MODELING THE SEA SURFACE WAVES IN HURRICANE BASING ON SELF-SIMILARITY CONCEPT

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- Model application to idealized situations: uniform wind, convergent, divergent wind
- Stationary cyclone-type wind field
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- TC parameterization
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Motivation

- Observation (satellites, buoy, aircraft) and modeling (e.g. WAWEWATCH III) of waves in TC is critical for forecasting and fundamental study

- Classical self-similar theory of wave development (Kitaigorodskii, 1962) demonstrated practical capabilities to reproduce surface wave characteristics, even under extreme wind conditions (e.g. Young, 1988, Young, 2013, Kudryavtsev et al., 2015).

- However, fields of surface waves under these extremes can rapidly become complex and characterized by multiple wave systems, limiting the direct use of the 1D self-similar fetch-laws (e.g. Hwang et al., 2017, Hwang and Walsh, 2018).

- In moving TC surface waves in the right sector obtain “unlimited” fetch (group velocity resonance) and can be “trapped” by TC

- It leads to typical asymmetry - appearance of waves, much larger than ones predicted by a “standard” fetch-law estimates using TC wind speed and its radius as a fetch

- Angular wave distributions in different TC parts are complicated : multimodal wave systems, some waves are moving windward

The aim of this work is to develop a 2D parametric model for wave evolution in non-uniform wind field, based on

1) energy and momentum conservation laws (Hasselmann et al., 1976)

and

2) self-similarity concept relating non-dimensional wave frequency, wave energy and fetch (Kitaigorodskii, 1962)

and to apply this model to TC conditions

Governing Equations

Energy and momentum conservation (Hasselmann et al., 1976; Phillips, 1977):

$$\partial E / \partial t + c_{gj} \partial E / \partial x_j = S^E$$

$$\partial M_i / \partial t + c_{gj} \partial M_i / \partial x_j = S_i^M$$

$E(\omega,\varphi) = A(\varphi - \varphi_p) F(\omega)$ - energy spectral density

$M_i = k_i E/\omega = \kappa_i \omega E/g$ - momentum spectral density

$\kappa_i = [\cos\varphi, \sin\varphi]$

$S^E = S_W - S_D + S_N$ - energy source

$S_i^M = \kappa_i \omega S^E / g$ - momentum source

c_{gj} - group velocity

Wind input:

$$S_W = \beta \omega A(\varphi - \varphi_p) F(\omega),$$

(Miles, 1957)

$\beta = c_\beta \left(u_*/c\right)^2 \cos^2(\varphi - \varphi_W)$ - growth rate

$c_\beta = (2 \div 6) \times 10^{-2}$ (Plant, 1982; Meirlink et al. 2003)

u_* - friction velocity

If wind projection is smaller than wave phase velocity $(u_k \cos(\varphi - \varphi_W) - c < 0)$ ⟹ **Sw = 0:**

$$\beta = c_\beta \left(u_*/c\right)^2 \cos^2(\varphi - \varphi_W) H_\beta(\varphi - \varphi_W, u_{10}/c)$$

$$H_\beta(\varphi - \varphi_W, u_{10}/c) = \frac{1}{2}\left[1 + \tanh\left(p\left(\cos(\varphi - \varphi_W)\frac{u_{10}}{c} - 1\right)\right)\right]$$

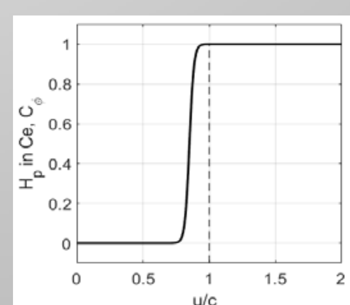
Dissipation:

Wave breaking (Longuet-Higgins, 1969)

$$D = \omega_p e (k_p{}^2 e / \varepsilon_T{}^2)^n$$

$$D = \int S_D \, d\varphi \, d\omega$$

$$e = \int E \, d\varphi \, d\omega$$



Non-linear interactions:

Four-wave interactions (Hasselmann, 1962)

Energy transfer towards low frequencies:

$$\partial S_N / \partial \omega < 0$$

$$\langle S_N \rangle \sim E^3$$

(Zakharov, 2010; Badulin et al., 2007)

Governing Equations

$$\partial E / \partial t + c_{gj} \partial E / \partial x_j = S^E$$

$$\partial M_i / \partial t + c_{gj} \partial M_i / \partial x_j = S_i^M$$

$$\iint d\varphi d\omega$$

and some algebra…

$$e = \int E d\varphi d\omega$$

$$\overline{c}_g = \int c_g F(\omega) d\omega \Big/ e$$

$$\overline{\omega} = \int \omega F(\omega) d\omega \Big/ e$$

$$\kappa_j^p = [\cos\varphi_p, \sin\varphi_p]$$

$$\omega_p \big/ \overline{\omega} = \overline{c}_g \big/ c_{gp} = r_g$$

$$\partial e / \partial t + \kappa_j^p \overline{c}_g \partial e / \partial x_j + e \partial (\kappa_j^p \overline{c}_g) / \partial x_j = \iint S^E d\varphi d\omega \approx \iint (S_W - S_D) d\varphi d\omega$$

- energy growth rate (wind+dissipation)

$$\frac{\partial}{\partial t}\omega_p + \kappa_j^p \overline{c}_g \frac{\partial}{\partial x_j}\omega_p = \frac{r_g}{e} \int (\omega - \overline{\omega}) S_O^E d\omega$$

- peak frequency (non-linear interactions)

$$\frac{\partial}{\partial t}\varphi_p + \kappa_j^p \overline{c}_g \frac{\partial}{\partial x_j}\varphi_p = \frac{1}{\overline{\omega} e} \iint \sin(\varphi - \varphi_p) \omega S^E d\varphi d\omega$$

- peak direction (wind)

Parameters in the right-hand side are derived using 1D equations (uniform wind: $\partial / \partial t = 0, \varphi_P = \varphi_W$), together with fetch-limited laws (Kitaigorodskii , 1962) and then generalized to 2D equations

Link to self-similarity

Energy

$$\partial e / \partial t + \kappa_j^p \overline{c}_g \partial e / \partial x_j + e\partial(\kappa_j^p \overline{c}_g) / \partial x_j = \iint S^E d\varphi d\omega$$

$$\approx \iint (S_W - S_D) d\varphi d\omega$$

Right hand side:

$$\iint (S_W - S_D) d\varphi d\omega = I_w - D$$

Model assumption:
Dissipation is proportional to the wind input:

$$D/I_w = \gamma \Rightarrow (k_p^2 e)^n \propto \alpha^2$$

$$\Rightarrow n = \frac{2q}{p+4q}$$

$$I_w - D = \omega_p e(\tilde{I}_w - \tilde{D})$$

$$\tilde{I}_w - \tilde{D} = C_e H_p \alpha^2 \cos^2(\varphi_p - \varphi_W) - \left(e k_p^2 / \varepsilon_T^2\right)^n$$

$$H_p = 1/2 \cdot \left\{1 + \tanh\left[p\left(\cos(\varphi_p - \varphi_W)\alpha - 1\right)\right]\right\}$$

$$\tilde{x} = xg / u^2$$
$$\tilde{e} = eg^2 / u^4$$
$$\tilde{\omega}_p = \omega_p u / g$$
$$\alpha = u / c_p$$

Fetch laws:

(Kitaigorodskii , 1962)

$$\tilde{\omega}_p \equiv \alpha = c_\alpha \tilde{x}^q, \qquad \tilde{e} = c_e \tilde{x}^p$$

For the case of uniform wind:

$$\partial e / \partial t = 0, \varphi_P = \varphi_W$$

$$\partial(\overline{c}_g e) / \partial x \propto (g/u)\alpha^3 e$$

$$q = -1/4$$

C_e and ε_T are constants calibrated on fetch-laws

Link to self-similarity

$$\tilde{x} = xg/u^2$$
$$\tilde{e} = eg^2/u^4$$
$$\tilde{\omega}_p = \omega_p u/g$$
$$\alpha = u/c_p$$

Peak frequency

$$\frac{\partial}{\partial t}\omega_p + \kappa_j^p \overline{c}_g \frac{\partial}{\partial x_j}\omega_p = \frac{r_g}{e}\int(\omega-\overline{\omega})S_O^E d\omega$$

Right hand side (NL interactions):

$$e^{-1}\int(\omega-\overline{\omega})S_O^N d\omega \approx$$

$$\approx e^{-1}\,\partial S_O^N/\partial\omega \int(\omega-\overline{\omega})^2 d\omega$$

$\delta\omega \propto \omega_p$, $\langle S_N\rangle \sim e^3$ →

$$\propto -e^{-1}\delta\omega^2\left\langle S_O^N\right\rangle$$

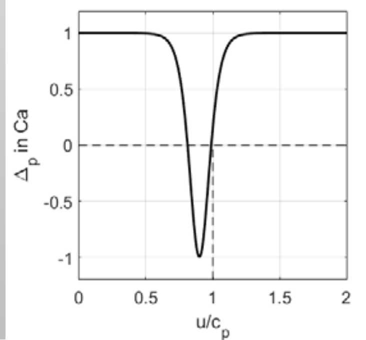
$$\propto -\omega_p^2 k_p^4 e^2 \propto -g^{-4}\omega_p^{10}e^2$$

$$e^{-1}\int(\omega-\overline{\omega})S_O^N d\omega = C_\alpha g^{-4}\omega_p^{10}e^2$$
$$= C_\alpha \omega_p^2\left(k_p^2 e\right)^2$$

Restriction for fully developed waves:

$$C_a = C_a \Delta_p$$
$$\Delta_p = 1\text{-}2\,\mathrm{sech}^2\left(10(\alpha-0.9)\right)$$

C_e calibrated on fetch-laws



Fetch laws:

(Kitaigorodskii, 1962)

$$\tilde{\omega}_p \equiv \alpha = c_\alpha \tilde{x}^q, \qquad \tilde{e} = c_e \tilde{x}^p$$

For the case of uniform wind:

$$\partial e/\partial t = 0, \varphi_P = \varphi_W$$

$$c_{gp}\partial\omega_p/\partial x \propto g^{-4}\omega_p^{10}e^2$$

$$2p + 10q + 1 = 0$$ - "magic relations" (Badulin et al., 2007; Zakharov, 2010)

$$p = 3/4$$
$$n = 2$$

$$q = -1/4$$
$$n = \frac{2q}{p+4q}$$

Spectral peak direction

$$\frac{\partial}{\partial t}\varphi_p + \kappa_j^p \overline{c}_g \frac{\partial}{\partial x_j}\varphi_p = \frac{1}{\overline{\omega} e}\iint \sin(\varphi - \varphi_p)\omega S^E d\varphi d\omega$$

Dissipation and four-wave interactions are functions of wave spectrum, while wind input is also a function of wave direction

Taylor series around $\varphi = \varphi_p$

Integral over azimuth vanishes for S_D and S_N.
Change of wave direction is caused by **wind**

$$\frac{\partial}{\partial t}\varphi_p + \kappa_j^p \overline{c}_g \frac{\partial}{\partial x_j}\varphi_p = -C_\varphi \alpha^2 \omega_p H_p \sin\left[2(\varphi_p - \varphi_W)\right]$$

$$C_\varphi = c_\beta c_D \delta\varphi^2 \frac{\int \omega^4 F(\omega) d\omega}{\omega_p^3 \int \omega F(\omega) d\omega} = 1.8\times 10^{-5}$$ for JONSWAP spectrum

Effect of Group Velocity Divergence (Ray Focusing)

Energy growth rate in ray characteristic form:

c_g divergence

$$\frac{de}{dt} = -e\left(\partial\bar{c}_g/\partial l + \bar{c}_g\,\partial\varphi_p/\partial n\right) + \omega_p e\left(\tilde{I}_w - \tilde{D}\right) \quad \Big|\, /e$$

$$\partial\bar{c}_g/\partial l \approx \Delta\bar{c}_g/(\bar{c}_g\Delta t) = \bar{c}_g^{-1}\,d\bar{c}_g/dt$$

$$\bar{c}_g\partial\varphi_P/\partial n \approx \bar{c}_g\Delta\varphi_P/\Delta n = \bar{c}_g G_n$$

Δn - distance between neighbor characteristics

$\Delta\varphi_p$ - direction difference between them

G_n, G_W – peak/wind direction gradient in cross-ray direction, $\Delta\varphi\ /\Delta n$

ray focusing/defocusing

$$\frac{d}{dt}\ln(\bar{c}_g e) = -\bar{c}_g G_n + \omega_p\left(\tilde{I}_w - \tilde{D}\right)$$

Caustic: $\Delta n \to 0$

Restriction for G_n (wave is not monochromatic):

$$G_n = \frac{\Delta\varphi_p}{\Delta n_0}\left[\frac{\Delta n/\Delta n_0}{(\Delta n/\Delta n_0)^2 + \left(1/2\cdot\Delta c_g/\bar{c}_g\right)^2}\right]$$

$$\left(\Delta c_g/\bar{c}_g\right)^2 = 4.6\times10^{-2} \quad \text{- JONSWAP}$$

from eq. for φ_p :

$$\frac{d\Delta\varphi_P}{dt} \approx T^{-1}(\Delta\varphi_W - \Delta\varphi_P) \quad \Big|\, /\Delta n$$

$$T^{-1} = 2C_\varphi H_p\alpha^2\omega_p\cos\left(2(\varphi_p - \varphi_W)\right)$$

$$d\Delta n/dt = \Delta\varphi_p\bar{c}_g$$

$\Downarrow$

$$dG_n/dt + G_n/T + \bar{c}_g G_n^2 = G_W^n/T$$

$\Downarrow$

$$G_n = \Delta\varphi_p/\Delta n$$

Complete System of Equations

System describes the development of surface waves under a varying wind field in both space and time, as well as the evolution of swell propagation in the absence of wind forcing

$$\frac{d}{dt}x_j = \kappa_j^p \bar{c}_g$$ - wave train position

$$\frac{d}{dt}\ln(\bar{c}_g e) = -\bar{c}_g G_n + \omega_p\left(\tilde{I}_w - \tilde{D}\right)$$ - modified energy

$$\frac{d}{dt}c_{gp} = -\frac{r_g C_\alpha}{2}\Delta_p g\left(k_p^2 e\right)^2$$ - spectral peak group velocity (from eq. for frequency)

$$\frac{d}{dt}\varphi_p = C_\varphi \alpha^2 \omega_p H_p \sin\left[2(\varphi_p - \varphi_W)\right]$$ - spectral peak direction

$$dG_n/dt + G_n/T + \bar{c}_g G_n^2 = G_W^n/T$$ - peak direction gradient (focusing term), $G_n = \Delta\varphi_p/\Delta n$

(or two eq. instead: for $\Delta\varphi_p$ and Δn)

Wave breaking of dominant waves:

(Phillips, 1985)

$$Q_p = \varepsilon k_p^{-1} L_p \qquad D = bg^{-1}c^5 L$$

$$\Downarrow$$

$$D = \omega_p e\left(k_p^2 e/\varepsilon_T^2\right)^n$$

$$\Longrightarrow \quad Q_P \propto \varepsilon_T{}^2 (ek_P{}^2/\varepsilon_T{}^2)^3$$

Method to solve the equations

- A wind field on a uniform grid and a initial locations of wave trains are set

- Wave characteristics at t=0: $\omega_0 = 3$, $\varphi_0 = \varphi_W$, $\Delta\varphi_p = 0$, $e_0 = U^4/g^2 c_e (a/c_a)^{p/q}$

