DLR
Deutsches Zentrum
für Luft- und Raumfahrt
German Aerospace Center

Recent progress and new methods for detecting causal relations in large nonlinear time series datasets

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DLR Institute of Data Science

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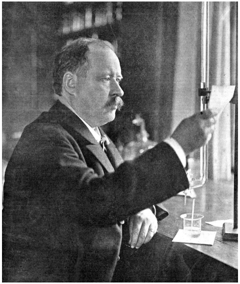
Inferring causality: Three strands of modern (Earth) science

Inferring causality: Three strands of modern (Earth) science

- Real experiments

Bertini fresco of Galileo Galilei and Doge of Venice



Svante Arrhenius, 1909.
Print Collector/Getty Images / Getty Images

Earth Science Experiments Class Kit

Inferring causality: Three strands of modern (Earth) science

- Real experiments
- Earth system simulation models

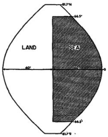


Fig. 1. Ocean-continent configuration of the model

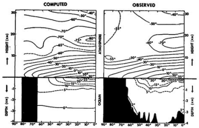
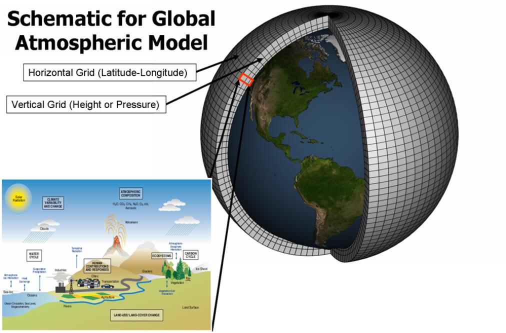


Fig. 2. Zonal mean temperature of the joint ocean-atmosphere system, left-hand side. This distribution, which is the average of two hemispheres, represents the time mean over two-sevenths of the period of the final stage of the time integration. The right-hand side shows the observed distribution in the Northern Hemisphere. The atmospheric part represents the zonally averaged, annual mean temperature. The oceanic part is based on a cross section for the western North Atlantic from Sverdrup *et al.* (1942).

First coupled climate model: Manabe, S., and K. Bryan, 1969: Climate calculations with a combined ocean-atmosphere model. J. Atmos. Sci., 26, 786–789



Inferring causality: Three strands of modern (Earth) science

- Real experiments
- Earth system simulation models
- Observational data analysis

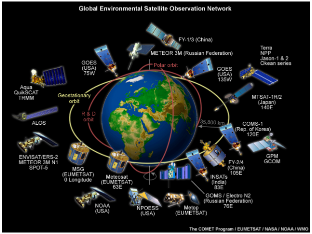
HONOLULU PRESSURE

	Years of data	December-February: Two quarters before Honolulu	December-February: One quarter before Honolulu	December-February: Same quarter	December-February: One quarter after Honolulu	December-February: Two quarters after Honolulu	March-May: Two quarters before Honolulu	March-May: One quarter before Honolulu	March-May: Same quarter	March-May: One quarter after Honolulu	March-May: Two quarters after Honolulu
		J-A	*S-N*	*D-F*	*M-M*	*J-A*	*S-N*	*D-F*	*M-M*	*J-A*	*S-N*
Iceland	37	0.00	−0.16	−0.18	−0.10	−0.04	+0.06	+0.06	−0.06	−0.16	+0.16
Alaska	19	−.14	−.06	−.70	−.26	−.04	−.08	−.14	−.52	−.30	+.50
C. Siberia	33	−.20	−.28	−.30	+.14	−.38	−.22	+.10	−.02	−.34	+.24
Vienna	35	−.20	+.08	−.04	+.10	−.14	−.10	−.14	+.18	−.40	+.08
Azores	38	+.16	−.14	+.11	+.16	+.20	−.34	−.06	−.14	+.06	−.14
Charleston	37	+.10	+.04	−.20	+.20	+.20	−.30	−.12	−.08	+.04	+.30
San Francisco	39	+.14	+.10	+.14	+.18	+.20	+.04	+.14	+.14	.00	+.40
Tokio	38	−.12	−.02	+.40	−.02	−.10	−.06	−.20	+.04	−.06	+.22
Cairo	36	+.18	−.02	+.20	+.34	+.10	−.20	−.10	+.28	+.08	+.18
Honolulu	39	−.16	+.12	+1.00	+.44	+.12	.00	+.44	+1.00	+.52	+.24
N. W. India	38	+.08	+.22	+.14	+.12	+.12	+.04	−.06	−.04	+.24	−.32
Port Darwin	38	+.46	+.48	+.38	+.02	+.12	+.14	+.08	−.06	−.38	−.44
Mauritius	38	+.30	−.02	−.26	−.06	−.18	+.08	−.30	+.10	−.22	−.12
Samoa	20	+.08	+.38	+.20	.00	+.04	+.02	+.36	+.48	+.10	+.60
S. E. Australia	38	+.28	+.30	+.32	−.10	+.20	+.06	+.14	−.14	−.16	−.30
Cape	38	+.16	−.20	+.30	+.10	.00	−.38	−.04	−.20	−.06	−.06
S. America	38	+.12	+.10	+.06	−.04	+.18	−.06	+.18	+.14	+.24	+.34
Temperature											
Dutch Harbor	35	.....	−.08	−.24	−.16	.....	.....	−.18	−.22	+.04	.....
Rain											
Peninsula (J-S)	39	−.42	.....	.....	.....	+.26	.....	.....	.....	+.30	.....
Java (O-F)	36	.....	.....	−.32	.....	.....	.....	−.06	.....	.....	.....

	Years of data	June-August: Two quarters before Honolulu	June-August: One quarter before Honolulu	June-August: Same quarter	June-August: One quarter after Honolulu	June-August: Two quarters after Honolulu	September-November: Two quarters before Honolulu	September-November: One quarter before Honolulu	September-November: Same quarter	September-November: One quarter after Honolulu	September-November: Two quarters after Honolulu
		D-F	*M-M*	*J-A*	*S-N*	*D-F*	*M-M*	*J-A*	*S-N*	*D-F*	*M-M*
Iceland	38	−0.06	+0.06	−0.34	−0.06	0.00	−0.10	[1] 0.20	−0.12	−0.20	−0.10
Alaska	20	−.12	−.14	−.28	+.42	+.22	+.16	−.24	−.22	−.14	−.02
C. Siberia	33	+.06	−.04	−.16	−.30	−.08	+.26	−.20	+.42	−.14	+.02
Vienna	35	+.10	+.12	−.26	+.12	+.10	+.16	−.03	−.02	−.04	+.26
Azores	38	+.08	−.24	+.20	+.14	−.08	−.04	−.04	+.20	−.02	−.02
Charleston	37	+.10	−.14	−.08	+.10	+.30	−.40	−.18	−.08	+.22	−.02
San Francisco	39	+.04	+.22	+.14	+.36	+.08	−.32	+.02	+.26	+.02	+.10
Tokio	38	−.22	+.08	+.40	−.06	−.24	+.24	+.30	−.02	+.04	−.02
Cairo	36	−.12	−.06	−.06	+.14	−.28	.00	+.08	+.02	−.20	−.10
Honolulu	39	+.12	+.52	+1.00	+.44	−.16	+.24	+.44	+1.00	+.12	.00
N. W. India	38	−.28	−.22	.00	−.42	−.30	+.30	+.10	−.32	−.22	−.14
Port Darwin	38	−.34	−.54	−.68	−.68	−.64	−.04	−.20	−.24	−.30	−.28
Mauritius	38	−.30	−.06	−.22	+.16	−.10	−.24	+.02	+.04	−.12	−.18
Samoa	20	+.46	+.42	+.74	+.22	+.50	+.20	+.44	+.42	+.56	+.28
S. E. Australia	38	−.24	−.46	−.28	−.38	−.54	−.26	−.04	−.04	−.30	−.18
Cape	38	−.28	−.16	−.08	+.04	−.42	−.10	+.02	−.14	−.22	+.06
S. America	38	.00	+.30	+.52	+.32	−.06	−.02	+.24	+.32	−.08	+.28
Temperature											
Dutch Harbor	35	.....	−.40	−.04	−.20	.....	+.02	−.12	−.22	−.02	.....
Rain											
Peninsula (J-S)	39	.....	.....	+.46	.....	.....	.....	+.02	.....	.....	.....
Java (O-F)	36	−.14	.....	.....	.....	+.40	.....	.....	.....	+.30	.....

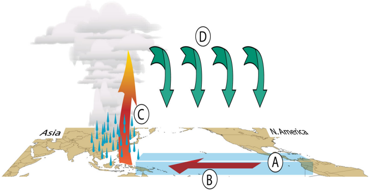
[1] Sign omitted in original.

Walker, G T. 1924. "Correlations in Seasonal Variations of Weather." IX. Mem. Ind. Metorol. Dept. 24: 53–84.



Observational (climate) data analysis: 1st attempt

- Walker circulation: Monthly surface pressure anomalies in the West Pacific (WPAC), surface air temperature anomalies in the Central Pacific (CPAC) and East Pacific (EPAC)



HONOLULU PRESSURE

	Years of data	December-February: Two quarters before Honolulu	December-February: One quarter before Honolulu	December-February: Same quarter	December-February: One quarter after Honolulu	December-February: Two quarters after Honolulu	March-May: Two quarters before Honolulu	March-May: One quarter before Honolulu	March-May: Same quarter	March-May: One quarter after Honolulu	March-May: Two quarters after Honolulu
		J-A	*S-N*	*D-F*	*M-M*	*J-A*	*S-N*	*D-F*	*M-M*	*J-A*	*S-N*
Iceland	37	0.00	−0.16	−0.18	−0.10	−0.04	+0.06	+0.06	−0.06	−0.16	+0.16
Alaska	19	−.14	−.06	−.70	−.26	−.04	−.08	−.14	−.52	−.30	+.50
C. Siberia	33	−.20	−.28	−.30	+.14	−.38	−.22	+.10	−.02	−.34	+.24
Vienna	35	−.20	+.08	−.04	+.10	−.14	−.10	−.14	+.18	−.40	+.08
Azores	38	+.16	−.14	+.14	+.16	+.20	−.34	−.06	−.14	+.06	−.14
Charleston	37	+.10	+.04	−.20	+.20	+.20	−.10	−.12	−.08	+.04	+.30
San Francisco	39	+.14	+.10	+.14	+.18	+.20	+.04	+.14	+.14	.00	+.40
Tokio	38	−.12	−.02	+.40	−.02	−.10	−.06	−.20	+.04	−.06	+.22
Cairo	36	+.18	−.02	+.30	+.34	+.10	−.20	−.10	+.28	+.08	+.18
Honolulu	39	−.16	+.12	+1.00	+.44	+.12	.00	+.44	+1.00	+.52	+.24
N. W. India	38	+.08	+.22	+.14	+.12	+.12	+.04	−.06	−.04	+.24	−.32
Port Darwin	38	+.46	+.48	+.38	+.02	+.12	+.14	+.08	−.06	−.38	−.44
Mauritius	38	+.30	−.02	−.26	−.06	−.18	+.08	−.30	+.10	−.22	−.12
Samoa	20	+.08	+.38	+.20	.00	+.04	+.02	+.36	+.48	+.10	+.60
S. E. Australia	38	+.28	+.30	+.32	−.10	+.20	+.06	+.14	−.14	−.16	−.30
Cape	38	+.16	−.20	+.30	+.10	.00	−.38	−.04	−.20	−.06	−.06
S. America	38	+.12	+.10	+.06	−.04	+.18	−.06	+.18	+.14	+.24	+.34
Temperature											
Dutch Harbor	35	......	−.08	−.24	−.16	......	......	−.18	−.22	+.04	......
Rain											
Peninsula (J-S)	39	−.42	......	......	......	+.26	......	......	......	+.30	......
Java (O-F)	36	......	......	−.32	......	......	......	−.06	......	......	......

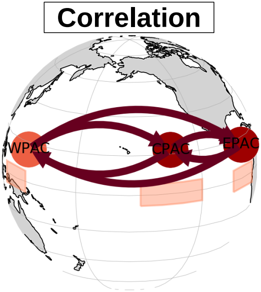
	Years of data	June-August: Two quarters before Honolulu	June-August: One quarter before Honolulu	June-August: Same quarter	June-August: One quarter after Honolulu	June-August: Two quarters after Honolulu	September-November: Two quarters before Honolulu	September-November: One quarter before Honolulu	September-November: Same quarter	September-November: One quarter after Honolulu	September-November: Two quarters after Honolulu
		D-F	*M-M*	*J-A*	*S-N*	*D-F*	*M-M*	*J-A*	*S-N*	*D-F*	*M-M*
Iceland	38	−0.06	+0.06	−0.34	−0.06	0.00	−0.10	[1]0.20	−0.12	−0.20	−0.10
Alaska	20	−.12	−.14	−.28	+.42	+.22	+.16	−.24	−.22	−.14	−.02
C. Siberia	33	+.06	−.04	−.16	−.30	−.08	+.26	−.20	+.42	−.14	+.02
Vienna	35	+.10	+.12	−.26	+.12	+.10	+.16	−.08	−.02	−.04	+.26
Azores	38	+.08	−.24	+.20	+.14	−.08	−.04	−.04	+.20	−.02	−.02
Charleston	37	+.10	−.14	−.08	+.10	+.30	−.40	−.18	−.08	+.22	−.02
San Francisco	39	+.04	+.22	+.14	+.36	+.08	−.32	+.02	+.28	+.02	+.10
Tokio	38	−.22	+.08	+.40	−.06	−.24	+.24	+.30	−.02	+.04	−.02
Cairo	36	−.12	−.06	−.06	+.14	−.28	.00	+.08	+.02	−.20	−.10
Honolulu	39	+.12	+.52	+1.00	+.44	−.16	+.24	+.44	+1.00	+.12	.00
N. W. India	38	−.28	−.22	.00	−.42	−.30	+.30	+.10	−.32	−.22	−.14
Port Darwin	38	−.34	−.54	−.66	−.68	−.64	−.04	−.20	−.24	−.30	−.26
Mauritius	38	−.30	−.06	−.22	+.16	−.10	−.24	+.02	+.04	−.12	−.18
Samoa	20	+.46	+.42	+.74	+.22	+.50	+.20	+.44	+.42	+.56	+.28
S. E. Australia	38	−.24	−.46	−.28	−.38	−.54	−.26	−.04	−.04	−.30	−.18
Cape	38	−.28	−.16	−.08	+.04	−.42	−.10	+.02	−.14	−.22	+.06
S. America	38	.00	+.30	+.52	+.32	−.06	−.02	+.24	+.32	−.08	+.28
Temperature											
Dutch Harbor	35	......	−.40	−.04	−.20	......	+.02	−.12	−.22	−.02	......
Rain											
Peninsula (J-S)	39	......	......	+.46	......	......	......	+.02	......	......	......
Java (O-F)	36	−.14	......	......	......	+.40	......	......	......	+.30	......

[1] Sign omitted in original.

