Probability estimation of a Carrington-like geomagnetic storm

A giant 'doomsday' solar geomagnetic storm

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IMAGE: GROUP PHOTO OF THE FOUR RESEARCHERS (FROM LEFT TO RIGHT: ÁLVARO CORRAL, DAVID MORIÑA, PEDRO PUIG, ISABEL SERRA). view more >

Three mathematicians and a physicist from the Universitat Autònoma de Barcelona (UAB), the Mathematics Research Centre (CRM) and the Barcelona Graduate School of Mathematics (BGSMath) propose a mathematical model which allows making reliable estimations on the probability of geomagnetic storms caused by solar activity.

Overview

1 - The Carrington Event

2 - The data

3 - The model

4 - The results

5 - Conclusions

1- The Carrington Event

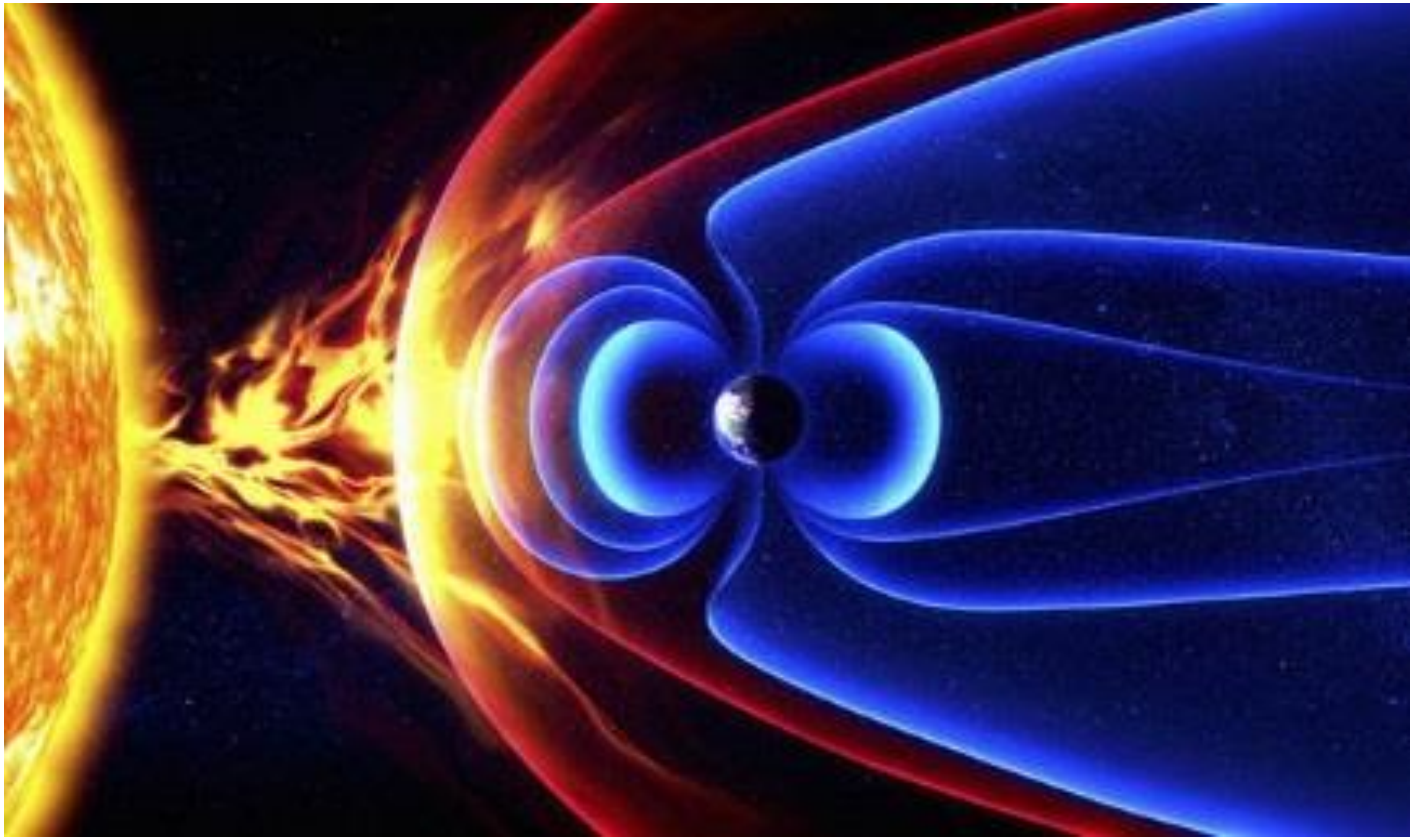
Richard C. Carrington was an English amateur astronomer who, on September 1st 1859, made the observation of a big solar flare, which produced a giant geomagnetic storm.

It was powerful enough to disrupt telegraph communications, shock telegraph operators, and even ignite telegraph paper due to discharges from the telegraph lines.

