

## Recent studies on the impact of troposphere delay modeling for Satellite Laser Ranging



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# Troposphere delay modeling in SLR solutions

**Current model** (no tropo parameters are estimated in SLR solutions):

**Wet delay:** based on water vapor pressure records and the position of an SLR station (latitude, height)

(Mendes and Pavlis, 2004)

$$d_{atm} = m_{fs}(d_h^z + d_{nh}^z)$$

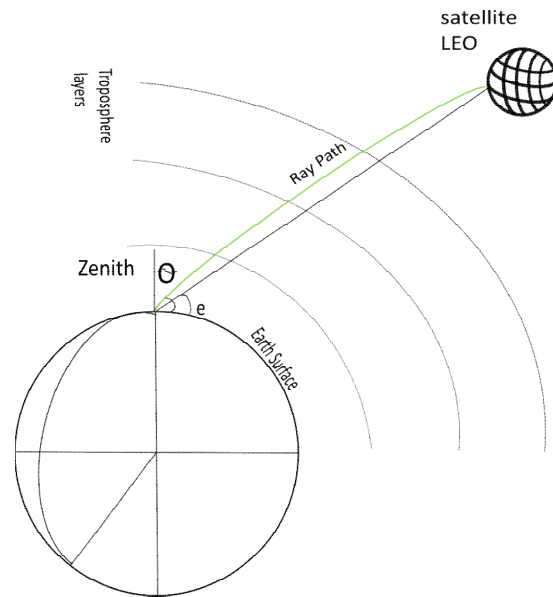
**Common mapping function:**  
based on temperature records and the position of an SLR station (latitude, height)

(Mendes et al., 2002)

**Hydrostatic delay:** based on pressure records and the position of an SLR station (latitude, height)

(Mendes and Pavlis, 2004)

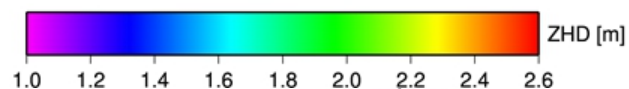
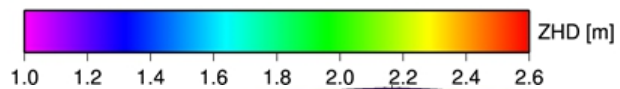
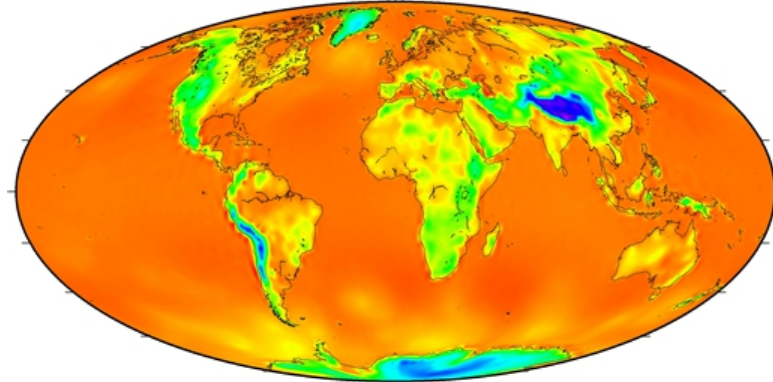
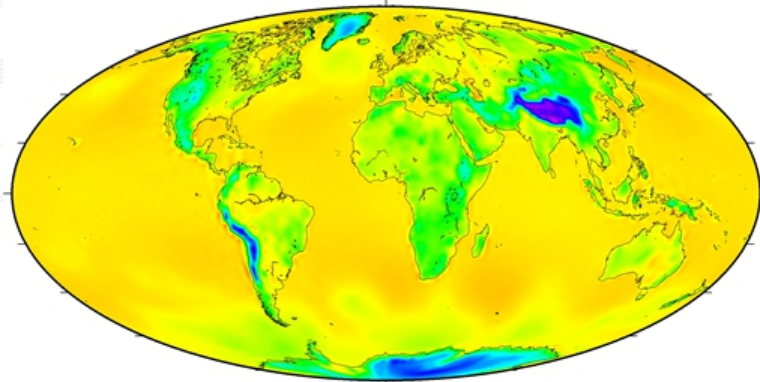
**A full symmetry of the atmosphere over SLR stations is assumed**



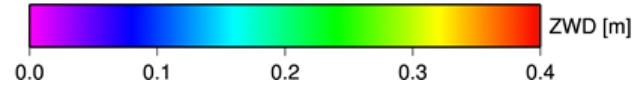
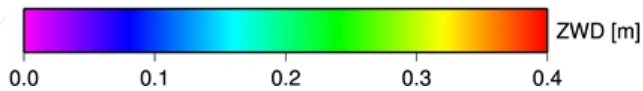
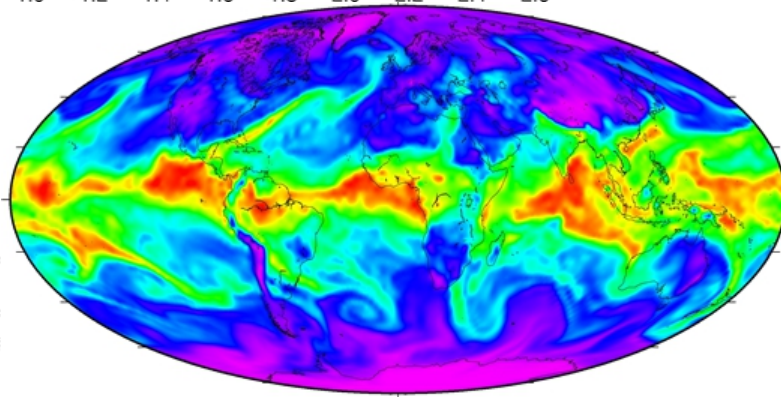
# GNSS