- Right-hand sides of every equation are calculated

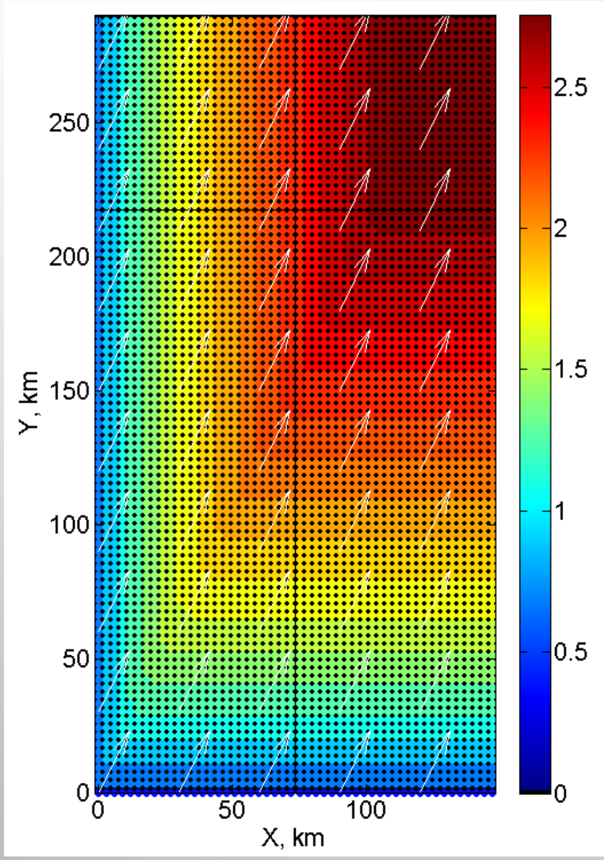
- Wave train coordinates and other parameters at $t_{i+1}=t_i+dt$ are obtained with the use of 4th order Runge-Kutta scheme

- Starting dt~1s slowly increases to 30 min to reduce calculation time and data amount

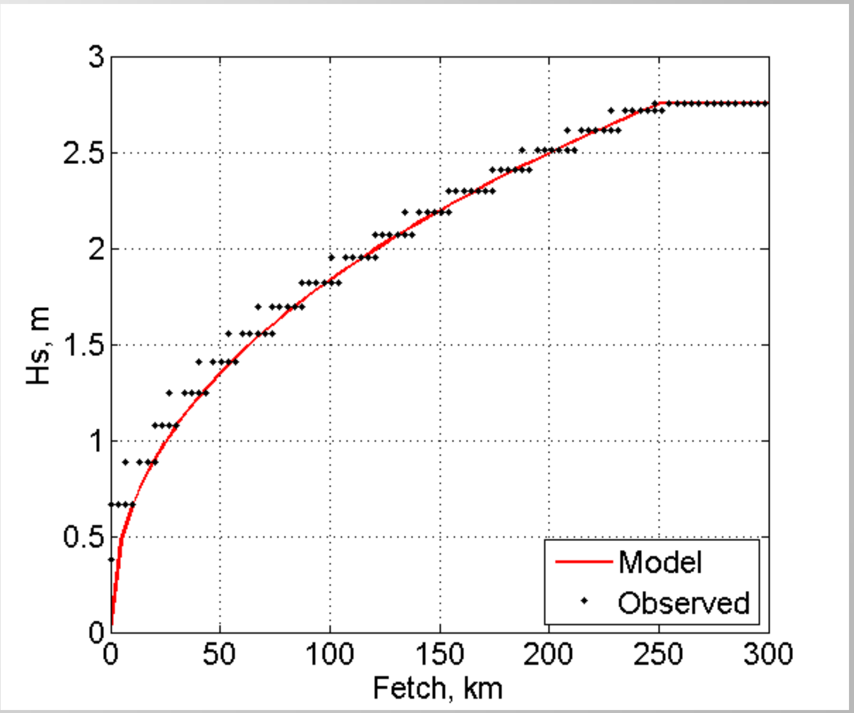
An array of wave train coordinates, peak frequency, energy, direction, wave age, distance between "neighbor" characteristics (focusing effect) at every discrete time point

Model Simulations: Uniform Wind

Constant wind from the shore.
Effective fetch: $U\cos(a)$



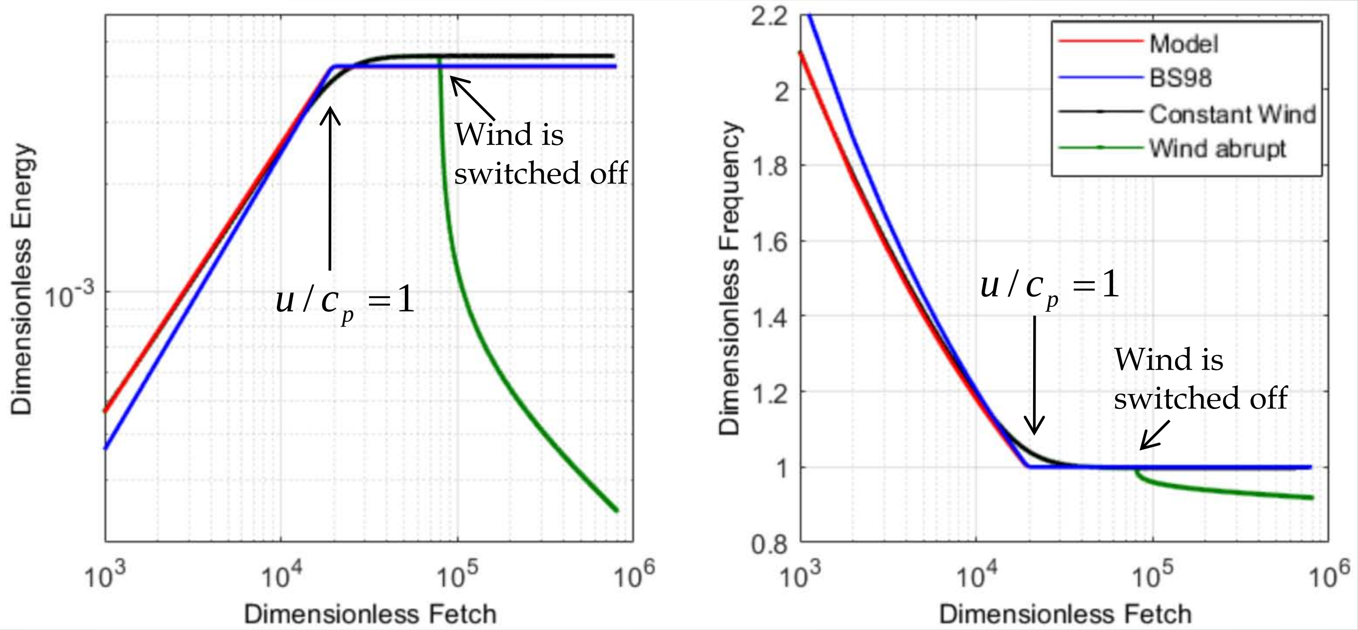
Hs field



Max Hs inside areas around grid points vs effective fetch in comparison with fetch law model, $\tilde{e} = c_e \tilde{x}^p$

(p=0.83, c_e=5.88e-7, Babanin&Soloviev, 1998)

Model Simulations: Uniform Wind



$$\tilde{x} = xg/u^2$$

$$\tilde{e} = eg^2/u^4$$

$$\tilde{\omega}_p = \omega_p u/g$$

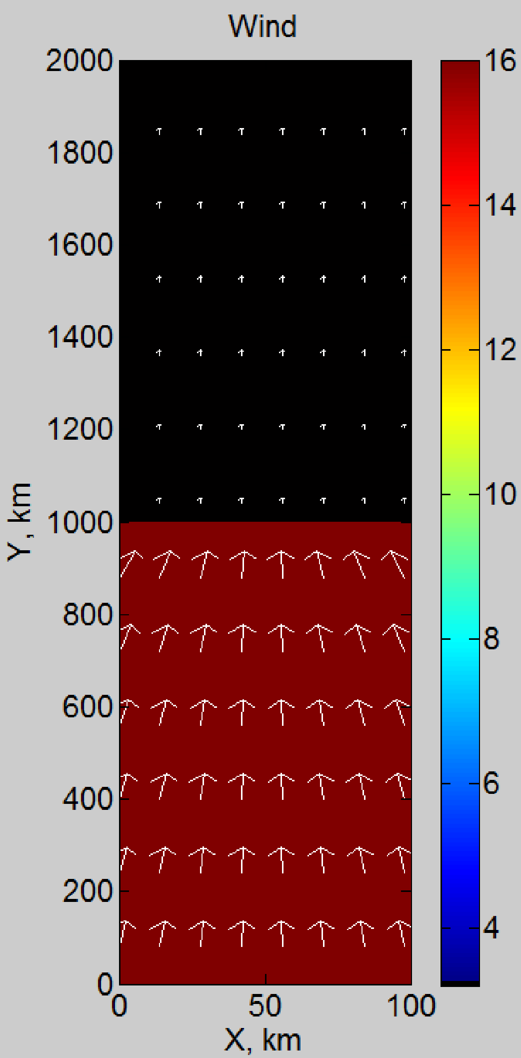
<u>Fetch laws:</u>

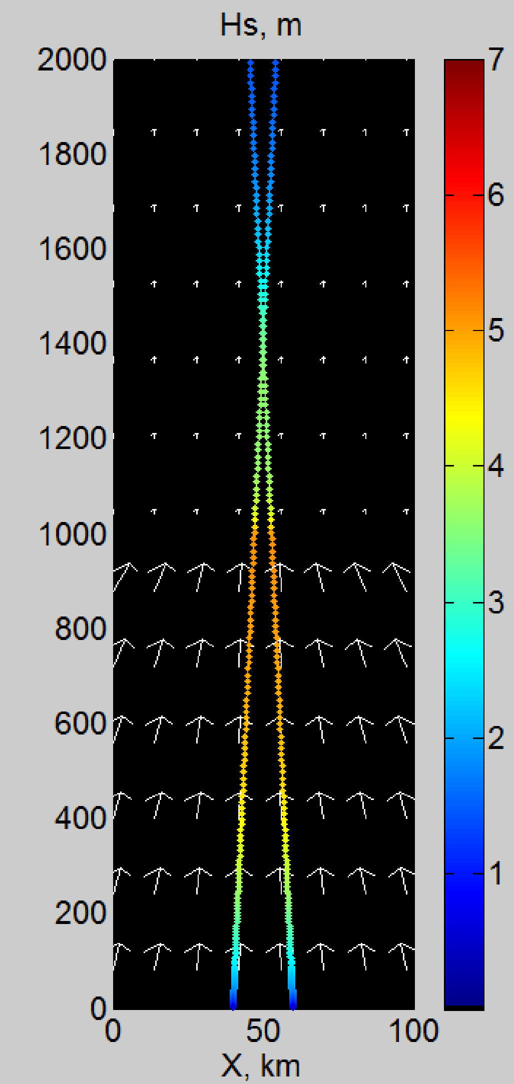
$$\tilde{\omega}_p \equiv \alpha = c_\alpha \tilde{x}^q, \qquad \tilde{e} = c_e \tilde{x}^p$$

Dimensionless energy and peak frequency vs dimensionless fetch.
Our model (red/black), Babanin&Soloviev, 1998 (blue).
Green line shows model evolution of energy and frequency after the wind suddenly drops at fetch $\tilde{x} = 8 \cdot 10^4$

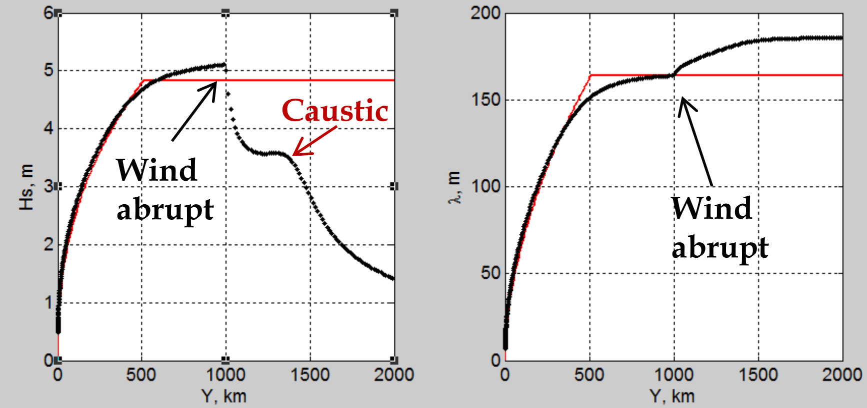
1. The model provides a smooth transition from developing to fully developed waves.
2. Swell: a rapid decay of the wave energy (much stronger than predicted by Zakharov and Badulin, 2017) under the weakly turbulent theory, $\tilde{x}^{-1/12}$)
3. Swell: a moderate downshift of the peak frequency

Convergent Wind Field. Caustic





Hs and wavelength profiles



1. Wind (16 m/s) abruptly decreases to 3m/s at fetch 1000 km
2. Fully developed waves turn to decaying focusing swell
3. Rays cross at fetch ~1300 km
4. In caustic point energy temporarily grows, than rays diverge with additional energy lose
5. Caustic effects are weakly/not manifested in wavelength

Convergent/Divergent Wind Effects

Wind is uniform in the main direction, but varying in the perpendicular direction, V<<U

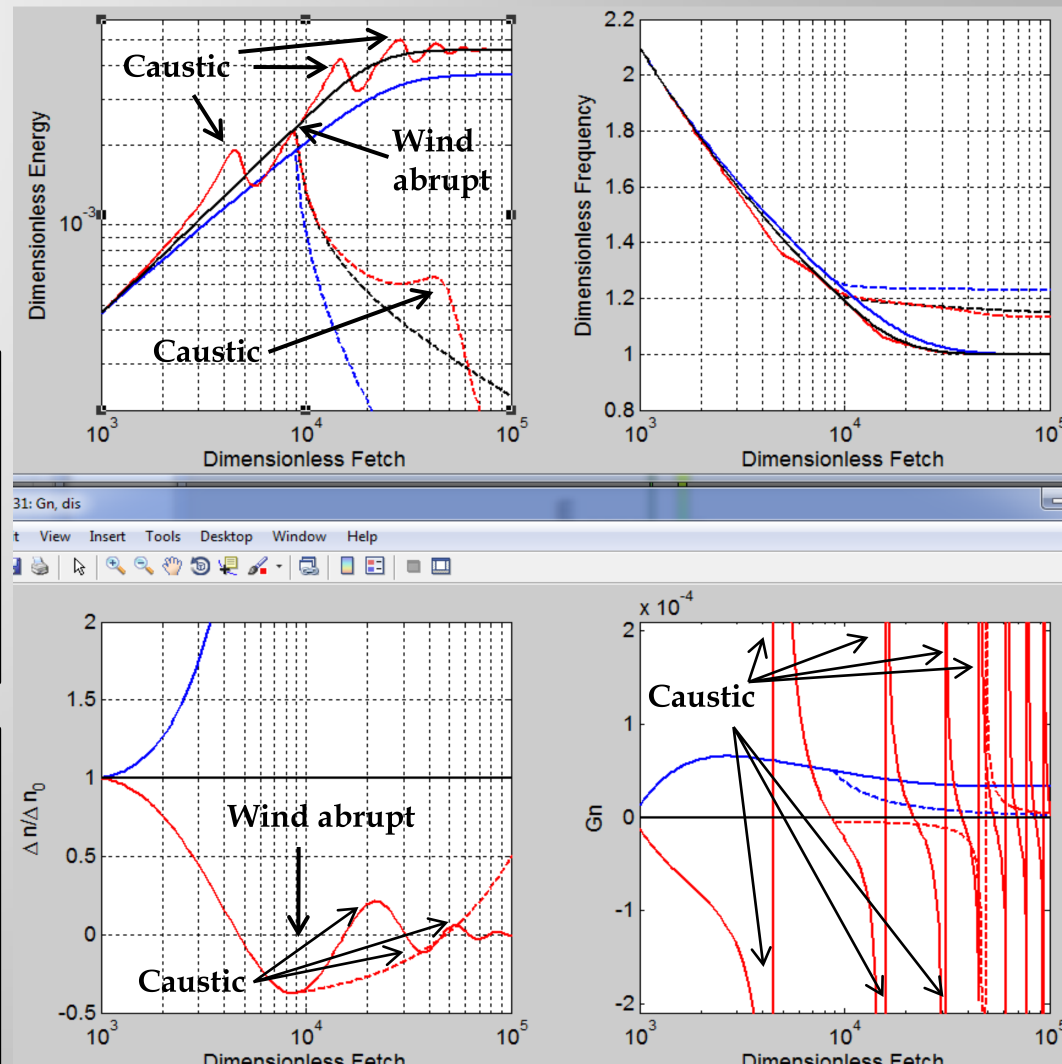
Gradient of wind direction

$$|G_W| = 10^{-4}\ \text{rad/m}:$$

negative – divergence (**blue**),
positive – convergence (**red**)
zero – **black**
dashed line ---- wind abrupt (swell)

