Full 3-D pseudo-transient finite difference modelling of stress distribution around continental plateaus

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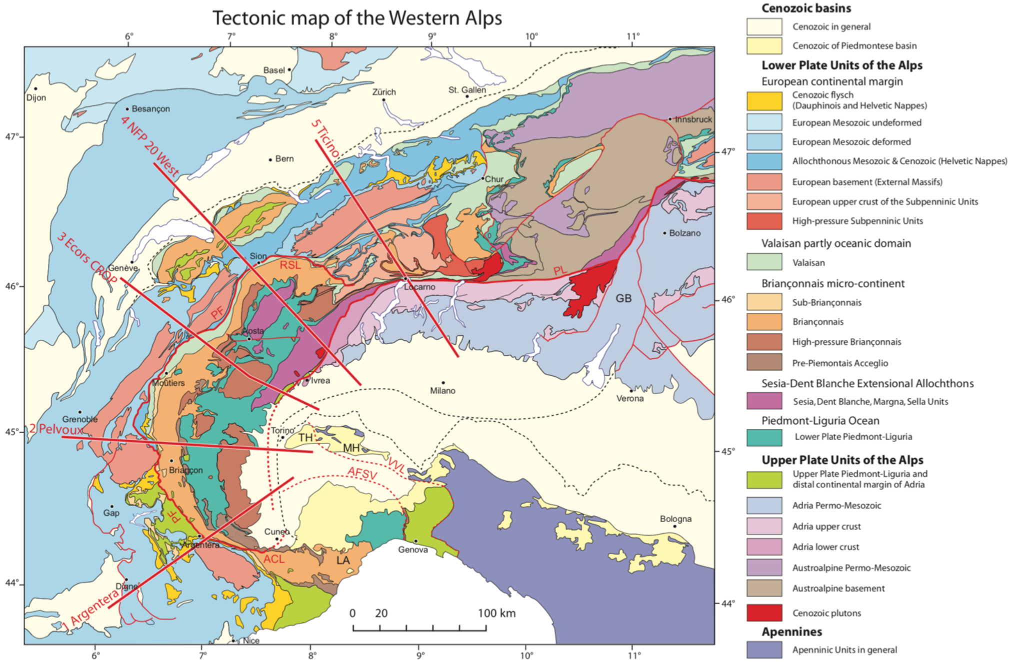
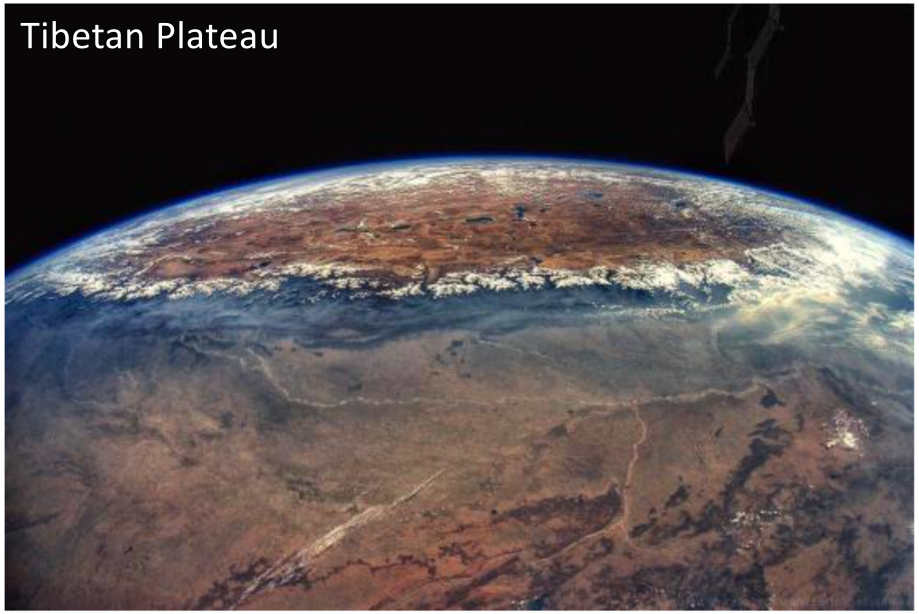
Motivation – Curvature of the Earth and of orogens

- What is the impact of the spherical geometry of tectonic plates on the stress field around continental plateaus?
- What is the impact of oblique collision on the stress field during continental collision?