# SLR

## ZHD



## ZWD



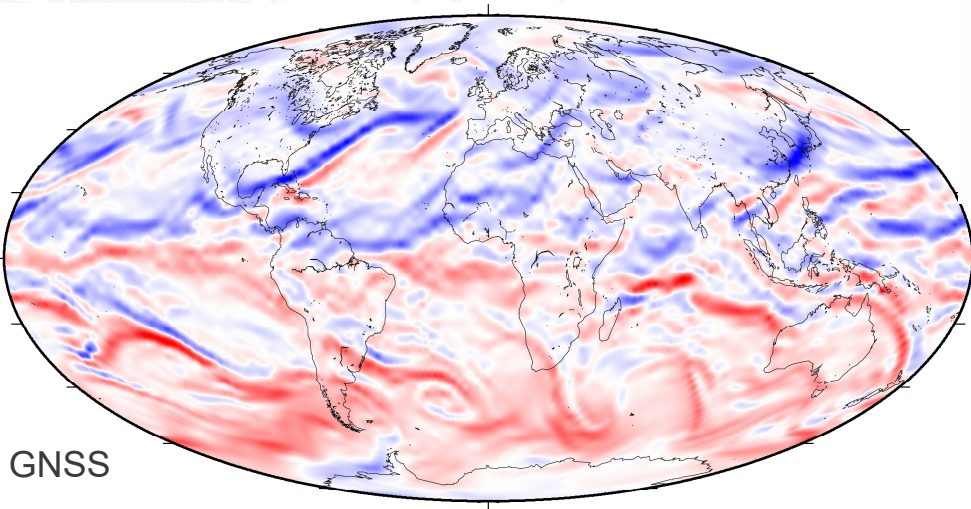
A large, faint watermark of the University of Wrocław seal is visible on the left side of the slide. The seal is circular and contains a central shield with various heraldic symbols, including a crowned eagle and a figure. The Latin text "UNIVERSITAS WROCLAVIENSIS" is inscribed around the perimeter of the seal.

# *Mapping functions and horizontal gradients dedicated for optical measurements*

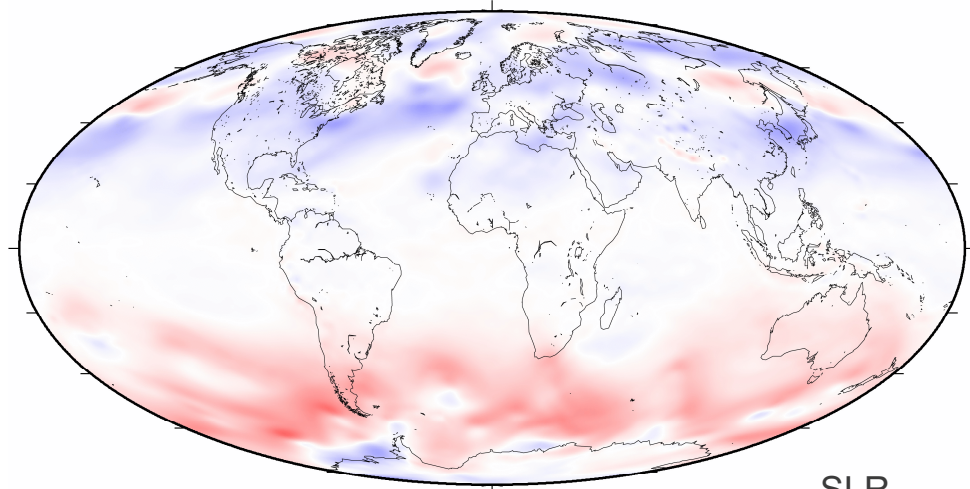
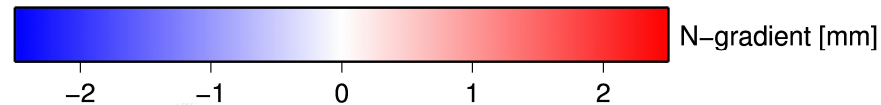


## Comparison of horizontal gradients

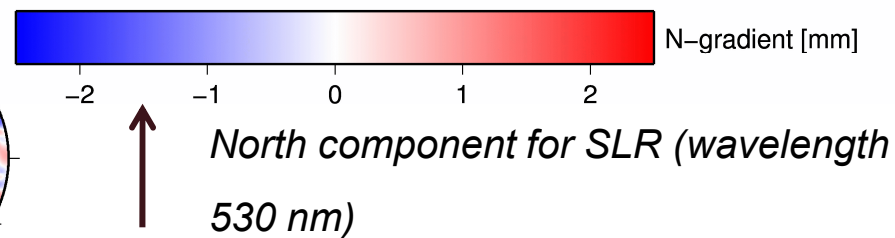
- Time resolution: 6h
- Spatial resolution: 0.5x0.5 degree



GNSS



SLR



North component for GNSS



# PMF troposphere delay models

Common mapping function:  
based on temperature records  
and the position of an SLR  
station (latitude, height)

(Mendes et al., 2002)

Hydrostatic delay: based on pressure records  
and the position of an SLR station (latitude, height)

(MP, Mendes and Pavlis, 2004)

Wet delay: based on water vapor pressure records  
and the position of an SLR station (latitude, height)

(MP, Mendes and Pavlis, 2004)

$$(1): d_{atm} \Rightarrow m_f(d_h + d_w)$$

Potsdam Mapping Function:

$$(2): d_{atm} \Rightarrow m_{PMF}(d_h + d_w)$$

O1: Linear horizontal gradients

$$(3): d_{atm} = m_{PMF}(d_h + d_w) + m_{CH-H}(G_N \cos A + G_E \sin A)$$

O1+O2: Nonlinear horizontal gradients:

$$(4): d_{atm} = m_{PMF}(d_h + d_w) + m_{CH-H}(G_N \cos A + G_E \sin A + G_{NN} \cos^2 A + G_{EE} \sin^2 A + G_{NE} \cos A \sin A)$$

