3D high order seismic imaging salt domes in the context of RTM algorithm using adjoint-based methods.

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1. General context

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Salt dome basins in Mexico.

Figure 1: Mexican salt basin [García, 1983]
Characteristics of salt structures.

- High impermeability
- Change shape and flow through adjacent rocks.
- Generally they form a trap for oil and natural gas. Many hydrocarbon reservoirs are located in a context of salt tectonics.
Characteristics of seismic images from salt domes.

- Bad quality below the salt structure.
- The base of the salt is not well defined.
- High concentration of noise attributed to multiples.

Figure 3: Example of a seismic image of a complex geology region including salt domes. Taken from [Buehnemann et al., 2002]
Seismic imaging.

Figure 4: Forward model. Seismic data acquisition

\[ \mathbf{d} = g(\mathbf{m}), \]  

where \( g \) is an operator that relates the data \( \mathbf{d} \) recorded by the receivers and the parameters from the subsoil \( \mathbf{m} \).

Inversion techniques:
- Migration approach.
- Direct inversion.
- Inverse problem theory.
Elastodynamic equations.

We have to solve the elastodynamic equations, that govern the wave propagation through the subsoil. For lack of space, I show the 2D elastodynamic equations.

\[
\begin{align*}
\rho \frac{\partial v_x}{\partial t} &= \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} + f_x, \\
\rho \frac{\partial v_z}{\partial t} &= \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{zz}}{\partial z} + f_z, \\
\frac{\partial \tau_{xx}}{\partial t} &= (\lambda + 2\mu) \frac{\partial v_x}{\partial x} + \lambda \frac{\partial v_z}{\partial z}, \\
\frac{\partial \tau_{zz}}{\partial t} &= (\lambda + 2\mu) \frac{\partial v_z}{\partial z} + \lambda \frac{\partial v_x}{\partial x}, \\
\frac{\partial \tau_{xz}}{\partial t} &= \mu(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x}).
\end{align*}
\]

* \( f_{x,z} \) could be point sources or moment tensors.
* Finite differences: Second order, staggered grids [Virieux, 1986].
* Absorbing boundary conditions (PML) [Komatitsch and Martin, 2007].
Reverse Time Migration based on adjoint theory.

Reverse Time Migration $\Rightarrow$ RTM

According to [Chang and McMechan, 1986], there are three stages in the algorithm:

- **Forward modeling:** Solution of the elastodynamic equation, we used point sources.
- **Adjoint modelling:** Solution of the elastodynamic equations in reverse time.
  *Adjoint source inversion:*
  \[
  f^\dagger(x, t) = \sum_{r=1}^{nr} (u(x_r, T - t) - d(x_r, T - t))
  \]
  *Adjoint source migration:*
  \[
  f^\dagger(x, t) = \sum_{r=1}^{nr} d(x_r, T - t)
  \]
- **Classical imaging condition**
  *Normalized by the forward wavefield:*
  \[
  I(x) = \sum_{n=1}^{N} u(x, N + 1 - n) u^\dagger(x, n)
  \]
  *Normalized by the adjoint wavefield:*
  \[
  I(x) = \frac{\sum_{n=1}^{N} u(x, N+1-n) u^\dagger(x, n)}{\sum_{n=1}^{N} (u(x, N+1-n)^2}
  \]
Sensitivity Kernels.

Misfit function \( \chi(m) = \frac{1}{2} \sum_{r=1}^{nr} \int_0^T \| u(x_r, t, m) - d(x_r, t) \|^2 dt \).

Fréchet derivatives [Tromp et al., 2005], [Monteiller et al., 2015]:

\[
\delta \chi = \sum_{r=1}^{nr} \int_0^T \left( u(x_r, t, m) - d(x_r, t) \right) \cdot \delta u(\overline{x}_r, t, m) dt
\]

Fréchet derivatives with respect to the logarithm of velocity, density and Lamé coefficients.

\[
K_{\rho}(x) = - \int_0^T \rho(x) u^\dagger(x, T-t) \cdot \partial_t^2 u(x, t) dt, \\
K_{\kappa}(x) = - \int_0^T (\lambda(x) + 2\mu(x)) \nabla \cdot u^\dagger(x, T-t) \nabla \cdot u(x, t) dt, \\
K_{\mu}(x) = -2 \int_0^T \mu(x) \nabla u^\dagger(x, T-t) : \nabla u(x, t) dt, \\
K_{\beta}(x) = 2(K_{\mu} - \frac{4}{3} \frac{\mu}{\kappa} K_{\kappa}), \quad K_{\alpha}(x) = 2(\frac{\kappa + \frac{4}{3} \mu}{\kappa}) K_{\kappa}.
\]
Parameters and characteristics of the code to solve the 3D simulation.

We use UniSolver to solve the elastodynamic equation showed previously, which is a FORTRAN, high order, finite differences-based code in 3 dimension and following a parallel approach. We use supercomputers and we also applied boundary conditions of the CPML type [Komatitsch and Martin, 2007], [Martin et al., 2010]. We use point sources.

Receptors \(\Rightarrow 500\).

Sources \(\Rightarrow 250\).

Frequency \(\Rightarrow 4\) [Hz]

Longitude \(x \Rightarrow 400\) (m)

Longitude \(y \Rightarrow 3600\) [m]

Longitude \(z \Rightarrow 9680.0\) [m]

Total time \(\Rightarrow 3.825\) (s)

\(\Delta x \Rightarrow 8.0\) [m], \(\Delta y \Rightarrow 8.0\) [m], \(\Delta z \Rightarrow 8.0\) [m], \(\Delta t \Rightarrow 0.00045\) [s]

\(nx \Rightarrow 50\), \(ny \Rightarrow 450\), \(nz \Rightarrow 1210\), \(nt \Rightarrow 8500\)
Velocity and density salt dome models.

Figure 5: P velocity model [m/s]

Figure 6: Density model [kg/m³].
Example of a 2D forward propagation.

Figure 7: Snapshots from the forward propagation.
Principal methods used to attenuate multiples.

- SRME methods [Verschuur and Berkhout, 1992], which use the predictability feature of surface related multiples to predict and attenuate them.
- Change of domain methods, sometimes the multiples can be separated from the data in another domain, the most common one is the $\tau - \rho$ domain, (time of intercept and slowness).
- There are some approaches which use multiples as extra information and they do not attenuate them [Liu et al., 2015].
2D Kernels using simultaneous sources.

- We used a method based on CPML boundary conditions to attenuate the multiples.
- SB, TS, BS, S, and B represent the Sea bottom, top of the salt, base of the salt, sandstone and basement interfaces respectively.
- The kernel with multiple attenuation shows better definition in interfaces below the salt structure.
A priori model.

We assumed that we can know the bathymetry, below it we used a gradient with the maximal and minimal value of the real model.

Figure 8: P velocity a priori model.
3D seismic sections with simultaneous sources.

The 3D simulations give more information of the model because they have a higher azimuth.

(a) Horizontal x component.  (b) Horizontal y component.

(c) Vertical component.
P velocity kernel simultaneous sources 3D.

Figure 9: P velocity model.

Figure 10: P velocity kernel
With separated sources the salt dome interfaces are reproduced almost as in the original model.

Figure 11: P velocity model.

Figure 12: P velocity kernel obtained with separated sources.
Perspectives.

- Continue the development of the Full Waveform Inversion (FWI) in the UniSolver code in the context of an OBC simulation in order of attenuate the multiples in a more effective way.
- Application of the FWI to synthetic data related to complex geology (salt domes) together with gravity inversion.
- Application of the FWI in real hydro-geophysics data.