Divergence of the wind velocity forces the wave rays to widen – additional energy sink (decrease ~30%, and deceleration of the frequency downshift)
Swell: rapid attenuation and shorter peak wavelength

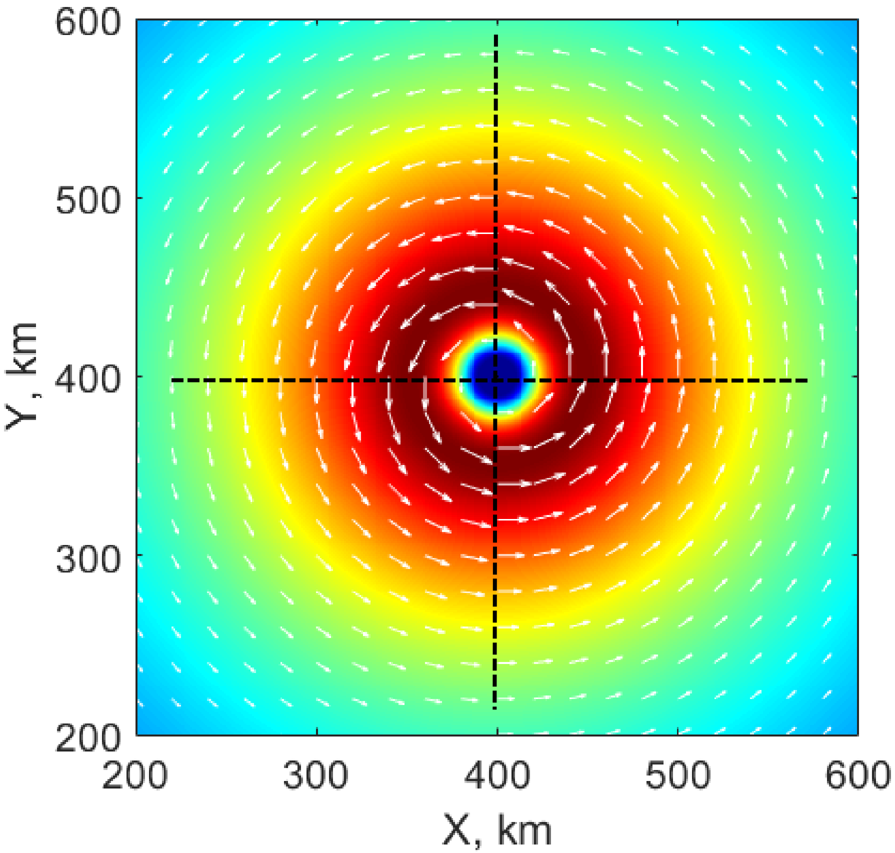
Wind velocity convergence forces wave rays to concentrate (caustic), then diverge and converge again under the action of wind gradient (recurrent process).
Swell: wave-ray thickening is terminated, rays diverge away from the last caustic zone, energy rapidly attenuates



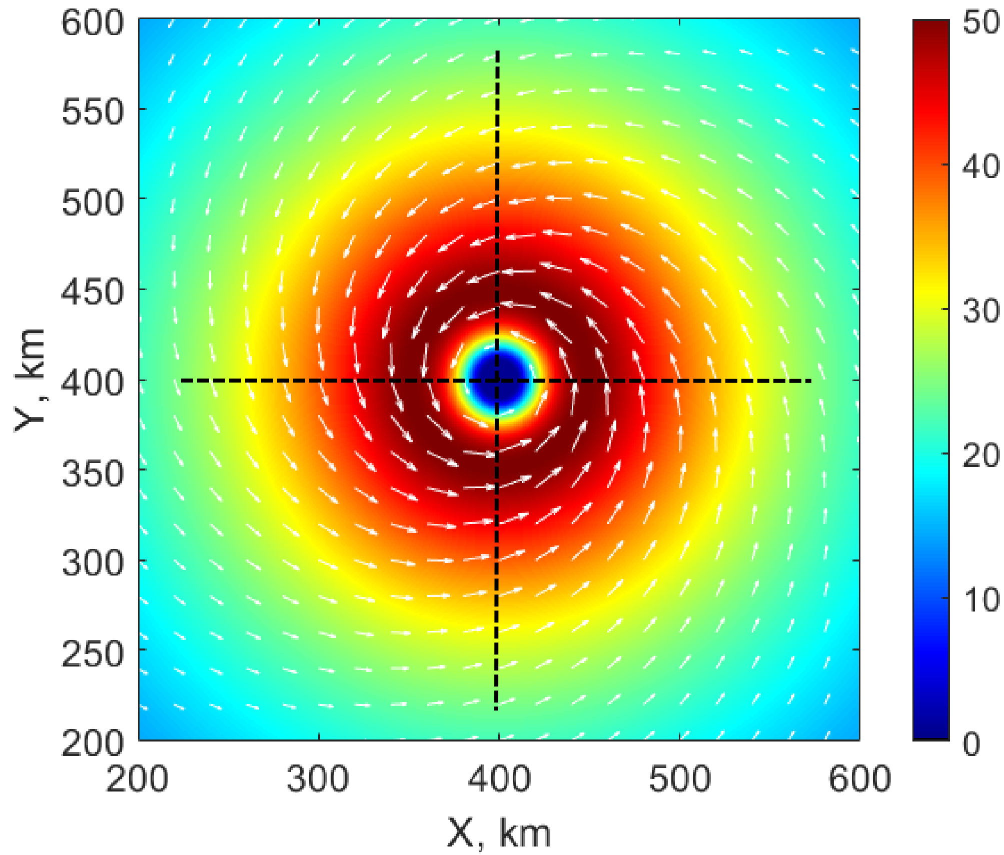
Stationary cyclone-type wind field

$$u(r) = \left[\left(u_m^2 + u_m r f\right) \left(\frac{R_m}{r}\right)^B \exp\left(-\left(\frac{R_m}{r}\right)^B + 1\right) + \left(\frac{rf}{2}\right)^2 \right]^{1/2} - \frac{rf}{2}$$

Holland, 1980



Wind field with zero inflow angle



Wind field with 20 deg inflow angle

(Shea and Gray, 1973)

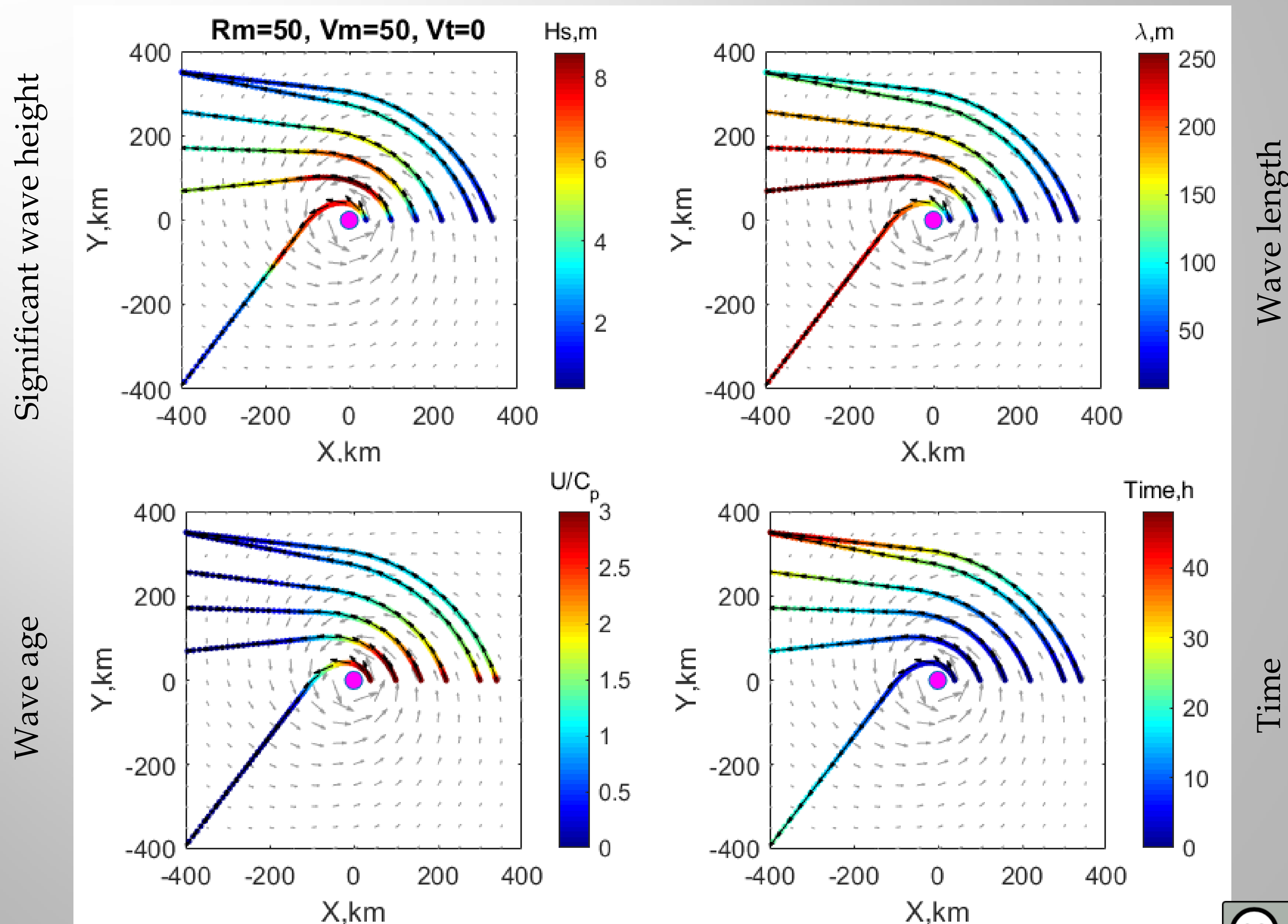
Wind direction gradient:

$|G_W| = 1/r$ with a direction tangent to the circle

Cross-ray projection: $G_w{}^n = 0$

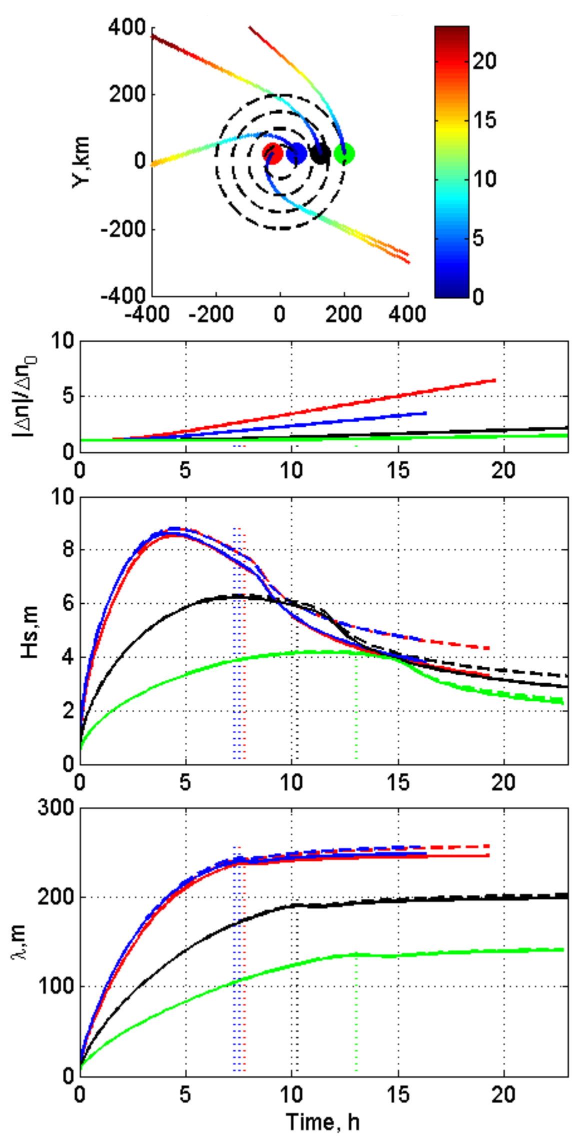
$G_W^n = -|G_W| \sin\varphi_{in}$ - wave-ray focusing

Typical Wave Rays in Stationary Cyclone (20 deg wind inflow angle)

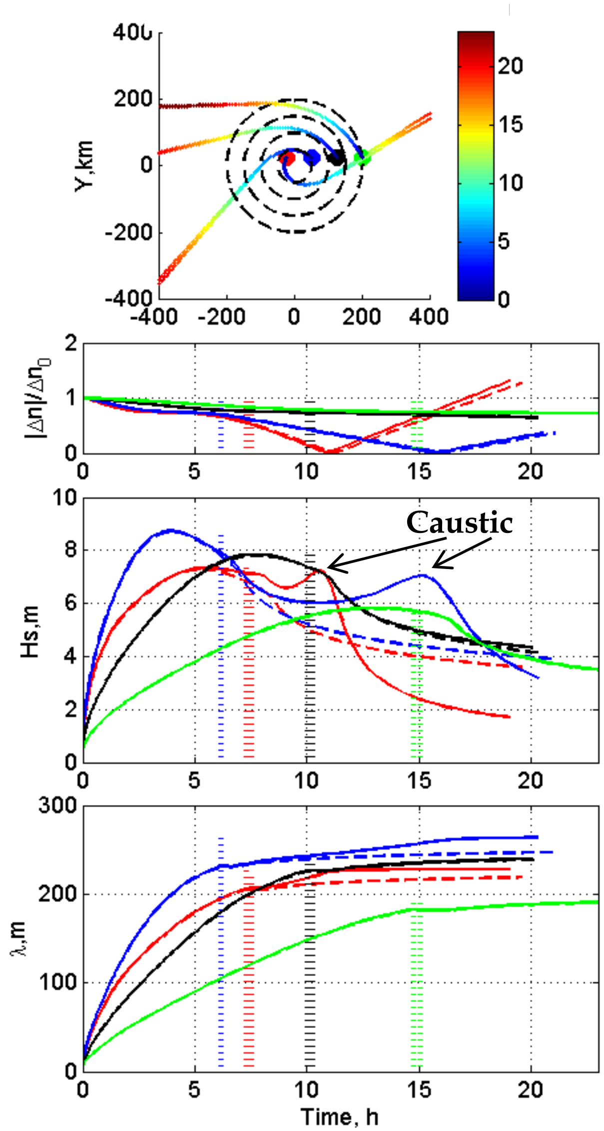


Along-ray Profiles

Zero inflow angle



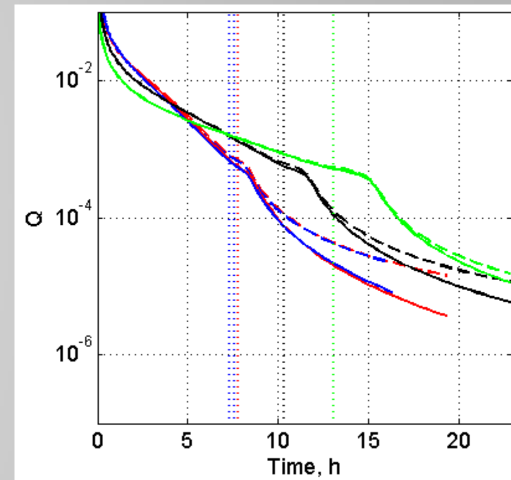
20 deg inflow angle



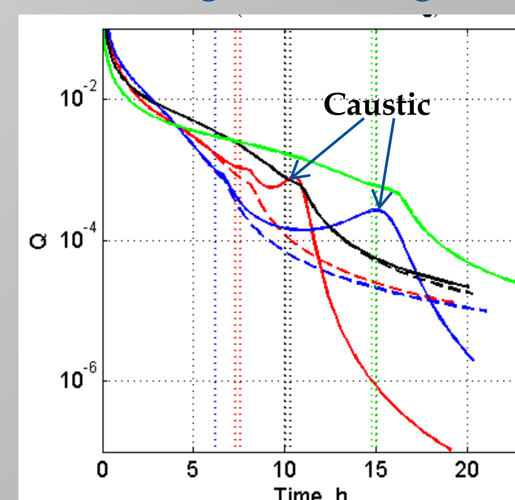
Wave breaking

(strong dependency on wave age)