# VMF3o troposphere delay model

Hydrostatic mapping function:

Wet mapping function:

Separate mapping function:  
based on NWM

(Boisits J., et al., 2018)

$$m(e)_{VMF3oh} = \frac{1 + \frac{a_h}{1 + \frac{b_h}{1 + c_h}}}{\sin e + \frac{a_h}{\sin e + \frac{b_h}{\sin e + c_h}}},$$

$$m(e)_{VMF3ow} = \frac{1 + \frac{a_w}{1 + \frac{b_w}{1 + c_w}}}{\sin e + \frac{a_w}{\sin e + \frac{b_w}{\sin e + c_w}}}$$

Hydrostatic delay:

$$d_{atm h} = d_h \cdot m(e)_{VMF3oh} + m_{gh}(G_{Nh} \cdot \cos A + G_{Eh} \cdot \sin A)$$

Wet delay:

$$d_{atm w} = d_w \cdot m(e)_{VMF3ow} + m_{gw}(G_{Nw} \cdot \cos A + G_{Ew} \cdot \sin A)$$

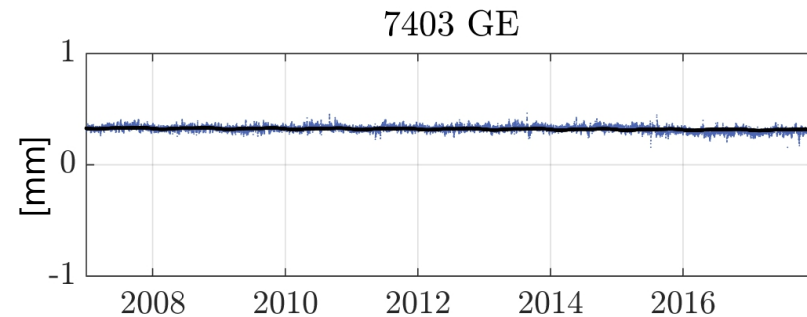
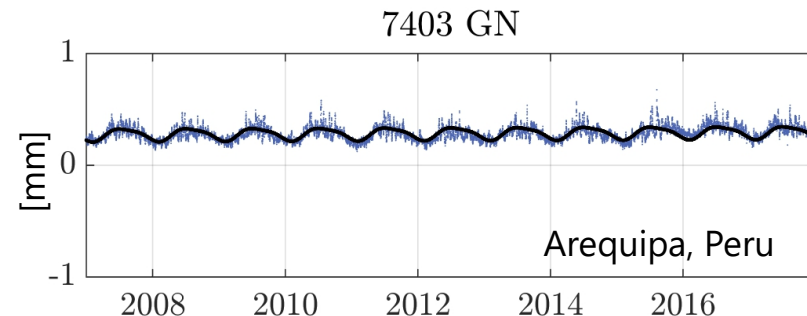
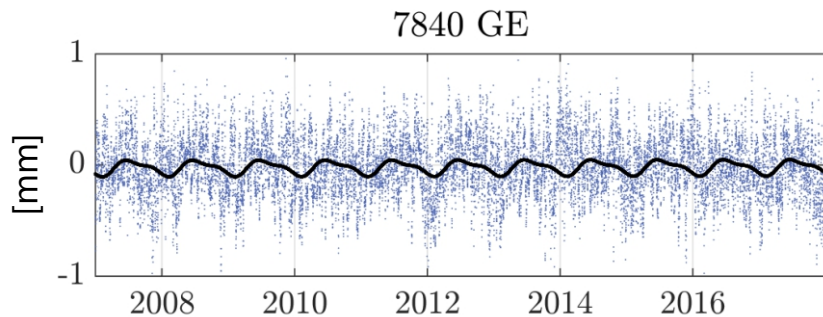
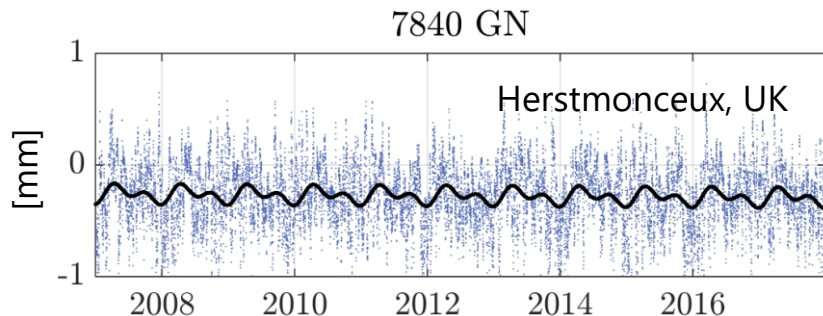
Hydrostatic delay:

Wet delay

Vienna Mapping Function for optical frequencies (VMF3o):