Alpine Arc



Schmid et al. (SJG, 2017)



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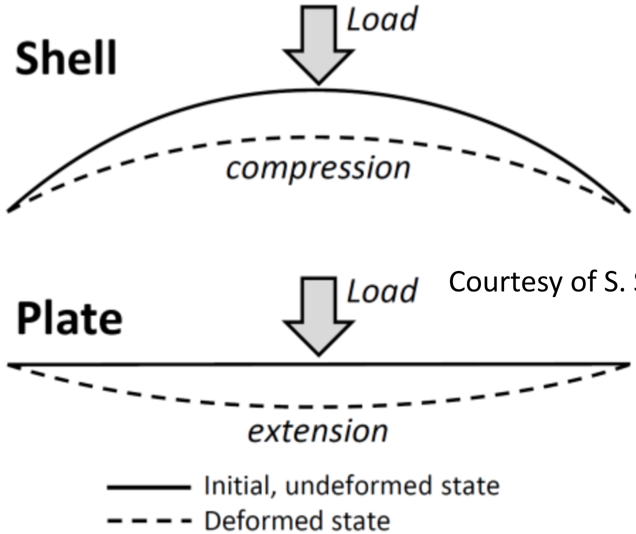
Goals:

- Quantification of spatio-temporal evolution of stress around continental plateaus
- Quantification of stress and strain during curved and oblique continental collision

Motivation - Plate vs Shell Tectonics

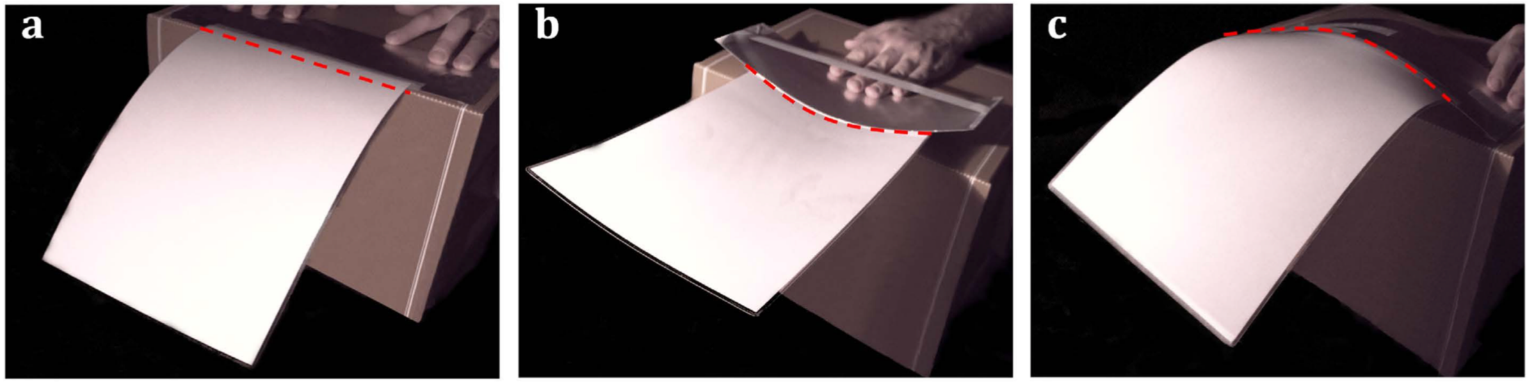
The mechanical behavior of a shell is fundamentally different to the one of a plate with respect to:

- Flexure due to lateral loading (e.g. Von Karman et al., JAS, 1940)
- Geometrical stiffening (e.g. Mahadevan et al., Tectonics, 2010)