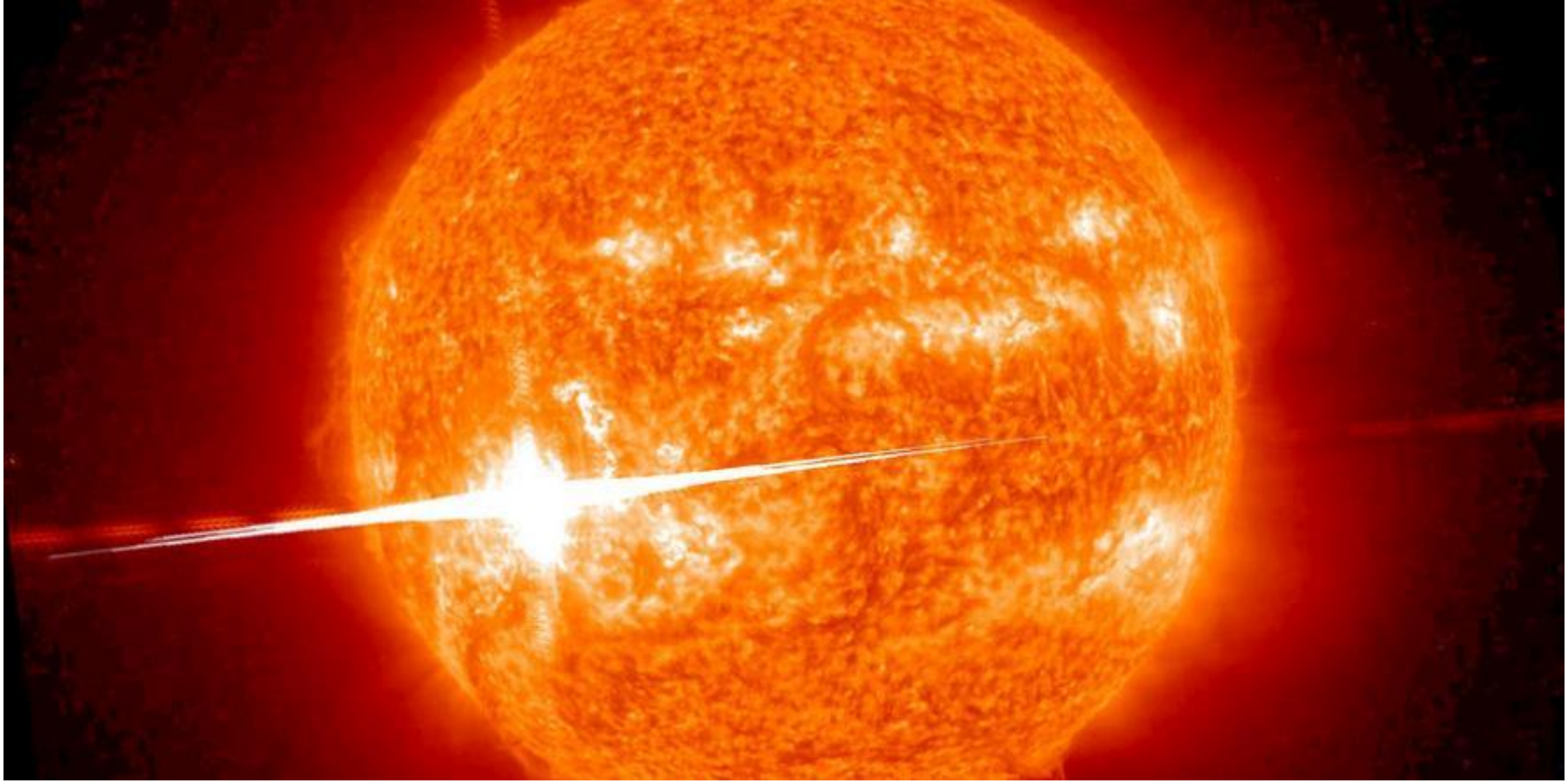
During the first display the whole of the northern hemisphere was as light as though the sun had set an hour before, and luminous waves rolled up in quick succession as far as the zenith, some a brilliancy sufficient to cast a perceptible shadow on the ground.
[**The Times London**, September 6, 1859].

Aurora appeared, illuminating the city so brightly as to draw crowds into the streets
[**New York Times**, September 5, 1859].

The French telegraph communications at Paris were greatly affected, and on interrupting the circuit of the conducting wire strong sparks were observed. The same thing occurred at the same time at all the telegraphic station in France
[**The Illustrated London News**, September 24, 1859].



A geomagnetic storm is a disturbance of the Earth's magnetosphere caused by a solar wind shock wave, generally caused by a coronal mass ejection (CME).