Walker (1924)

Observational (climate) data analysis: 1st attempt

- Walker circulation: Monthly surface pressure anomalies in the West Pacific (WPAC), surface air temperature anomalies in the Central Pacific (CPAC) and East Pacific (EPAC)
- All three regions are strongly lag-correlated with each other 'in all directions'



Walker (1924)

Causal inference: 1st attempt

S. Wright, Correlation and Causation, J. of Agricultural Res. 10(7), 1921

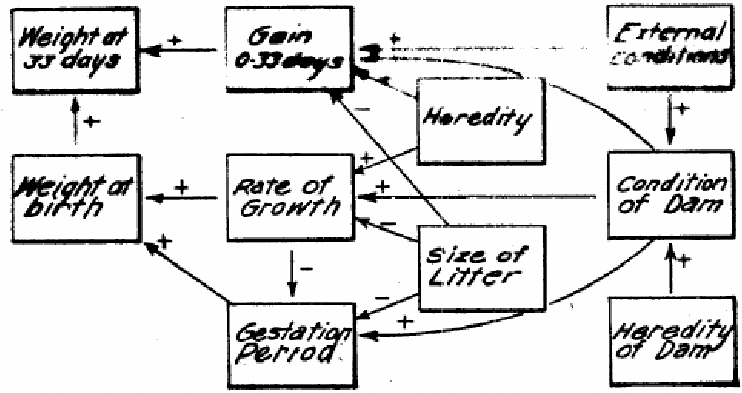
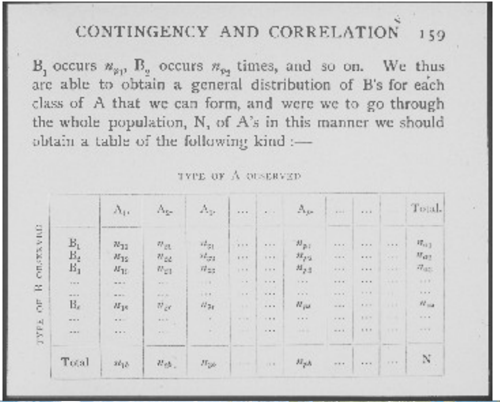


FIG. 1.—Diagram illustrating the interrelations among the factors which determine the weight of guinea pigs at birth and at weaning (33 days).

Causality and statistics

Karl Pearson's "Grammar of Science" (1911): "Beyond such discarded fundamentals as 'matter' and 'force' lies still another fetish amidst the inscrutable arcana of modern science, namely, the category of cause and effect."



CONTINGENCY AND CORRELATION 159

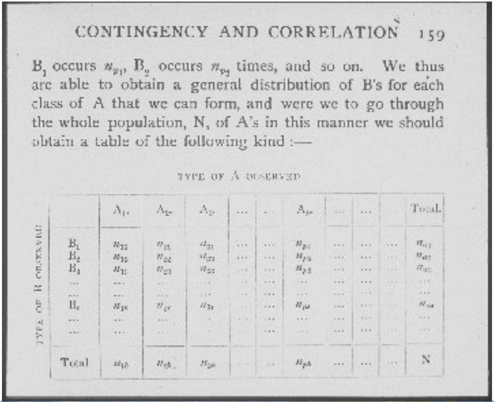
B_1 occurs n_{p1}, B_2 occurs n_{p2} times, and so on. We thus are able to obtain a general distribution of B's for each class of A that we can form, and were we to go through the whole population, N, of A's in this manner we should obtain a table of the following kind :—

TYPE OF A OBSERVED

TYPE OF B OBSERVED	A_1	A_2	A_3	…	…	A_p	…	…		Total
B_1	n_{11}	n_{21}	n_{31}	…	…	n_{p1}	…	…	…	n_{a1}
B_2	n_{12}	n_{22}	n_{32}	…	…	n_{p2}	…	…	…	n_{a2}
B_3	n_{13}	n_{23}	n_{33}	…	…	n_{p3}	…	…	…	n_{a3}
…	…	…	…	…	…	…	…	…	…	…
…	…	…	…	…	…	…	…	…	…	…
B_s	n_{1s}	n_{2s}	n_{3s}	…	…	n_{ps}	…	…	…	n_{as}
…	…	…	…	…	…	…	…	…	…	…
…	…	…	…	…	…	…	…	…	…	…
Total	n_{1b}	n_{2b}	n_{3b}	…	…	n_{pb}	…	…	…	N

Causality and statistics

Karl Pearson's "Grammar of Science" (1911): "Beyond such discarded fundamentals as 'matter' and 'force' lies still another fetish amidst the inscrutable arcana of modern science, namely, the category of cause and effect." **Correlation is not causation!**



CONTINGENCY AND CORRELATION 159

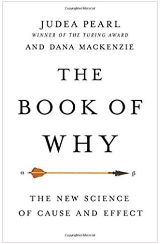
B_1 occurs n_{s1}, B_2 occurs n_{s2} times, and so on. We thus are able to obtain a general distribution of B's for each class of A that we can form, and were we to go through the whole population, N, of A's in this manner we should obtain a table of the following kind :—

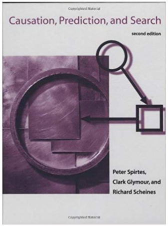
TYPE OF A OBSERVED

TYPE OF B OBSERVED	A_1.	A_2.	A_3.	...	...	A_p.	...	...		Total.
B_1	n_{11}	n_{21}	n_{31}	...	...	n_{p1}	...	...	...	n_{o1}
B_2	n_{12}	n_{22}	n_{32}	...	...	n_{p2}	...	...	...	n_{o2}
B_3	n_{13}	n_{23}	n_{33}	...	...	n_{p3}	...	...	...	n_{o3}
...	...	...	...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...	...	...	...
B_s	n_{1s}	n_{2s}	n_{3s}	...	...	n_{ps}	...	...	...	n_{os}
...	...	...		...	...	...	...	...	...	...
...	...	...		...	...	...	...	...	...	...
Total	n_{1o}	n_{2o}	n_{3o}	...	...	n_{po}	...	...	...	N

Causality and statistics

Correlation is not causation! Well... not generally, but...
[Pearl, 2000, Pearl and Mackenzie, 2018, Spirtes et al., 2000]

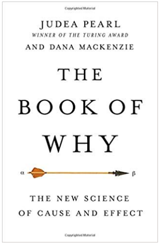


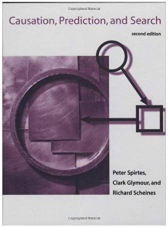





Causality and statistics

Causal inference is about identifying assumptions and methods that enable to learn causal relations from observational data





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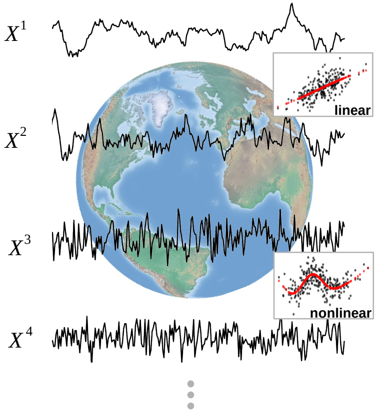
Causal discovery problem

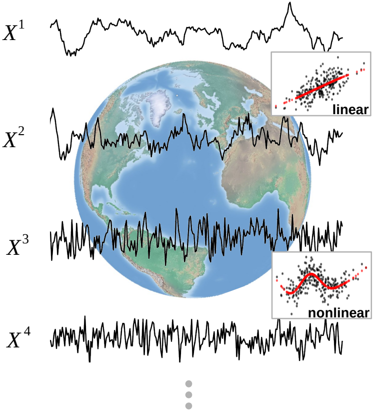
Causal discovery problem

Time series dataset



Causal discovery problem

Time series dataset



Causal model

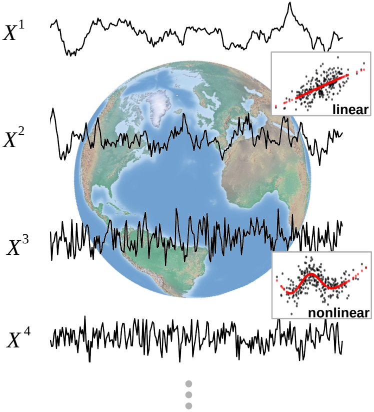
$$X_t^1 = f^1(\mathbf{X}_t^-, \eta_t^1)$$

$$\mathbf{X}_t^- = (\mathbf{X}_{t-1}, \mathbf{X}_{t-2}, \ldots)$$

Noise η

Causal discovery problem

Time series dataset



Causal model

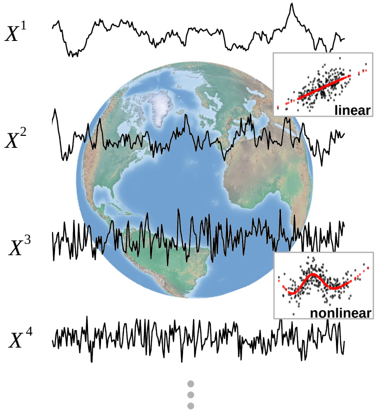
$$X_t^1 = f^1(\mathbf{X}_t^-, \eta_t^1)$$
$$X_t^2 = f^2(\mathbf{X}_t^-, \eta_t^2)$$
$$X_t^3 = f^3(\mathbf{X}_t^-, \eta_t^3)$$
$$X_t^4 = f^4(\mathbf{X}_t^-, \eta_t^4)$$
$$\vdots$$

$$\mathbf{X}_t^- = (\mathbf{X}_{t-1}, \mathbf{X}_{t-2}, \ldots)$$

Noise η

Causal discovery problem

Time series dataset



Causal model

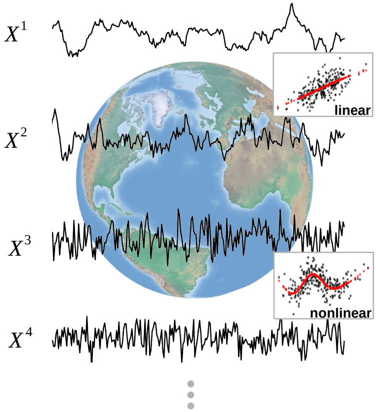
$$X_t^1 = f^1(\mathbf{X}_{t+1}^-, \eta_t^1)$$
$$X_t^2 = f^2(\mathbf{X}_{t+1}^-, \eta_t^2)$$
$$X_t^3 = f^3(\mathbf{X}_{t+1}^-, \eta_t^3)$$
$$X_t^4 = f^4(\mathbf{X}_{t+1}^-, \eta_t^4)$$
$$\vdots$$

$$\mathbf{X}_{t+1}^- = (\mathbf{X}_t, \mathbf{X}_{t-1}, \ldots)$$

Noise η

Causal discovery problem

Time series dataset



Causal model

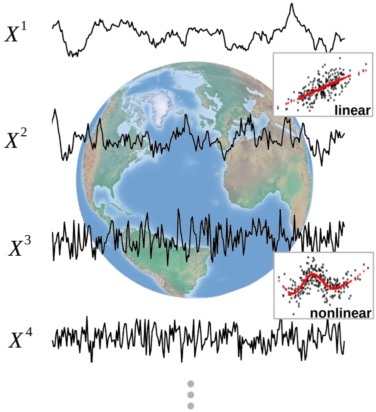
$$X_t^1 = f^1(\mathcal{P}_{X_t^1}, \eta_t^1)$$
$$X_t^2 = f^2(\mathcal{P}_{X_t^2}, \eta_t^2)$$
$$X_t^3 = f^3(\mathcal{P}_{X_t^3}, \eta_t^3)$$
$$X_t^4 = f^4(\mathcal{P}_{X_t^4}, \eta_t^4)$$
$$\vdots$$

Parents $\mathcal{P}_{\cdot} \subset \mathbf{X}_{t+1}^-$

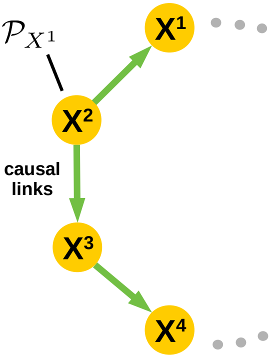
Noise η

Causal discovery problem

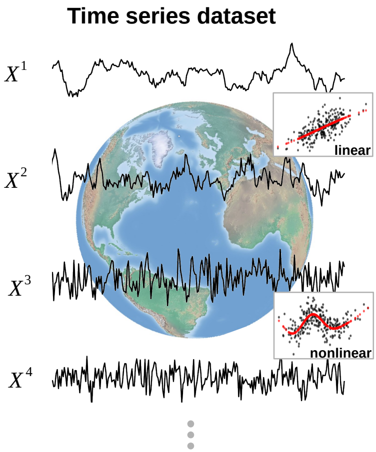
Time series dataset

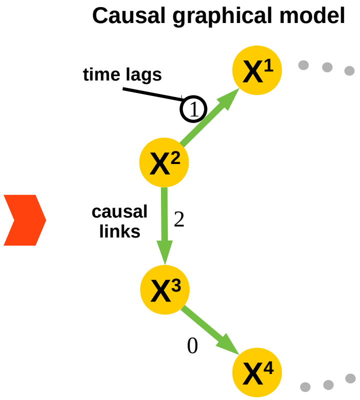


Causal graphical model

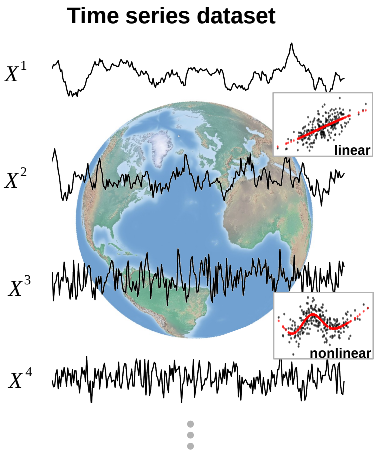


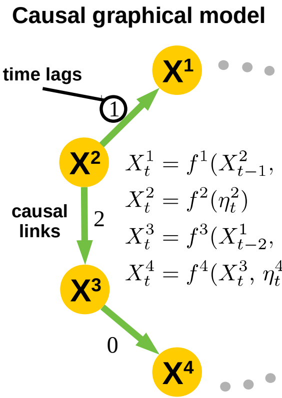
Causal discovery problem





Causal discovery problem

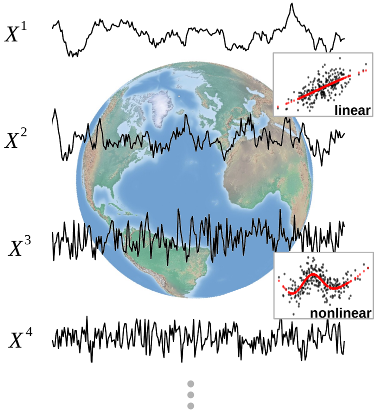




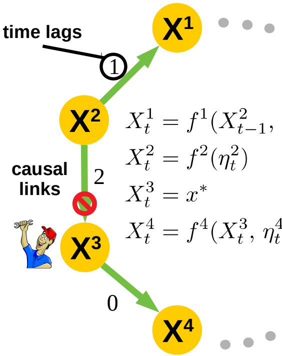
$$X^1_t = f^1(X^2_{t-1}, \eta^1_t)$$
$$X^2_t = f^2(\eta^2_t)$$
$$X^3_t = f^3(X^1_{t-2}, \eta^3_t)$$
$$X^4_t = f^4(X^3_t, \eta^4_t)$$

Causal discovery problem

Time series dataset



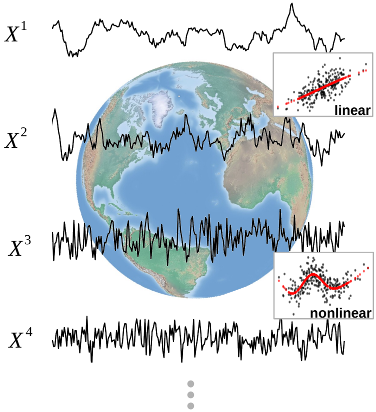
Causal effect (Pearl)



$$X_t^1 = f^1(X_{t-1}^2, \eta_t^1)$$
$$X_t^2 = f^2(\eta_t^2)$$
$$X_t^3 = x^*$$
$$X_t^4 = f^4(X_t^3, \eta_t^4)$$