$$d_{atm} = (d_{atm h} + d_{atm w})$$

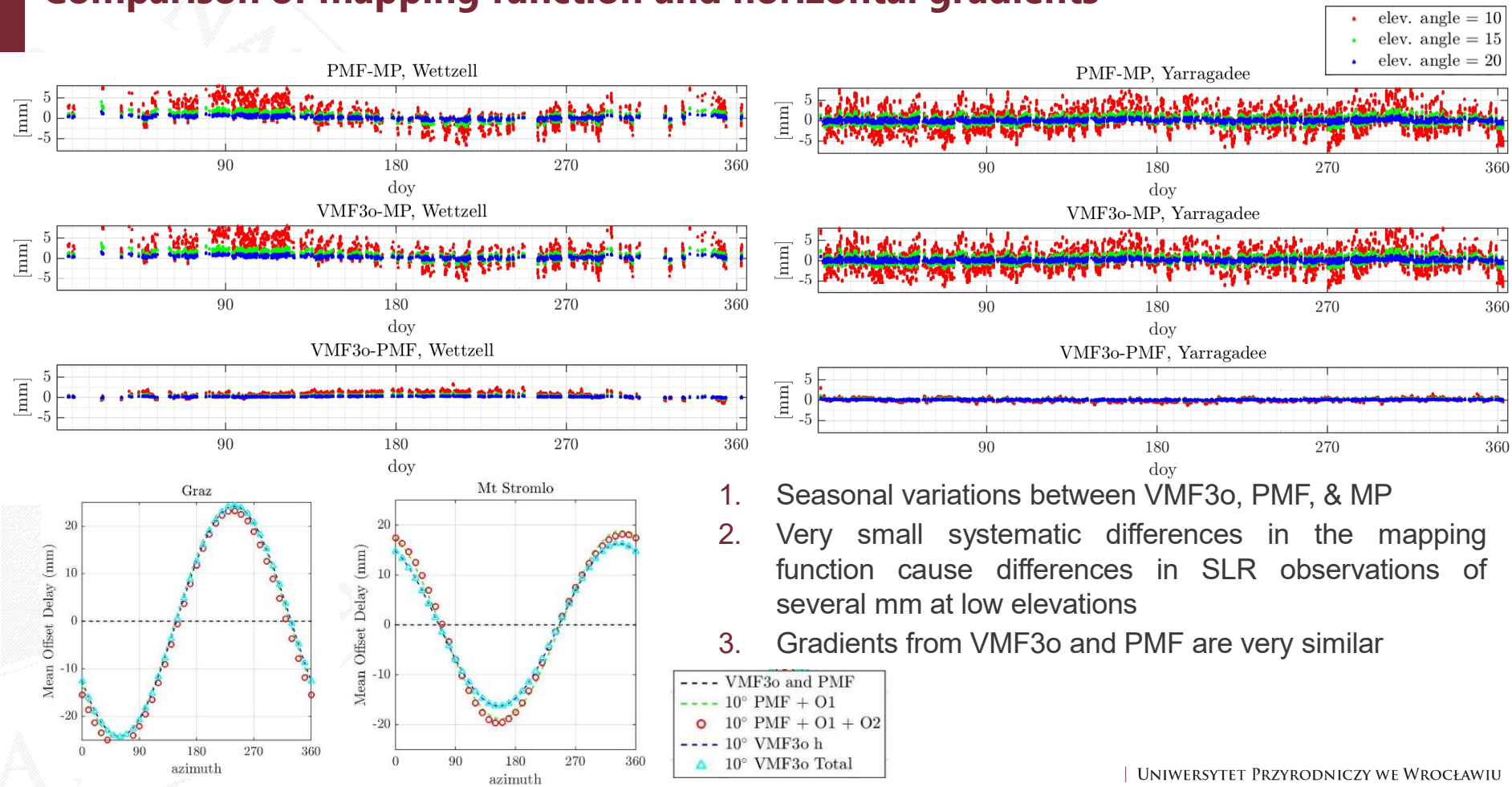
## Simple model of gradients for SLR?



$$f(t) = a_0 + a_1 t + a_{s1} \sin\left(\frac{2\pi}{T} t\right) + a_{c1} \cos\left(\frac{2\pi}{T} t\right) + a_{s2} \sin\left(\frac{4\pi}{T} t\right) + a_{c2} \cos\left(\frac{4\pi}{T} t\right)$$

Offset + drift + annual signal + semi-annual signal for each component for each SLR station

# Comparison of mapping function and horizontal gradients



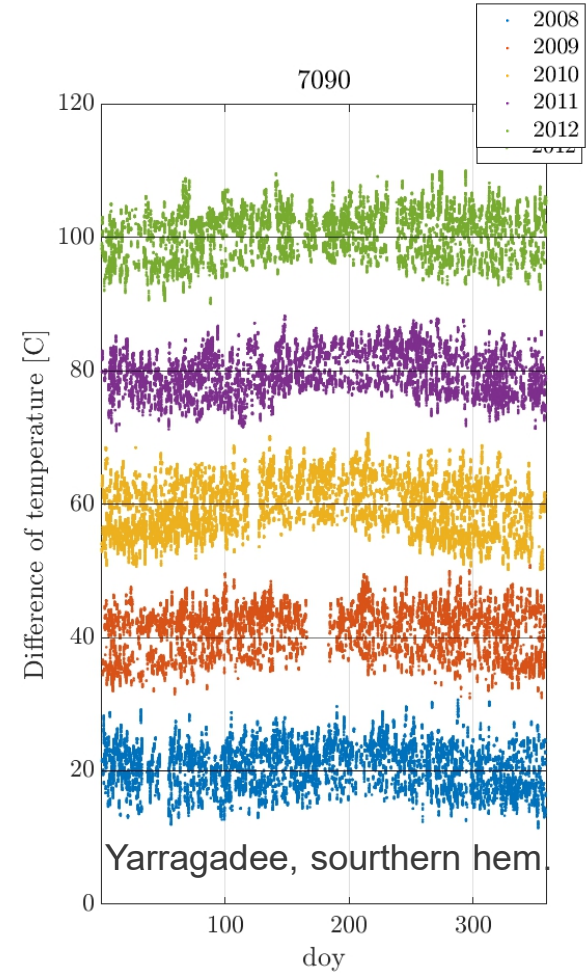
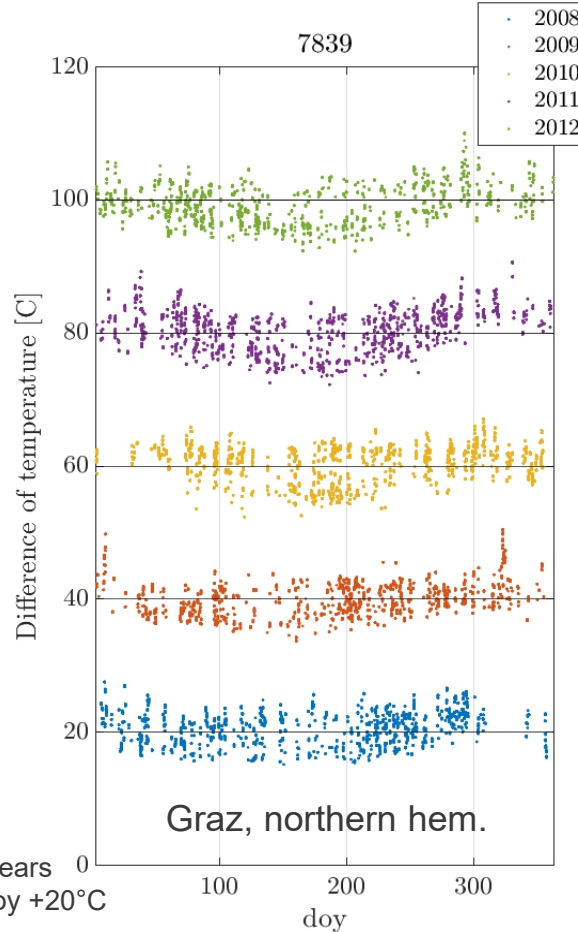
1. Seasonal variations between VMF3o, PMF, & MP
2. Very small systematic differences in the mapping function cause differences in SLR observations of several mm at low elevations
3. Gradients from VMF3o and PMF are very similar