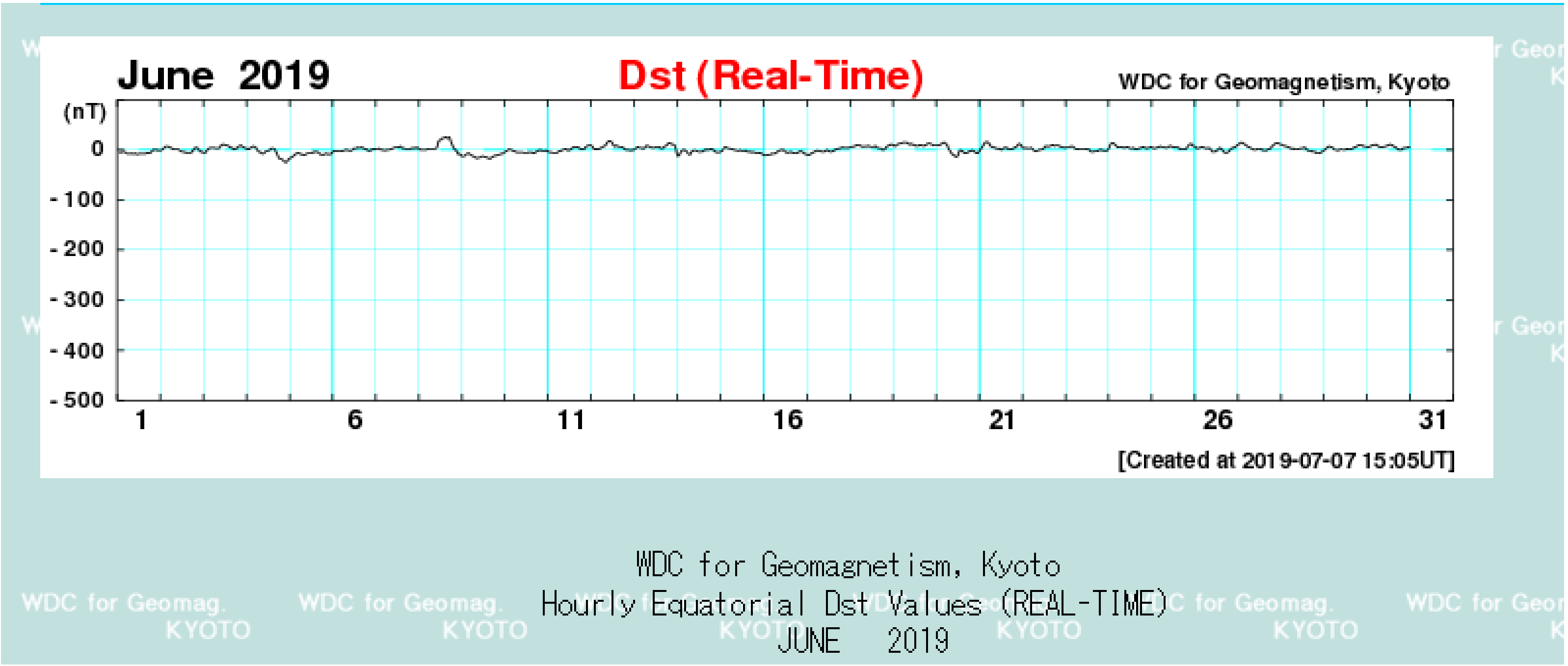


Nowadays, a Carrington-like geomagnetic storm would be catastrophic for electrical systems and communications.

