How many grains do we need for tracer thermochronology?

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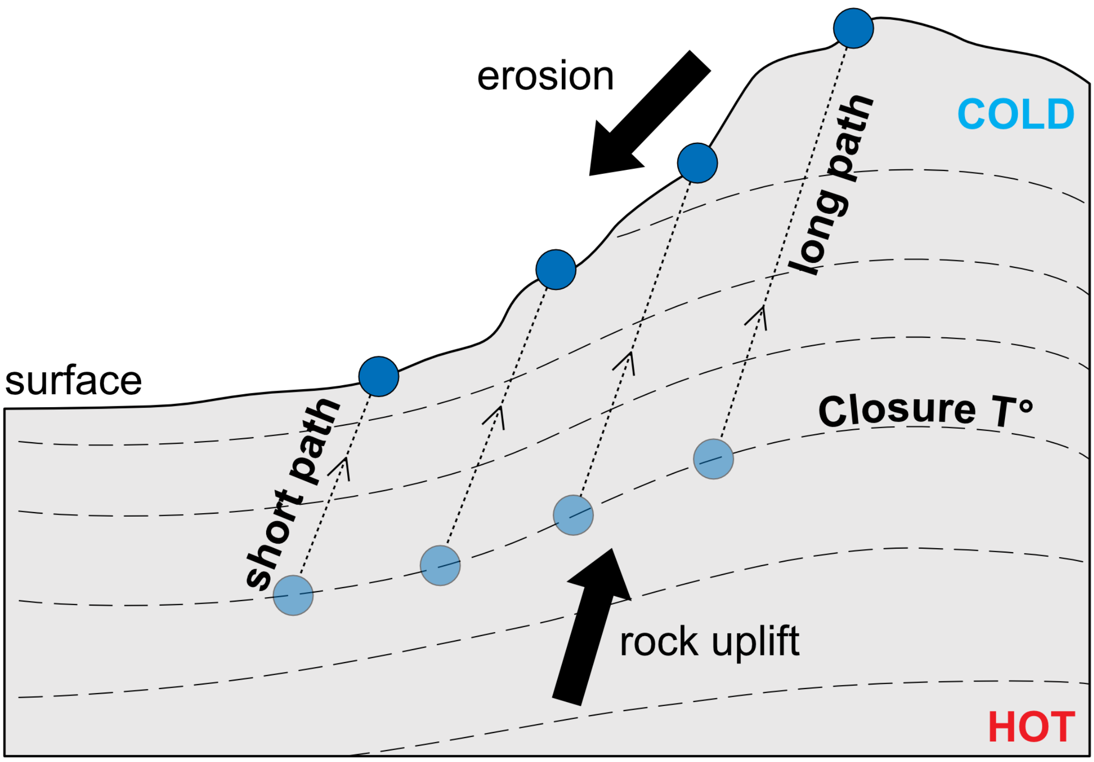
Introduction