Causal discovery problem

Time series dataset



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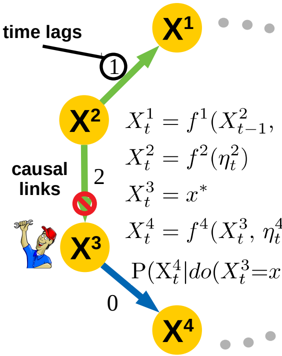
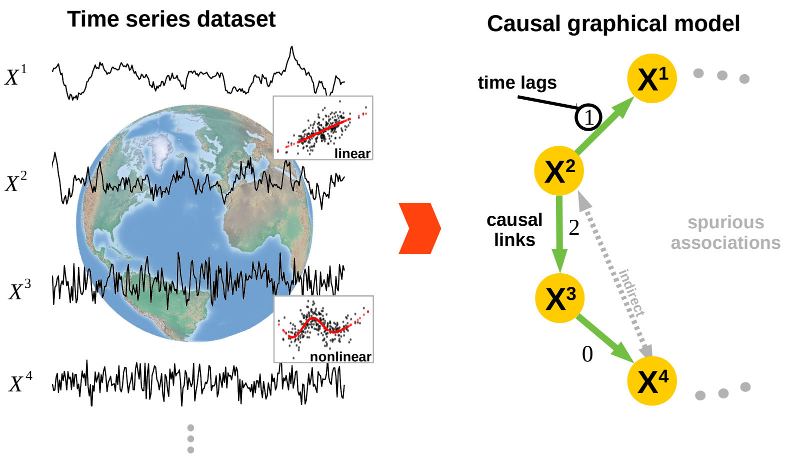


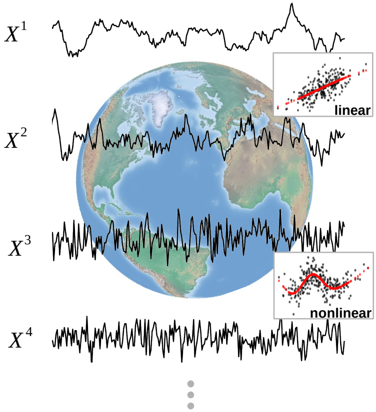
$$X_t^1 = f^1(X_{t-1}^2, \eta_t^1)$$
$$X_t^2 = f^2(\eta_t^2)$$
$$X_t^3 = x^*$$
$$X_t^4 = f^4(X_t^3, \eta_t^4)$$
$$\mathrm{P}(X_t^4 | do(X_t^3 = x^*))$$

Causal discovery problem

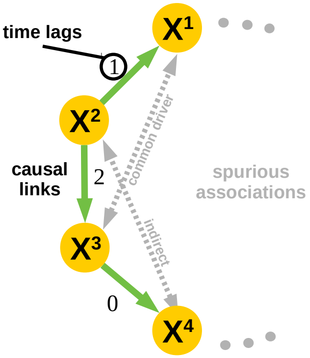


Causal discovery problem

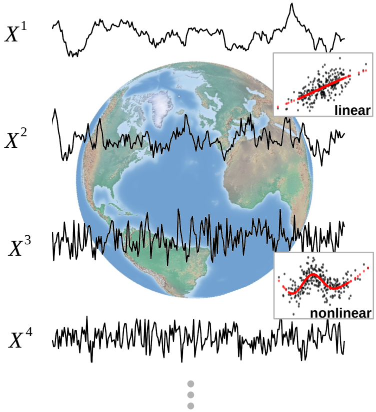
Time series dataset

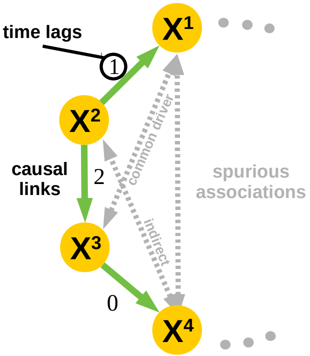


Causal graphical model



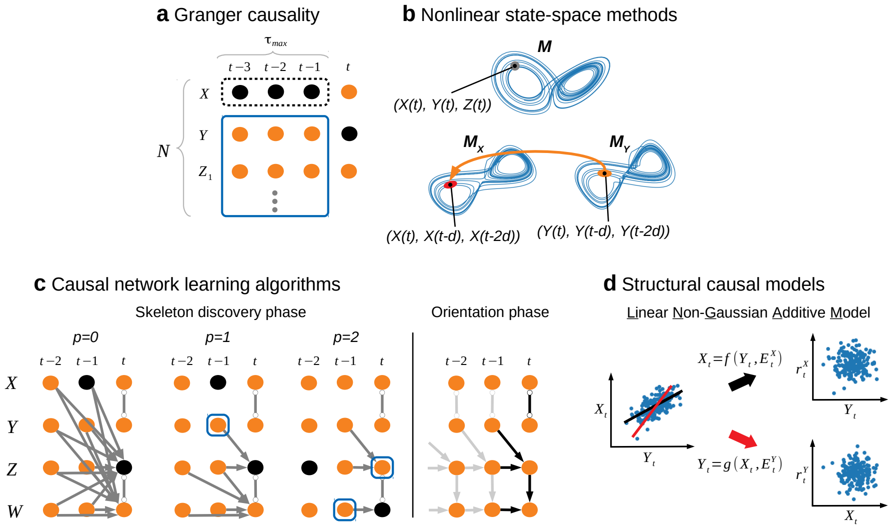
Causal discovery problem





State of the art

State of the art: Runge et al., NatComm Perspective 2019



Challenges for causal inference

Challenges for causal inference: Runge et al., NatComm 2019

Challenges

Process:

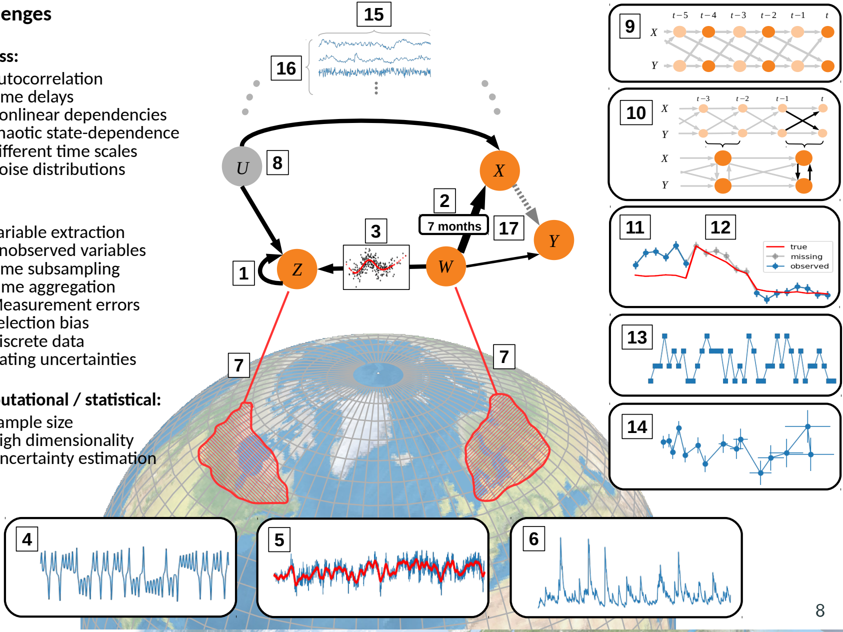
1 Autocorrelation
2 Time delays
3 Nonlinear dependencies
4 Chaotic state-dependence
5 Different time scales
6 Noise distributions

Data:

7 Variable extraction
8 Unobserved variables
9 Time subsampling
10 Time aggregation
11 Measurement errors
12 Selection bias
13 Discrete data
14 Dating uncertainties

Computational / statistical:

15 Sample size
16 High dimensionality
17 Uncertainty estimation

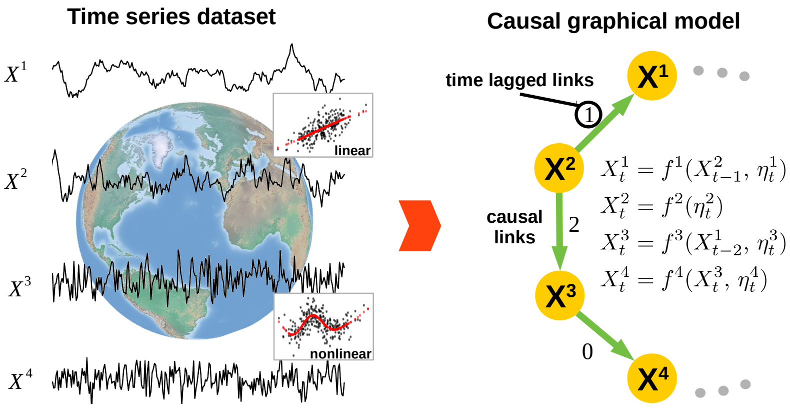


PCMCI causal discovery framework

PCMCI causal discovery framework

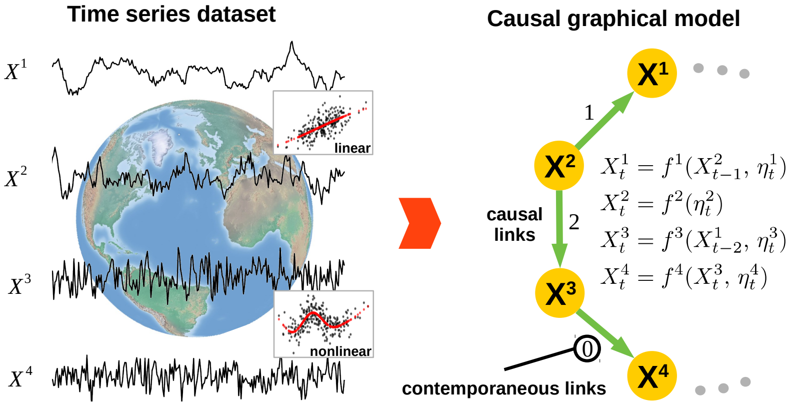
PCMCI: Assumes time-lags (Runge et al. *Science Advances* 2019)

`Tigramite` 4.2 python package



PCMCI causal discovery framework

PCMCI+: Allows time-lags and contemporaneous links (Runge (2020) https://arxiv.org/abs/2003.03685)



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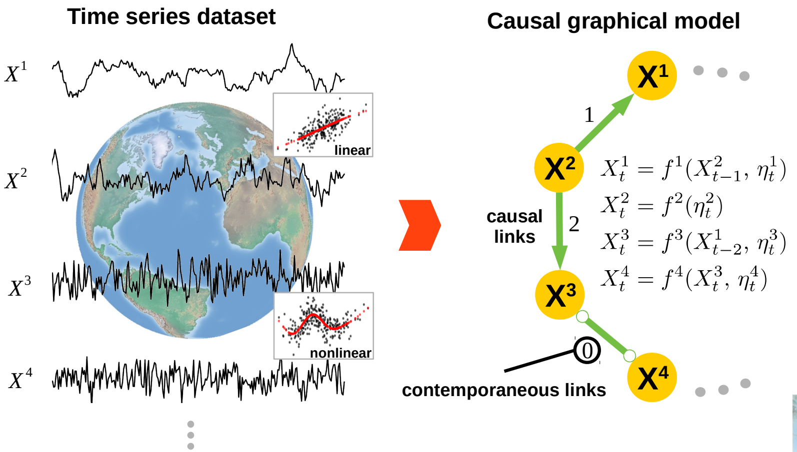

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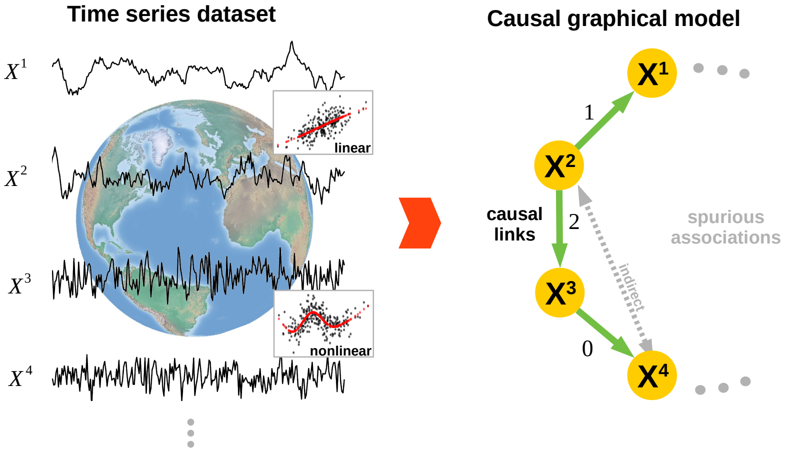

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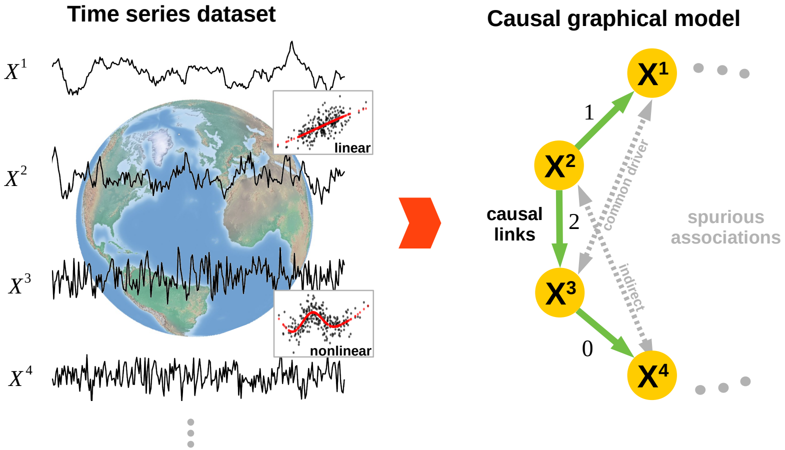

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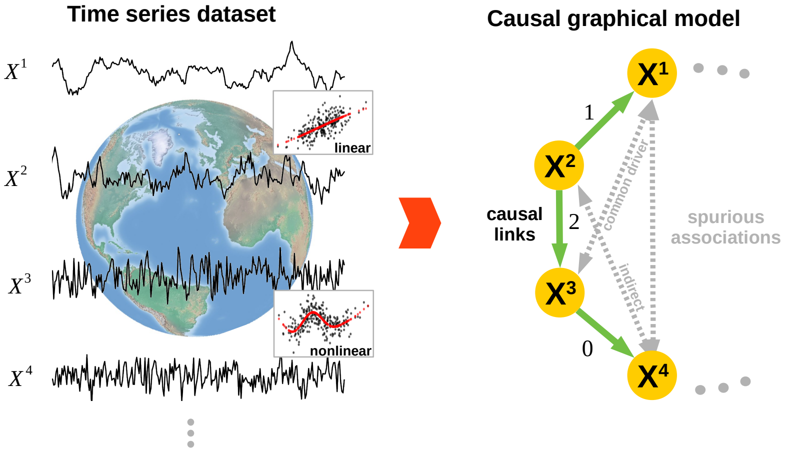
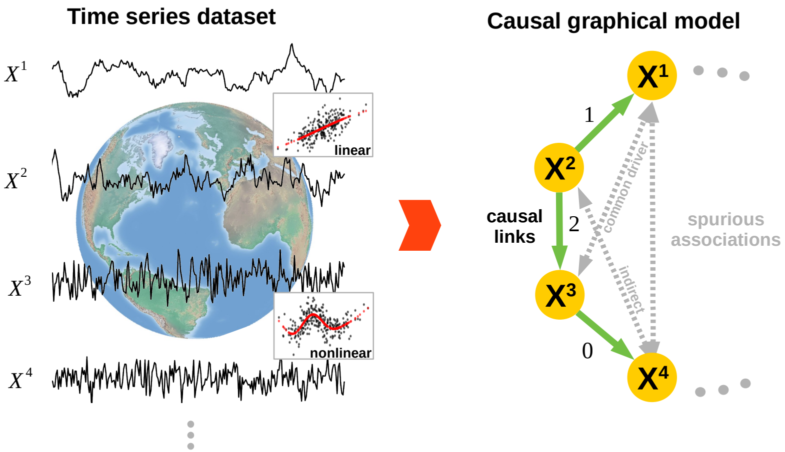