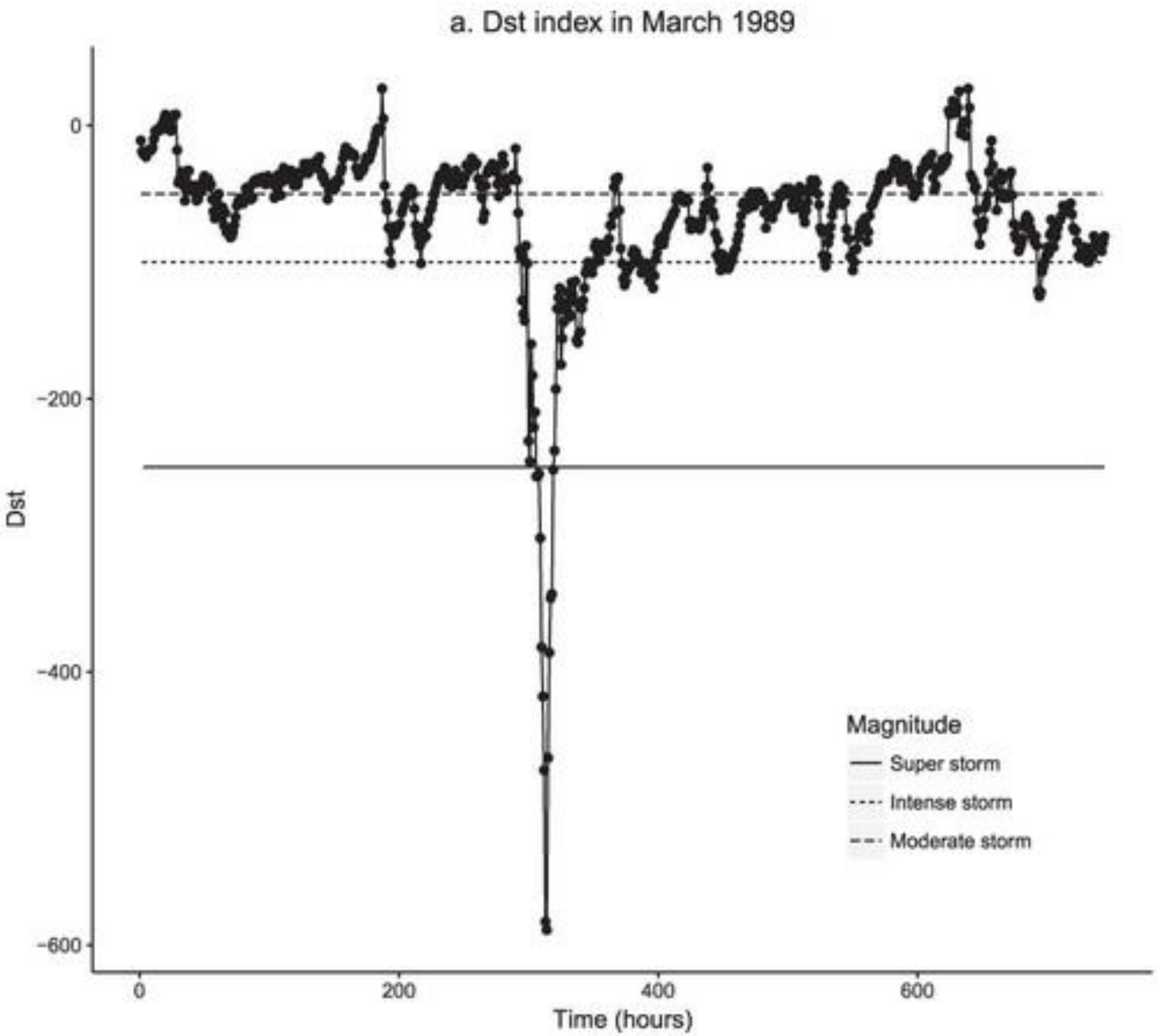
2- The Data

To analyse the process of temporal occurrence of Geomagnetic storms we use the Dst index, recorded hourly from 1957-01-01 to 2017-12-31, from the World Data Center for Geomagnetism in Kyoto

Dst (disturbance-storm time) index measures the globally averaged change of the horizontal component of the Earth's magnetic field at the magnetic equator.

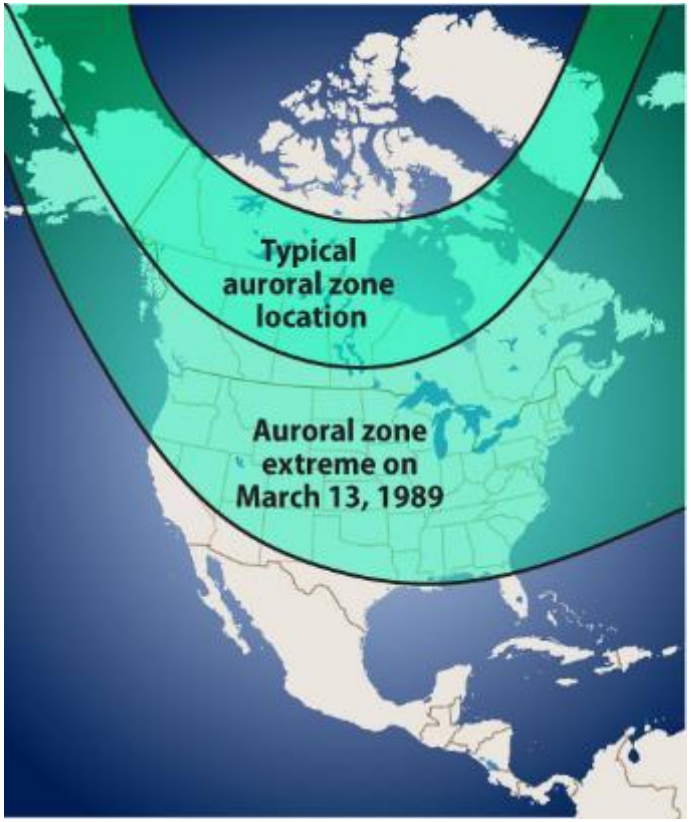


During quiescent times, the Dst index varies between -20 and +20 nT



Three categories of geomagnetic storms:

- super-storms
  < -250 nT
- intense
  [- 250nT, -100nT)
- moderate
  [-100nT, -50nT)



This geomagnetic storm caused a nine-hour power outage for the majority of the Quebec region and for parts of the northeastern United States

... but Carrington event was at least 1.5 times greater !