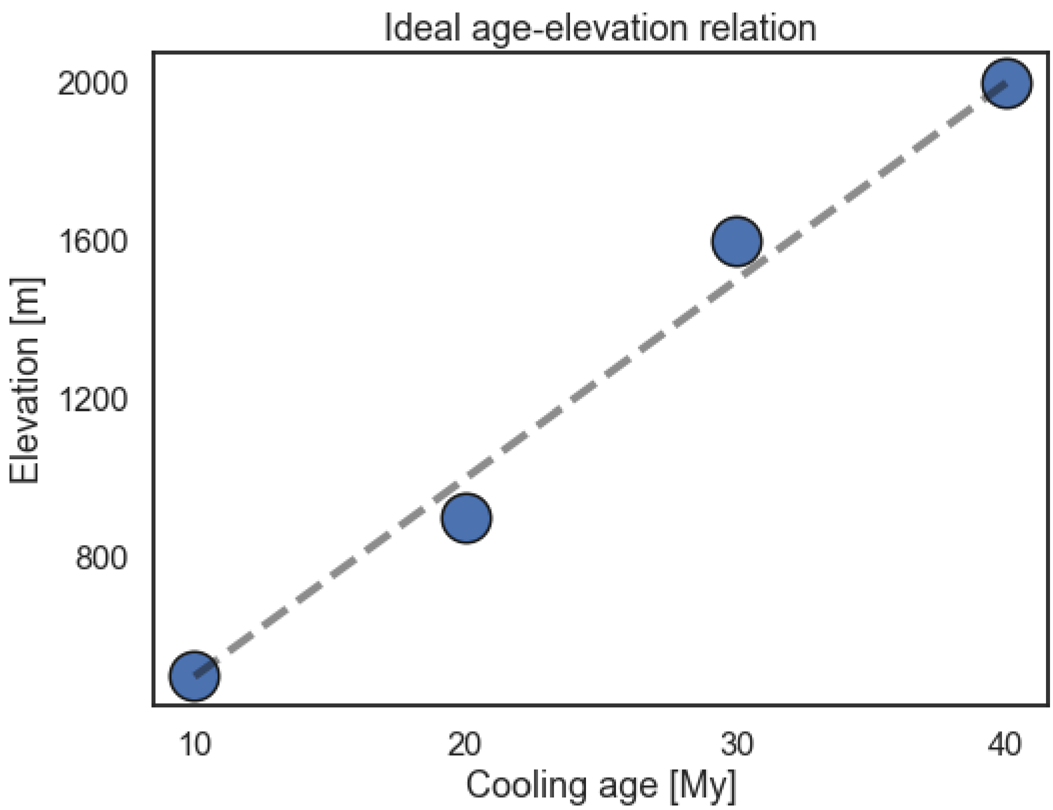
Tracer thermochronology uses the relationship between bedrock cooling age and elevation to infer the pattern of catchment erosion from the detrital grain age distribution.

Let's consider the following example:

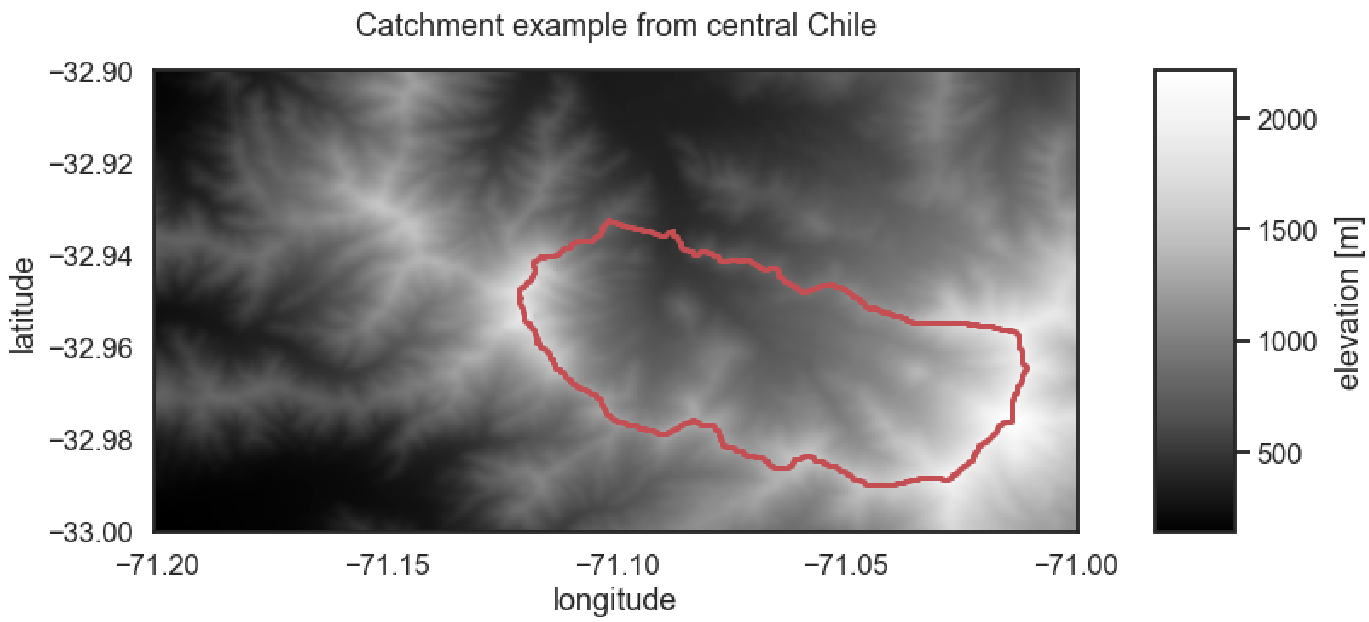


In the illustration above, **higher samples have taken longer to travel from the closure isotherm to the surface**.