PCMCI causal discovery framework

PCMCI+: Allows time-lags and contemporaneous links (Runge (2020) https://arxiv.org/abs/2003.03685)

Enabling assumptions: Faithfulness, Markovity, Causal Sufficiency, ~~no contemporaneous effects~~, and stationarity

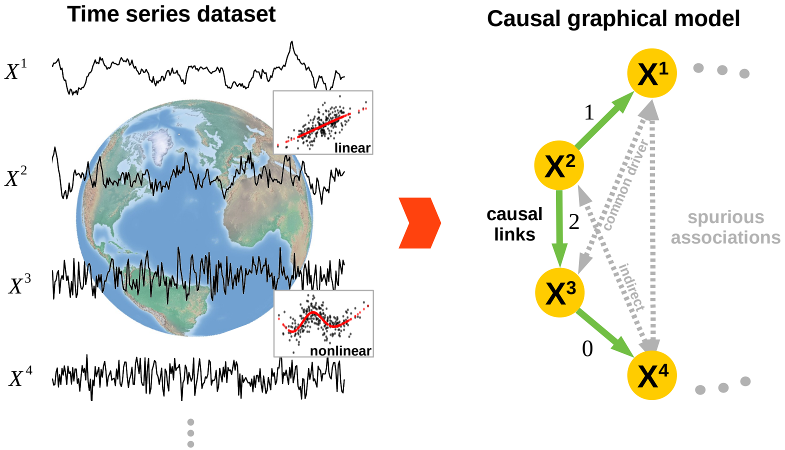


PCMCI causal discovery framework

PCMCI+: Allows time-lags and contemporaneous links (Runge (2020) https://arxiv.org/abs/2003.03685)

Enabling assumptions: Faithfulness, Markovity, Causal Sufficiency, ~~no contemporaneous effects~~, and stationarity

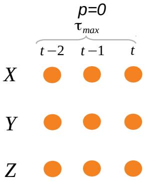
Nonlinearity and noise distributions handled by flexible conditional inde-



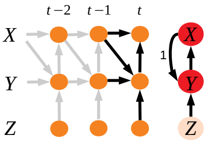
Problems with PC algorithm

PC algorithm skeleton discovery phase can use different conditional independence (CI) tests: Partial Correlation $\rho(X; Y|\mathbf{S})$, Conditional Mutual Information $I(X; Y|\mathbf{S})$, etc.

Skeleton discovery phase



True process

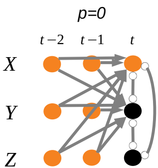


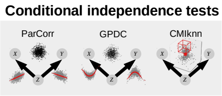
Problems with PC algorithm

Detection power for detecting $X \not\perp\!\!\!\perp Y \mid \mathbf{S}$ depends on:

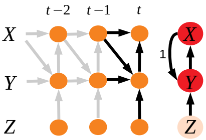
Sample size

Skeleton discovery phase





True process



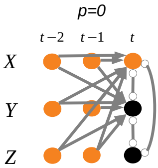
Problems with PC algorithm

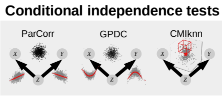
Detection power for detecting $X \not\perp\!\!\!\perp Y \mid \mathbf{S}$ depends on:

Sample size

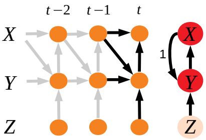
Significance level $\alpha_{\rm PC}$

Skeleton discovery phase





True process



Problems with PC algorithm

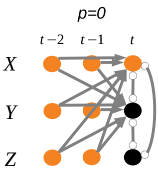
Detection power for detecting $X \not\perp\!\!\!\perp Y \mid \mathbf{S}$ depends on:

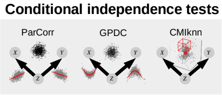
Sample size

Significance level α_{PC}

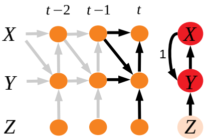
Condition dimension, cardinality of $|\mathbf{S}|$

Skeleton discovery phase





True process



Problems with PC algorithm

Detection power for detecting $X \not\perp\!\!\!\perp Y \mid \mathbf{S}$ depends on:

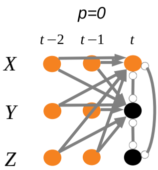
Sample size

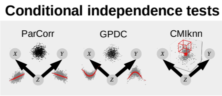
Significance level α_{PC}

Condition dimension, cardinality of $|\mathbf{S}|$

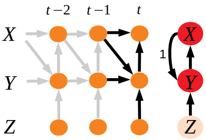
Effect size, i.e., magnitude of $I(X; Y|\mathbf{S})$

Skeleton discovery phase





True process



Problems with PC algorithm

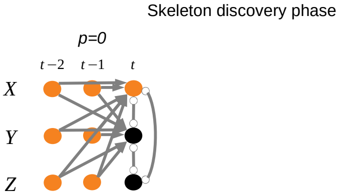
Detection power for detecting $X \not\perp\!\!\!\perp Y \mid \mathbf{S}$ depends on:

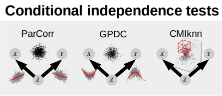
Sample size (given by dataset)

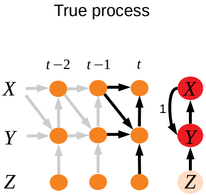
Significance level α_{PC} (hyperparameter, difficult to tune)

Condition dimension, cardinality of $|\mathbf{S}|$ (PC optimizes this)

Effect size, i.e., magnitude of $I(X; Y|\mathbf{S})$ (Problem addressed here)





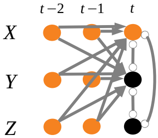


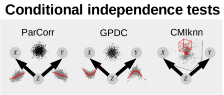
Problems with PC algorithm

Consider underlying linear model, here $I(Y_t; Z_t) = \frac{1}{2}\ln\left(1 + \frac{cVar(Z)}{Var(Y)}\right)$

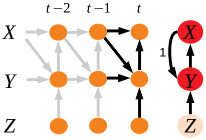
Skeleton discovery phase

p=0



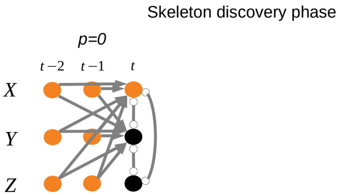


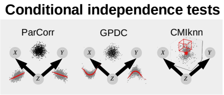
True process

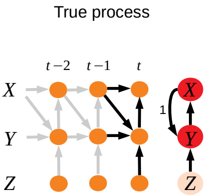


Problems with PC algorithm

Consider underlying linear model, here $I(Y_t; Z_t) = \frac{1}{2} \ln \left(1 + \frac{cVar(Z)}{Var(Y)}\right)$
$\Longrightarrow$ Effect size is small if $Var(Y) \gg Var(Z)$, and lead to false removal.







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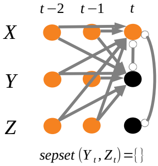
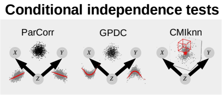
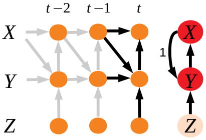
Skeleton discovery phase

p=0



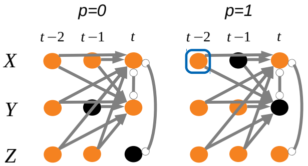


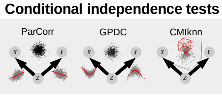
True process



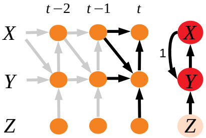
Problem: PC iterates through **all** adjacent conditions $\mathbf{S}$ and link is removed if $\boxed{\min_{\mathbf{S}} I(X; Y|\mathbf{S}) < I_{\alpha_{\mathrm{PC}}}}$

Skeleton discovery phase





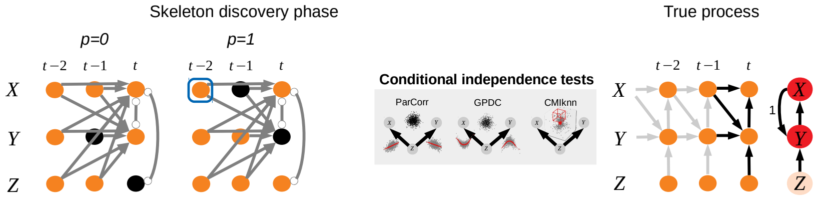
True process



Problems with PC algorithm

Problem: PC iterates through **all** adjacent conditions **S** and link is removed if $\boxed{\min_{\mathbf{S}} I(X;Y|\mathbf{S}) < I_{\alpha_{\rm PC}}}$

Generally: Effect size for a link $X_{t-\tau} \to Y_t$ is small when conditioning on parents of $X_{t-\tau}$ and large when conditioning on parents of Y_t, i.e., $I(X;Y|\mathcal{P}(X)) \ll I(X;Y|\mathcal{P}(Y))$ [Runge et al., 2012a]



Problems with PC algorithm

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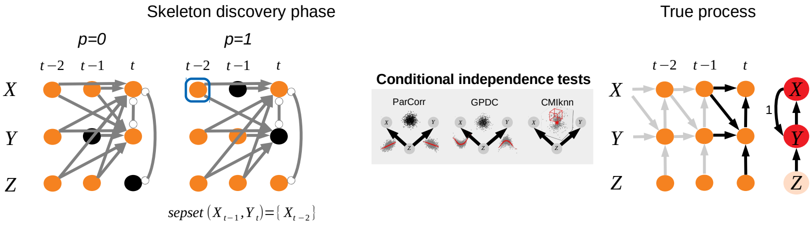
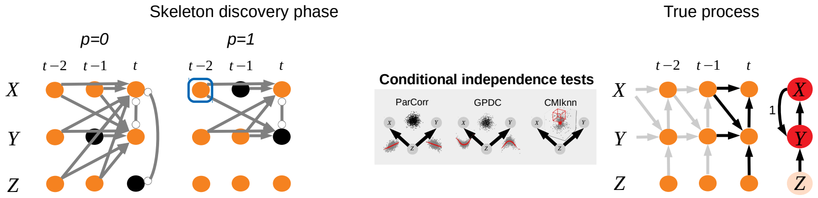


Problems with PC algorithm

Problem: PC iterates through **all** adjacent conditions $\mathbf{S}$ and link is removed if $\boxed{\min_{\mathbf{S}} I(X;Y|\mathbf{S}) < I_{\alpha_{\rm PC}}}$

Generally: Effect size for a link $X_{t-\tau} \to Y_t$ is small when conditioning on parents of $X_{t-\tau}$ and large when conditioning on parents of Y_t, i.e., $I(X;Y|\mathcal{P}(X)) \ll I(X;Y|\mathcal{P}(Y))$ [Runge et al., 2012a]



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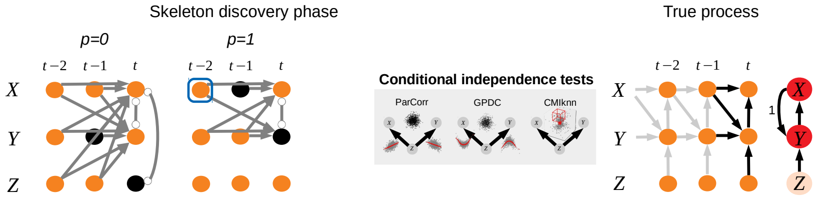
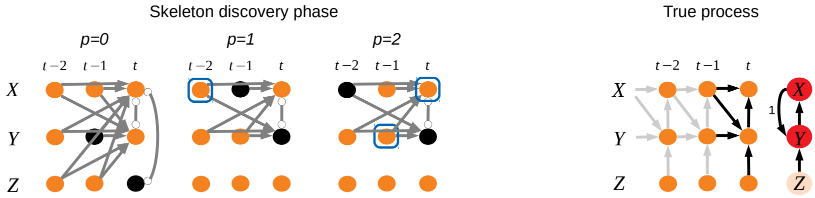
PC likely iterates through such conditions and removes true links.



Problems with PC algorithm

Removed links are not used as conditions for larger p.



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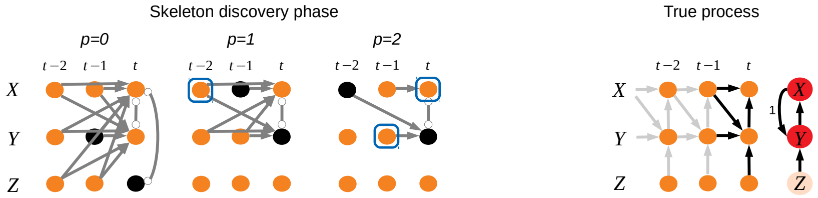
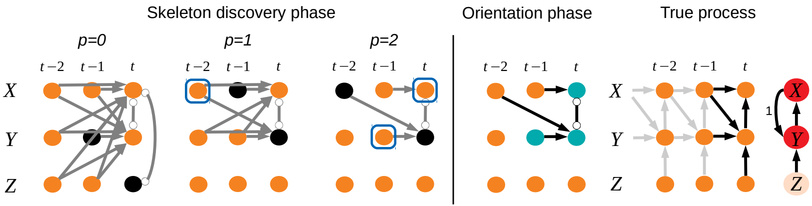


Problems with PC algorithm

$\implies$ False positives (incorrect links)! Then orientation phase also suffers from wrong sepsets.



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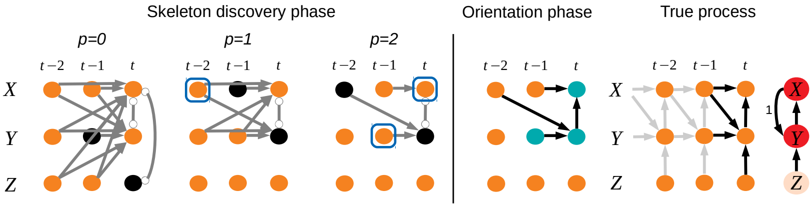
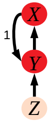


PCMCI$^+$ causal discovery

Consider underlying true process graph.

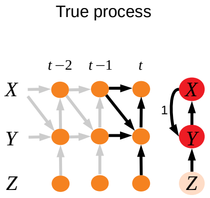
True process



PCMCI$^+$ causal discovery

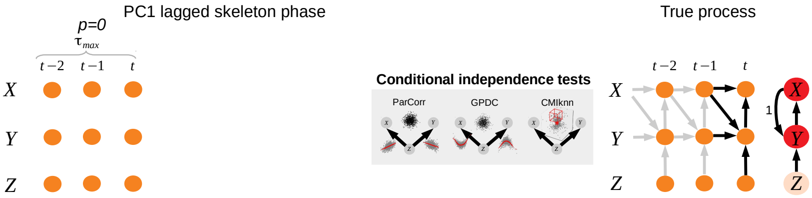
Consider underlying true process graph.

Associated time series graph.