# Differences of temperatures (temperature derived from the numerical weather model and temperature measured at SLR stations)

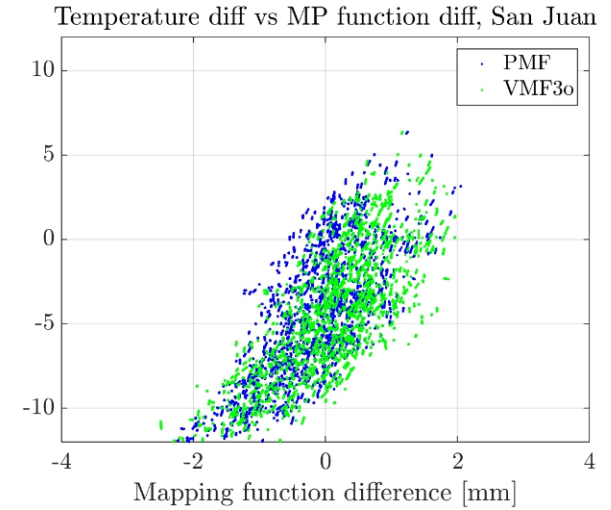
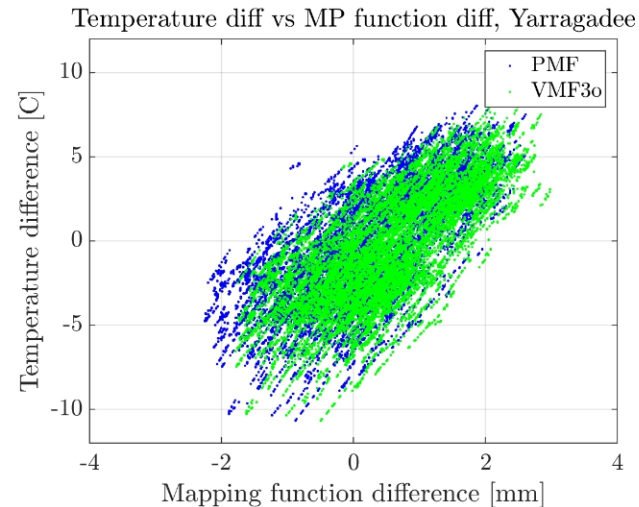
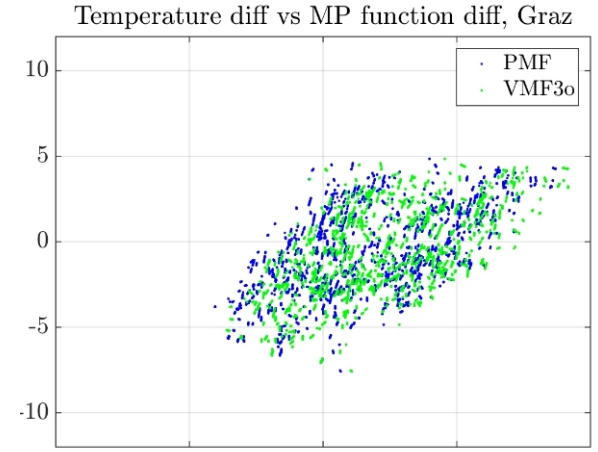
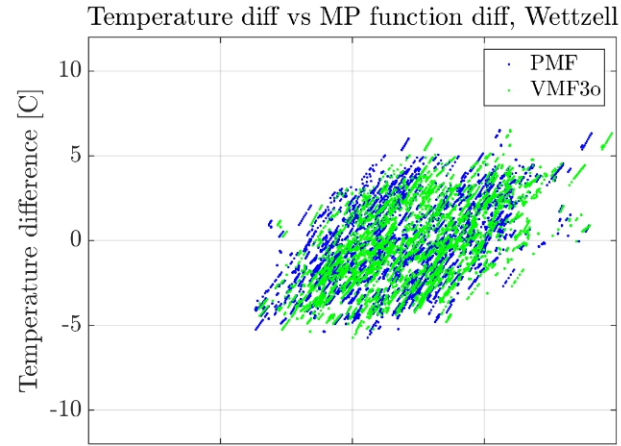
A characteristic difference of the temperature for the north and the south hemispheres can be observed (a seasonal variation).