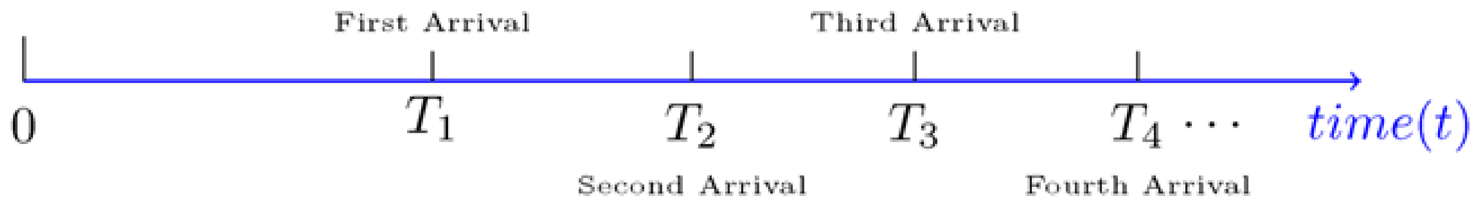
Dst ~ -850nT

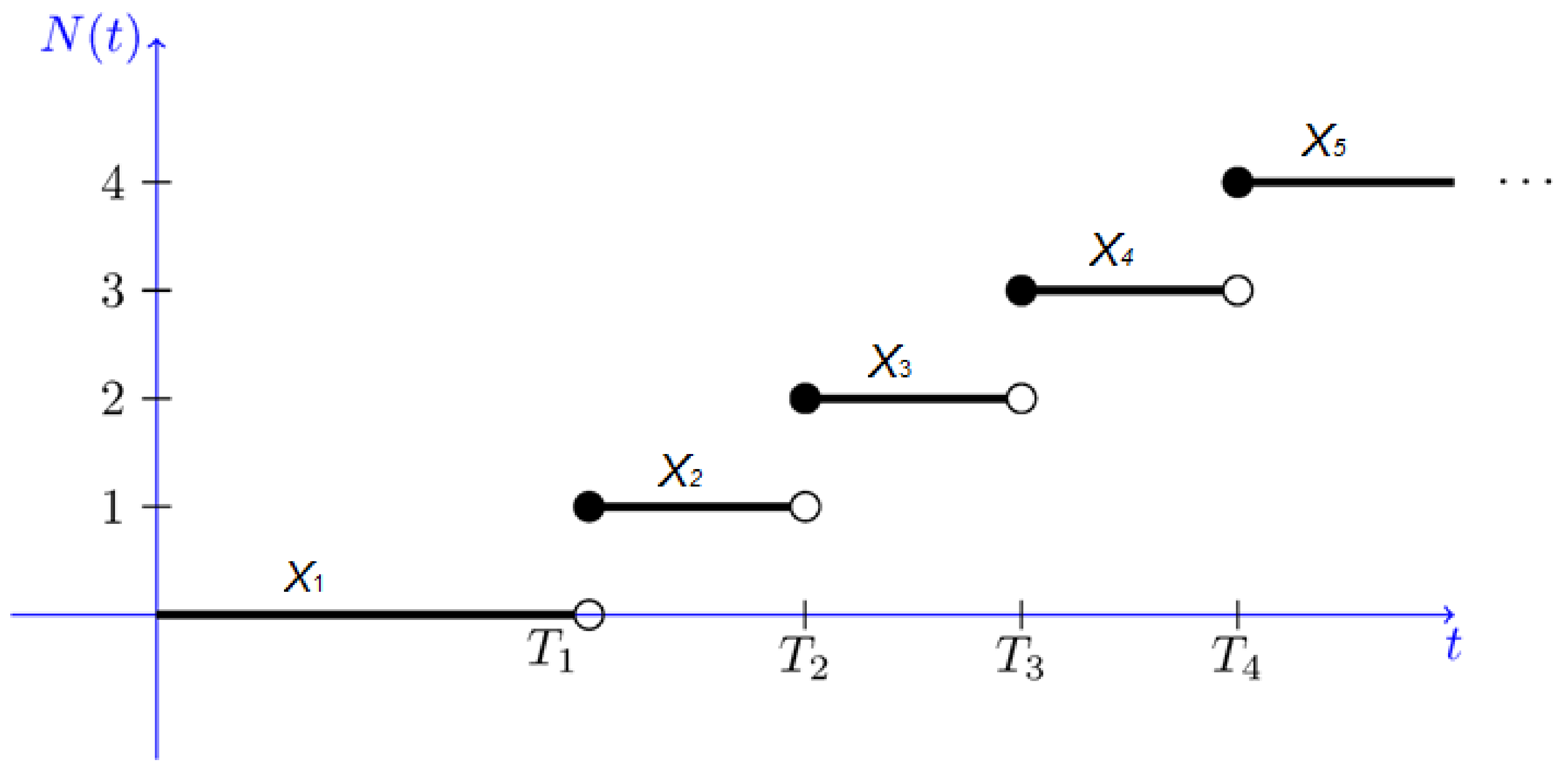
3- The model

The time of occurrence of geomagnetic storms, given a threshold, has been analysed as a homogeneous Poisson process (Riley, 2012).

An arrival process is a sequence T_n of nonnegative and increasing random variables, representing the times at which some phenomenon occurs, called the *ocurrence times*.

The inter-ocurrence times are defined as,
$X_1=T_1$, and $X_i=T_i-T_{i-1}$
for i>1. The process starts at time 0 and multiple ocurrences cannot happen at the same time.


First Arrival
Third Arrival
0
T_1
T_2
T_3
T_4 ⋯
$time(t)$
Second Arrival
Fourth Arrival


$N(t)$
4
3
2
1
X_1
X_2
X_3
X_4
X_5
⋯
T_1
T_2
T_3
T_4
t

A renewal process is an arrival process such that the inter-occurrence times X_i are iid random variables with distribution *F*.

