Large organized structures in stably stratified
turbulent shear flows

Andrey Glazunov\textsuperscript{1}, Evgeny Mortikov\textsuperscript{1,2}, Grigory Zasko\textsuperscript{3}, Yuri Nechepurenko\textsuperscript{1},
and Sergej Zilitinkevich\textsuperscript{4,5}

\textsuperscript{1}Marchuk Institute of Numerical Mathematics, Russian Academy of Sciences, Moscow, Russia
\textsuperscript{2}Research Computing Center of Lomonosov Moscow State University, Moscow, Russia
\textsuperscript{3}Keldysh Institute of Applied Mathematics, Russian Academy of Sciences, Moscow, Russia
\textsuperscript{4}Finnish Meteorological Institute, Helsinki, Finland
\textsuperscript{5}Institute for Atmospheric and Earth System Research, University of Helsinki, Helsinki, Finland
DNS and LES of a stably stratified turbulent Couette flow


Vertical cross-section. Temperature isolines.
Large scale irregular inclined layers with weak stratification, separated by very thin layers with large gradients.
Large eddy simulation

Vertical cross-section. Temperature isolines.
The existence of such layered structures in nature?

The specific shape of PDFs of temperature gradients is indicative of its jumplike changes along the wind direction. This agrees with a representation of the temperature field as inclined layers with weak stratification, separated by thin sublayers with strong stratification.

SMEAR II (Station for Measuring Ecosystem-Atmosphere Relations), Finland

Measurements $T'(t)$
Height of sensor – 67 m
Frequency - 10 Hz

Other confirmations of the existence of layered structures


Stably stratified Ekman layer (LES, ~10^9 grid points)


Figure 10: Contours of the temperature difference $\theta - \theta_u$ in an $x$-$z$ plane at $y = 200$ m at $t \sim 9$ hrs. Upper panel stratification $z_u/L = 1.7$ (the same slice as in the upper panel of Fig. 9) with 71 equally spaced contour levels spanning the range $[-2.0]$ K. Lower panel $z_u/L = 6.0$ with 101 contour levels spanning the range $[-8.0, 0.5]$ K. Notice how the tilt angle of the fronts is reduced with stronger stratification.
Conclusion:

Large-scale layered structures are a common feature of stably stratified turbulent flows

Subjects for further investigations:

Mechanisms of structures arising

Spatial and temporal scales associated with structures

Influence of the large layered structures on the turbulence and on the heat and momentum transport
Turbulence at the large gradient Richardson number?

\[ Ri < 0.25 \]

Miles–Howard criterion

\[ Rf < 1 \]

TKE budget

Increasing of \( Pr \) with \( Ri \) growth should be an indicator of maintaining turbulence

\[ Pr = \frac{K_m}{K_h} = \frac{\phi_h}{\phi_m} = \frac{Ri}{Rf} \]

~10^8 grid nodes

Supercomputer «Lomonosov 1»

MSU

~ 10^2 hours

~ 1000 cores

(Zilitinkevich et al., 2013)

Numerical simulations are still here

Increasing stability (Ri growth)
The DNS and LES results demonstrate an increase in the turbulent Prandtl number with increasing Ri.

It can be hypothesized that turbulence is self-organized and produces multilayered stratified fluid, so that:

the exchange of momentum between layers occurs mainly through its redistribution by pressure fluctuations,
while

the turbulent heat exchange, which requires air-mass mixing, is largely blocked.

---

![Diagram showing momentum and heat fluxes](image-url)
Simplified linearised model of large scale disturbances in Couette flow:

\[
\begin{align*}
\frac{\partial u'}{\partial t} + \bar{U} \frac{\partial u'}{\partial x} + \frac{d\bar{U}}{dy} v' + \frac{\partial p'}{\partial x} - \Delta_v u' &= 0 \\
\frac{\partial v'}{\partial t} + \bar{U} \frac{\partial v'}{\partial x} + \frac{\partial p'}{\partial y} - \Delta_v v' - \text{Ri} T' &= 0 \\
\frac{\partial w'}{\partial t} + \bar{U} \frac{\partial w'}{\partial x} + \frac{d\bar{T}}{dy} v' - \Delta_v w' &= 0 \\
\frac{\partial T'}{\partial t} + \bar{U} \frac{\partial T'}{\partial x} + \frac{d\bar{T}}{dy} v' - \Delta_\mu T' &= 0 \\
\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z} &= 0
\end{align*}
\]

Turbulent viscosity and diffusivity coefficients (obtained from DNS data):

\[
\bar{\nu}(y) = -\tau / \left( \frac{d\bar{U}}{dy} \right), \quad \bar{\mu}(y) = -F_T / \left( \frac{d\bar{T}}{dy} \right)
\]

\[
\tau = U'V' - \frac{1}{\text{Re}} \frac{d\bar{U}}{dy}, \quad F_T = T'V' - \frac{1}{\text{PrRe}} \frac{d\bar{T}}{dy}
\]
disturbances:

\[ u' = e^{i\alpha x + i\gamma z} u_{\alpha \gamma}, \quad v' = e^{i\alpha x + i\gamma z} v_{\alpha \gamma}, \quad w' = e^{i\alpha x + i\gamma z} w_{\alpha \gamma} \]

\[ p' = e^{i\alpha x + i\gamma z} p_{\alpha \gamma}, \quad T' = e^{i\alpha x + i\gamma z} T_{\alpha \gamma} \]

Energy norm:

\[ \mathcal{E}_t = \frac{1}{2} \int_{-1}^{1} \left( |u_{\alpha \gamma}|^2 + |v_{\alpha \gamma}|^2 + |w_{\alpha \gamma}|^2 + \frac{\text{Ri}}{dT/\text{dy}} |T_{\alpha \gamma}|^2 \right) \, dy \]

The maximal possible growth:

\[ \Gamma_{\alpha \gamma}^{\max} = \max_{t \geq 0} \max \frac{\mathcal{E}_t}{\mathcal{E}_0} \]

The disturbances for which \( \Gamma_{\alpha \gamma}^{\max} \) are attained - the optimal disturbances
Large-scale harmonics obtained from DNS temperature field:

Correspondent optimal disturbances at the moment of maximal growth:


Conclusion:

Layered structures observed in the stably stratified turbulent flow have spatial scales and configurations coinciding with the scales and configurations of optimal disturbances of the linear model.

Subjects for further investigations:

Mechanisms of arising of thin layers with large temperature gradients. Nonlinearity?

Temporal statistic of large-scale optimal disturbances generation in multiscale turbulent flow. (additional specialized hi-Re DNS runs are needed)