The average value of the temperature difference is at the level of 5 deg [°C]

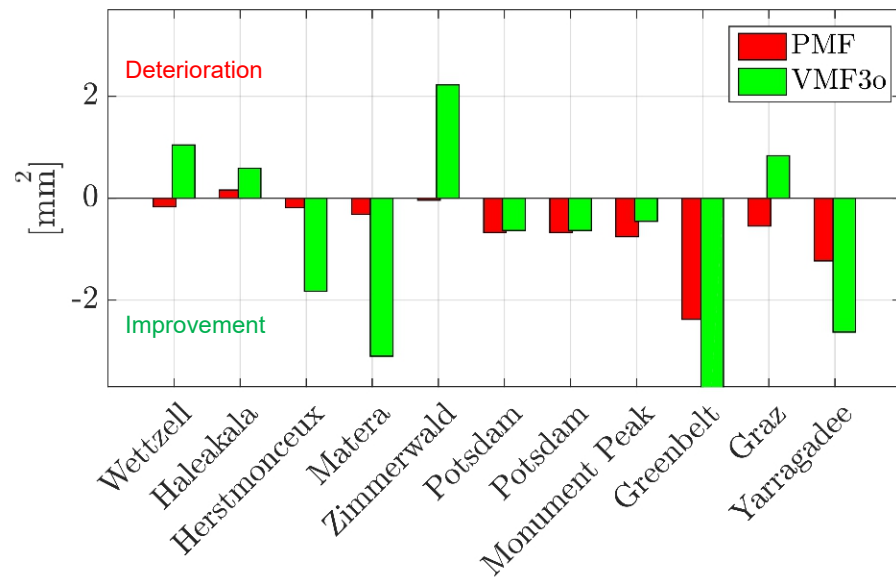


# Effect of the difference of the site temperature and the numerical weather model temperature

Station	Correlation coefficient	
	PMF	VMF3o
7080	0.58	0.58
7090	0.66	0.69
7105	0.72	0.73
7119	0.76	0.76
7124	0.73	0.69
7249	0.69	0.72
7358	-0.32	-0.26
7405	0.73	0.76
7406	0.73	0.75
7501	0.93	0.94
7810	0.17	0.31
7820	0.27	0.36
7821	-0.37	-0.34
7824	0.36	0.39
7825	0.62	0.63
7832	0.67	0.66
7839	0.58	0.61
7840	0.28	0.32
7841	0.35	0.38
7845	0.75	0.63
7848	0.82	0.82
7941	0.49	0.53
8834	0.5	0.55



## SLR observation residuals to SENTINEL 3a



Average elevation angle of observations: 27° degrees

Analyzed period 2017.0 – 2018.0



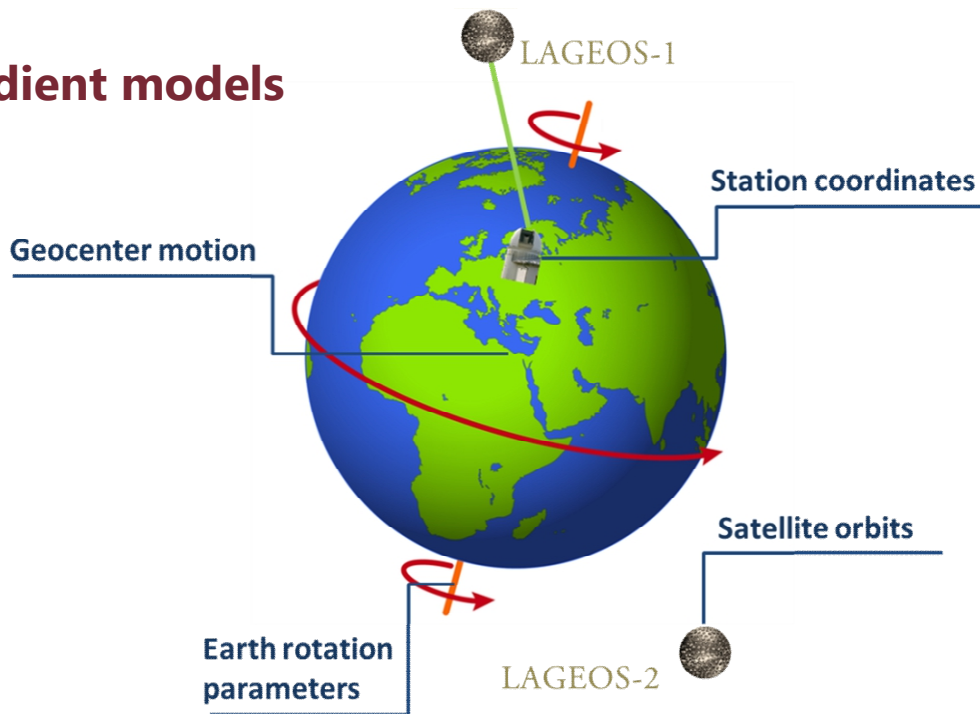
Differences the of observations residuals with respect to standard FCULa mapping function

The total improvement of variance for analyzed stations is at the level of 6.8 [ $\text{mm}^2$ ] for PMF, and 11.5 [ $\text{mm}^2$ ] for VMF3o mapping function