- ***F*** **exponential**: Poisson process

- ***F*** **Gamma**: introduced by Winkelmann (1995), analyzing the distribution of the number of births per woman in Germany. Zeviani et al. (2014) use this model to describe the number of cotton bolls wit respect the defoliation level and growth stage.

- ***F*** **Weibull**: McShane et al. (2008) reanalize the number of births per woman in Germany.

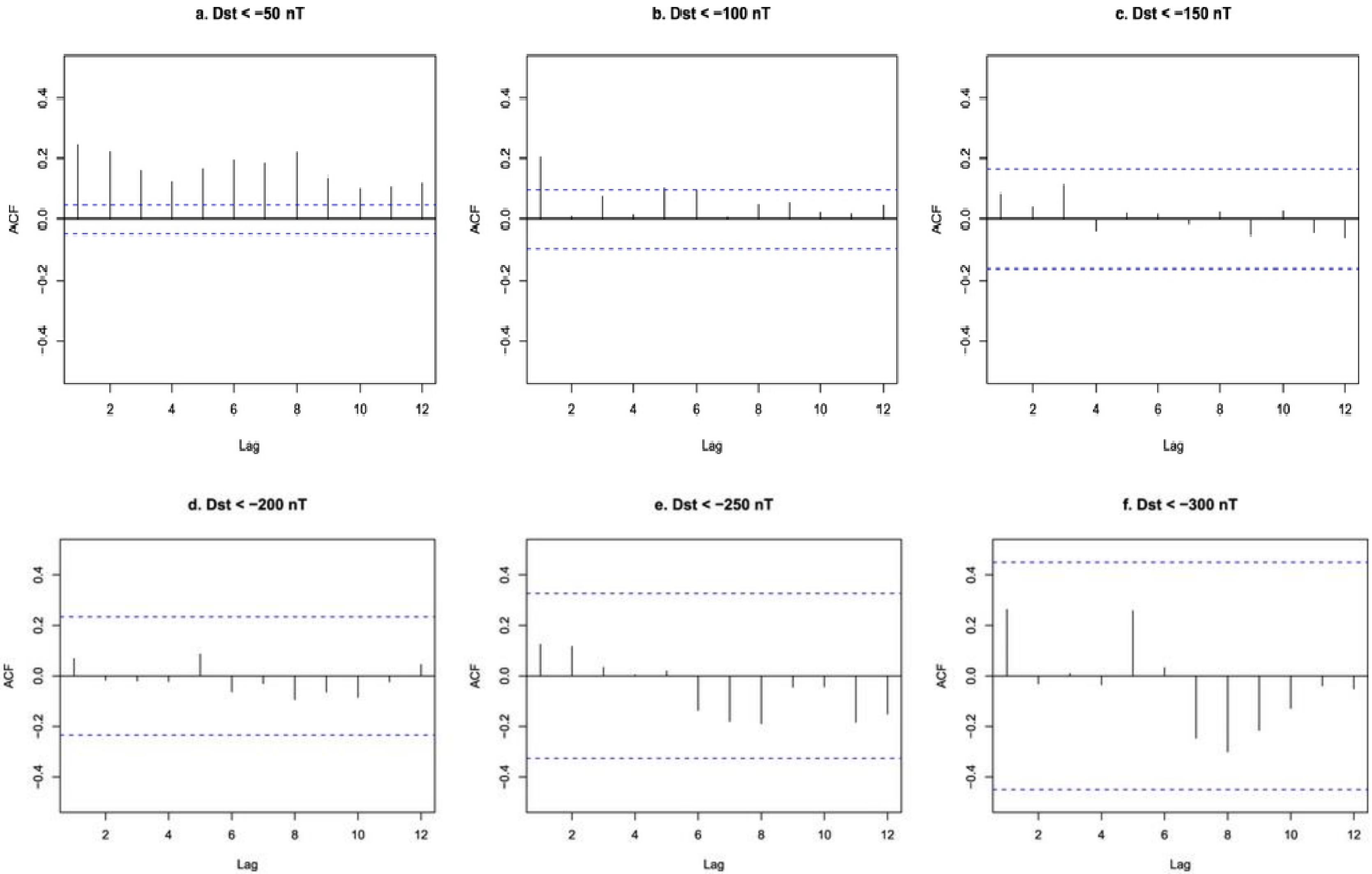
Given a threshold, we propose a Weibull renewal process to model magnetic storm inter-occurrence times.

