpha_\kappa l^2 + m^2, \quad K^2_{\nu} = \pi^2 + \alpha_\nu l^2 + m^2, \quad K^2_{\eta} = \pi^2 + \alpha_\eta l^2 + m^2 \quad \text{in BM} \]
Critical Rayleigh numbers in stationary SA (partial, full) anisotropy

From now on, for simplicity we assume that in partial and full anisotropy:

\[ \alpha_\kappa = \alpha_\nu = \alpha, \quad \alpha_\kappa = \alpha_\nu = \alpha_\eta = \alpha \]

\[ \lambda = 0, \quad 1_\alpha = (K_\alpha/K)^2 \]

\[ R^s_p = \frac{\pi (\pi^2 + a^2)^{1/2}}{a^2} K_\alpha^2 \left( C_p + \frac{1}{C_p} \right), \quad R^s = \frac{\pi (\pi^2 + a^2)^{3/2}}{a^2} \left( C_\alpha + \frac{1^2}{C_\alpha} \right). \]

where \( C_p = \frac{E_z K^2 K_\alpha^2 + \Lambda m^2}{\pi K} \) and \( C_\alpha = \frac{E_z K_\alpha^4 + \Lambda m^2}{\pi K} \)

in SA \( K_\alpha^2 = \pi^2 + \alpha a^2 \) and

\[ C_p = \frac{E_z (\pi^2 + a^2)(\pi^2 + \alpha a^2) + \Lambda m^2}{\pi (\pi^2 + a^2)^{1/2}}, \quad C_\alpha = \frac{E_z (\pi^2 + \alpha a^2)^2 + \Lambda m^2}{\pi (\pi^2 + a^2)^{1/2}} \] (3)

\[ \frac{\partial R^s_p}{\partial C_p} = 0, \quad \frac{\partial R^s_p}{\partial a^2} = 0 \quad \text{and} \quad \frac{\partial R^s}{\partial C_\alpha} = 0, \quad \frac{\partial R^s}{\partial a^2} = 0 \]

we have critical \( C_p, a^2 \) and \( C_\alpha, a^2 \) in case of partial and full anisotropy respectively thus

\[ C_{pc} = 1, \quad a^2_c = \pi^2 2_\alpha = \pi^2 \frac{1 + \sqrt{1 + 8\alpha}}{2\alpha} \quad \text{and} \quad C_{\alpha c} = 1_\alpha, \quad a^2_c = \pi^2 2_\alpha \] (4)

by the (3) and (4) it is possible to obtain \( m_c \) and \( l_c \)
Critical Rayleigh numbers in stationary SA $m$-case anisotropy

\[ \alpha_\nu = \alpha_\kappa = 1, \; \alpha_\eta = \alpha \]

\[ R_m^s = \frac{\pi}{a^2} K^2 K_\alpha \left( \frac{C_m}{1_\alpha} + C_m^{-1} \right) \]

where

\[ C_m = \frac{K^2_\alpha K^2 E + \Lambda z m^2}{\pi K_\alpha} \]

\[ \frac{\partial R_m^s}{\partial C_m} = 0 \quad \text{and} \quad \frac{\partial R_m^s}{\partial a^2} = 0 \]

\[ C_{mc} = \sqrt{1_\alpha} \quad \text{and} \quad a^2_c = 2\pi^2 \]

\( a_c \) in pure-\( m \) anisotropy is identical to the isotropic one
Critical Rayleigh numbers in stationary heat transport and partial-\( q \) SA anisotropy

From now on, for simplicity we assume that in heat transport and partial-\( q \) anisotropy:

\[
\alpha_\kappa = \alpha, \alpha_\nu = \alpha_\eta = 1, \quad \alpha_\nu = 1, \alpha_\kappa = \alpha_\eta = \alpha
\]

\[
\lambda = 0, \quad 1_\alpha = \left(K_\alpha/K\right)^2
\]

\[
R^s_h = 1_\alpha \frac{\pi (\pi^2 + a^2)^{3/2}}{a^2} \left(C + \frac{1}{C}\right), \quad R^s_q = \frac{\pi K_\alpha^3}{a^2} \left(\frac{C_q}{1_\alpha} + C_q^{-1}\right).
\]

in SA \( K_\alpha^2 = \pi^2 + \alpha a^2 \) and

\[
C = \frac{EK^4 + \Lambda m^2}{\pi K}, \quad C_q = \frac{K_\alpha^2 K^2 E + \Lambda z m^2}{\pi K_\alpha}
\]

\[
\frac{\partial R^s_h}{\partial C} = 0, \quad \frac{\partial R^s_h}{\partial a^2} = 0 \quad \text{and} \quad \frac{\partial R^s_q}{\partial C_q} = 0, \quad \frac{\partial R^s_q}{\partial a^2} = 0
\]

\[
C_c = 1, \quad a_c^2 = \pi^2 2\alpha \quad \text{and} \quad C_{qc} = \sqrt{1_\alpha}, \quad a_{qc}^2 = \pi^2 2\alpha
\]
The orientation of the rolls of convection

