Mathematical Reconstruction of Land Carbon Models From Their Numerical Output: Computing Soil Radiocarbon From ^{12}C Dynamics

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Key Points

- Numerical output of land carbon models can be used to reconstruct a compartmental dynamical system.
- The reconstructed model can be used to compute isotope dynamics such as radiocarbon.
- We demonstrate the approach using the land component of the E3SM model.

Plain Language Summary

Models representing ecosystem carbon dynamics are generally complex and difficult to analyze. Comparing different models with different structures is even more challenging due to the variety of processes represented in the models. However, it is possible to use the numerical output of models to reconstruct the original structure using a common mathematical framework. We demonstrate this approach and apply it to compute radiocarbon dynamics of a land carbon model. The proposed approach can reconstruct the carbon and radiocarbon dynamics of the original model very accurately and can be used to study system-level dynamics of complex contrasting models.

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Some land carbon models **explicitly model the** ^{14}C component

- compare model output with radiocarbon measurements
- use radiocarbon measurements to parameterize models
- infer soil ages of the model
- additional implementation costs
- additional sources for errors

Many models do **not** model any ^{14}C

- no additional implementation costs
- no additional error sources

? compare model output with radiocarbon measurements

? use radiocarbon measurements to parameterize models

? infer soil ages of the model

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Mathematical representation as compartmental system

$$\frac{\mathrm{d}}{\mathrm{d}t}\,\mathbf{C}(t) = \mathrm{B}(\mathbf{C}(t), t)\,\mathbf{C}(t) + \mathbf{u}(\mathbf{C}(t), t), \quad t \geq t_0,$$
$$\mathbf{C}(t_0) = \mathbf{C}_0$$

$\mathbf{C}(t)$	vector of carbon stocks at time t
B	compartmental matrix
	governs internal cycling and external outputs
$\mathbf{u}$	vector of internal inputs
$\mathbf{C}_0$	initial vector of carbon stocks

Assume model piecewise constant on time intervals dependent on data's time resolution.

Linear time-independent model

$$\frac{\mathrm{d}}{\mathrm{d}t}\,\mathbf{C}(t) = \mathrm{B}\,\mathbf{C}(t) + \mathbf{u}, \quad t \geq t_0,$$
$$\mathbf{C}(t_0) = \mathbf{C}_0$$

Radiocarbon component

$$\frac{\mathrm{d}}{\mathrm{d}t}\,{}^{14}\mathbf{C}(t) = (\mathrm{B} - \lambda\,\mathrm{Id})\,{}^{14}\mathbf{C}(t) + {}^{14}\mathbf{u}, \quad t \geq t_0,$$
$${}^{14}\mathbf{C}(t_0) = {}^{14}\mathbf{C}(t_0)$$

λ radiocarbon decay rate
Id identity matrix

From ^{12}C to radiocarbon

- identify $\mathbf{C}$, B, $\mathbf{u}$, and $\mathbf{C}_0$ from model output

→ ^{12}C - component of model

- replace compartmental matrix B by $\mathrm{B} - \lambda\,\mathrm{Id}$

→ radiocarbon component of model

ELMv1-ECA (land component of E3SM) (Riley et al., 2018; Zhu et al., 2019)

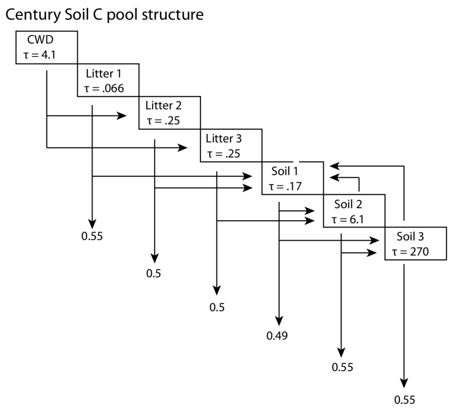

Figure: (Koven et al., 2013)

10 depth layers

B is 70×70 - dimensional matrix

$\mathbf{u}$ is 70 - dimensional vector

Provided data on yearly basis

- last 100 years of spinup: 1801 to 1901
- 110 years of transient model run: 1901 to 2011
- initial ^{12}C stocks and initial ^{14}C stocks (instantaneous)
- for **all time steps all ^{12}C fluxes** (mean)
  - external inputs and outputs
  - internal fluxes within and between layers