$$S(t) = P(X > t) = e^{(-t/\tau)^{\gamma}}$$

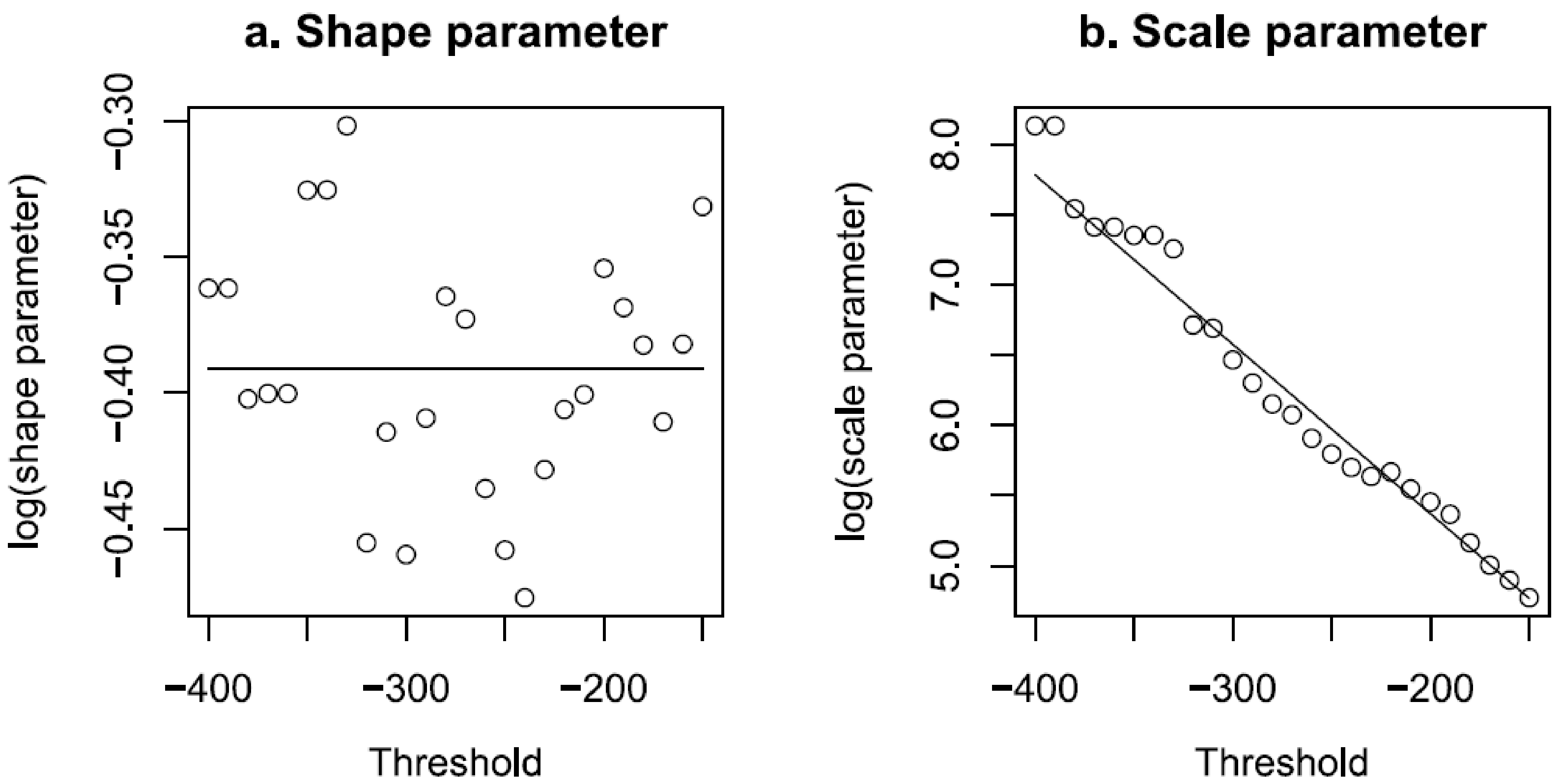
Why Weibull?

- We have fitted the data with other distributions (exponential, Gamma, Inverse Gaussian) and the results are worst (AIC, BIC).
- The hazard rate is just a power-law function: this is invariant under change of scale, which is physically suggestive, and the Weibull distribution is the only one having this property.
- Has been used to model inter-occurrence times of earthquakes (de Arcangelis et al., 2016) and solar flares (Paczuski et al., 2005).

Given a threshold, are the inter-occurrence times iid?



Relationship between Dst threshold and the estimated shape and scale parameters.



Therefore, the inter-occurrence times were fitted using a Weibull regression model where the scale parameter changes with the threshold of the storm, T, according to