Zero inflow angle



20 deg inflow angle



Dash lines – focusing term is off (Gn=0)

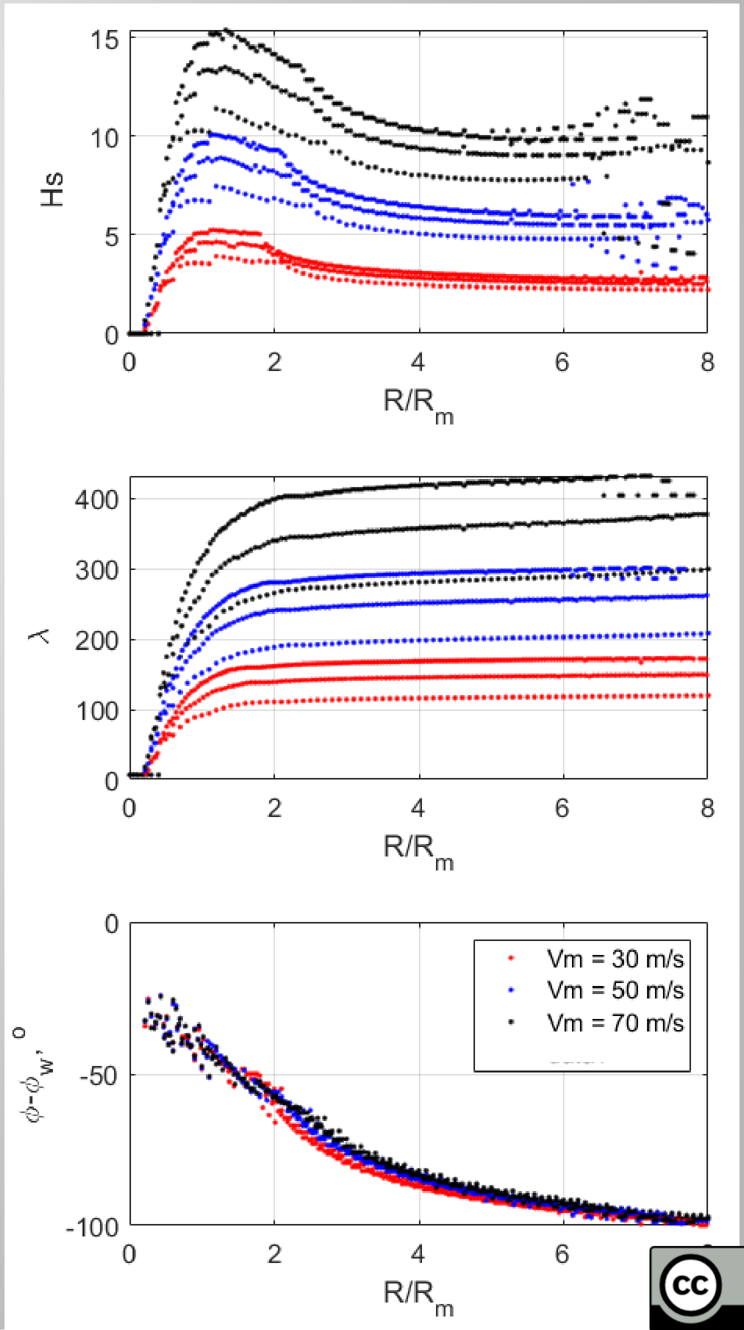
Vertical dash lines - a wave-train passes from wind force regime to swell one

Radial Distributions for Different Wind Fields

Different combinations of TC radii and maximum wind speeds:
Rm=30, 50, 70 km,
Um=30, 50, 70 m/s

For the **same radius**:
the larger the wind speed, the larger the wavelength and Hs

For the **same maximum wind speed**:
the larger the radius, the larger the wavelength and Hs



Radial Distributions (Scaled)

Different combinations of TC radii and maximum wind speeds (Rm=30, 50, 70 km, Um=30, 50, 70 m/s)

Radial distributions are scaled:

$$Hs_d = Hs(g/U_m^2)/\tilde{Hs},$$

$$\lambda_d = \lambda(g/U_m^2)/\tilde{\lambda}$$

where

$$\tilde{Hs} \sim \sqrt{\tilde{e}} = \tilde{R}^{p/2}$$

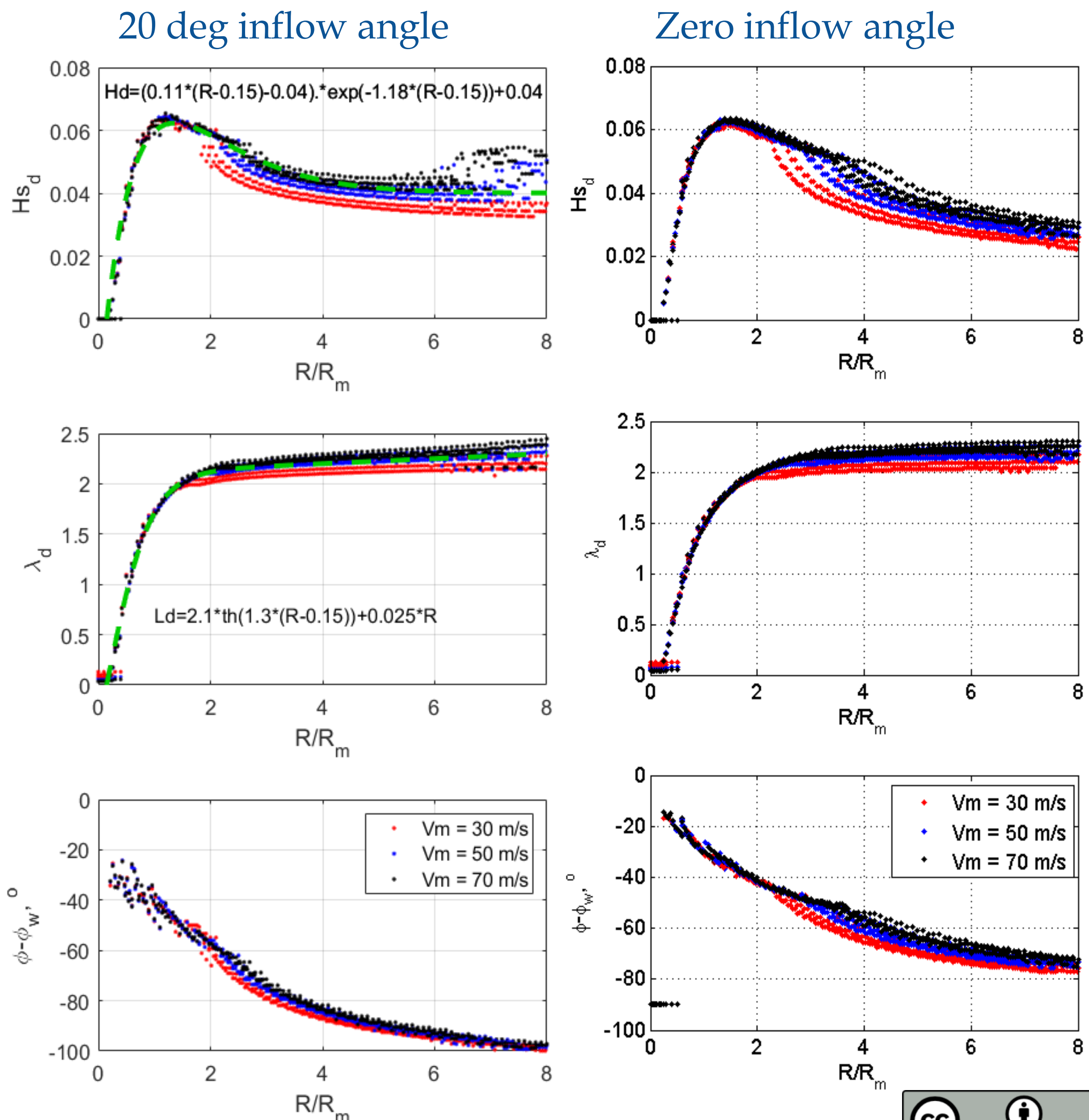
$$\tilde{\lambda} \sim \tilde{\omega}_p^{-2} = \tilde{R}^{-2q}$$

$$\tilde{R} = R_m g/U_m^2$$

$$p = 3/4, q = -1/4$$

Profiles almost collapse, exhibiting self-similar shapes for distances smaller than $2Rm$

Inflow angle modifies Hs profiles (extra energy pumping due to cross-ray convergence)
These effects on wavelength are less strong

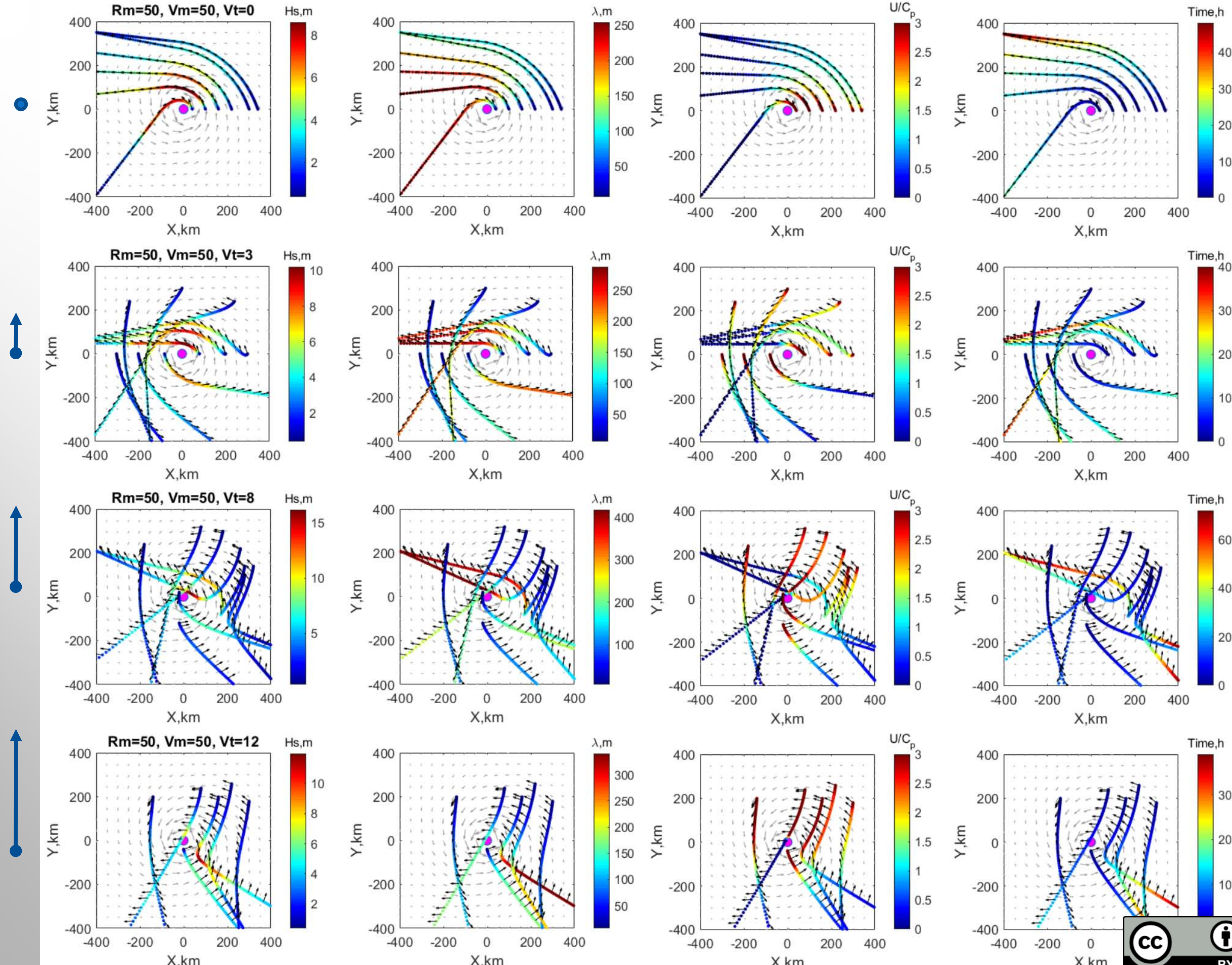


Waves in Moving TC

In coordinate system related to TC,
wave train position:

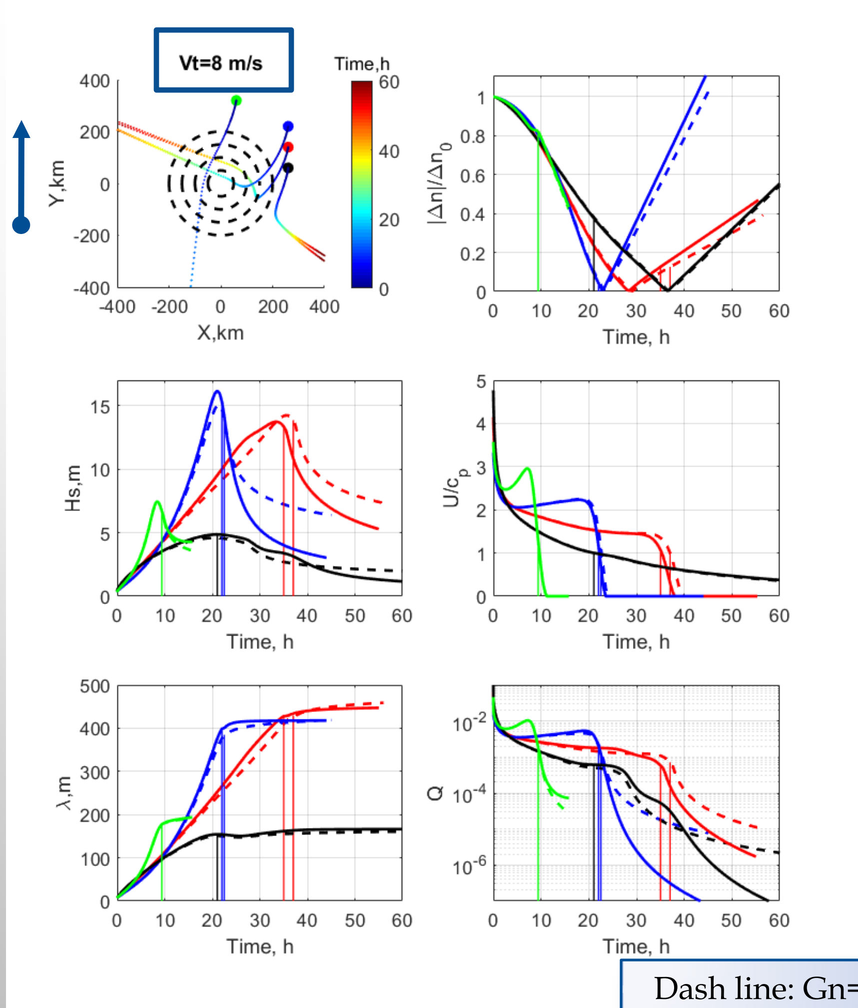
$$\frac{d}{dt} x_j = \kappa_j^p \overline{c}_g - Vt_i$$