Roberts and Jones (2000)

Cross rolls (stationary convection SC mode)

Oblique roll (stationary convection SO mode)

Parallel rolls (stationary convection P mode)

\[ \gamma = \arctan \left( \frac{m}{l} \right) \] is the angle between the axis of rolls and the magnetic field \( B \).

\[ l = 0 \Rightarrow \gamma = \frac{\pi}{2} \] rolls \( \perp B \),

\[ m = 0 \Rightarrow \gamma = 0 \] rolls \( \parallel B \)

graphical results about partial and full \( S_a \), \( S_o \) and \( B_M \) anisotropy
Comparison of all anisotropic rotating magnetoconvection models studied

Several models of anisotropic magnetoconvection studied: isotropy \((i)\), heat transport \((h)\), partial-\(p\) \((p)\), partial-\(q\) \((q)\) and full \((f)\) anisotropies

\(i - \) case \(\Rightarrow N_i = (K^2 + \lambda)(K^2 + p\lambda)E + \Lambda m^2,\)

\(h - \) case \(\Rightarrow N_h = (K^2 + \lambda)(K^2 + p\lambda)E + \Lambda m^2,\)

\(m - \) case \(\Rightarrow N_m = (K^2 + \lambda)(K^2 + p\lambda)E + \Lambda z m^2,\)

\(p - \) case \(\Rightarrow N_p = (K^2 + \lambda)(K^2 + p\lambda)E_z + \Lambda m^2,\)

\(q - \) case \(\Rightarrow N_q = (K^2 + \lambda)(K^2 + p\lambda)E + \Lambda z m^2,\)

\(f - \) case \(\Rightarrow N_f = (K^2 + \lambda)(K^2 + p\lambda)E_z + \Lambda z m^2,\)

\[
R \frac{q_z (K^2 + \lambda)}{(q_z K^2 + \lambda)} a^2 = \frac{K^2 N_i^2 + \pi^2 (K^2 + \lambda)^2}{N_i}
\]

\[
R \frac{q_z (K^2 + \lambda)}{(q_z K^2 + \alpha)} a^2 = \frac{K^2 N_h^2 + \pi^2 (K^2 + \lambda)^2}{N_h}
\]

\[
R \frac{q_z (K^2 + \lambda)}{(q_z K^2 + \alpha)} a^2 = \frac{K^2 N_m^2 + \pi^2 (K^2 + \lambda)^2}{N_m}
\]

\[
R \frac{q_z (K^2 + \lambda)}{(q_z K^2 + \alpha)} a^2 = \frac{K^2 N_p^2 + \pi^2 (K^2 + \lambda)^2}{N_p}
\]

\[
R \frac{q_z (K^2 + \lambda)}{(q_z K^2 + \alpha)} a^2 = \frac{K^2 N_q^2 + \pi^2 (K^2 + \lambda)^2}{N_q}
\]

\[
R \frac{q_z (K^2 + \lambda)}{(q_z K^2 + \alpha)} a^2 = \frac{K^2 N_f^2 + \pi^2 (K^2 + \lambda)^2}{N_f}
\]

some regime diagrams about \(h, p\) and \(f\) anisotropy, \(m, q\) and \(f\) anisotropy and all cases.
\( \Lambda E_z \) Regime diagrams in SA anisotropy (\( h, p, f \) with \( \alpha = 0.1, 10 \))

The \( \Lambda E_z \) regime diagrams for steady convection in SA anisotropy in \( h, p \) and \( f \) cases compared with isotropy. In the left figure atmospheric anisotropy (Sa), in the right figure oceanic anisotropy (So).
\( \Lambda E_z \) Regime diagrams in SA anisotropy \((m, q, f \text{ with } \alpha = 0.1, 10)\)

The \( \Lambda E_z \) regime diagrams for steady convection in SA anisotropy in \( m, q, f \) cases compared with isotropy. In the left figure atmospheric anisotropy (Sa), in the right figure oceanic anisotropy (So).

[Diagram showing \( \Lambda E_z \) regime diagrams for steady convection in SA anisotropy in \( m, q, f \) cases compared with isotropy. In the left figure atmospheric anisotropy (Sa), in the right figure oceanic anisotropy (So).]
Some $\Lambda E_z$ Regime diagrams in SA anisotropy with $\alpha = 0.1, 10$
Bibliography


Anisotropic diffusive coefficients in SA and BM anisotropy (partial and full)

Assumption 1: following anisotropy in the viscosity and thermal diffusivity holds

\[ \nu = \begin{pmatrix} \nu_{xx} & 0 & 0 \\ 0 & \nu_{yy} & 0 \\ 0 & 0 & \nu_{zz} \end{pmatrix}, \quad \kappa = \begin{pmatrix} \kappa_{xx} & 0 & 0 \\ 0 & \kappa_{yy} & 0 \\ 0 & 0 & \kappa_{zz} \end{pmatrix}. \]