Courtesy of S. Schmalholz

Illustration of geometrical stiffening of a sheet of paper



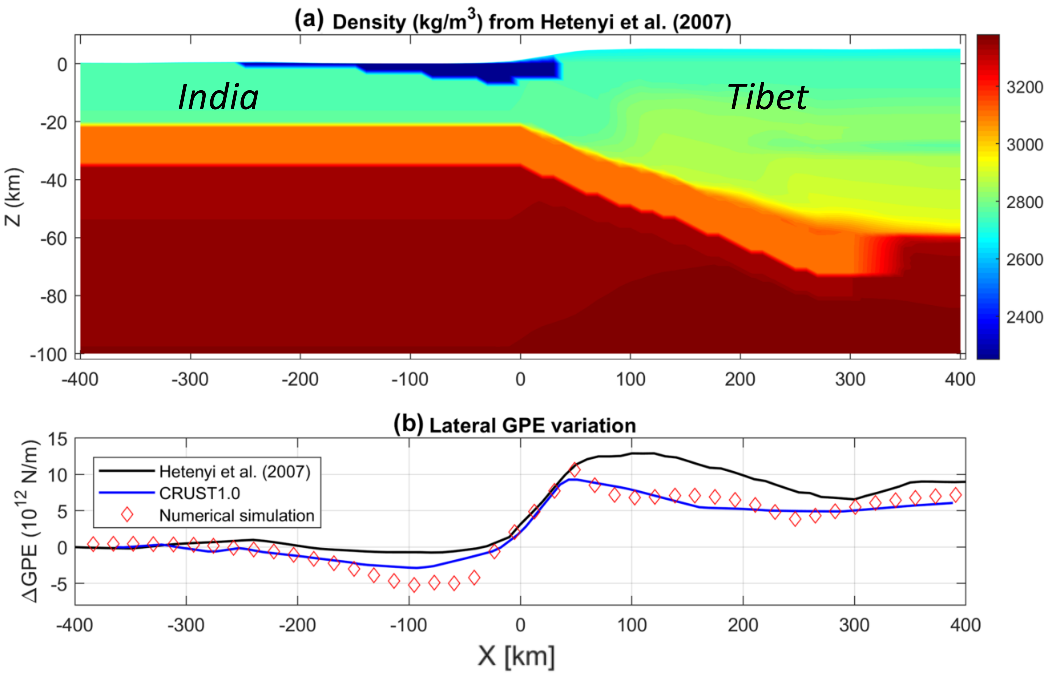
Pini et al. (Scientific Reports, 2016)

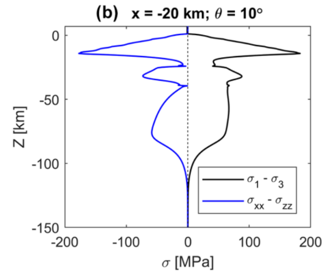
Flexure and associated stresses

Lateral variation of Gravitational Potential Energy per unit area (GPE) between plateau and lowland causes not only horizontal crustal flow (gravitational collapse) and associated inplane stresses, but also differential vertical motion and associated flexural stresses (Schmalholz et al., 2019).

How does a spherical geometry of the plateau and lowland impact these flexural stresses in 3D?