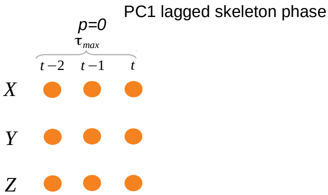
PCMCI$^+$ causal discovery

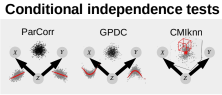
PCMCI$^+$ has 3 phases: PC_1 lagged phase, MCI contemporaneous phase, Orientation phase.

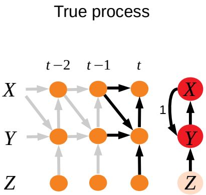


PCMCI$^+$ causal discovery

PC$_1$ lagged phase differs from PC algorithm twofold:







PCMCI$^+$ causal discovery

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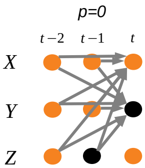
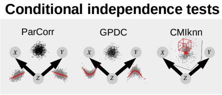
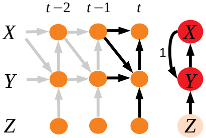
(1) **S** iterates through **lagged** links only,

PC1 lagged skeleton phase





True process



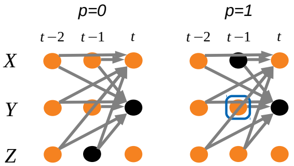
PCMCI$^+$ causal discovery

PC_1 lagged phase differs from PC algorithm twofold:

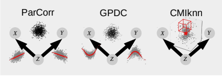
(1) $\mathbf{S}$ iterates through **lagged** links only,

(2) $\mathbf{S} = \{\mathcal{A}(X_t^j)\}_{l=1}^{p}$ for every cardinality p: lagged conditions with largest association with X_t^j.

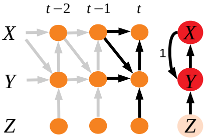
PC1 lagged skeleton phase



Conditional independence tests



True process



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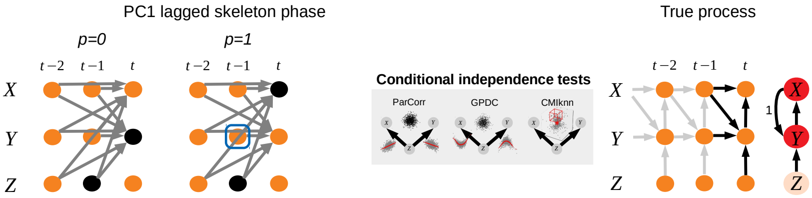
$\Longrightarrow$ much less likely to condition on "effect-size weakening" parents of $X^i_{t-\tau}$



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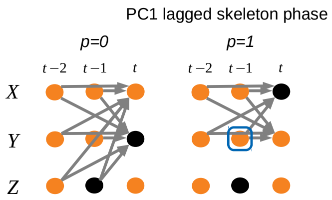
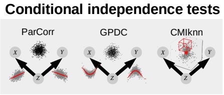
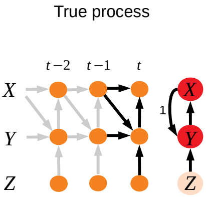
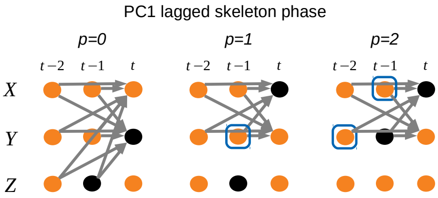


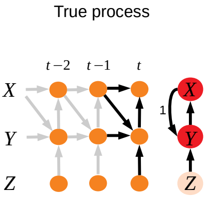




PCMCI^{+} causal discovery

PC$_1$ converges to lagged parents plus parents of contemporaneous ancestors: $\widehat{\mathcal{B}}_t^-(X_t^j)$.





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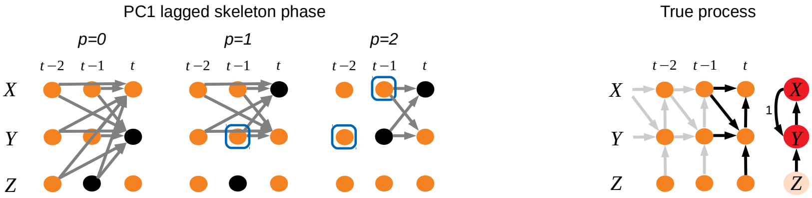
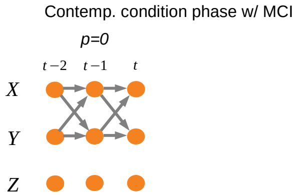
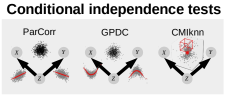
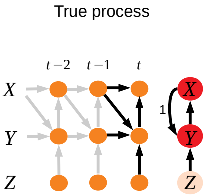


PCMCI$^+$ causal discovery

MCI contemporaneous phase is first initialized with lagged links $\widehat{\mathcal{B}}_t^-(X_t^j)$ and all contemporaneous links







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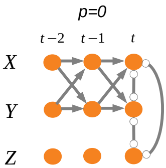
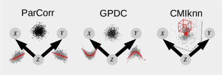
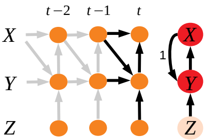
Contemp. condition phase w/ MCI



Conditional independence tests



True process

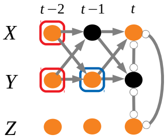


MCI phase iterates through **contemporaneous conditions** $\mathbf{S} \subseteq \mathcal{A}_t(X^j_t)$ with MCI tests:

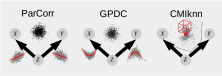
$$X^i_{t-\tau} \perp\!\!\!\perp X^j_t \mid \mathbf{S}, \widehat{\mathcal{B}}^-_t(X^j_t) \setminus \{X^i_{t-\tau}\}, \widehat{\mathcal{B}}^-_{t-\tau}(X^i_{t-\tau})$$

Contemp. condition phase w/ MCI

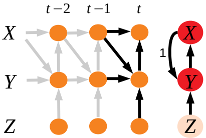
p=0



Conditional independence tests



True process



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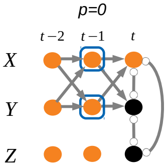
Contemp. condition phase w/ MCI



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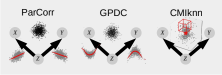

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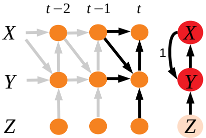
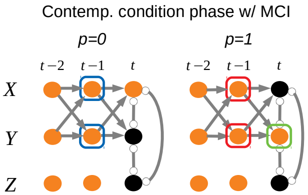
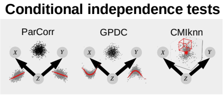
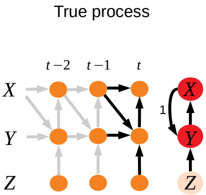


PCMCI^{+} causal discovery

Conditioning on **both** $\widehat{\mathcal{B}}^{-}_{t}(X^{j}_{t})$ and $\widehat{\mathcal{B}}^{-}_{t-\tau}(X^{i}_{t-\tau})$ has two important implications: (1) MCI effect size larger than PC effect size, (2) MCI tests well-calibrated (both discussed in paper)

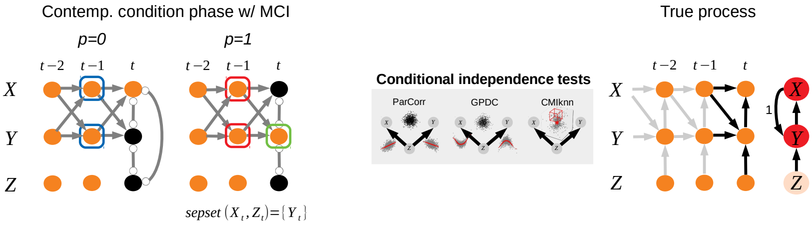






PCMCI$^+$ causal discovery

Spurious links due to contemporaneous drivers are removed and sepsets stored; converges much faster than PC algorithm, shorter runtimes.



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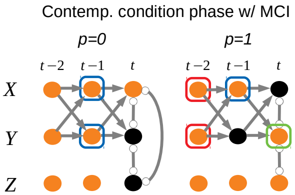

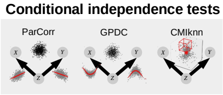

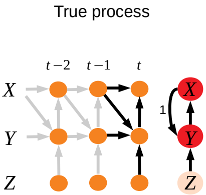

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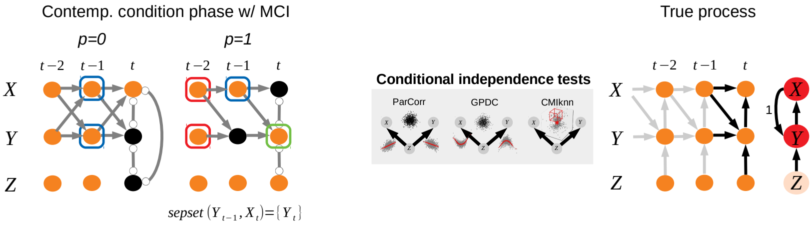
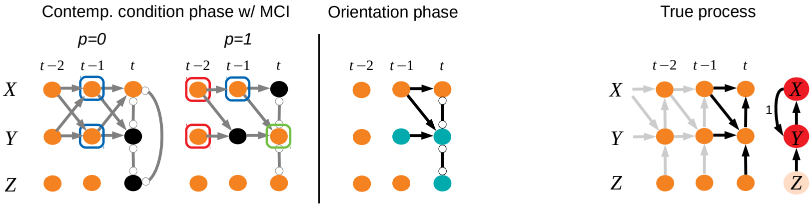


PCMCI$^+$ causal discovery

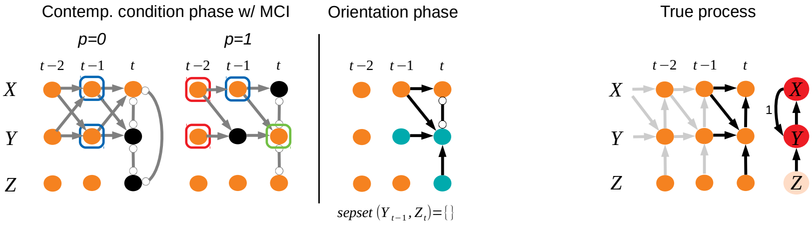
Orientation phase as for PC algorithm.



PCMCI$^+$ causal discovery

Orientation phase as for PC algorithm.

Collider/unshielded triple rule: Y_t is <u>not</u> in $sepset(Y_{t-1}, Z_t) \implies$ orient $Z_t \to Y_t$



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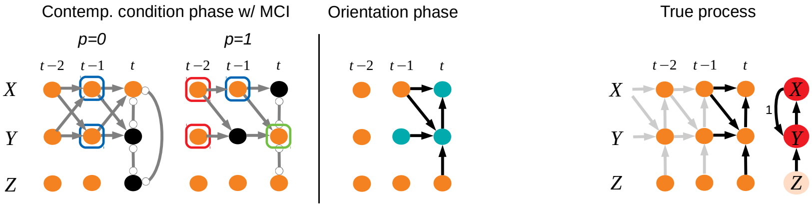
Rule R1: Orient remaining unshielded tripls in other direction



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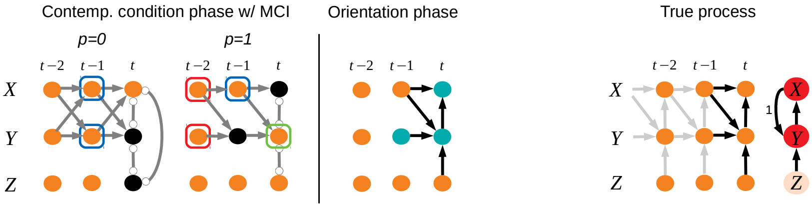
Further rules that make use of acyclicity assumption (see paper).



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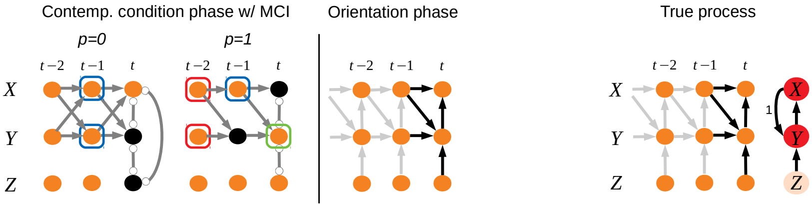
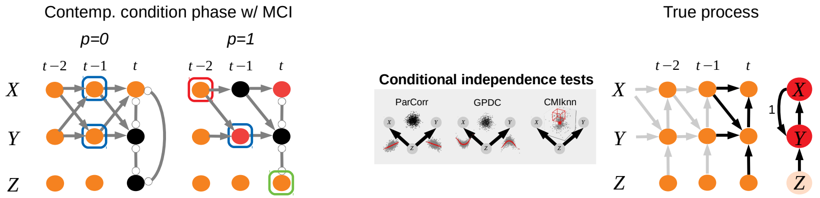
Further rules that make use of acyclicity assumption (see paper). PCMCI$^+$ converges, links are repeated by assuming stationarity,



PCMCI$^+$ causal discovery

In paper:

- Asymptotical consistency: PCMCI$^+$ is sound and complete



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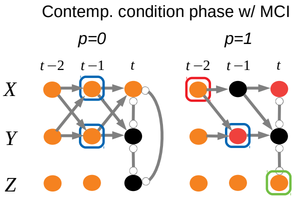
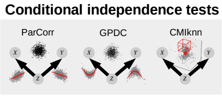
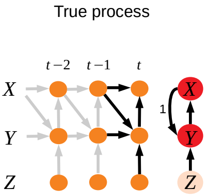
- Order independence (with majority rule in collider phase and conflict resolution)







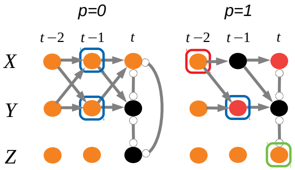
PCMCI$^+$ causal discovery

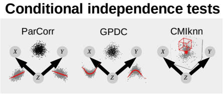
In paper:

- Asymptotical consistency: PCMCI$^+$ is sound and complete
- Order independence (with majority rule in collider phase and conflict resolution)
- Conjecture: Effect size is always greater than that of PC algorithm

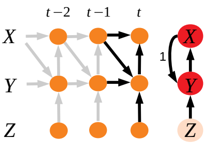
$$\min_{\mathbf{S}\ \text{in PCMCI}^+} I(X^i_{t-\tau}\ ;\ X^j_t \mid \mathbf{S}, \widehat{\mathcal{B}}_j, \widehat{\mathcal{B}}_i) > \min_{\mathbf{S}'\ \text{in PC}} I(X^i_{t-\tau}\ ;\ X^j_t \mid \mathbf{S}')$$

Contemp. condition phase w/ MCI





True process



PCMCI$^+$ causal discovery

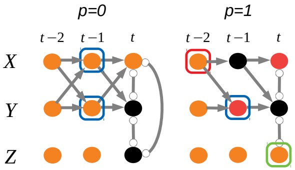
In paper:

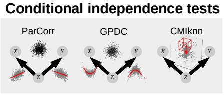
- Asymptotical consistency: PCMCI$^+$ is sound and complete
- Order independence (with majority rule in collider phase and conflict resolution)
- Conjecture: Effect size is always greater than that of PC algorithm

$$\min_{\mathbf{S}\ \text{in PCMCI}^+} I(X^i_{t-\tau}\ ;\ X^j_t \mid \mathbf{S}, \widehat{\mathcal{B}}_j, \widehat{\mathcal{B}}_i) > \min_{\mathbf{S}'\ \text{in PC}} I(X^i_{t-\tau}\ ;\ X^j_t \mid \mathbf{S}')$$