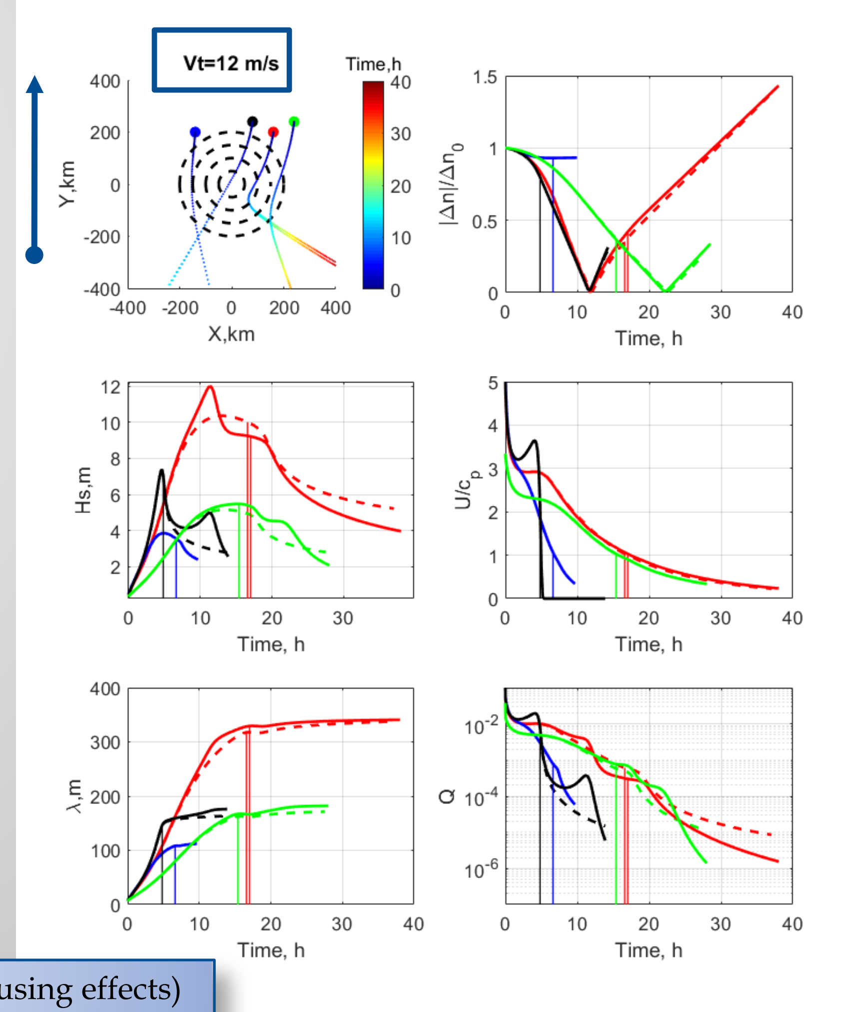
Typical wave train parameters for different hurricane translation velocities, *Vt*
(TC is moving upwards)



Rm=50, Vm=50, Vt=0
Rm=50, Vm=50, Vt=3
Rm=50, Vm=50, Vt=8
Rm=50, Vm=50, Vt=12
Hs,m
λ,m
U/Cp
Time,h
X,km
Y,km

Along-ray Profiles





Dash line: Gn=0 (no focusing effects)

Waves from up-right sector (red, blue) are "trapped" by TC central part and shifted to the left (high energy).

Extra energy pumping is due to ray focusing effect.

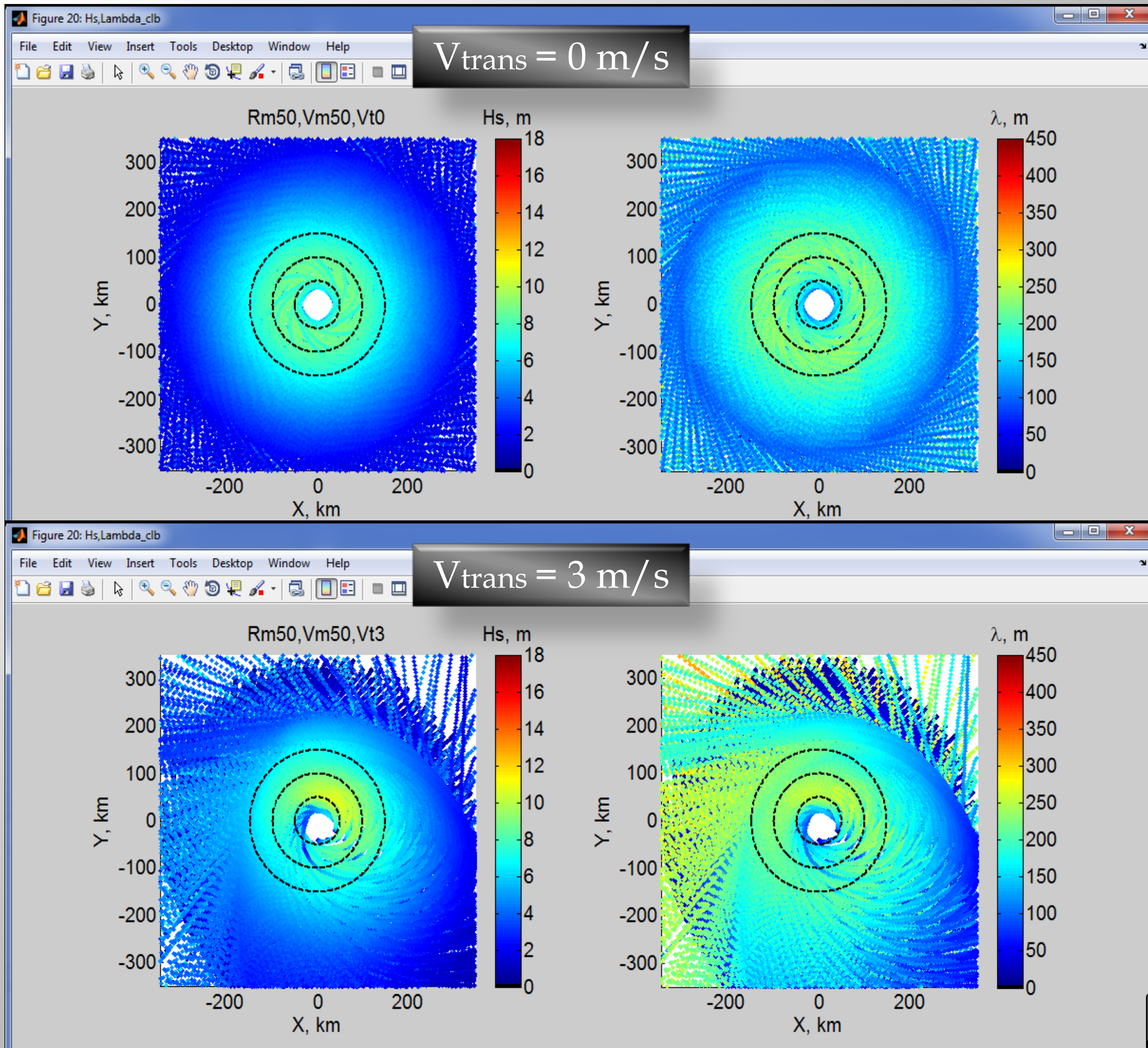
After caustic ⟶ stronger dissipation.

In very fast hurricanes most waves pass downwards.

Energy intensification is in down-right TC sector.