→ ^{12}C - and ^{14}C - reconstructions with minimal errors

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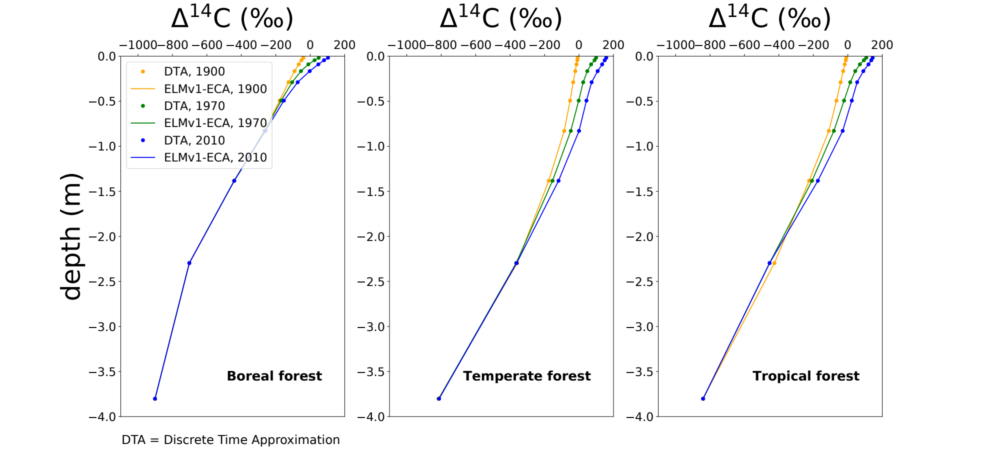
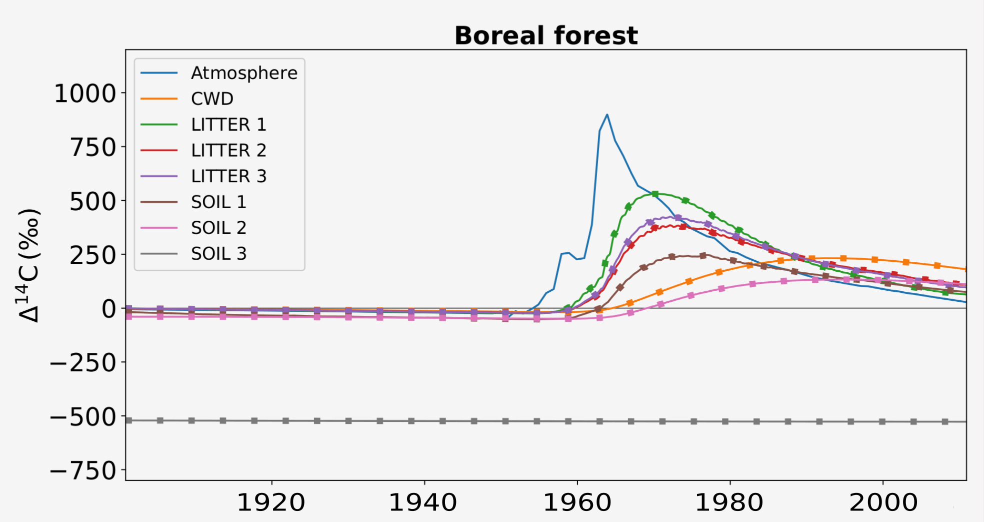
Δ^{14}C reconstruction in depth



$\Delta^{14}C$ reconstruction in time



Challenges

- CMIP6 output is not sufficient, mostly because models are not yet required to output all fluxes.
- We need to identify an initial vector for ^{14}C stocks, i.e., at the interface of spinup and transient data.
  → This is easy only if model is in equilibrium at the end of the spinup.
- We need to identify an external ^{14}C input vector, i.e., the amount of radiocarbon moving from the vegetation to the soil component of the model.

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Even if models **do not provide radiocarbon, we can** (possibly)

- estimate an end-of-spinup radiocarbon structure,
- compute radiocarbon through time,
- compute the Δ^{14}C depth structure,
- compare models based on their Δ^{14}C structure,
- parameterize models with radiocarbon as additional constraint.

- CMIP models are asked to provide fluxes instead of radiocarbon.
- This requires more storage but provides more information.

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Special Section:
The Energy Exascale Earth System Model (E3SM)

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Abstract Radiocarbon (^{14}C) is a powerful tracer of the global carbon cycle that is commonly used to assess carbon cycling rates in various Earth system reservoirs and as a benchmark to assess model performance. Therefore, it has been recommended that Earth System Models (ESMs) participating in the Coupled Model Intercomparison Project Phase 6 report predicted radiocarbon values for relevant carbon pools. However, a detailed representation of radiocarbon dynamics may be an impractical burden on model developers. Here, we present an alternative approach to compute radiocarbon values from the

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- so far only three sites as proof of concept (boreal, temperate, and tropical forest)
- currently ongoing work on global results