# SLR solutions for the validation of gradient models

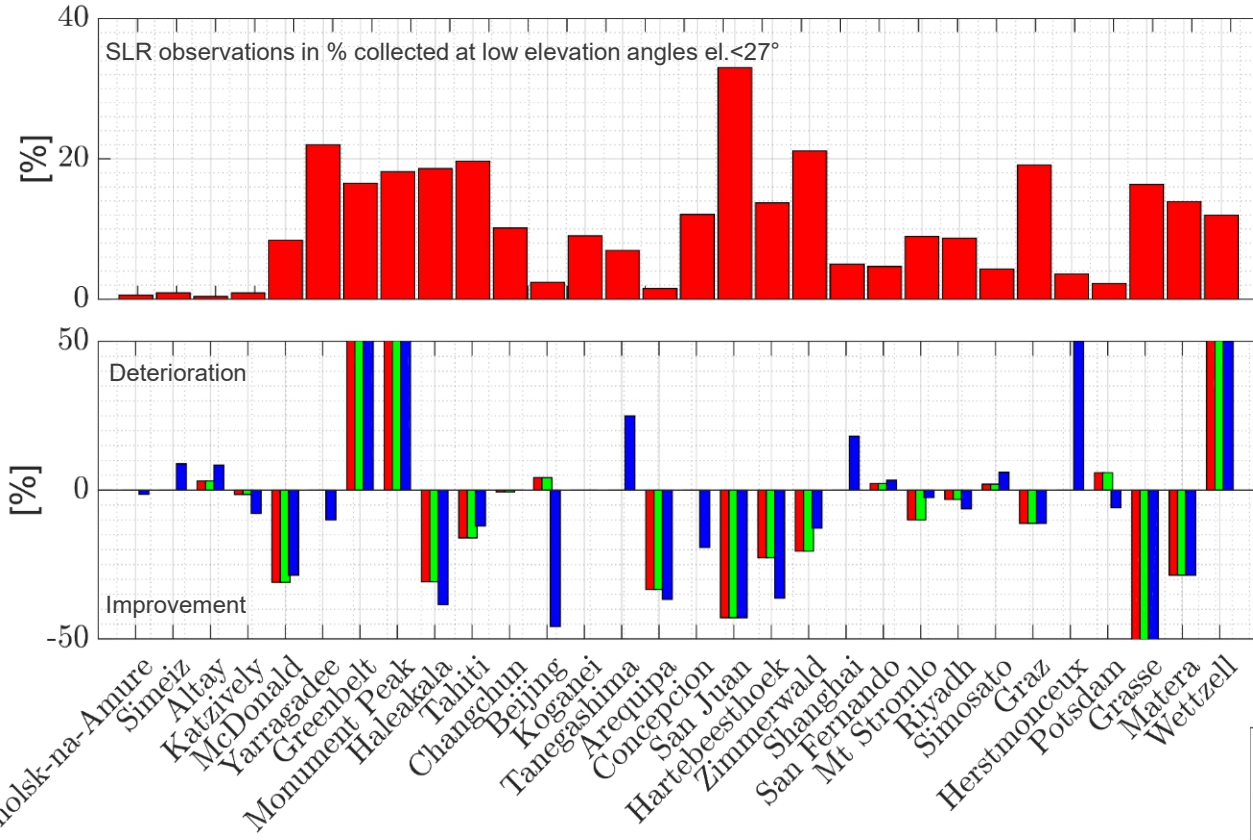
## Estimated parameters:

- Station coordinates (7-day)
- Orbit parameters:
  - 6 Keplerian + 5 empirical (7-day)
- Geocenter coordinates (7-day)
- Range biases for selected stations (1-3 per week)
- X-pole, Y-pole (8 par per 7-day)
- UT1-UTC (8 par per 7-day, 1 parameter fixed)





# Observation SLR residuals – impact from including gradients



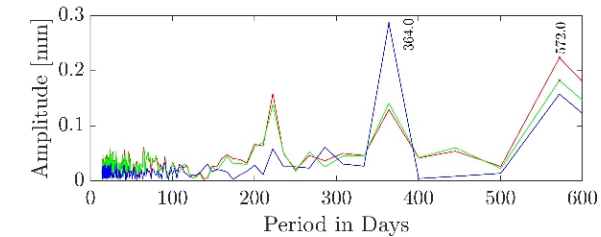
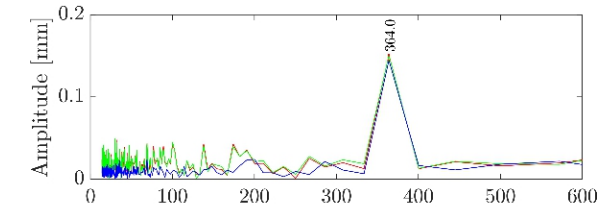
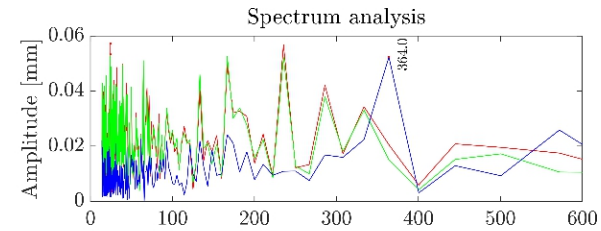
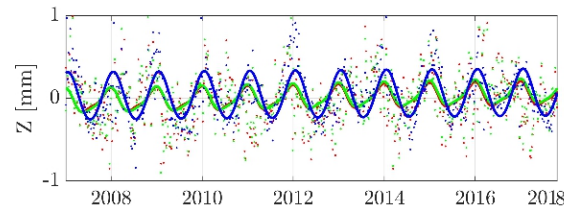
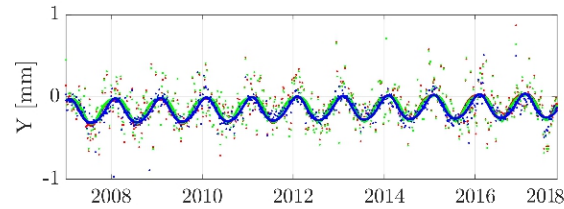
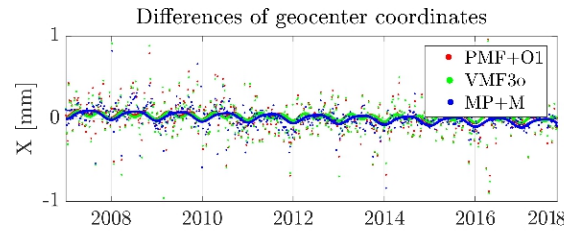
1. Observations below 27 degrees of the elevation angle constitute on average **10 %** of SLR observations to LAGEOS-1/2
2. The negative values correspond to a reduction of median residuals for solutions based on PMF, VMF3o or MP + simple model of horizontal gradients model with respect to the standard approach (MP with no gradients).