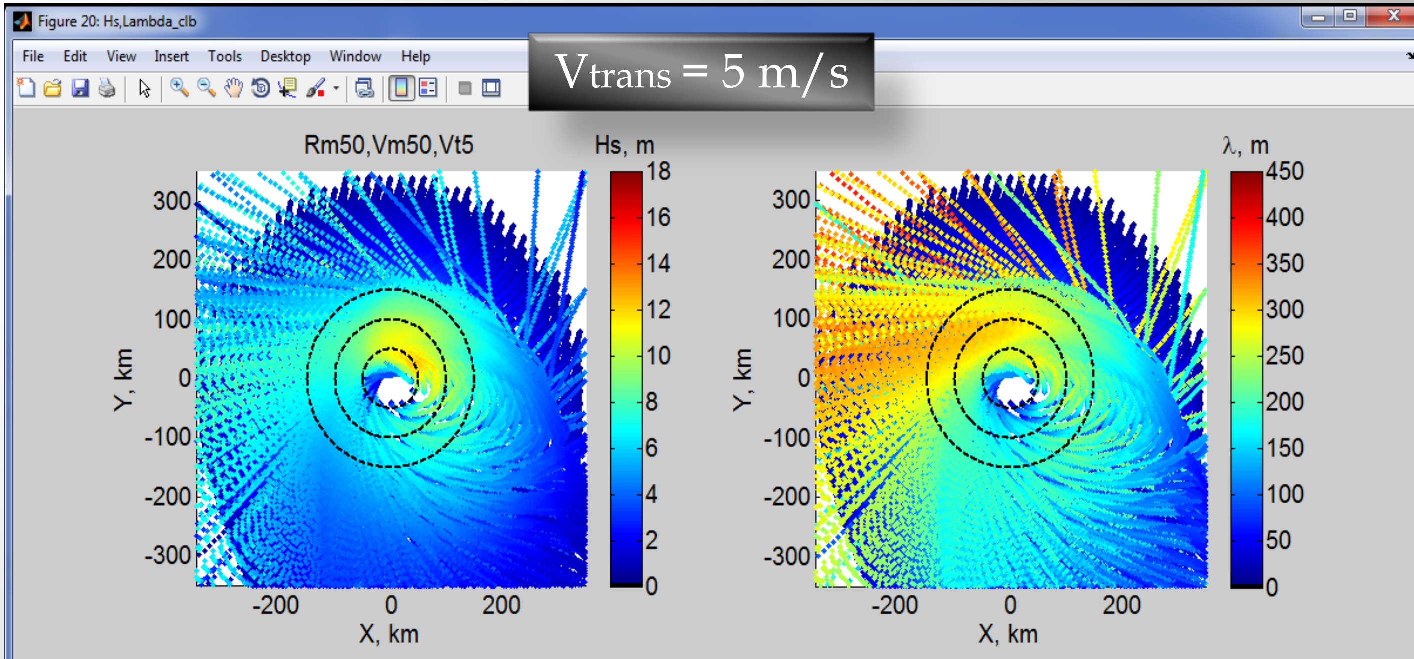
Wave Parameters from All Trajectories

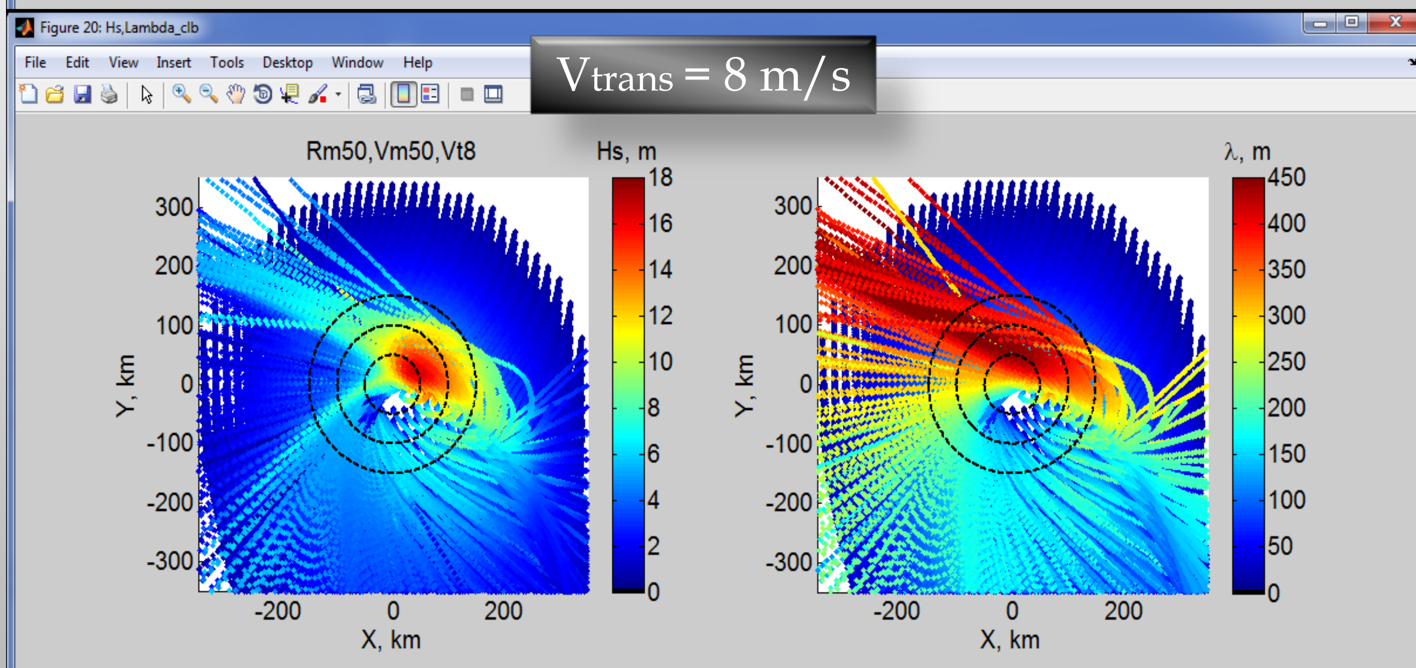
Hs and wavelength



Wave Parameters for All Trajectories

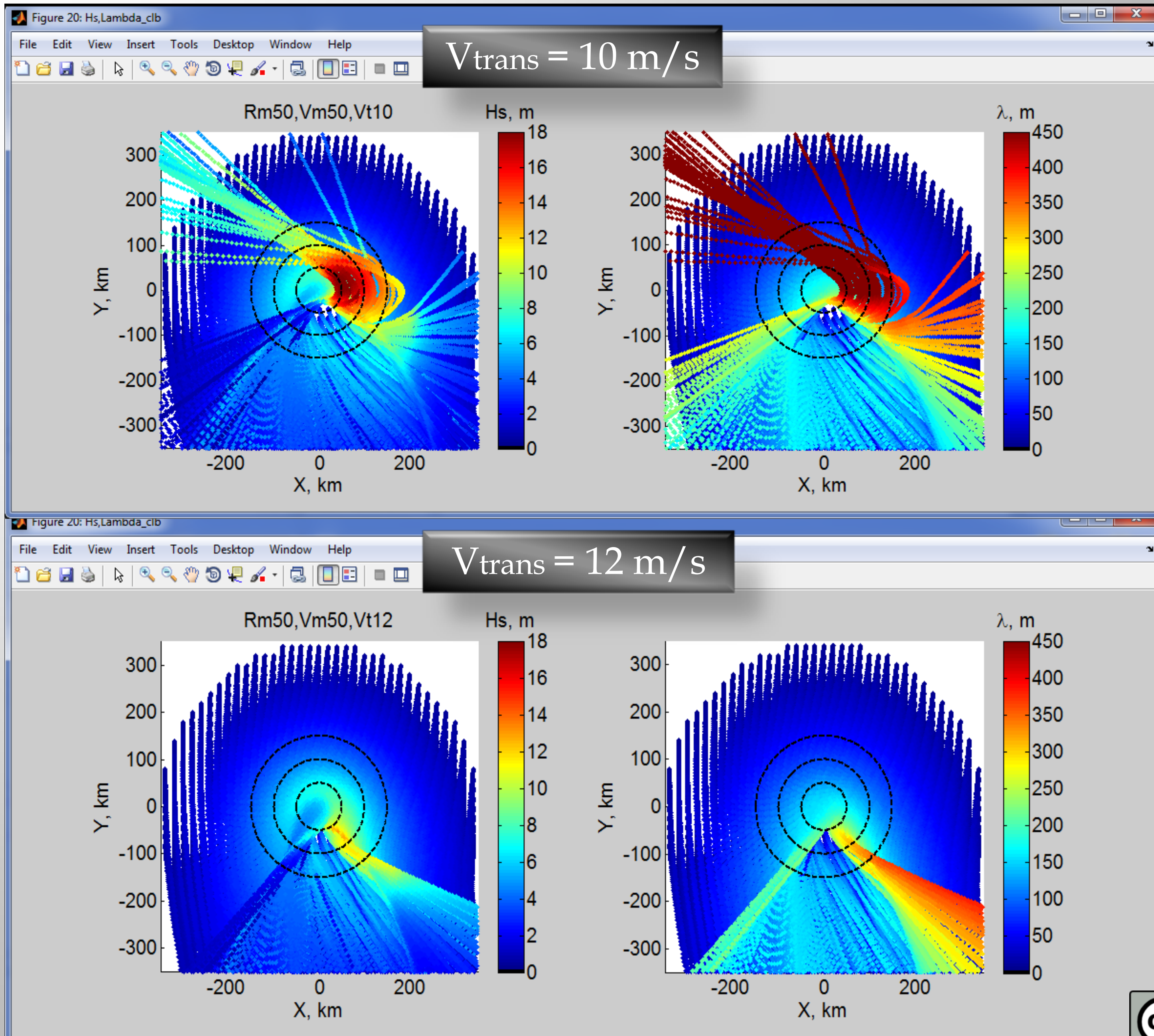
Hs and wavelength





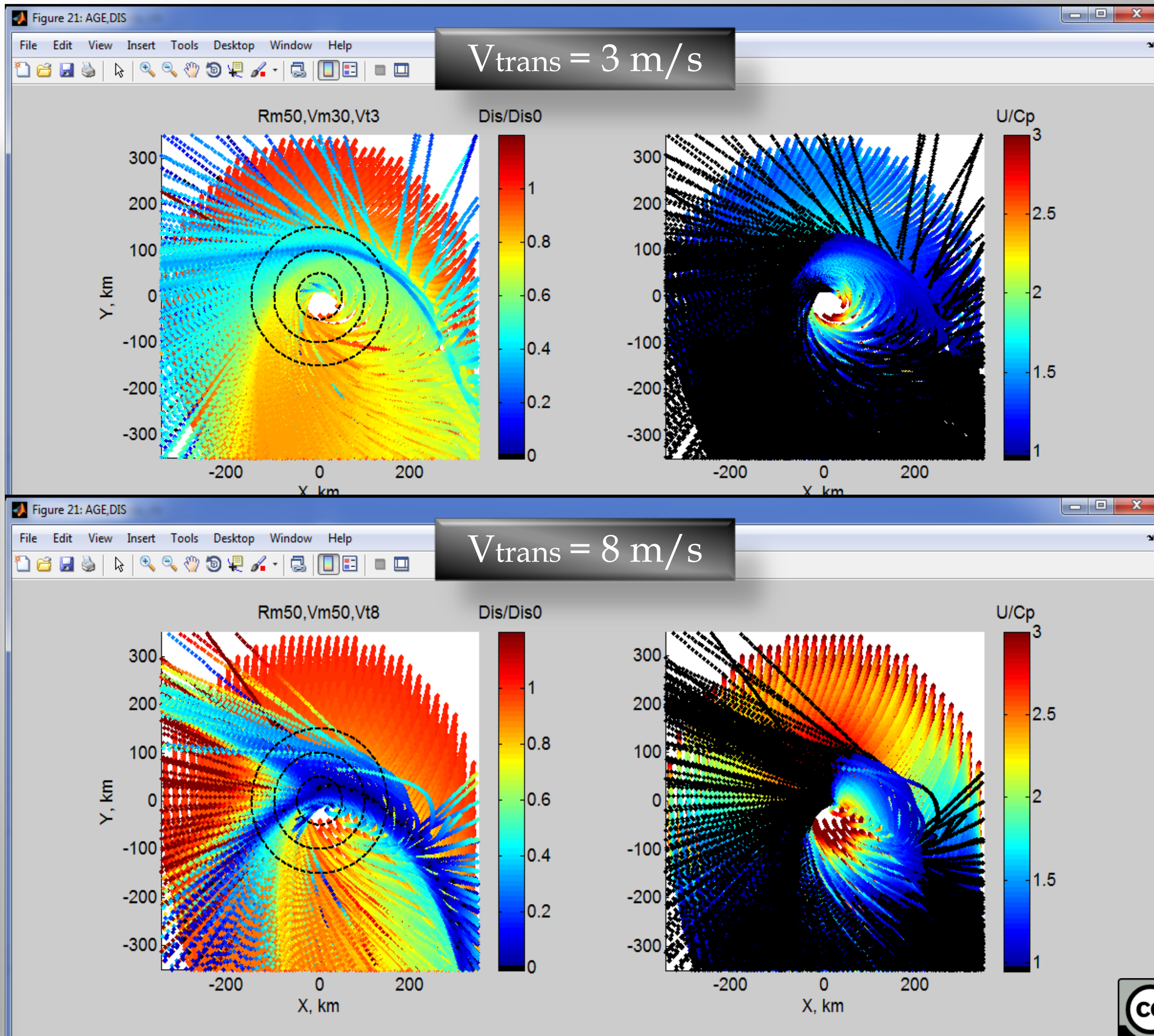
Wave Parameters for All Trajectories

Hs and wavelength



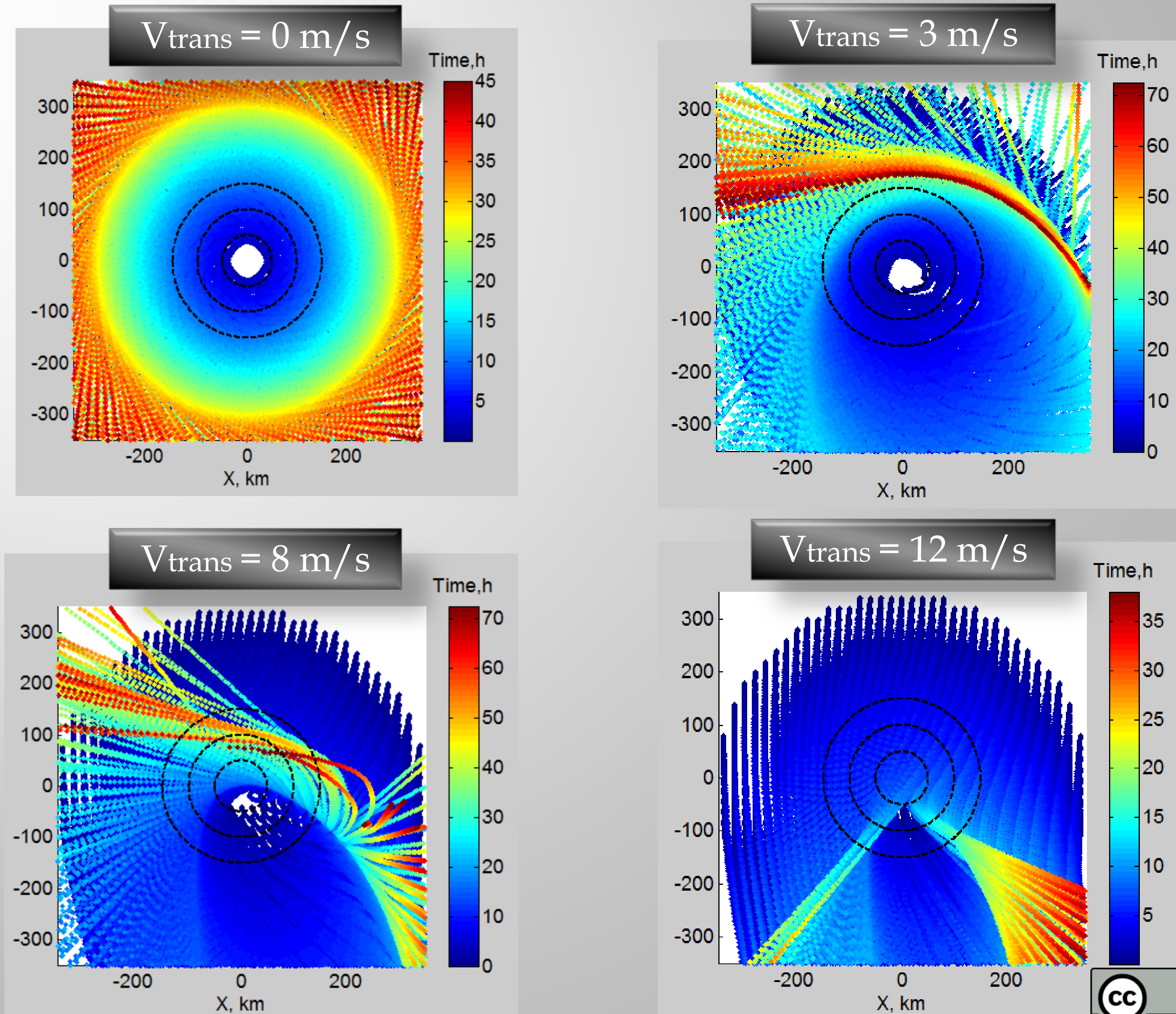
Wave Parameters for All Trajectories

Distance between neighbor characteristics (caustic) and wave age



Wave Parameters for All Trajectories

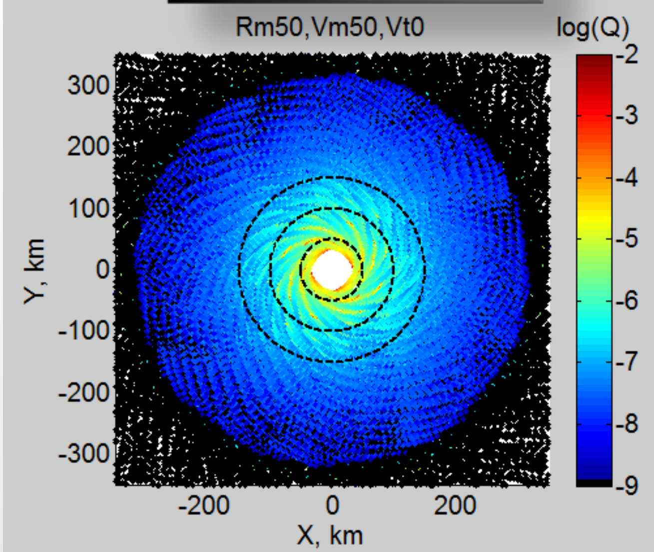
Time of wave development



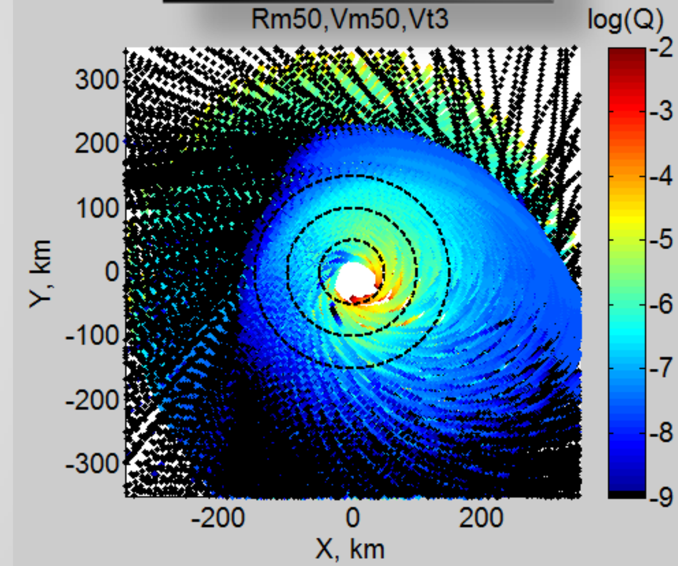
Wave Parameters for All Trajectories

Wave breaking of peak waves

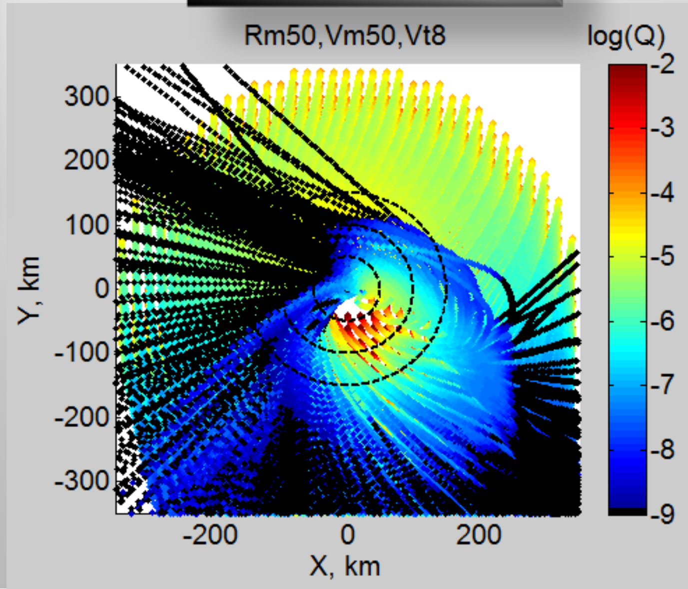
Vtrans = 0 m/s



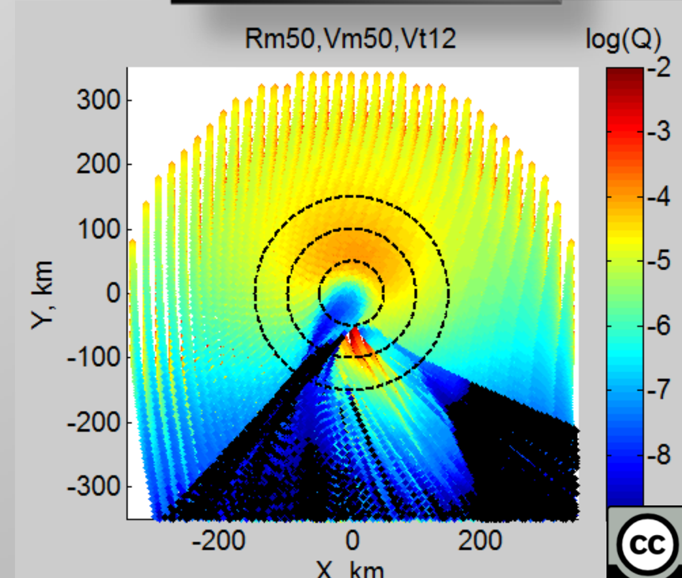
Vtrans = 3 m/s



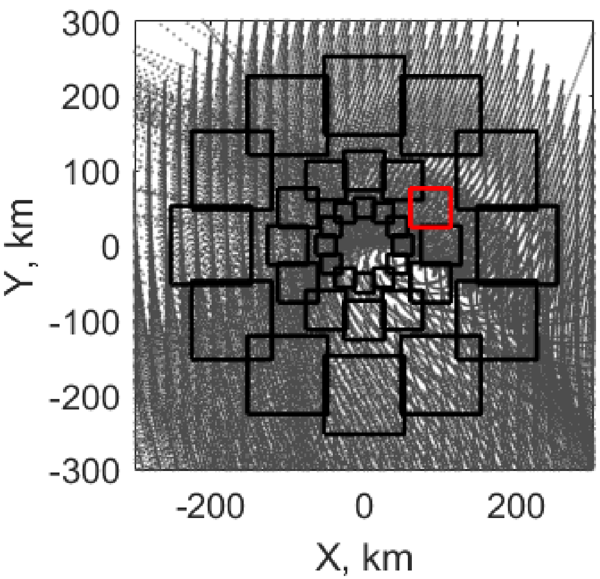
Vtrans = 8 m/s

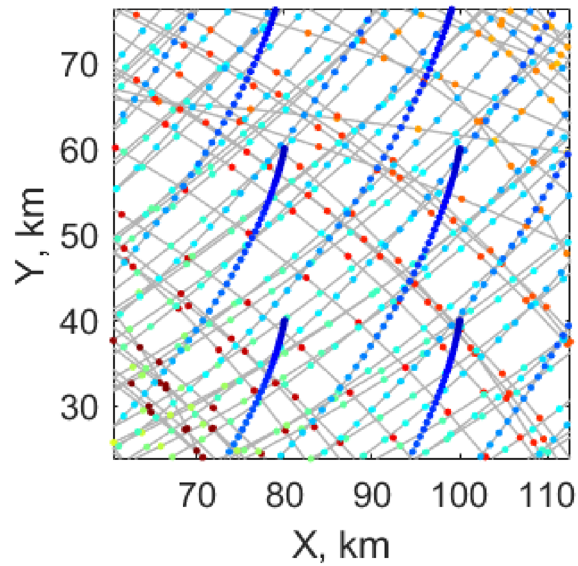


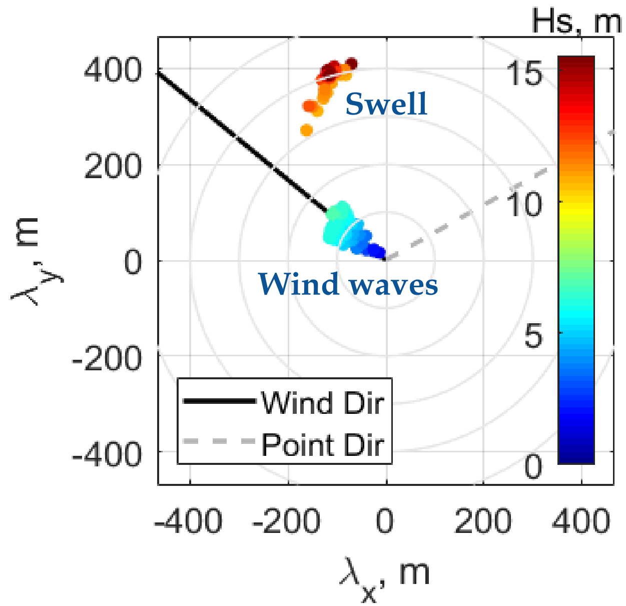
Vtrans = 12 m/s



Hs-Wavelength Angular Distributions







Vtrans=8m/s

Square areas with ~30 deg angular size are taken at distances Rm, 2Rm, 4Rm from TC center.