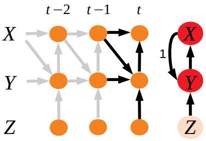
- MCI tests are well-calibrated also for autocorrelated data [Runge et al., 2019b]

Contemp. condition phase w/ MCI



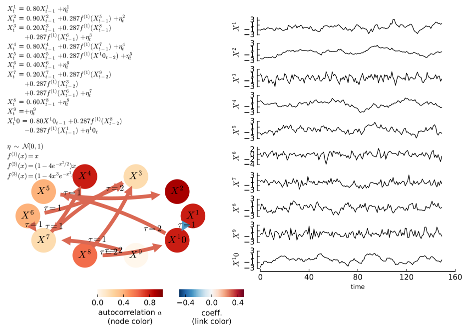


True process

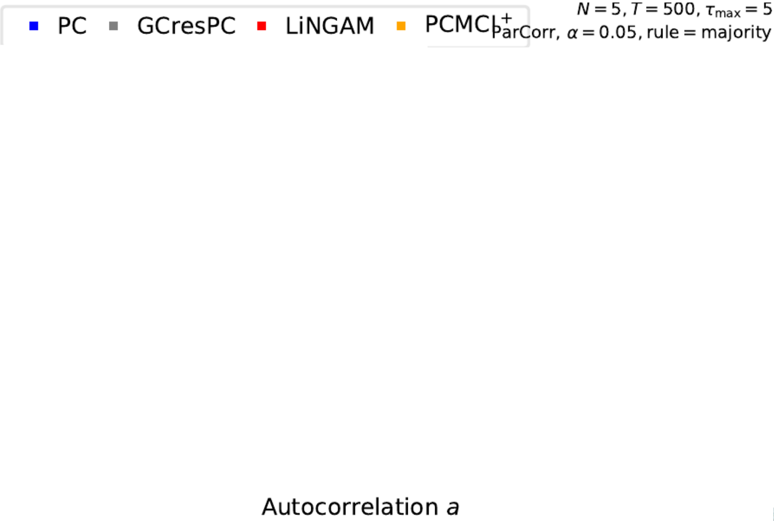


Numerical experiments

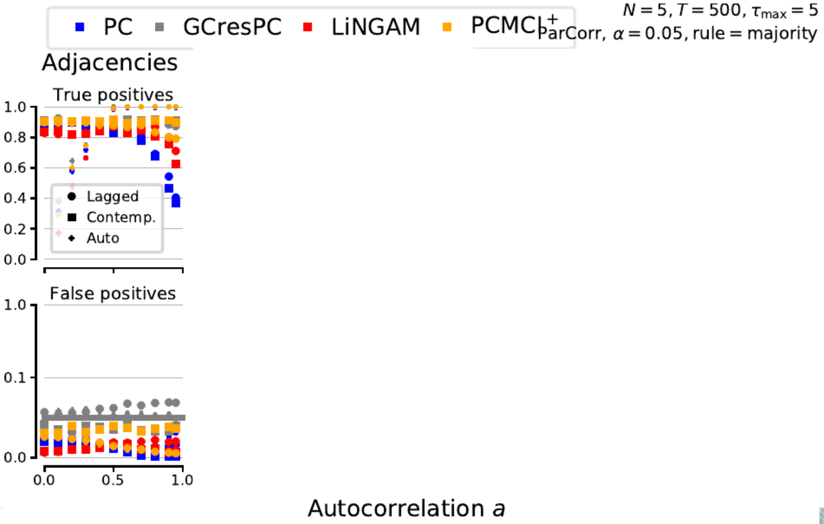
- random coupling topologies, time lags, linear/nonlinear
- 30% contemporaneous links, coefficients $\pm[0.1..0.5]$
- different autocorrelations for variables
- $\tau_{\max} = 5$, $T = 500$, varying $N = 2..40$
- α_{PC} fixed, can be chosen via AIC



Comparing with PC algo, Granger causality + PC on residuals (GCresPC), LiNGAM

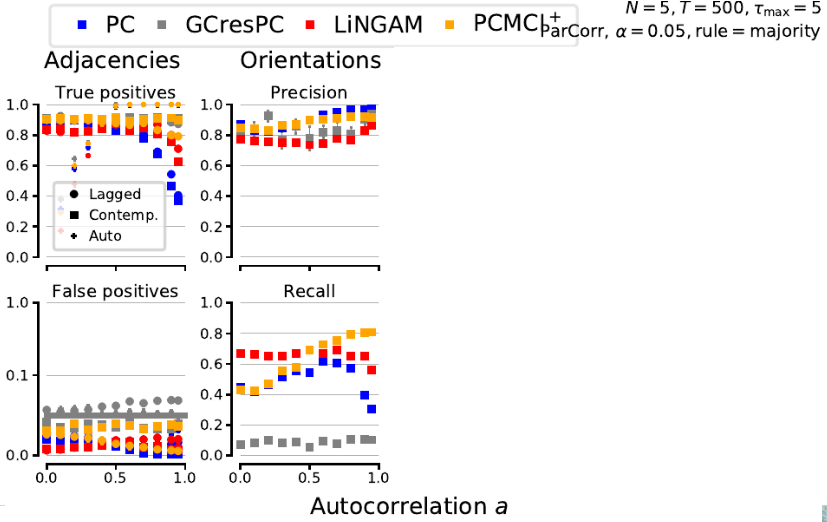


High adjacency detection rate, well-controlled false positives



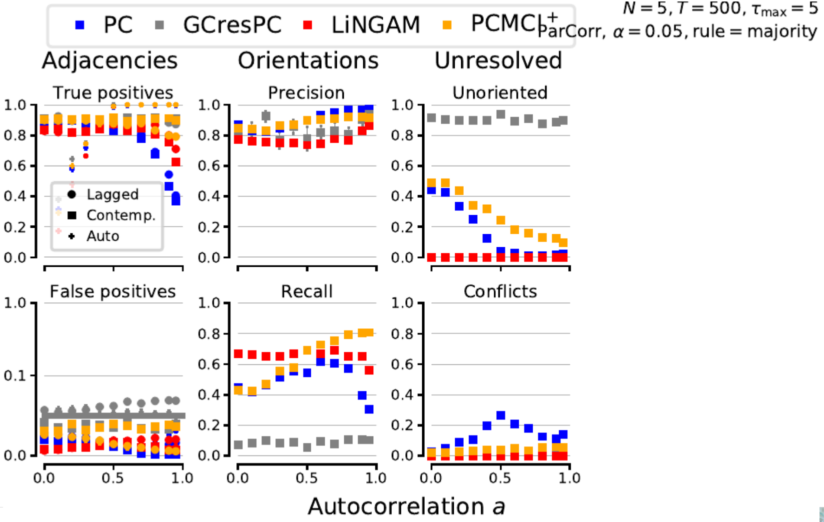
Numerical experiments

High precision and high recall for strong autocorrelation; LinGAM makes use of non-Gaussianity here, fails for Gaussians



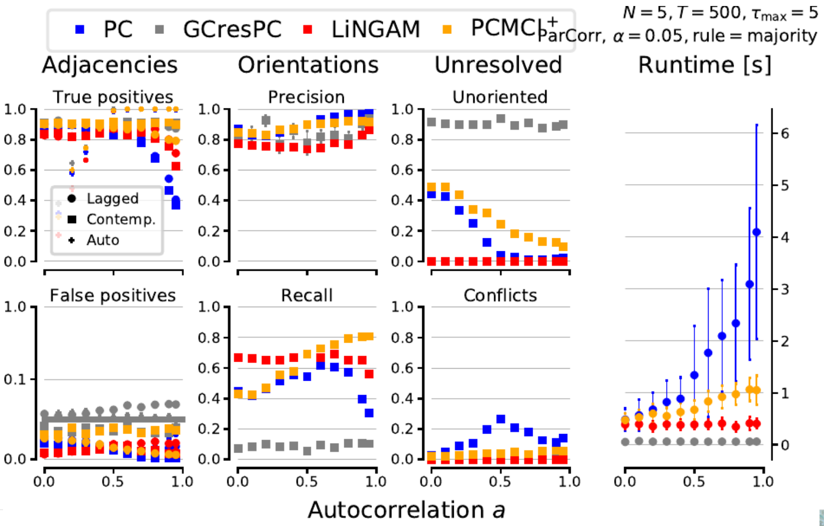
Numerical experiments

Slightly more unoriented, but also fewer conflicts (majority rule and conflict resolution enabled)



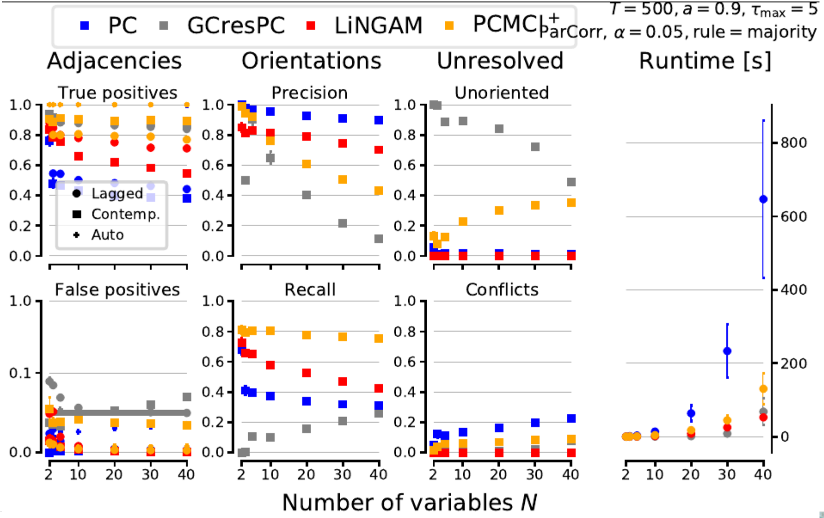
Numerical experiments

PC takes longer and is more variable



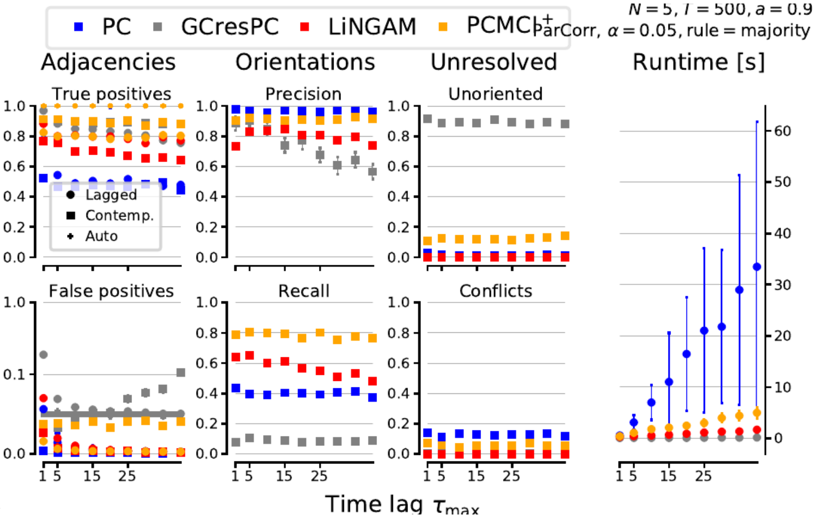
Numerical experiments

High dimensionality: Still well-calibrated, high recall, less precision (at this α_{PC})



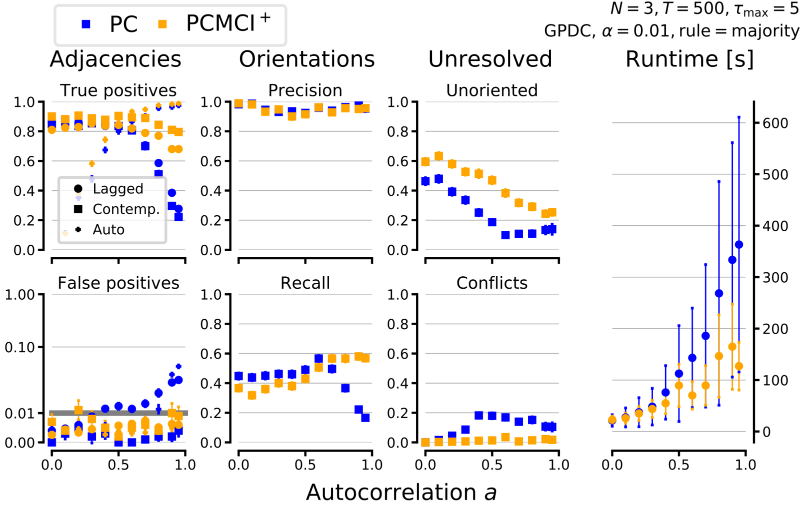
Numerical experiments

Large time lags: Almost no effect on precision and recall



Numerical experiments

Nonlinear GPDC test: Higher recall than PC for high autocorrelation



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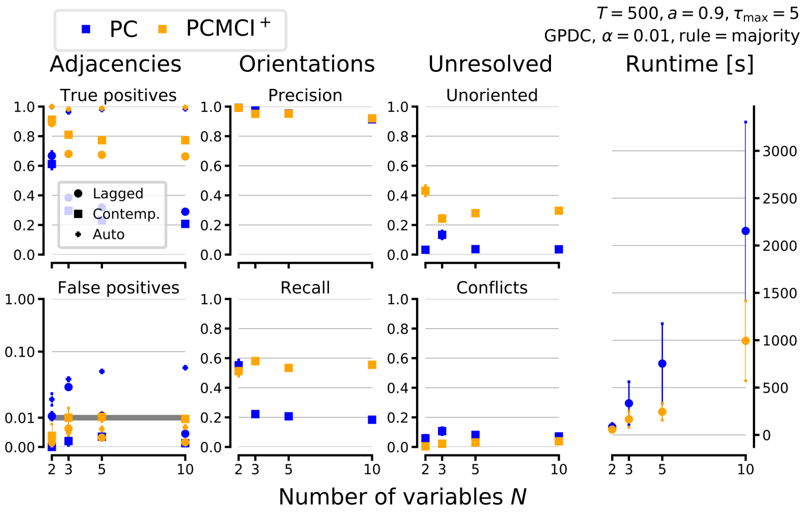
Nonlinear GPDC test: Higher recall than PC for high dimensionality



Application examples

Application cases

- Testing causal hypotheses
  [Runge et al., 2014, Runge et al., 2015b, Kretschmer et al., 2016, Runge et al., 2019b, Kretschmer et al., 2018, Runge et al., 2018, Runge et al., 2019a, Krich et al., 2019]

Application cases

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- Optimal statistical prediction schemes
  [Runge et al., 2015a, Kretschmer et al., 2017, Di Capua et al., 2019]

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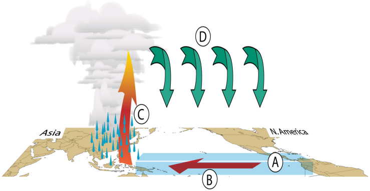
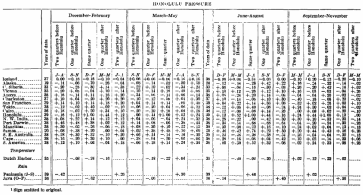
- Optimal statistical prediction schemes
  [Runge et al., 2015a, Kretschmer et al., 2017, Di Capua et al., 2019]
- Evaluating climate/physical models
  [Schleussner et al., 2014, Nowack et al., 2019]

- Monthly surface pressure anomalies in the West Pacific (WPAC), surface air temperature anomalies in the Central Pacific (CPAC) and East Pacific (EPAC)