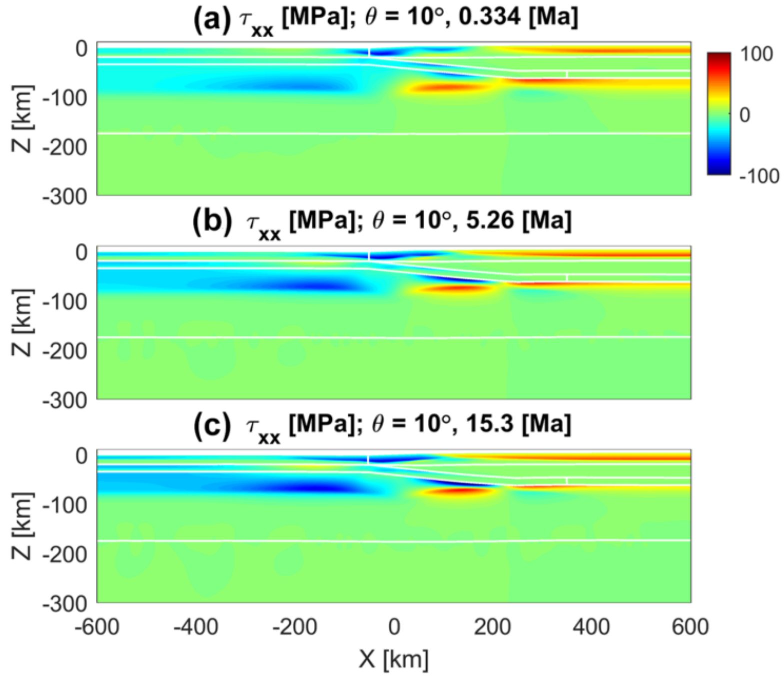
Density profile across Tibetan plateau





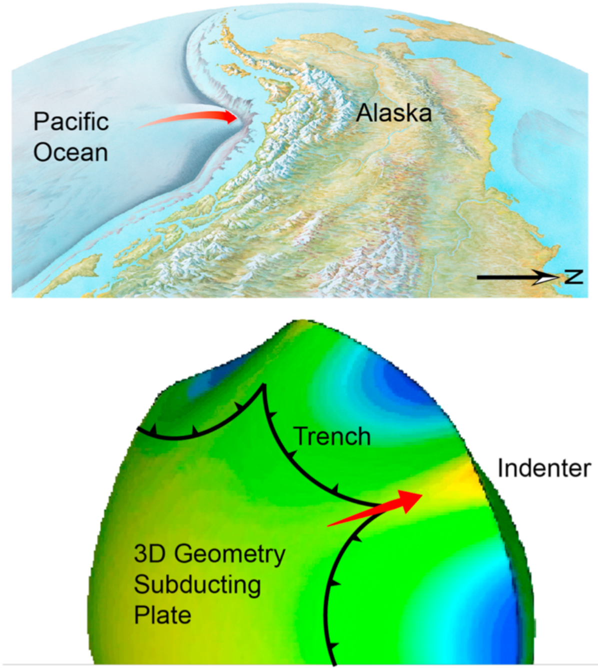
all images from Schmalholz et al. (GJI, 2019)

Numerical simulation of stress field



Existing models, some examples

- Explanation for curved subduction zones: *Frank (Nature, 1968), Mahadevan et al. (Tectonics, 2010)*
- Bending stresses due to plate curvature: *Tanimoto (GJI, 1997, 1998)*
- Rapid exhumation due to geometrical stiffening: *Bendick and Ehlers (GRL, 2014), Flesch et al. (GRL, 2018), Bischoff and Flesch (JGR, 2019), Koptev et al. (Tectonics, 2019)*



Bendick and Ehlers (GRL, 2014)

We aim to quantify systematically the impact of curved geometries and oblique indenters on stress evolution

Method – Numerical simulation

Thermo-mechanical model:

- Visco-elastic rheology
- Gravity
- Compressible deformation
- Heat transfer

Numerical method:

- Pseudo-transient finite difference method (e.g. Duretz et al., GJI, 2019)
- Iterates until the pseudo-time derivative (τ_V) approaches zero, then a steady-state is found

Example: Force balance equation

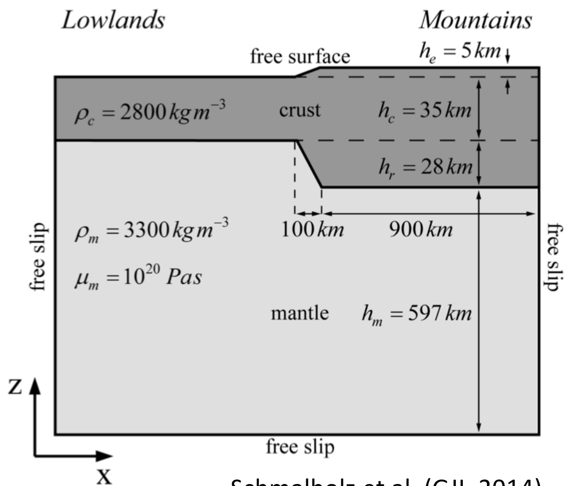
Force balance equation for steady state without inertial forces	Adding pseudo-transient time derivative for velocity
$0 = \nabla_{ij}\left(\tau_{ij} - P\delta_{ij}\right) - \rho g_i$	$\rho \dfrac{\partial V_i}{\partial \tau_V} = \nabla_{ij}\left(\tau_{ij} - P\delta_{ij}\right) - \rho g_i$