Ray trajectories inside one of the areas (red one).

High point concentration corresponds to starting stage of wave development (*dt* is small).

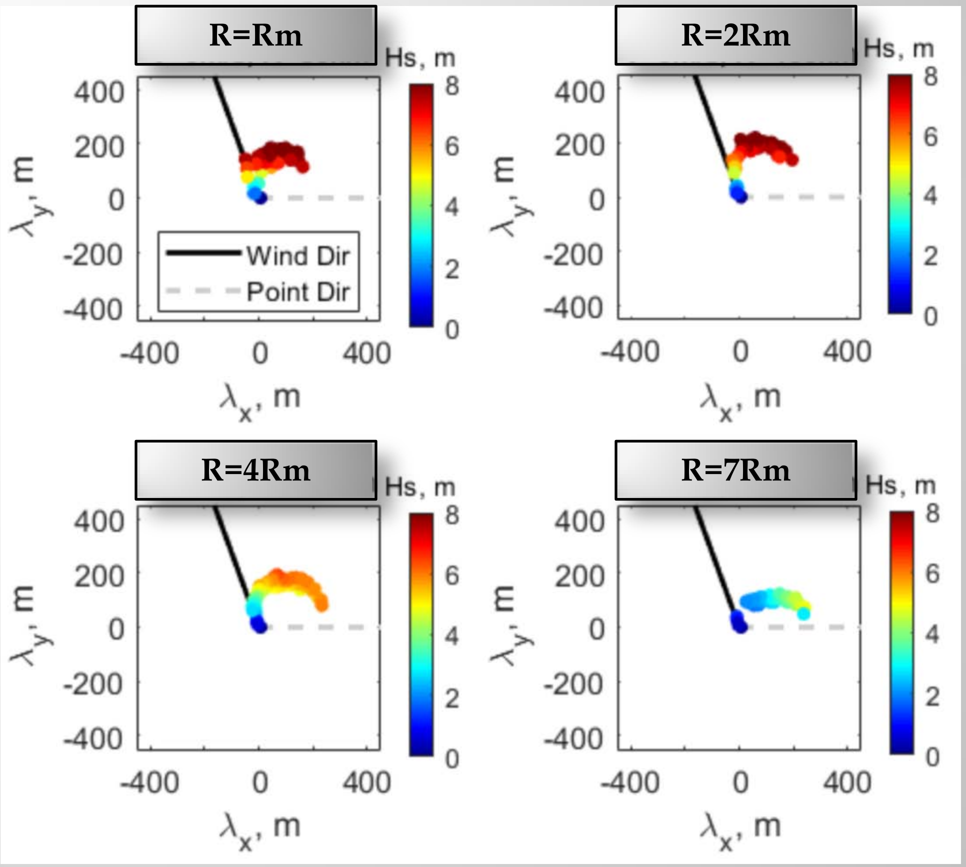
Color is Hs.

Hs "spectrum".

Every point is mean Hs along trajectory part inside square area, in mean wave direction.

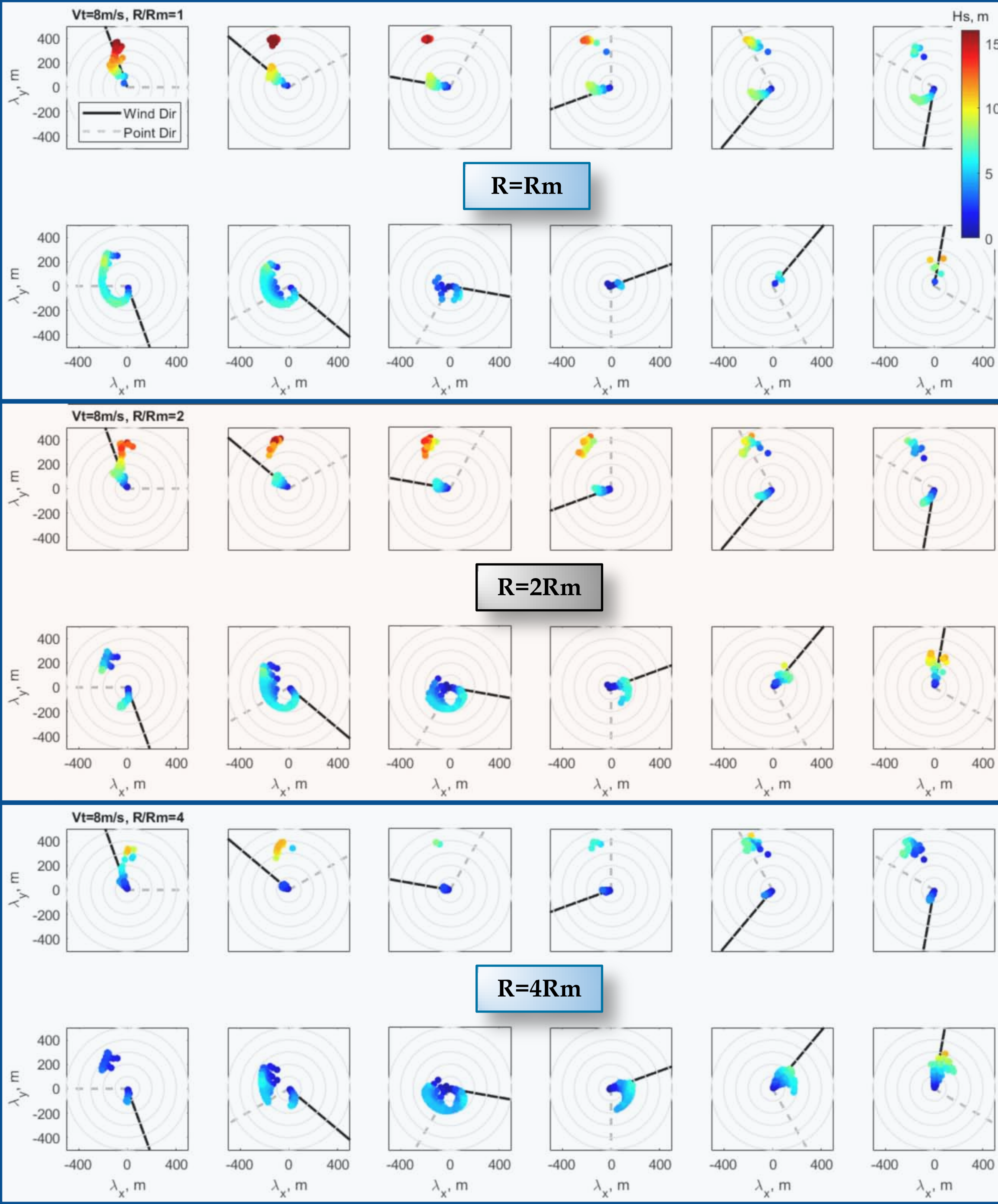
NB In moving coordinate system, wave peak direction does not coincide with tangent to trajectory!

Hs-Wavelength Angular Distributions In Stationary Cyclone



Distributions are "tracing" wave evolution

"Spectra" in moving TC



Wave direction in TC from field measurements

Young, 2006

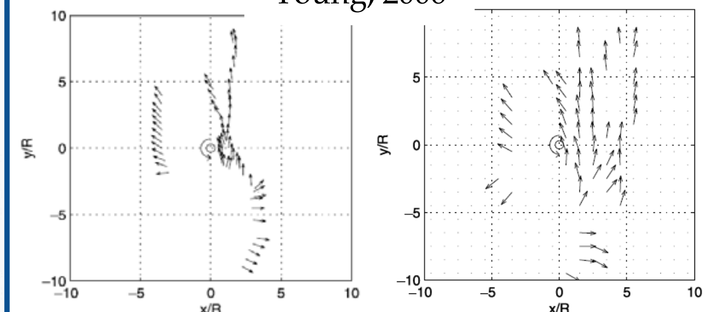
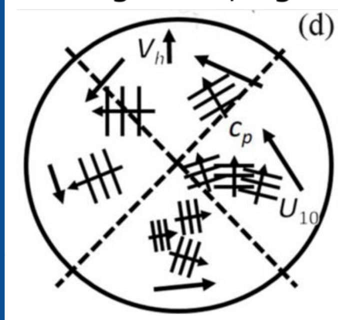


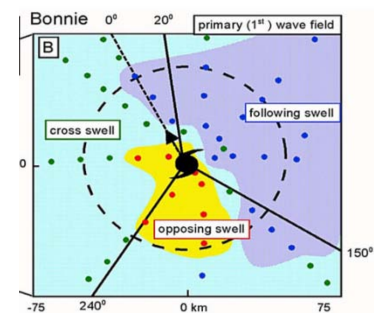
Figure 3. Vectors showing the dominant wave direction for three of the hurricanes. The data are shown in a transformed co-ordinate system moving with the hurricane, with the direction of propagation towards the top of the page. The storm centre is shown by the open circle at co-ordinates 0,0. The spatial co-ordinates are shown scaled by the radius to maximum winds, *R*. The system is shown for the Northern Hemisphere (i.e. anti-clockwise circulation).

Figure 4a. A composite of all storms in the database showing the mean values of the dominant wave direction in squares of size 1*R* × 1*R*. Areas with no vectors shown correspond to squares where there were insufficient measurements to form a reliable estimate of dominant wave direction. The hurricane centre is shown by the open circle at co-ordinates 0,0. The system is shown for the Northern Hemisphere (i.e. anti-clockwise circulation).

Hwang 2017, fig .11

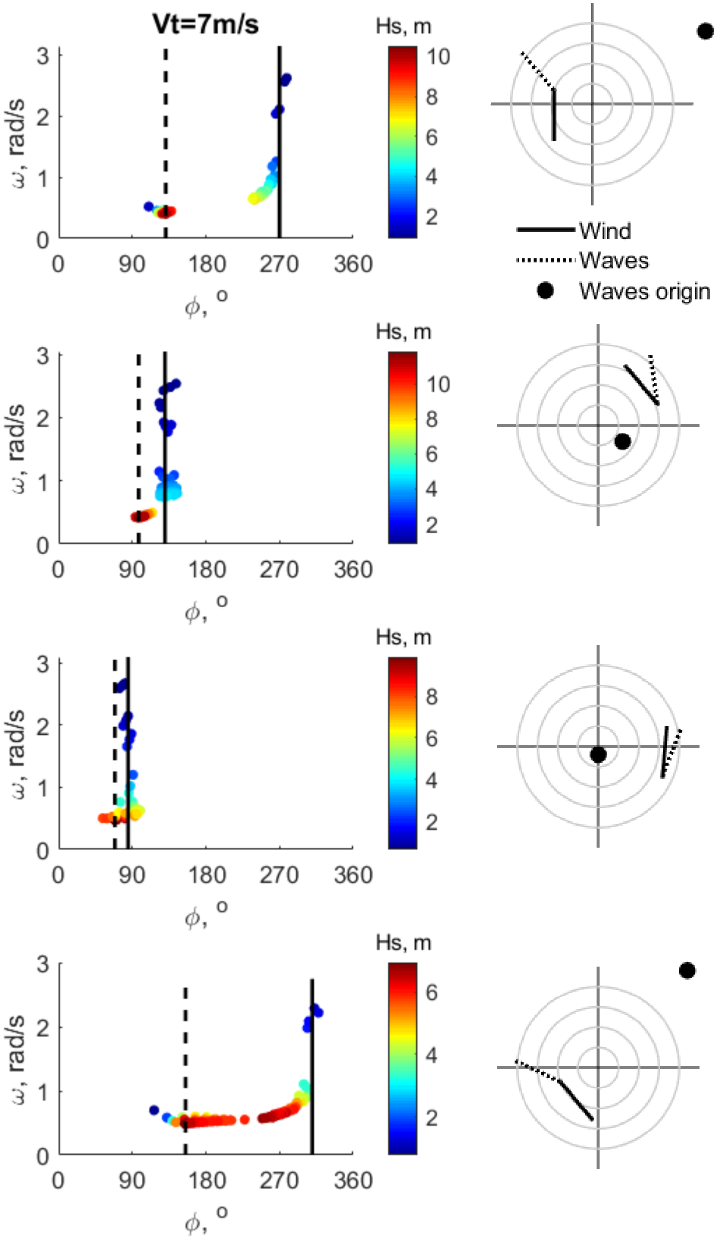


Holthuijsen 2012, fig .4



Model is consistent with observations

Wave Angular distribution. Comparison with Buoy Data (Young, 2006)