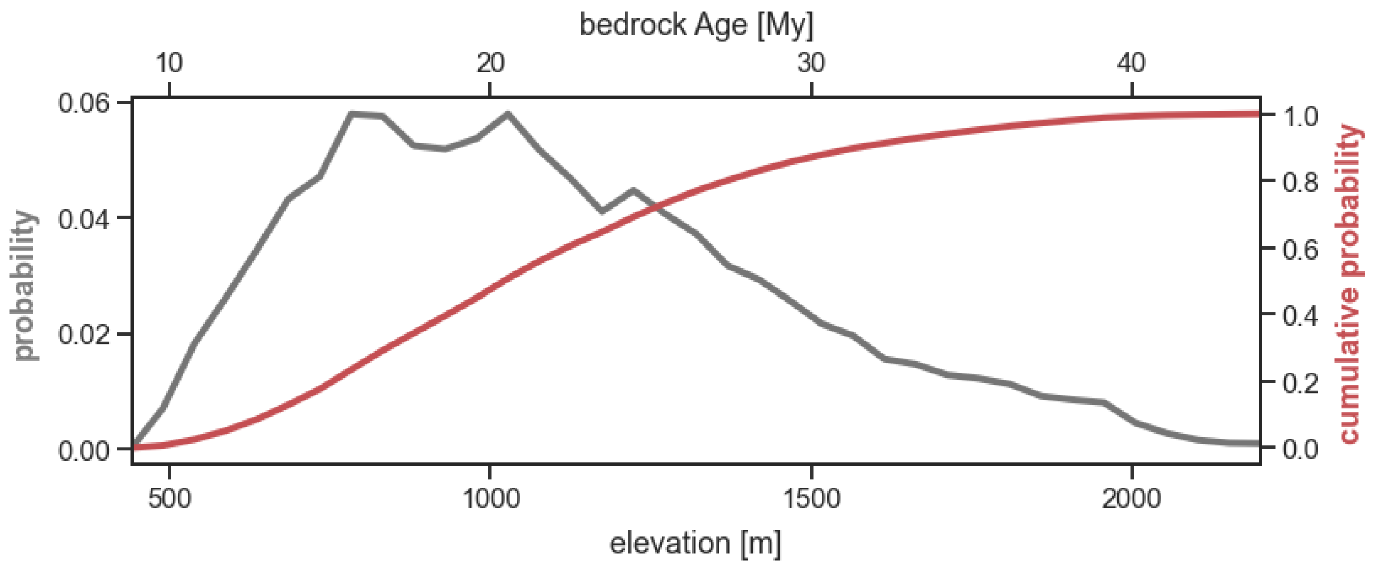
The age-elevation plot of these 4 samples looks something like this:



Now, if the relation above holds for an entire catchment, the probability distribution of bedrock ages should reflect the catchment hypsometry. For instance, considering this catchment:



The probability distribution of bedrock ages is expected to follow the distribution of bedrock elevation, like so:



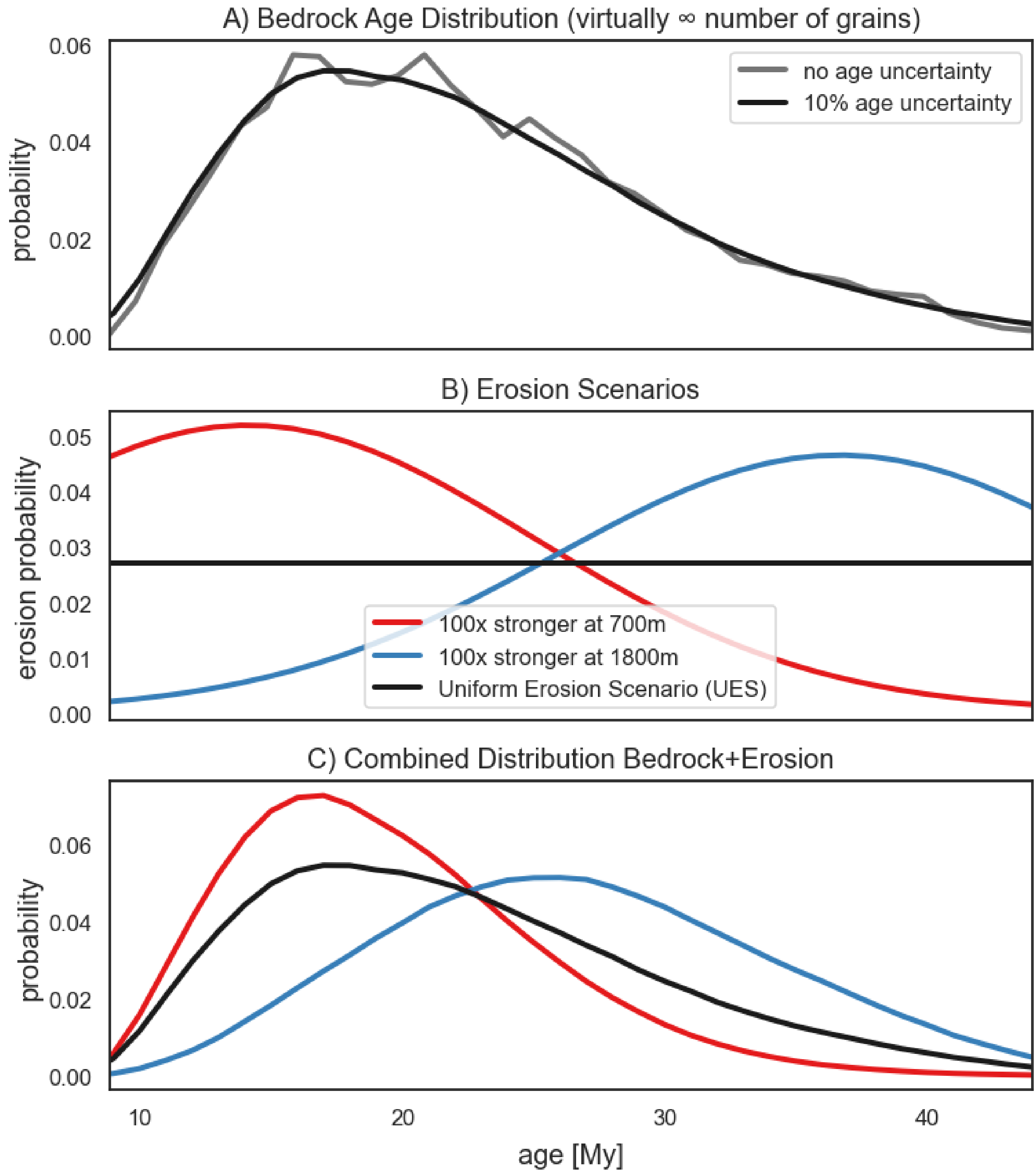
At the catchment outlet, the detrital age distribution will depend on the bedrock age distribution above.

However, **the probability to sample an age in the detritus is additionally affected by:**

1. Spatial changes in **mineral fertility** of the bedrock.
2. Spatial changes in **erosion**.

For this example, we assume constant mineral fertility, and rather focus on variations in erosion.

Different erosion scenarios will affect the detrital age distribution in different ways:



A) The gray line equals the hypsometric curve, the black line accounts for bedrock age analytical error.

B) The black line represents the reference Uniform Erosion Scenario (*UES*); blue and red lines represent a Gaussian

