Continuum modelling of grain-size segregation in bedload transport

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Objective of this work

Bridge the gap between

Particle scale forces for size segregation (Guillard et al. 2016)

Continuum modelling of size segregation
Segregation forces on a single particle in a bath of small particles

Vertical Lagrangian equation of the large intruder:

\[ \rho^p V_l \frac{dw^l}{dt} = P - \Pi_f + f_d^f + f_d^p - f_{seg} \]

- **Solid drag force:**
  \[ f_{seg} = V_l \mathcal{F}(\mu) \frac{\partial P^s}{\partial z} \]
  Tripathi and Khakhar (2011)  \quad \sim \quad \text{Stokesian drag force}

- **Segregation force:**
  \[ f_d^p = c \pi \eta^p d_l \left(w^s - w^l\right) \]
  Guillard et al. (2016)  \quad \sim \quad \text{Buoyancy force}
How to express $F(\mu)$?

2D DEM Simulations:

Segregation force:

$$f_{seg} = V_i F(\mu) \frac{\partial P_s}{\partial z}$$

From Guillard et al. (2016)
Upscaling to numerous particles...necessity of a continuum model

3 continuum phases:
- Fluid
- Large particles
- Small particles
Multi-phase flow model equations

Fluid momentum balance:

\[ \rho_f \left( \frac{\partial \epsilon w_f^f}{\partial t} + \frac{\partial \epsilon w_f w_f^f}{\partial z} \right) = -\epsilon \frac{\partial p_f}{\partial z} - \rho_f g \cos \theta - n_l < f_{d \rightarrow f} > - n_s < f_{d \rightarrow s} > \]

Small particles momentum balance:

\[ \rho_p \left( \frac{\partial \Phi^s w^s}{\partial t} + \frac{\partial \Phi^s w^s w^s}{\partial z} \right) = -\frac{\partial p^s}{\partial z} - \Phi^s \frac{\partial p_f}{\partial z} - \rho_p g \cos \theta + n_s < f_{d \rightarrow s} > + n_s < f_{l \rightarrow s} > \]

Large particles momentum balance:

\[ \rho_p \left( \frac{\partial \Phi^l w^l}{\partial t} + \frac{\partial \Phi^l w^l w^l}{\partial z} \right) = -\frac{\partial p^l}{\partial z} - \Phi^l \frac{\partial p_f}{\partial z} - \rho_p g \cos \theta + n_l < f_{d \rightarrow f} > + n_l < f_{s \rightarrow f} > \]

\[ n_l < f_{s \rightarrow f} > = \frac{\rho_p \Phi^l}{t_{ls}} (w^s - w^l) + \Phi^l \mathcal{F}(\mu) \frac{\partial p^m}{\partial z} \]
The small particles momentum balance is made dimensionless:

\[
\frac{\partial \phi^s \bar{w}^s}{\partial \tilde{t}} + \frac{\partial \phi^s \bar{w}^s}{\partial \tilde{z}} = - \frac{\tilde{p}^m}{\Phi^s} \frac{\partial \phi^s}{\partial \tilde{z}} + \frac{\phi^s}{St_f} (\bar{w}^f - \bar{w}^s) - \frac{(\bar{w}^s - \bar{w}^m)}{St_p} + \phi^l \mathcal{F}(\mu) \frac{\partial \tilde{p}^m}{\partial \tilde{z}}
\]

with

\[
St_p = \frac{\rho p d_l W}{6c_m p}
\]

\[
\phi^s \bar{w}^s = - \frac{\phi^s}{\Phi^s} \tilde{p}^m St_p \frac{\partial \phi^s}{\partial \tilde{z}} + \phi^l \phi^s \mathcal{F}(\mu) St_p \frac{\partial \tilde{p}^m}{\partial \tilde{z}}
\]

\[
\frac{\partial \phi^s}{\partial \tilde{t}} + \frac{\partial}{\partial \tilde{z}} \left( \phi^l \phi^s S_r \right) = \frac{\partial}{\partial \tilde{z}} \left( D \frac{\partial \phi^s}{\partial \tilde{z}} \right)
\]

\[
S_r = \mathcal{F}(\mu) St_p \frac{\partial \tilde{p}^m}{\partial \tilde{z}}
\]

\[
D = \frac{\phi^s \tilde{p}^m St_p}{\Phi^s}
\]
Results against DEM simulations of Chassagne et al. 2020

- Multi-phase flow model
- DEM

- Small particle dynamics is qualitatively reproduced
- Too much diffusion of small particles concentration
Comparison between the multi-phase flow model and the advection-diffusion equation
Comparison of the coefficients with the DEM

\[ S_r = \mathcal{F}(\mu)St^p \frac{\partial \tilde{p}^m}{\partial \tilde{z}} \]

\[ D = \frac{\phi^s \tilde{p}^m St^p}{\bar{\Phi}} \]

- Advection-diffusion
- DEM simulation