2D Pseudo-transient method Plateau benchmark

Reproduce results from Schmalholz et al. (GJI, 2014)

Results from Schmalholz et al. (GJI, 2014) are calculated by a finite element code with a Lagrangian mesh

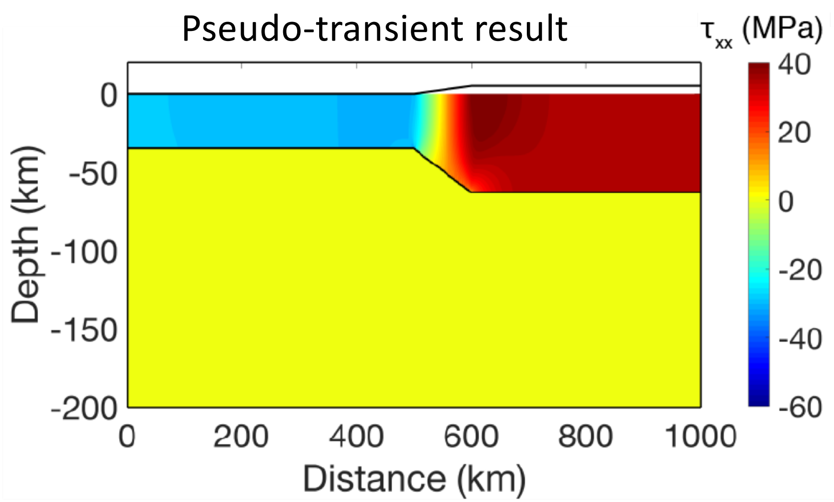
Model configuration



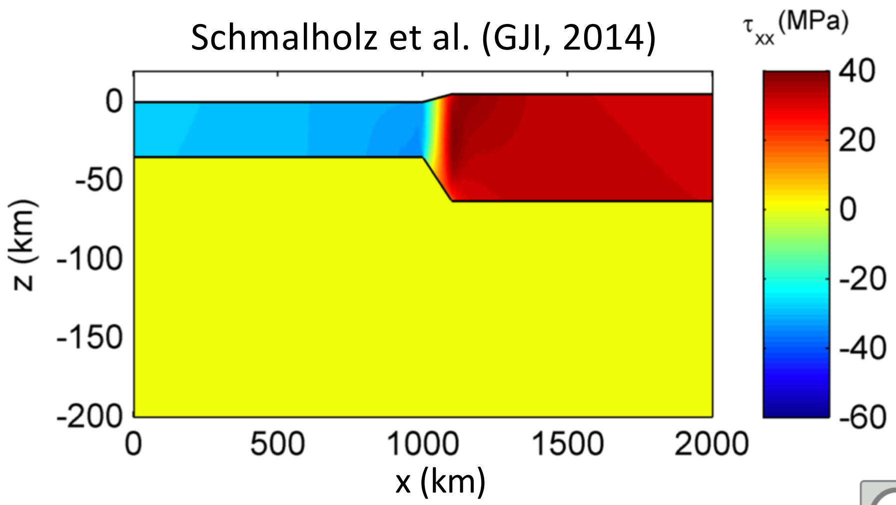
Schmalholz et al. (GJI, 2014)

Horizontal deviatoric stress (compression – extension)