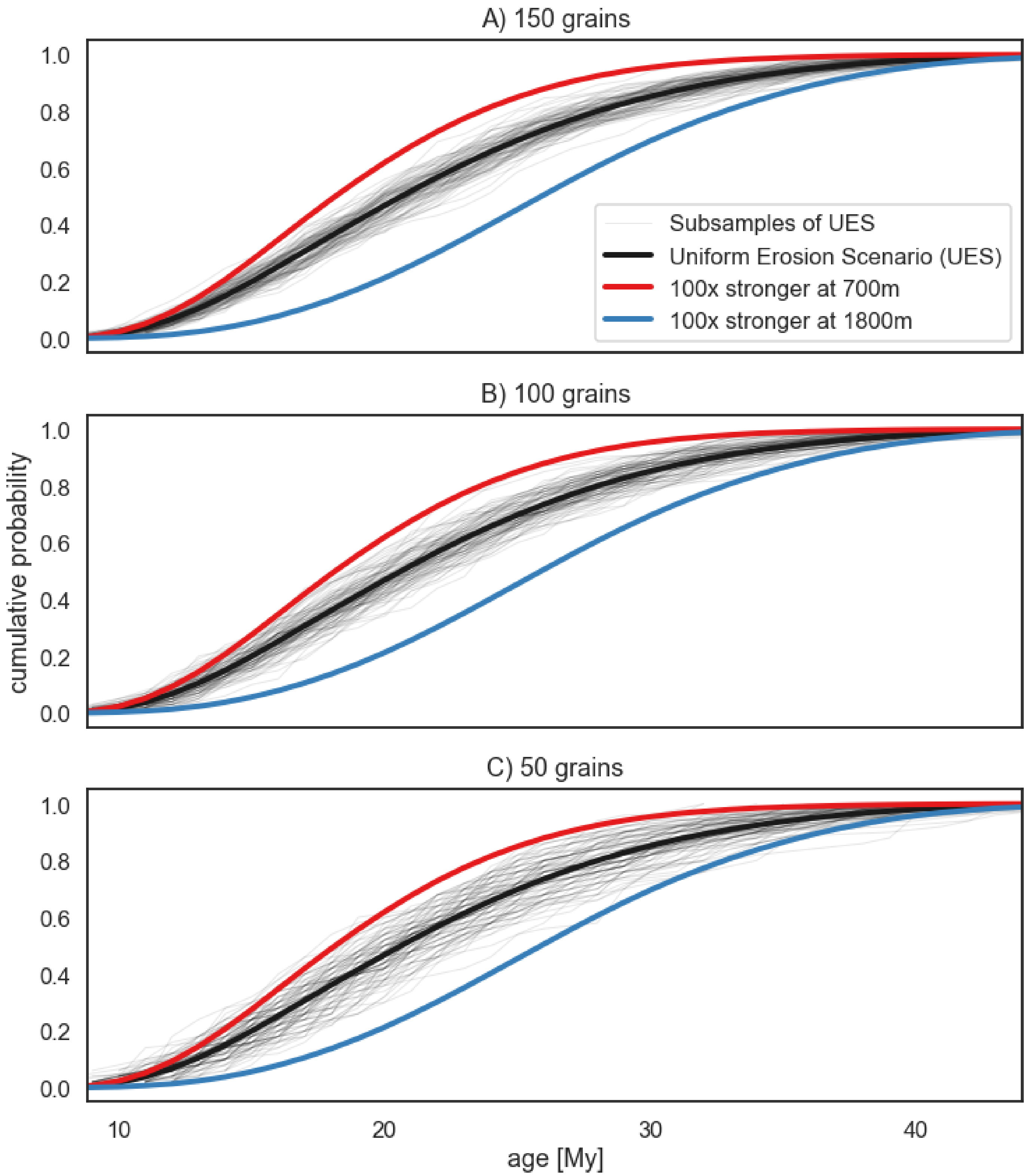
peak in erosion centered at the specified elevation, where erosion is 100 times the minimum found in the catchment.

C) Each of the scenarios in (B) affects the expected detrital age distribution differently.

The scope of tracer thermochronology is to find the scenario that best explains the measured detrital age distribution.

However, if not enough grains are measured, it might be statistically impossible to differentiate

between the best-fit scenario and uniform erosion. For instance:



Here scenarios are visually compared to random subsamples of the black curve with size 150 (A), 100 (B), 50 (C).

Discerning between red and black curves requires more detrital grains than between blue and black.