# Geocenter coordinates

Mean offset at the level of 0.12 mm for Y component for solutions with horizontal Gradients.

Occurrence of periodic components  
At the level of 0.16 mm for Y and Z Component.



Mean values of geocenter coordinates [mm]

	X [mm]		Y [mm]		Z [mm]	
	mean	$\sigma$	mean	$\sigma$	mean	$\sigma$
<b>VMF3o</b>	-0.039	0.004	-0.010	0.003	-0.007	0.008
<b>PMF+O1</b>	0.039	0.006	-0.122	0.009	-0.006	0.017
<b>MP+M</b>	0.035	0.013	-0.126	0.009	-0.038	0.017

# Pole coordinates

Improvement of mean offset value

At the level of 20  $\mu\text{as}$  for X component

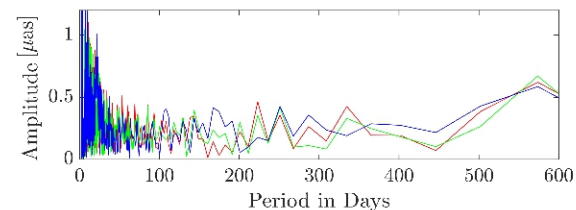
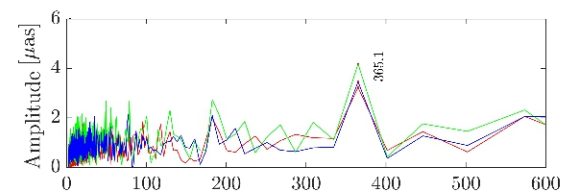
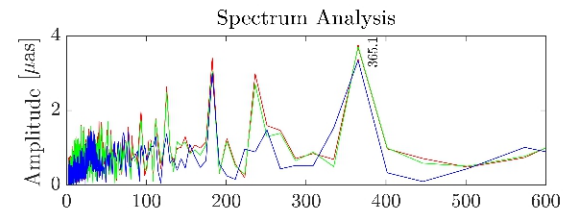
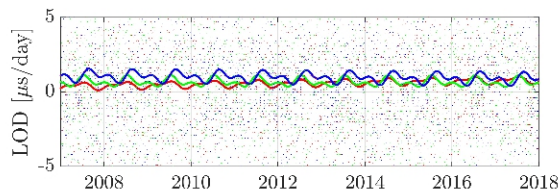
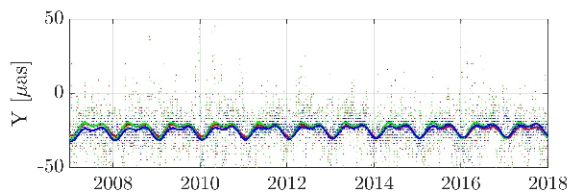
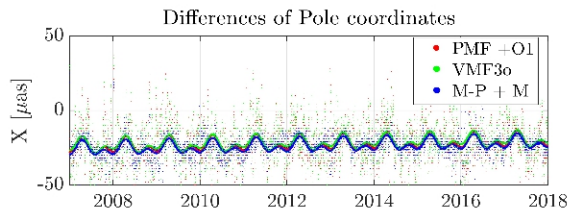
Improvement of mean offset between

combined solution C04 and SLR

at the level of 24  $\mu\text{as}$  for Y component

30  $\mu\text{as}$  = 1 mm

on the Earth surface



	X-POLE ( $\mu\text{as}$ )		Y-POLE ( $\mu\text{as}$ )		LOD ( $\mu\text{s/day}$ )		Number of epochs
	mean	$\sigma$	mean	$\sigma$	mean	$\sigma$	
<b>Standard sol.</b>	22	7.5	38	7.6	-77	5.2	574
<b>PMF + O1</b>	2	7.5	14	7.6	-77	5.2	574
<b>VMF3o</b>	10	7.5	12	7.6	-76	5.2	574
<b>M-P + M</b>	7	7.5	11	7.6	-75	5.2	574

# Conclusions

1. Modeling troposphere delay with horizontal gradients in SLR solutions improves observation residuals, especially for low elevation angles.
2. SLR solutions become more consistent with the IERS-14-C04 combined series when considering gradients which means that SLR solutions become more consistent with other techniques of space geodesy.
3. A simple model of horizontal gradients for SLR consisting of the offset, drift, annual and semi-annual signals captures most of the systematic effects (85-95%) in the Earth rotation parameters caused by the horizontal gradients w.r.t. numerical weather models.
4. Differences between site temperatures and temperatures from numerical weather models were detected. These may affect the mapping function coefficients.



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AND LIFE SCIENCES



Thank you

## Literature:

More on troposphere delay in SLR solutions:

Drożdżewski M., Sośnica K., Zus F., Balidakis K. (2019)

***Troposphere delay modeling with horizontal gradients for satellite laser ranging***

Journal of Geodesy, Vol. 93 No. 10, Berlin Heidelberg, Germany 2019, pp. 1853-1866

DOI: [10.1007/s00190-019-01287-1](https://doi.org/10.1007/s00190-019-01287-1)

URL: <https://link.springer.com/article/10.1007/s00190-019-01287-1>

Drożdżewski M., Sośnica K. (2018)

***Satellite laser ranging as a tool for the recovery of tropospheric gradients***

Atmospheric Research, Vol. 212 No. , 2018, pp. 33-42

DOI: [10.1016/j.atmosres.2018.04.028](https://doi.org/10.1016/j.atmosres.2018.04.028)

URL: <https://www.sciencedirect.com/science/article/pii/S0169809517313108>