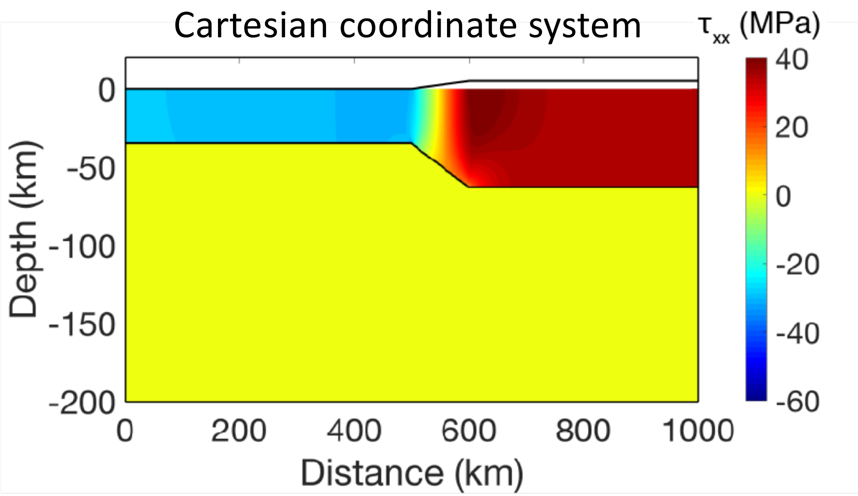
Pseudo-transient result

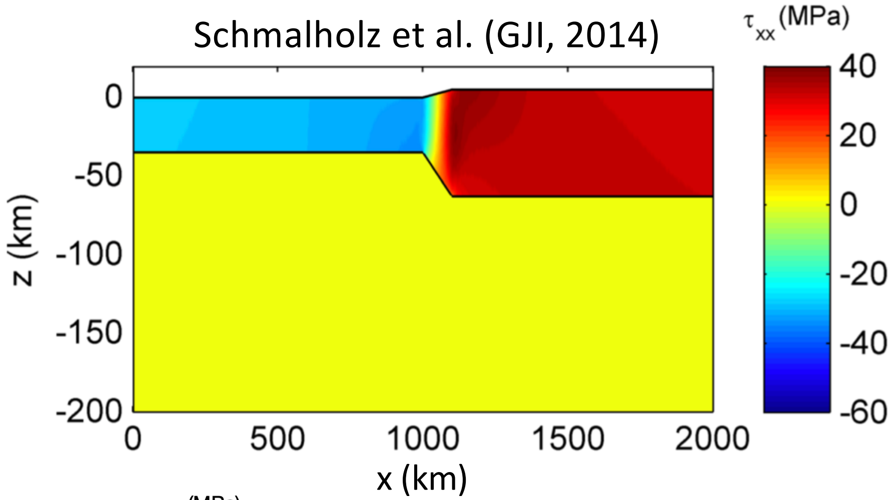


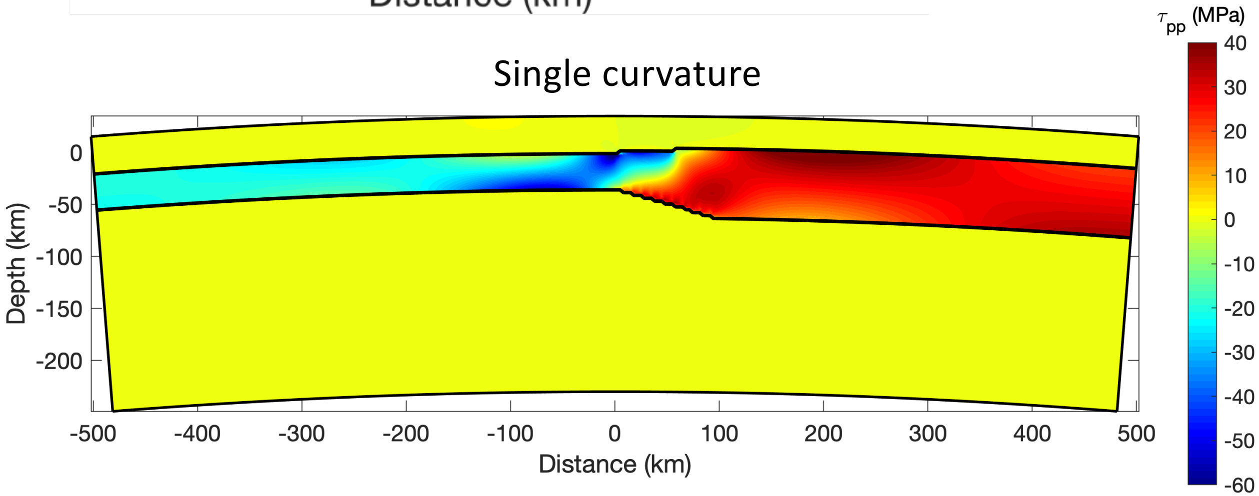
Schmalholz et al. (GJI, 2014)