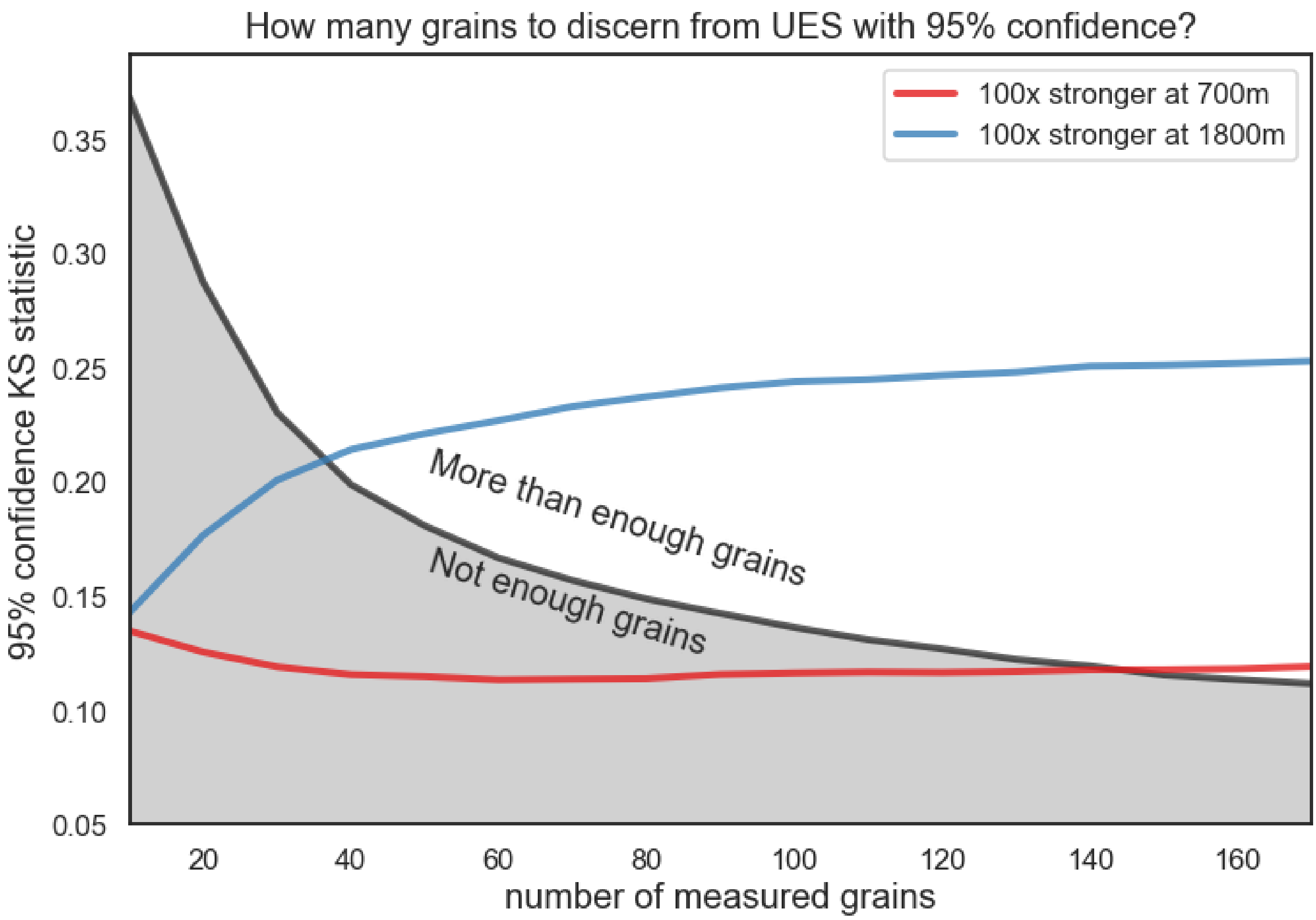
But how many grains?

Our Approach

To quantify this number, we do so:

1. We pick several random subsamples (*UES_sub*) of the reference scenario *UES*.
2. We compute the Kolmogorov-Smirnov (KS) statistic between the distributions of each *UES_sub* and *UES*.
3. We pick several random subsamples of a different erosion scenario (*S1_sub*).
4. We compute the KS statistic between distributions of each *S1_sub* and the *UES*.
5. We compare the 95% largest KS stats of (2) to the 95% smallest KS stats of (4).

We repeat all points for different subsample sizes and plot the results below:



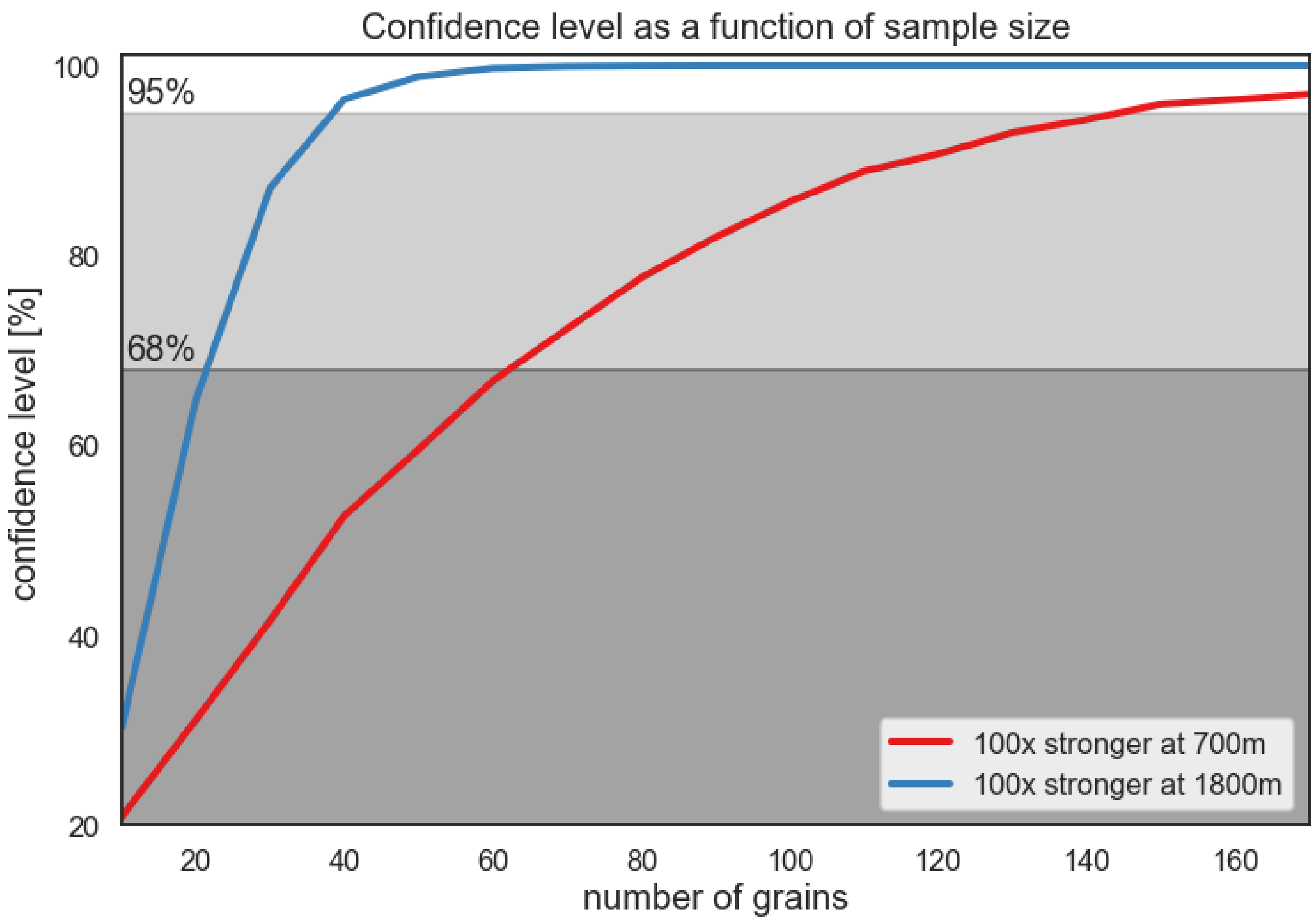
In the gray field, the difference from *UES* is smaller than between *UES_sub* and *UES*, more than 5% of the times.

Although both blue and red scenarios represent a local 100-times-increase in erosion,

the elevation range of the erosional peak is critical

for the determination of a distinct erosion scenario (here 100-times higher erosion).

With the same approach we can predict the confidence level as a function of sample size:



Again, note the striking difference in required grains to distinguish from UES.

Already 40 grains are enough to discern between UES and blue scenario with a 95% confidence.

In contrast, 140 grains do not suffice for the red scenario.

Take home message

The choice of detrital sample size should not only be a function of grain availability, but evaluated in advance based on given (geologic, geomorphic and thermochronological) boundary conditions and research question.

Our approach can inform this choice by stacking the following layers of information:

1. Catchment hypsometry.
2. Bedrock age-elevation data.
3. Mineral fertility.

4. Any hypothesized pattern of erosion.

If the grain number is limited for some reason, our approach helps quantifying the confidence level of the interpretations.

Outlook

- We plan to provide the script used to apply this method.
- A 2D-implementation is possible, for areas where age-elevation data cannot be explained by a linear function.