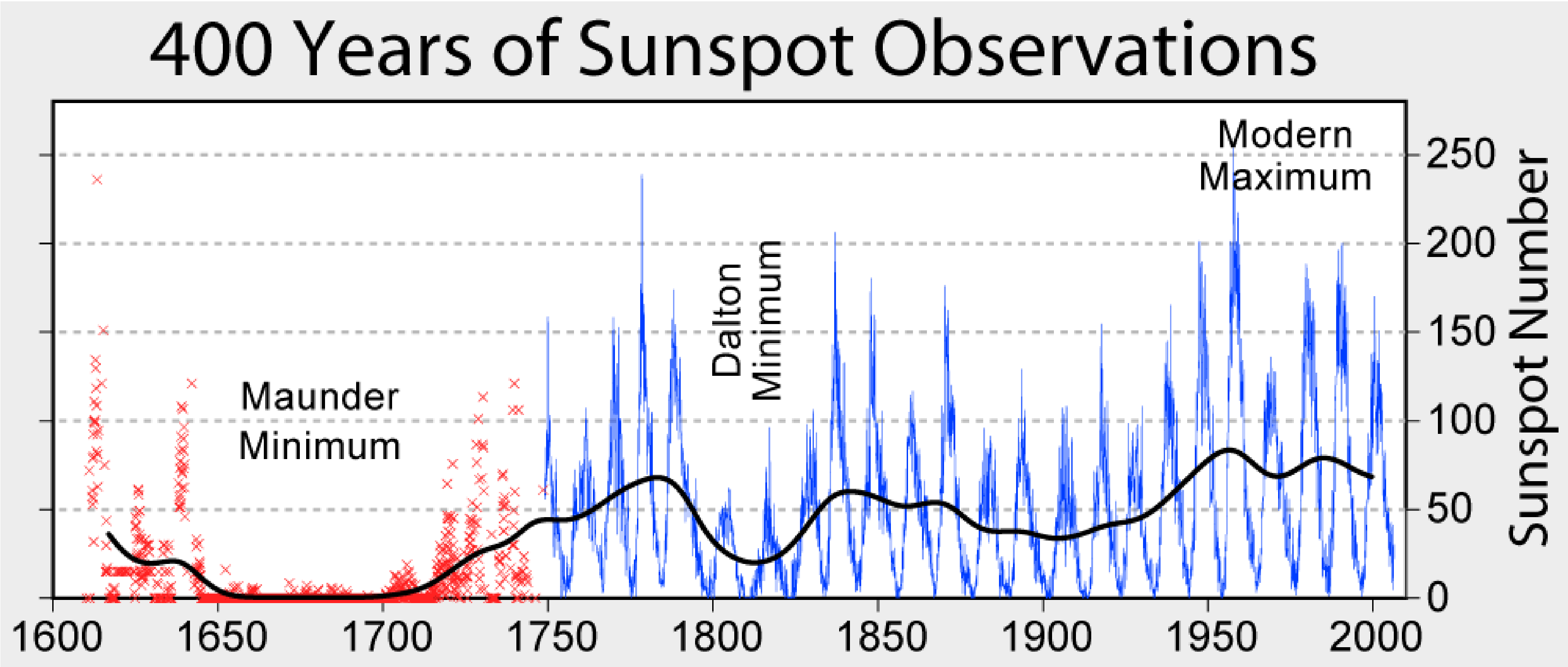
$$\log(\tau) = \beta_0 + \beta_1 T$$

and the shape parameter γ is constant.

$$S(t;T) = P(X > t) = e^{-t^{\gamma} e^{-\gamma\beta_0 - \gamma\beta_1 T}}$$

Is this model reliable?

- We want to extrapolate the model to thresholds never observed. Dst data starts in 1957.
- There are not good physical models for explaining the behavior of the sun.
- Most knowledge on the behavior of the sun is empirical (or statistical)
- Look what happened with sunspots …



Maunder minimum period
is known as
"little ice age".

4- The Results

The estimates are,

$$\log(\hat{\gamma}) = -0.39 \text{ (SE} = 0.023),$$
$$\beta_0 = 2.96 \text{ (SE} = 0.17)$$
$$\beta_1 = -0.0121 \text{ (SE} = 0.0008).$$

The negative sign of $\log(\hat{\gamma})$ implies that for each threshold the inter-occurrence times are DHR (decreasing hazard rate).

Consequence

The longer since the last super-storm happened, the less the chance to happen another one in the next unit of time.

Computing the probability of having a Carrington or more intense event during the next decade:

$$P(X \leq t_c + t_d \mid X \geq t_c) = \frac{S(t_c; -850) - S(t_c + t_d; -850)}{S(t_c; -850)} = 0.0092,$$

$t_c = 58000$ days, because Carrington event happened in 1859, approximately 58000 days ago.
$t_d = 3652$ days (10 years).

The number of geomagnetic storms in the interval (0,t], denoted as N(t), is a count variable of interest.

We have empirically seen that N(t) is overdispersed

A remarkable result for renewal processes (Barlow and Proschan, 1965):

If F is DHR(Decreasing Hazard Rate)

↓

The distribution of N(t) is overdispersed

We also estimate the expected number of magnetic storms for a period of time t, E(N(t)), for different thresholds.
It can be done using the asymptotic approximation,

$$E(N(t)) = t/\mu$$

where $\mu = \tau\Gamma(1 + 1/\gamma)$.

Threshold (nT)	Frequency
−100	4.95 per 1 year
−200	1.78 per 1 year
−400	1.63 per 10 years
−800	1.37 per 1,000 years
−1600	0.19 per 1,000,000 years

5- Conclusions

- ✓ The probability of occurrence on the next decade of Carrington event-type storm is estimated to be about 1% [0.46%, 1.88%] (95% CI)
- ✓ This value is not insignificant. Governments should have action protocols to react to such disasters, in order to inform and calm the population left without electrical energy and no way to communicate.
- ✓ There will be very little time of reaction before the unforeseen arrival of this type of storm

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We hope to see you soon!!