2D Pseudo-transient: First results of plate vs shell





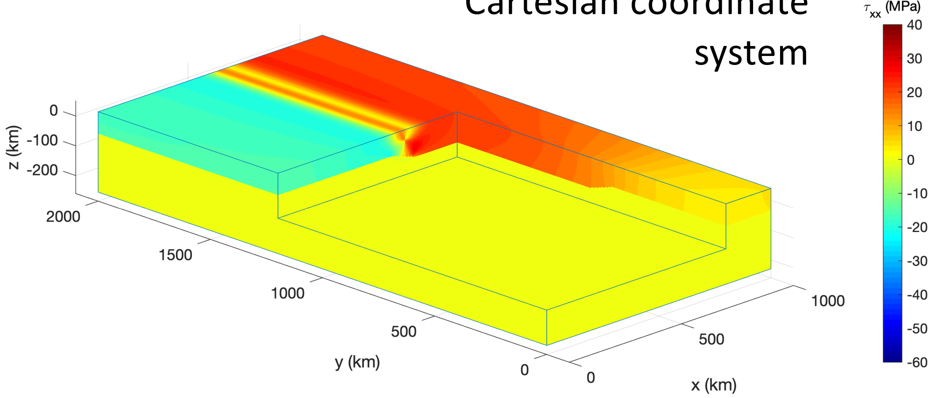


Horizontal deviatoric stress (compression – extension)

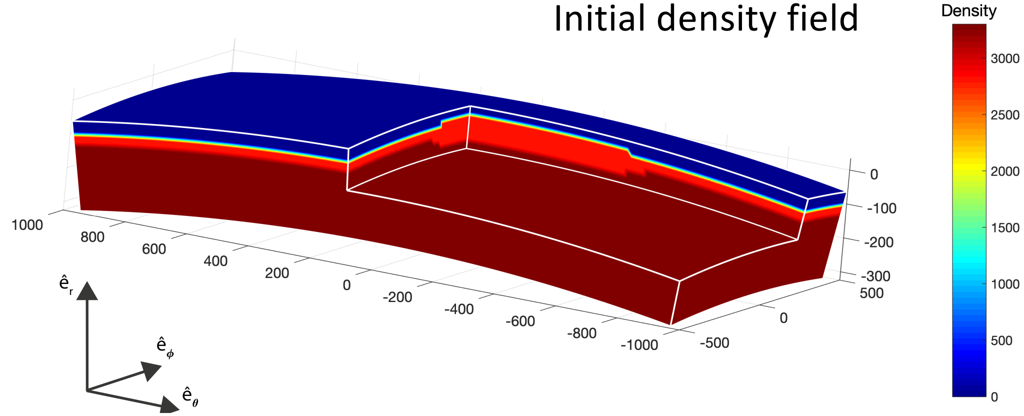
First results of stress field in cylindrical coordinates. Here with a sticky air approach.