HONOLULU PRESSURE

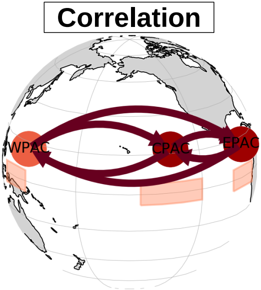
		December-February					March-May						June-August					September-November				
	Years of data	Two quarters before Honolulu	One quarter before Honolulu	Same quarter	One quarter after Honolulu	Two quarters after Honolulu	Two quarters before Honolulu	One quarter before Honolulu	Same quarter	One quarter after Honolulu	Two quarters after Honolulu	Years of data	Two quarters before Honolulu	One quarter before Honolulu	Same quarter	One quarter after Honolulu	Two quarters after Honolulu	Two quarters before Honolulu	One quarter before Honolulu	Same quarter	One quarter after Honolulu	Two quarters after Honolulu
		J-A	*S-N*	*D-F*	*M-M*	*J-A*	*S-N*	*D-F*	*M-M*	*J-A*	*S-N*		*D-F*	*M-M*	*J-A*	*S-N*	*D-F*	*M-M*	*J-A*	*S-N*	*D-F*	*M-M*
Iceland	37	0.00	−0.16	−0.18	−0.10	−0.04	+0.06	+0.06	−0.06	−0.16	+0.16	38	−0.06	+0.06	−0.34	−0.06	0.00	−0.10	[1] 0.20	−0.12	−0.20	−0.10
Alaska	19	−.14	−.06	−.70	−.26	−.04	−.08	−.14	−.52	−.30	+.50	20	−.12	−.14	−.28	+.42	+.22	+.16	−.24	−.22	−.14	−.02
C. Siberia	33	−.20	−.28	−.30	+.14	−.38	−.22	+.10	−.02	−.34	+.24	33	+.06	−.04	−.16	−.30	−.08	+.26	−.20	+.42	−.14	+.02
Vienna	35	−.20	+.08	−.04	+.10	−.14	−.10	−.14	+.18	−.40	+.08	35	+.10	+.12	−.26	+.12	+.10	+.16	−.03	−.02	−.04	+.26
Azores	38	+.16	−.14	+.14	+.16	+.20	−.34	−.06	−.14	+.06	−.14	38	+.08	−.24	+.20	+.14	−.08	−.04	−.04	+.20	−.02	−.02
Charleston	37	+.10	+.04	−.20	+.20	+.20	−.10	−.12	−.08	+.04	+.30	37	+.10	−.14	−.08	+.10	+.30	−.40	−.18	−.08	+.22	−.02
San Francisco	39	+.14	+.10	+.14	+.18	+.20	+.04	+.14	+.14	.00	+.40	39	+.04	+.22	+.14	+.36	+.08	−.32	+.02	+.26	+.02	+.10
Tokio	38	−.12	−.02	+.40	−.02	−.10	−.06	−.20	+.04	−.06	+.22	38	−.22	+.08	+.40	−.06	−.24	+.24	+.30	−.02	+.04	−.02
Cairo	36	+.18	−.02	+.20	+.34	+.10	−.20	−.10	+.28	+.08	+.18	36	−.12	−.06	−.06	+.14	−.28	.00	+.08	+.02	−.20	−.10
Honolulu	39	−.16	+.12	+1.00	+.44	+.12	.00	+.44	+1.00	+.52	+.24	39	+.12	+.52	+1.00	+.44	−.16	+.24	+.44	+1.00	+.12	.00
N. W. India	38	+.08	+.22	+.14	+.12	+.12	+.04	−.06	−.04	+.24	−.32	38	−.28	−.22	.00	−.42	−.30	+.30	+.10	−.32	−.22	−.14
Port Darwin	38	+.46	+.48	+.38	+.02	+.12	+.14	+.08	−.06	−.38	−.44	38	−.34	−.54	−.66	−.68	−.64	−.04	−.20	−.24	−.30	−.26
Mauritius	38	+.30	−.02	−.26	−.06	−.18	+.08	−.30	+.10	−.22	−.12	38	−.30	−.06	−.22	+.16	−.10	−.24	+.02	+.04	−.12	−.18
Samoa	20	+.08	+.38	+.20	.00	+.04	+.02	+.36	+.48	+.10	+.60	20	+.46	+.42	+.74	+.22	+.50	+.20	+.44	+.42	+.56	+.28
S. E. Australia	38	+.28	+.30	+.32	−.10	+.20	+.06	+.14	−.14	−.16	−.30	38	−.24	−.46	−.28	−.38	−.54	−.26	−.04	−.04	−.30	−.18
Cape	38	+.16	−.20	+.30	+.10	.00	−.38	−.04	−.20	−.06	−.06	38	−.28	−.16	−.08	+.04	−.42	−.10	+.02	−.14	−.22	+.06
S. America	38	+.12	+.10	+.06	−.04	+.18	−.06	+.18	+.14	+.24	+.34	38	.00	+.30	+.52	+.32	−.06	−.02	+.24	+.32	−.08	+.28
Temperature																						
Dutch Harbor	35	......	−.08	−.24	−.16	......	......	−.18	−.22	+.04	......	35	......	−.40	−.04	−.20	......	+.02	−.12	−.22	−.02	......
Rain																						
Peninsula (J-S)	39	−.42	......	......	......	+.26	......	......	......	+.30	......	39	......	......	+.46	......	......	......	+.02	......	......	......
Java (O-F)	36	......	......	−.32	......	......	......	−.06	......	......	......	36	−.14	......	......	......	+.40	......	......	......	+.30	......

[1] Sign omitted in original.

Runge et al. *Nat. Comm.* (2019)

Reconstructing Walker Circulation

- Monthly surface pressure anomalies in the West Pacific (WPAC), surface air temperature anomalies in the Central Pacific (CPAC) and East Pacific (EPAC)
- Correlation analysis gives a completely connected graph



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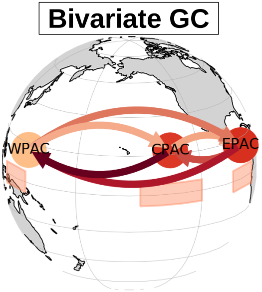

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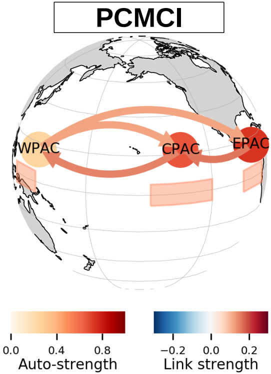
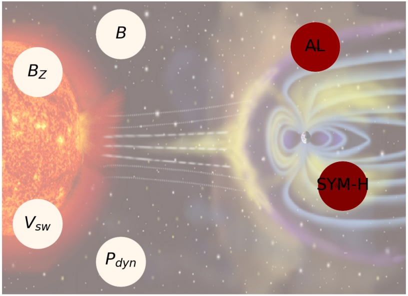
- Correlation analysis gives a completely connected graph
- Also bivariate Granger Causality cannot remove indirect and common driver links
- PCMCI [Runge et al., 2019b] better identifies the Walker circulation



Runge et al. *Nat. Comm.* (2019)

Space physics

- Hypothesis on interaction between magnetospheric Auroral Electrojet index (AL), magnetospheric ring current strength (SYM-H), and solar wind parameters



Runge et al. *Sci. Rep.* (2018), $\Delta t = 20$min resolution

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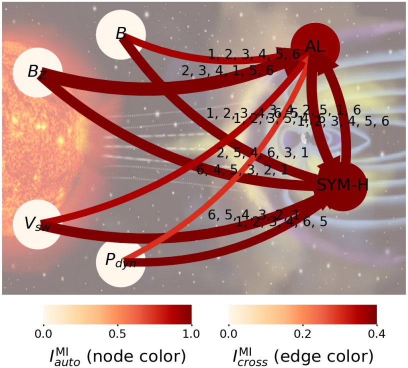
Mutual information



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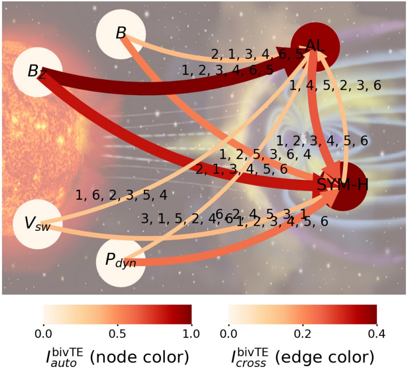
Transfer Entropy



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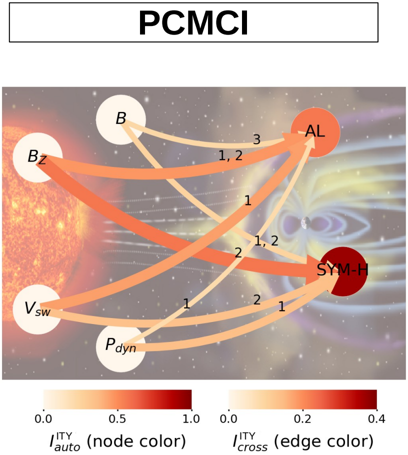
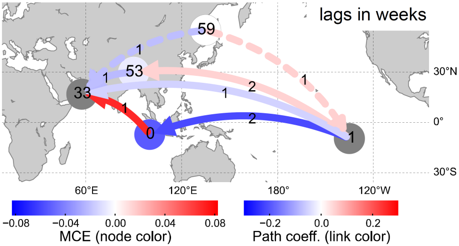


Runge et al. *Sci. Rep.* (2018), $\Delta t = 20$min resolution

Causal mediation analysis

- Pathway mechanisms by which El Nino influences Indian monsoon through sea-level pressure system



Runge et al. *Nat. Comm.* (2015)

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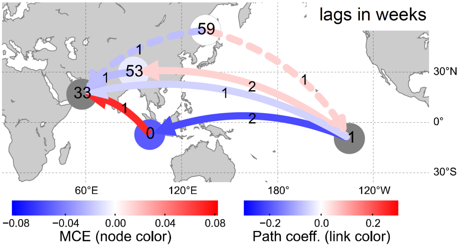
- Mediated Causal Effect (MCE) quantifies how much an intermediate variable (node) contributes to a causal effect



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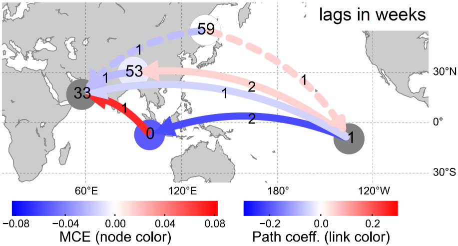

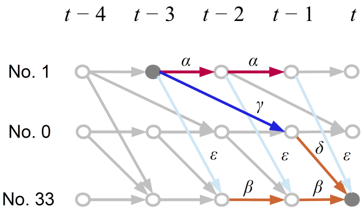

Runge et al. *Nat. Comm.* (2015)

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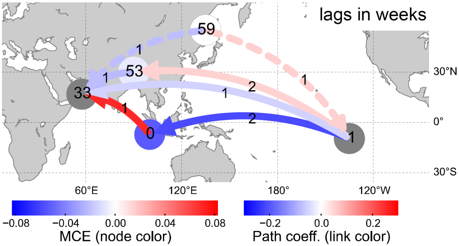
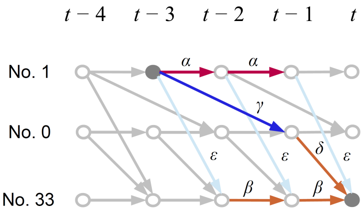
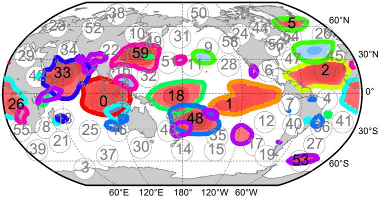
Runge et al. *Nat. Comm.* (2015)





Causal complex network analysis

- Complex network measures based on extracted causal network from sea-level pressure system



Runge et al. *Nat. Comm.* (2015)

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- Complex network measures based on extracted causal network from sea-level pressure system
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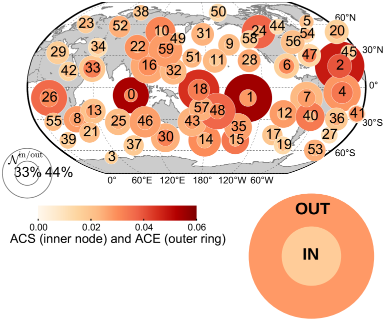

Runge et al. *Nat. Comm.* (2015)

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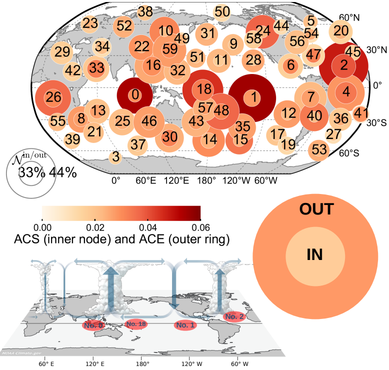

Runge et al. *Nat. Comm.* (2015)

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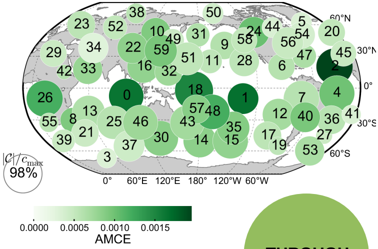
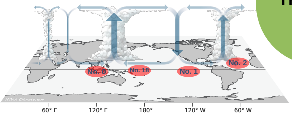
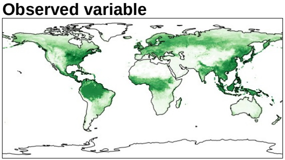
Runge et al. *Nat. Comm.* (2015)

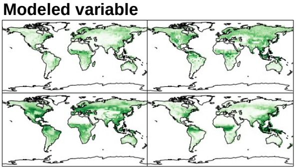




Causal model evaluation (Nowack et al., 2020)

Motivation: Simple statistics (e.g. mean, variance, trends) can be right for the wrong reasons





Real world processes

Modeled processes

Causal model evaluation (Nowack et al., 2020)

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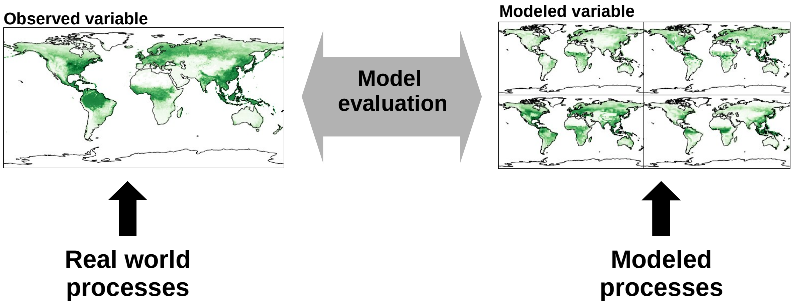
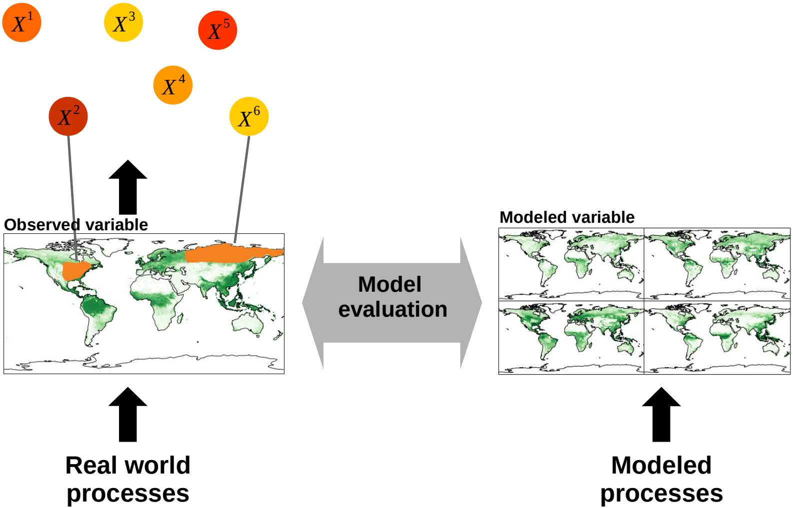