Assumption 2: same diffusion in two directions, but a different diffusion in the third one. This is called partial anisotropy for reasons which will be clearer later. However, in a more general case we need to deal also with the anisotropy in the magnetic diffusion also due to turbulence

\[ \eta_{xx} = \eta_0 + \beta_{xx}, \quad \eta_{yy} = \eta_0 + \beta_{yy}, \quad \eta_{zz} = \eta_0 + \beta_{zz}. \]

We call this situation full anisotropy (Filippi et al., 2019) considering magnetic diffusivity also anisotropic

SA, Stratification anisotropy:

\[ \nu_{xx} = \nu_{yy} \neq \nu_{zz}, \quad \kappa_{xx} = \kappa_{yy} \neq \kappa_{zz}, \quad \eta_{xx} = \eta_{yy} = \eta_{zz} = \eta_0 \quad \text{and} \quad \eta_{xx} = \eta_{yy} \neq \eta_{zz}. \]

Gravity or/and the Archimedean buoyancy force lead the dynamics of turbulent eddies

BM by Braginsky and Meytlis (1990):

\[ \nu_{xx} < \nu_{yy} = \nu_{zz}, \quad \kappa_{xx} < \kappa_{yy} = \kappa_{zz}, \quad \eta_{xx} = \eta_{yy} = \eta_{zz} = \eta_0 \quad \text{and} \quad \eta_{xx} < \eta_{yy} = \eta_{zz}. \]

Rotation and magnetic field lead the dynamics of turbulent eddies

There is “horizontal isotropy” in SA, but not in BM
Method of solution in the most general case

We look for a solution by using some methods developed, e.g., in Chandrasekhar (1961): \( u \) and \( B \) are divergenceless, therefore

\[
\begin{align*}
\mathbf{u} &= a^{-2}[\nabla \times (\nabla \times \mathbf{w})] + \nabla \times \mathbf{\omega}, \\
\mathbf{b} &= a^{-2}[\nabla \times (\nabla \times \mathbf{b})] + \nabla \times \mathbf{j}.
\end{align*}
\]

All perturbations \((w, \omega, b, j \) and \( \vartheta \)) have a form

\[
f(x, y, z, t) = \Re\{F(z) \exp(ilx + imy) \exp(\lambda t)\}
\]

\[
a = \sqrt{l^2 + m^2}, \quad \lambda = i\sigma \in \mathbb{C}, \quad F(z) = W(z), \quad \Omega(z), \quad B(z), \quad J(z), \quad \text{and} \quad \Theta(z)
\]

\[
[E_z D_\nu - R_o \lambda] \Omega + DW + im\Lambda_z J = 0, \quad (1)
\]

\[
(D^2 - a^2)[E_z D_\nu - R_o \lambda] W - D\Omega + im\Lambda_z (D^2 - a^2) B = a^2 R\Theta, \quad (2)
\]

\[
(D_\eta - \lambda) J + im\Omega = 0, \quad (3)
\]

\[
(D_\eta - \lambda) B + im W = 0, \quad (4)
\]

\[
(D_\kappa - \zeta \lambda) \Theta + W = 0, \quad (5)
\]

where \( \zeta = q_z^{-1} \), \( D = d/dz \) and \( D_\kappa, D_\nu, D_\eta \) are equal to \( D^2 - \alpha_\kappa l^2 - m^2 \), \( D^2 - \alpha_\nu l^2 - m^2 \), \( D^2 - \alpha_\eta l^2 - m^2 \) and \( D^2 - \alpha_\kappa a^2 \), \( D^2 - \alpha_\nu a^2 \), \( D^2 - \alpha_\eta a^2 \) for BM and SA types of anisotropies, respectively.
Regime diagrams in Sa anisotropy, $\alpha < 1$

$\Lambda E_z$ diagrams for several cases of Sa anisotropy compared with isotropic case.
Regime diagrams in So anisotropy, $\alpha > 1$

$\Lambda E_z$ diagrams for several cases of So anisotropy compared with isotropic case.

- full So anisotropy $\alpha=1000$
- partial So anisotropy $\alpha=1000$
- full So anisotropy $\alpha=100$
- partial So anisotropy $\alpha=100$
- full So anisotropy $\alpha=10$
- partial So anisotropy $\alpha=10$
- isotropy

...back to rolls
Anisotropic turbulent diffusivities and rotating magnetoconvection problems

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\(\Lambda E_z\)-Regime diagrams in BM anisotropy

The regime diagrams for steady convection in various cases of partial and full BM anisotropy in two different \(\Lambda\)-axis scales. For comparison cases of isotropy, \(\alpha = 1\), and of strong \(Sa\), \(\alpha = 0.1\), are added; there are three asymptotes at \(\Lambda \to \infty\), represented by the dotted vertical lines, for six SO/P lines for \(\alpha = 0.1, 0.5, 0.9\).

...back to rolls

Outline  Conclusion  Anisotropic diffusion  Several models  Main equations  Compared anisotropies