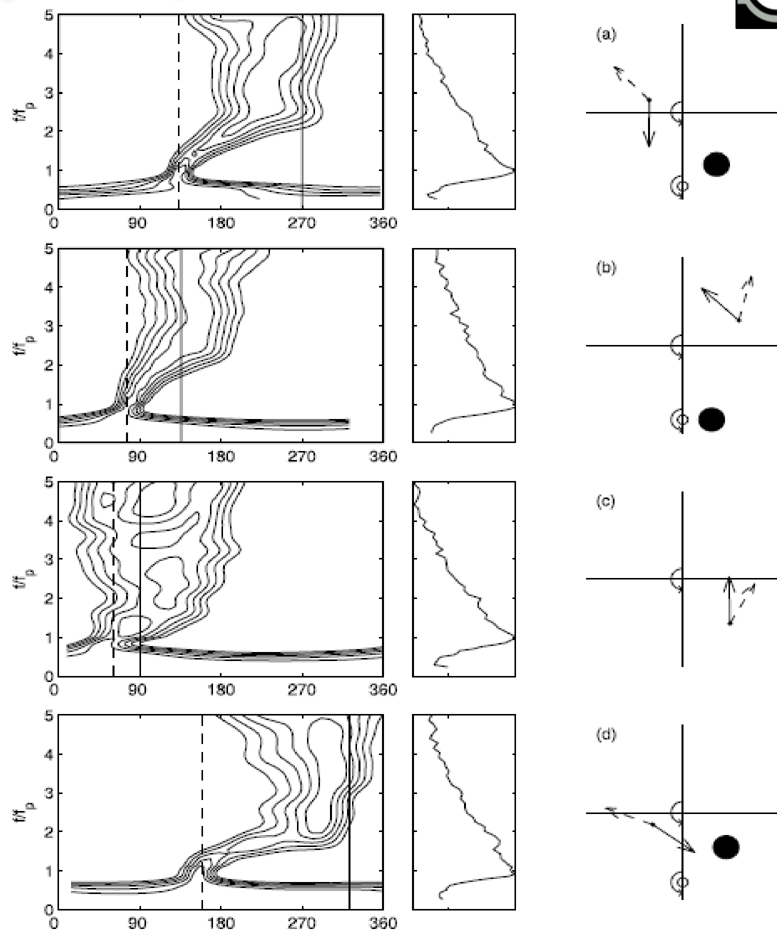
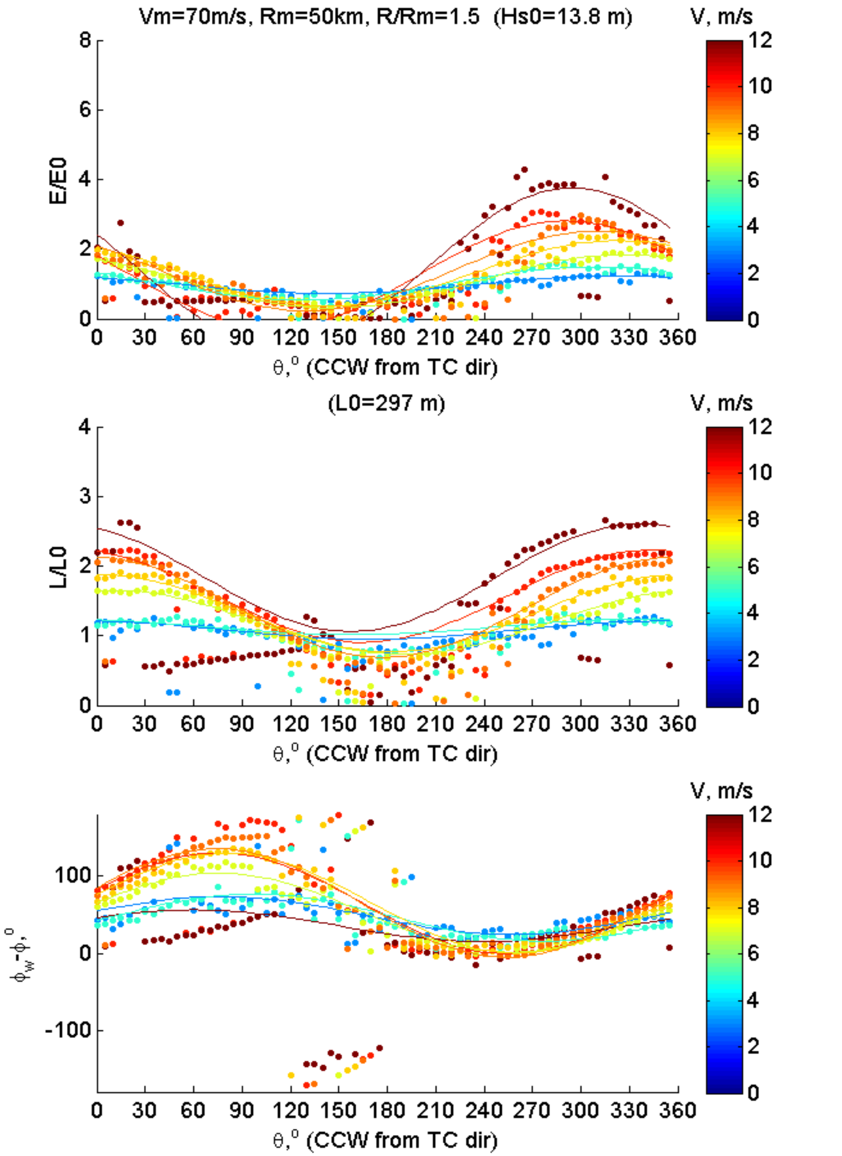


Figure 5. Examples of the directional spreading function, $D(f, \theta)$ for each quadrant of a hurricane. For each quadrant, the panel to the left shows D contoured in f, θ cartesian space. At each frequency, D has been normalized to have a maximum value of one. Contours are drawn 0.9, 0.8, 0.7, 0.6 and 0.5. The vertical dashed line shows the dominant wave direction and the vertical solid line the local wind direction. To the right of the spreading function the one-dimensional frequency spectrum is shown. For these one-dimensional spectra, the energy scale is logarithmic, with the maximum ordinate being 1.0 and the minimum 10^{-3}. The panels to the extreme right show the corresponding position of the measurement in the quadrant under consideration (small solid dot). The dominant wave direction is shown by the dashed arrow and the local wind direction by the solid arrow. At the time of the measurement, the hurricane is located at the centre of the "cross" shown on the panel. The open circle below the cross shows the estimated position of the centre of the hurricane at the time when the dominant waves at the measurement location were generated. The large solid circle to the right of this point shows the approximate region in which these dominant waves were generated. The system is shown for the Northern Hemisphere (i.e. anti-clockwise circulation).

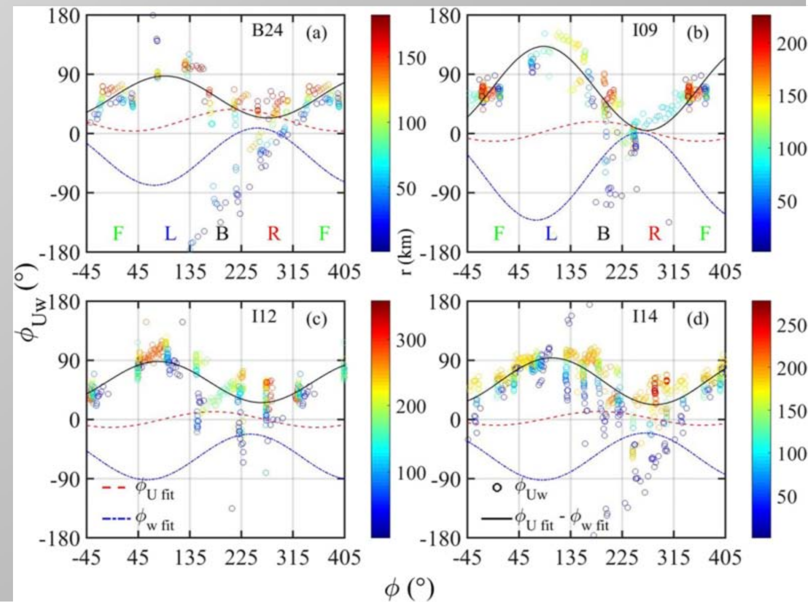
Fitting of Wave Parameters

Fitting example at radius R=1.5Rm

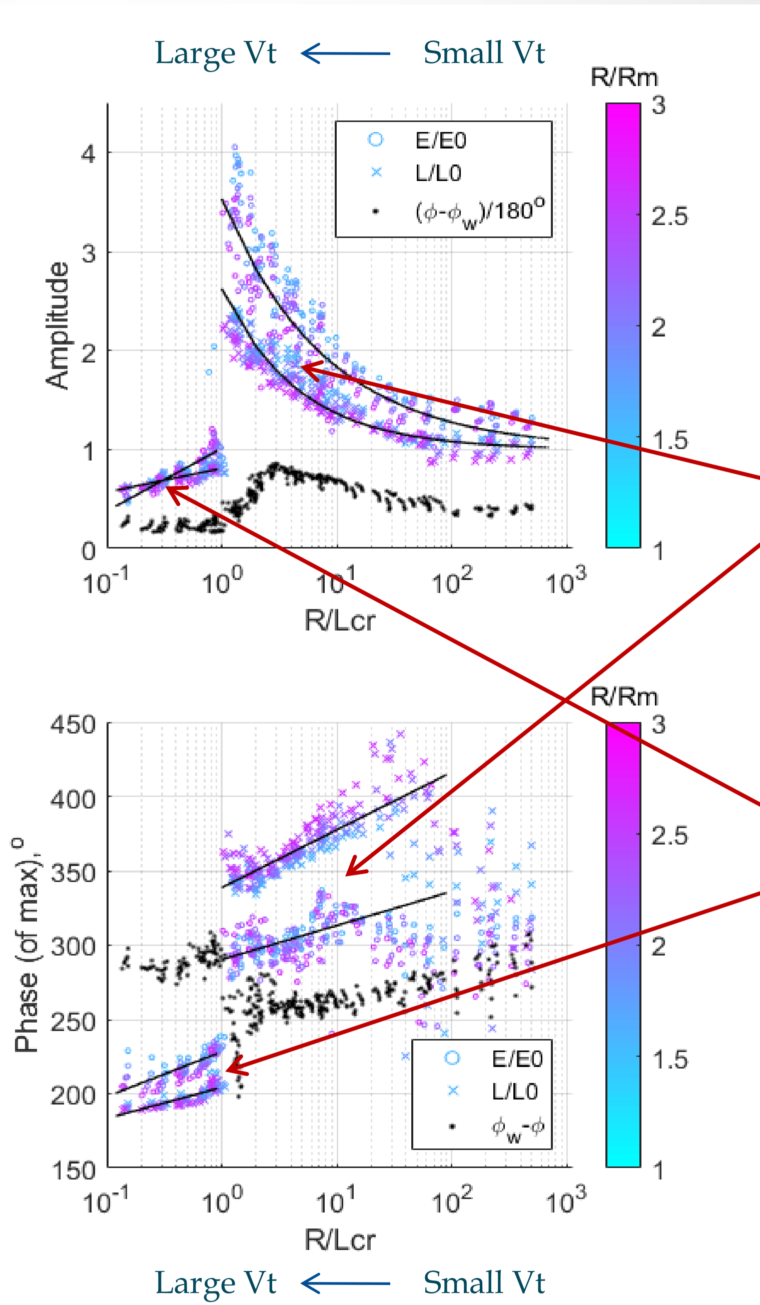


Wave energy, wavelength and wave direction at different distances from TC center are fitted with cosine functions for all considered cases (different TC radii, Umax and translation speed)

Hwang 2017, fig .8



Parameterization of Fitting Functions



Magnitude (A) and location (P) of fitting function's maxima **vs** dimensionless parameter:

$$Xd = R / Lcr$$

$$Lcr = -q c_a^{-1/q} (U^2 / g)(U / 2V_t)^{1/q} / (1+q)$$

Two regimes:

1. Slow TC (Xd>1):

*AE1=1+2.3*Xd.^(-0.46)*
*AL1=1+1.4*Xd.^(-0.62)*
*PE1=8.5*log(Xd)+293*
*PL1=19*log(Xd)+340*

2. Fast TC (Xd<1),
$V_t >> C_p$, waves are not trapped:

*AE2=0.26*log(Xd)+1.03*
*AL2=0.093*log(Xd)+0.815*
*PE2=12*log(Xd)+229*
*PL2=8.7*log(Xd)+205*

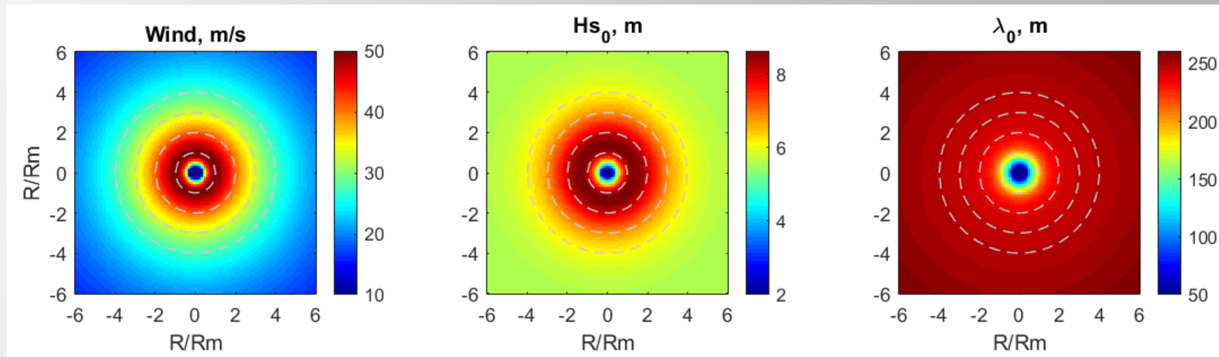
TC Reconstruction from Analytical Functions

Wind: Holland, 1980

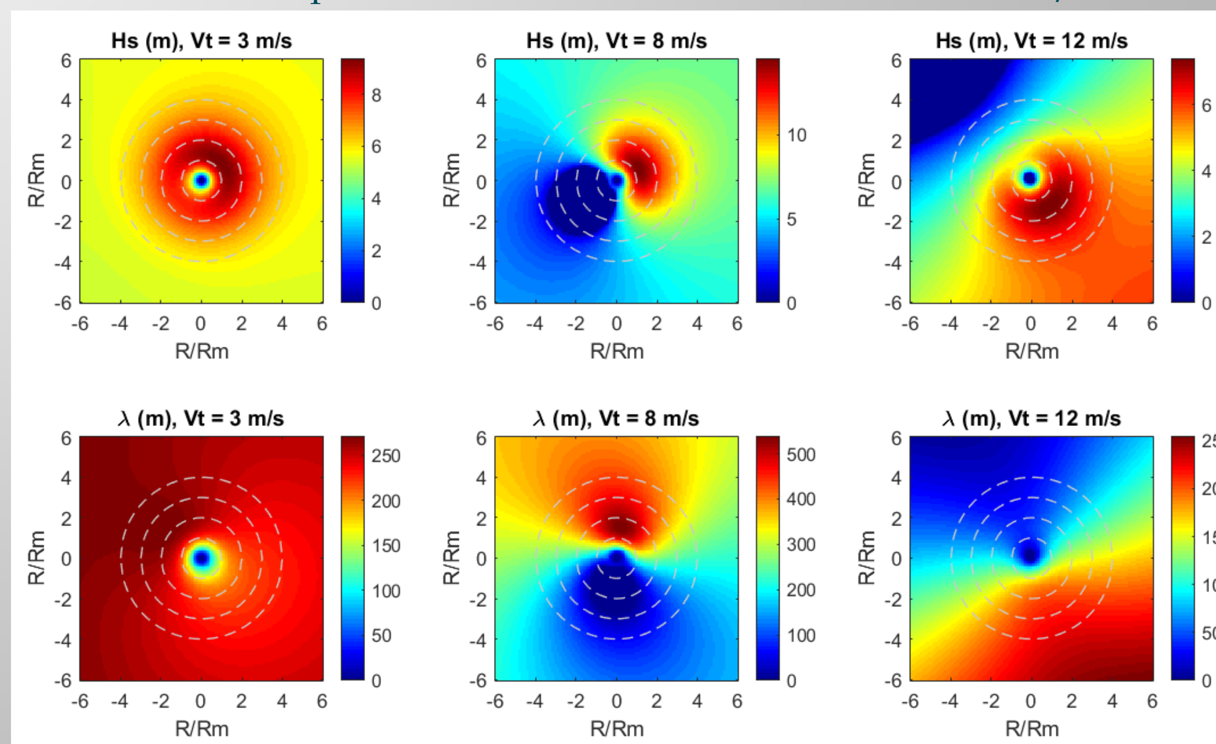
Stationary cyclone: Hs and wavelength from 1D fit

Moving TC: angular distributions from parameterized cosine functions (depend on Rm, Um, Vt and distance from TC center)

Parameters of stationary cyclone (Rm=50 km, Um=50 m/s)



Example of TC reconstruction for V_{trans}=3, 8, 12 m/s



V_{trans}

Conclusions

- A model for waves development and propagation under spatially and time varying winds, is suggested.
- The model is based on energy and momentum conservation laws. Wind energy input and wave breaking dissipation are the main sources to govern the wave energy conservation equation, while non-linear interactions are essential to control the peak frequency downshift of the energy-containing part of the spectrum.
- 1D self-similar fetch-laws are used to derive fully consistent parametric solutions for 2D surface wave development.
- Calculations were carried out for the case of the uniform wind field and for an inhomogeneous cyclonic wind field with different hurricane translation velocities.
- The calculations reproduce the anisotropy of the energy distribution inside the hurricane and the effect of wave trapping by a moving cyclone.
- As shown, varying winds can lead to the divergence of group velocities (focusing/defocusing wave groups), to significantly affect the energy balance.
- The results are in line with field measurements and existing knowledge about TC dynamics
- The model can provide practical means to rapidly map and assess the energy, frequency and peak wave direction distributions. Applications can serve to provide prior-information to analyze high-resolution satellite measurements and to improve remote sensing algorithm developments.

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Thank you for your attention!