3D – First results in Matlab for Cartesian and spherical coordinates

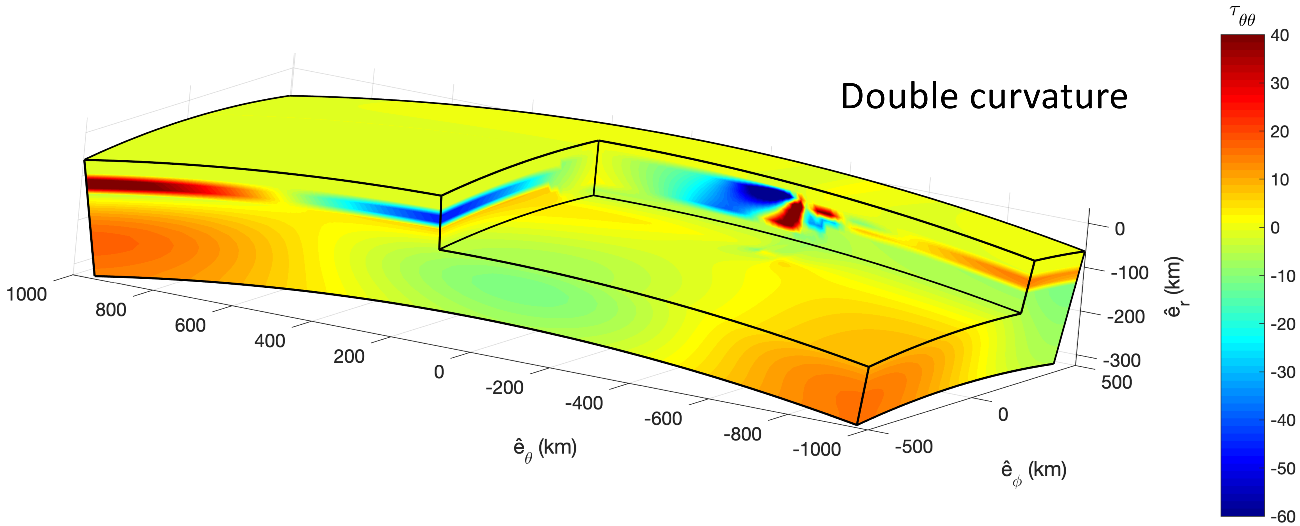
Cartesian coordinate system



Initial density field



Double curvature



Solution with standard algorithm in Matlab is slow and needs increase in computational performance.

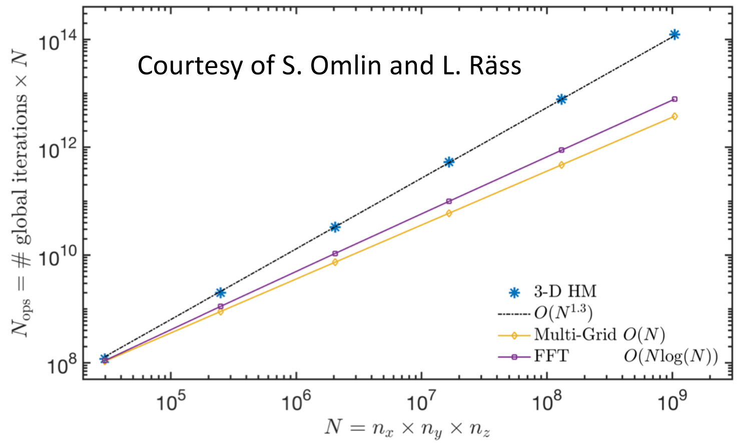
Current work – Speed up with Julia and GPU computation

- Julia is a high level language that allows high performance computing, in particular GPU-parallel computation (julialang.org)
- Transformation from Matlab codes into Julia. 2-D Julia version is already developed; see right
- Packages to use GPU massively parallel method with Julia for very high performance computation. Documentation for example on CSCS webpage: https://user.cscs.ch/tools/interactive/julia/

Part of the first 2D Julia version (force balance part)

```
# -------------------------------------------------------------
# momentum (or force) balance
Rhog   = Rhog0;
grav_z = -1*(Rhog[2:end-1,2:end-1,2:end  ] +
             Rhog[2:end-1,2:end-1,1:end-1])/2;
dVx    = dVx*(1-dmp/nx)+(diff(sig_xx,dims=1)/dx +
                         diff(tau_xy,dims=2)/dy +
                         diff(tau_xz,dims=3)/dz) .*
                         min.(dt_rho_iner[1:end-1,2:end-1,2:end-1],
                              dt_rho_iner[2:end  ,2:end-1,2:end-1]);
dVy    = dVy*(1-dmp/ny)+(diff(tau_xy,dims=1)/dx +
                         diff(sig_yy,dims=2)/dy +
                         diff(tau_yz,dims=3)/dz         ) .*
                         min.(dt_rho_iner[2:end-1,1:end-1,2:end-1],
                              dt_rho_iner[2:end-1,2:end  ,2:end-1]);
dVz    = dVz*(1-dmp/nz)+(diff(tau_xz,dims=1)/dx +
                         diff(tau_yz,dims=2)/dy +
                         diff(sig_zz,dims=3)/dz + grav_z) .*
                         min.(dt_rho_iner[2:end-1,2:end-1,1:end-1],
                              dt_rho_iner[2:end-1,2:end-1,2:end  ]);

# -------------------------------------------------------------
```



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