Causal model evaluation (Nowack et al., 2020)

Idea: Compare climate models and observations in terms of causal relationships



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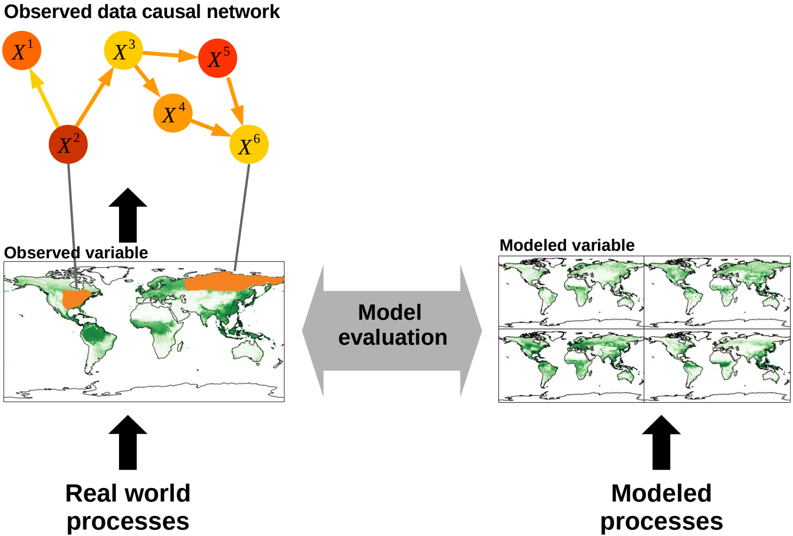

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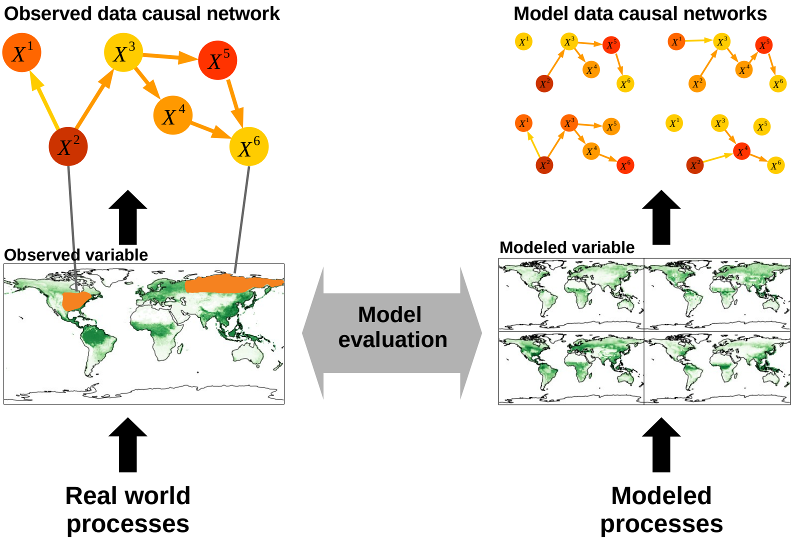

Causal model evaluation (Nowack et al., 2020)

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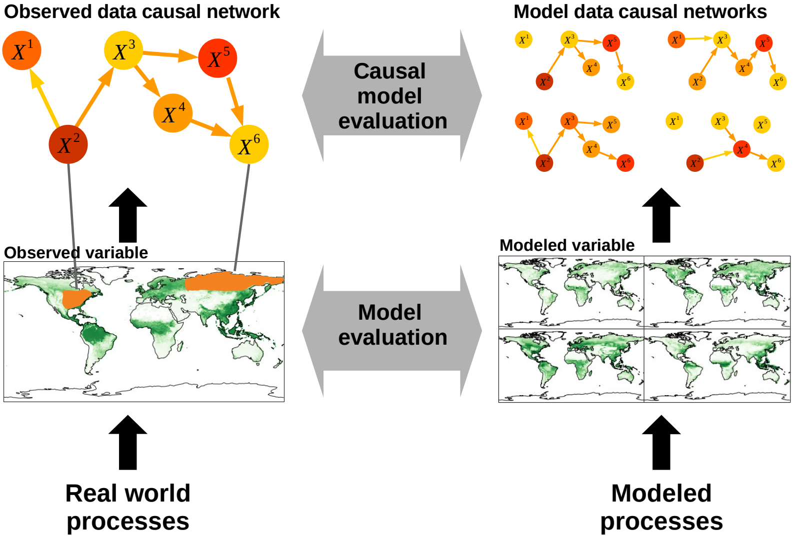
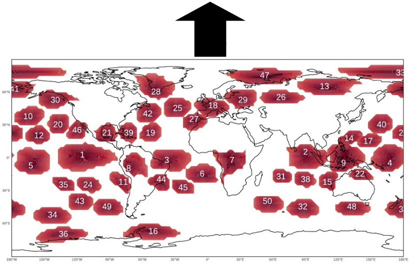


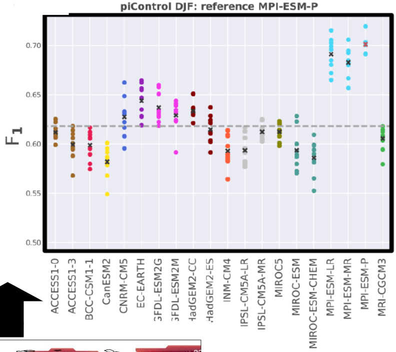
Causal model evaluation (Nowack et al., 2020)

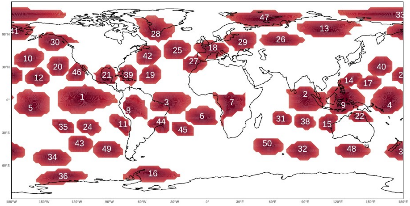
First results: CMIP5 simulations (historical and preindustrial) vs NCEP/NCAR reanalysis data of regional 3-day-mean sea level pressure



Causal model evaluation (Nowack et al., 2020)

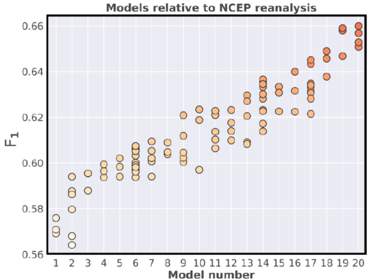
Validation: Similar climate models have similar causal networks; F-score as network comparison metric

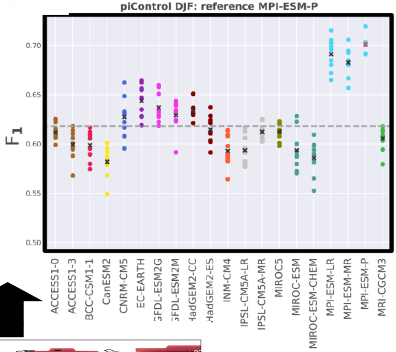


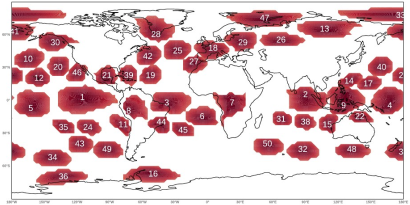


Causal model evaluation (Nowack et al., 2020)

Model evaluation: Significant differences in comparison to reanalysis



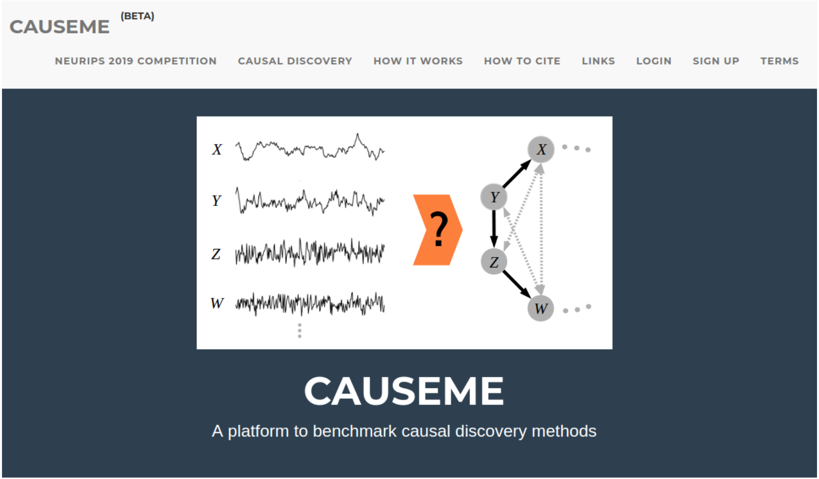




Causality benchmark platform

Causality benchmark platform `CauseMe.net`

Joint work with Jordi Munoz-Mari and Gustau Camps-Valls (U Valencia)



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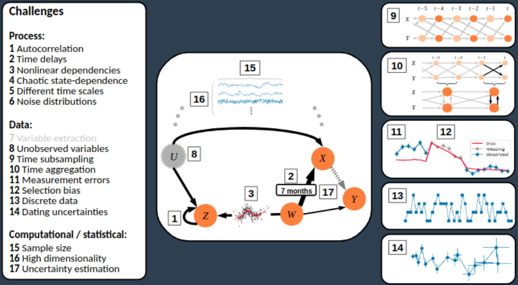
Joint work with Jordi Munoz-Mari and Gustau Camps-Valls (U Valencia)

CAUSEME (BETA)

NEURIPS 2019 COMPETITION | CAUSAL DISCOVERY | HOW IT WORKS | HOW TO CITE | LINKS | LOGIN | SIGN UP | TERMS

HOW IT WORKS

Causeme currently covers a wide range of synthetic model data mimicking a number of real world challenges. These cover time delays, autocorrelation, nonlinearity, chaotic dynamics, extreme events, measurement error, and will be extended by many more. Method developers can upload their predictions (matrices of causal connections) and the platform evaluates and ranks the methods according to different metrics of performance. After registering and logging in, more information, datasets, and example code snippets are given.



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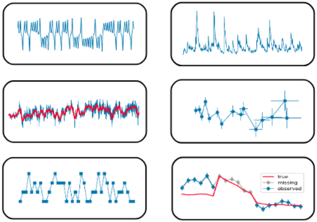
JAKOB RUNGE DATA AND MODELS METHODS ↓RANKING HOWTO MY RESULTS LOGOUT

DATA AND MODELS



Below you find a list of available datasets. Currently, they come from dynamical model systems featuring different challenges for causal discovery from time series as discussed in the accompanying Nature Communications Perspective paper. At the end of this page you find information on how to contribute real world datasets or model systems. Clicking on the model name will bring you to a description of the model and a list of experimental datasets. Please see the CauseMe workflow description in HowTo on how to upload your results for these experiments.

You can search through the database by name, description or tags.

Filter models:

Name	**Long name**	**Type**	**Tags**

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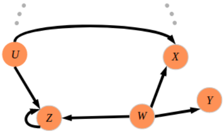
JAKOB RUNGE — DATA AND MODELS · METHODS · ↓↑RANKING · HOWTO · MY RESULTS · LOGOUT

METHODS



Below you find a list of methods applied by users of this platform. Clicking on the name will bring you to a description of the method. You can search through the database by name, user, and tags. Register your own methods on My Results!

Show 10 entries

Filter methods:

Name	User	Tags
adaptive-lasso	Jakob runge	Linear, time delays, high-dimensional
correlation	Jakob runge	Linear, time series, non-conditional
distance-correlation	Jakob runge	Time delays, nonlinear, non-conditional
FullCI-CMIknn	Jakob runge	Time delays, nonlinear
FullCI-GPDC	Jakob runge	Time delays, nonlinear
FullCI-ParCorr	Jakob runge	Linear, time delays

DLR

Causality benchmark platform CauseMe.net

Joint work with Jordi Munoz-Mari and Gustau Camps-Valls (U Valencia)

JAKOB RUNGE   DATA AND MODELS   METHODS   RANKING   HOWTO   MY RESULTS   LOGOUT

RANKING

The table below presents a ranking of methods for different experiments and can be sorted according to the different metrics in columns. Optionally, the table can be filtered by metric values above or below a certain threshold. For example, one can display only methods with a FPR below 6% and sort these by TPR in decreasing order. In addition, the search field can be used on the whole table to select only particular experiments or particular methods (or both). For example, "varmodel N-10 T-150" will list all methods with 'varmodel' in the string and all experiments with N=10 variables and sample length T=150. See here for a description of metrics: AUC is based on scores, while F-measure, FPR, and TPR are based on binary link predictions by thresholding uploaded p-values at 0.05 (only available if p-values were uploaded). TLR requires lag predictions.

Filter: FPR < 0.06 Go ☐ Paper ☐ Code ☑ Validated

Show 100 entries   Search: linear-VAR_multirealizations_N-

Id	User	Experiment	Method (params)	Paper	Code	Valid.	Time	AUC	AUC-PR	F-measure	FPR	TPR	TLR	Boxplot FPR	Boxplot TPR
37	Jakob Runge	linear-VAR_multirealiz	PCMCI-ParCorr (tau_max=5,pc_	✔	✔	✔	2.97	0.98	0.89	0.56	0.05	0.92	0.98	0 100	0 100
145	Jakob Runge	linear-VAR_multirealiz	adaptive-lasso (tau_max=5,)	✔	✖	✔	18.99	0.96	0.86	0.75	0.02	0.92	0.99	0 100	0 100
215	Jakob Runge	linear-VAR_multirealiz	varmodel (maxlags=5,)	✔	✔	✔	0.48	0.95	0.69	0.50	0.05	0.76	0.98	0 100	0 100
249	Jakob Runge	linear-VAR_multirealiz	FullCI-ParCorr (tau_max=5,)	✔	✔	✔	11.24	0.94	0.70	0.51	0.05	0.74	0.98	0 100	0 100

Showing 1 to 4 of 4 entries (filtered from 1,604 total entries)

Previous 1 Next

DLR

Discussion and Conclusions

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- And to indicate how conclusions are altered for different assumptions

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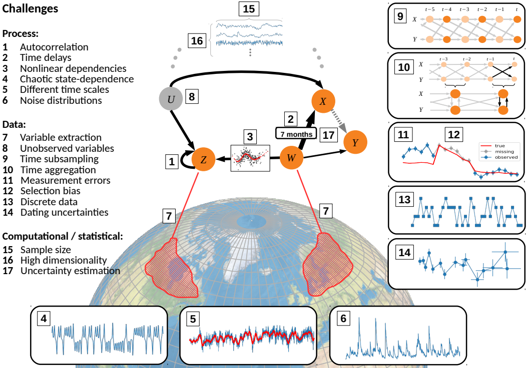
  - Causal evaluation of physical climate models (Nowack et al., 2020)

Thank you! Questions?

- PCMCI [Runge et al., 2019b] in Science Advances
- PCMCI$^+$ Runge (2020) `https://arxiv.org/abs/2003.03685`
- Conditional independence testing based on CMI [Runge, 2018b] in AISTATS
- Nature Comm. Perspective [Runge et al., 2019a]
- My software: `jakobrunge.github.io/tigramite`



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