

# Variance based sensitivity analysis of the RUSLE model in the E.U. parameter space

Balugani Enrico\*, Andrea Rava, Diego Marazza



# Modelling soil water erosion: the RUSLE semi-empirical model

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RUSLE (**R**evised **U**niversal **S**oil **L**oss **E**quation) defined as:

$$A = R \cdot K \cdot LS \cdot C \cdot P$$

*A*: annual soil loss per acre

*LS*: slope steepness (mostly physically based)

*R*: rainfall erosivity (mostly physically based)

*C*: vegetation cover (empirical)

*K*: soil erodibility (mostly physically based)

*P*: erosion control practices (empirical)

- developed in the US for the plot scale
- widely used in various places around the globe, at different scales, due to low data requirement and ease of applicability
- Does not account gully erosion or soil displacement and sedimentation

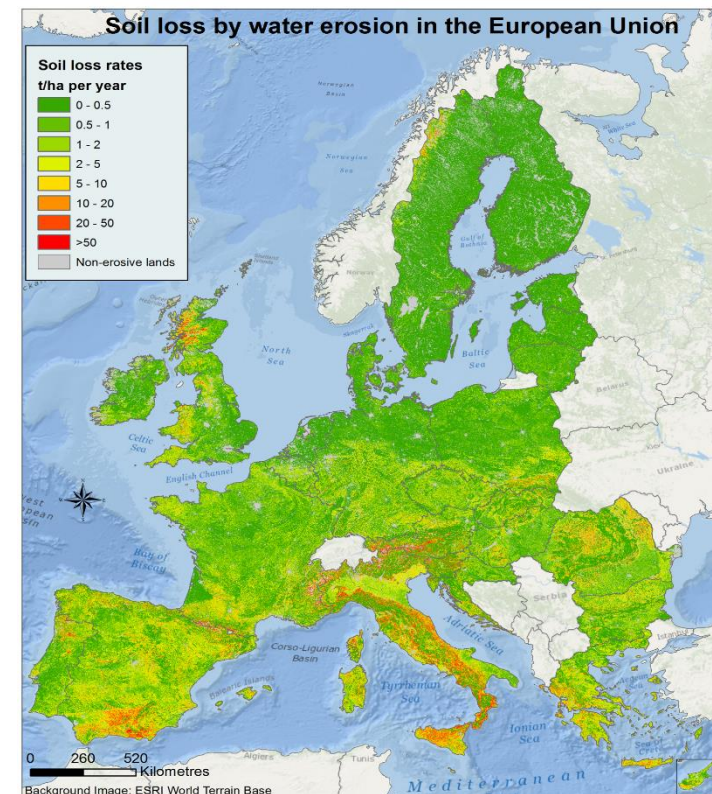


# Estimates of soil water erosion in the European Union with RUSLE by the JRC

- RUSLE already used by all countries participating to the EU soil loss dataset collection
- Simple and comparable estimates of potential soil loss

Factors used by Joint Research Centre (JRC):

- $R$  ~ Rainfall intensity
- $K_{st}$  ~ Soil properties and stoniness
- $LS$  ~ morphology
- $C$  ~ for agricultural land, vegetation type, agricultural practices (tillage, natural mulching)
- $P$  ~ contouring, stone walls and grass margins aimed at erosion control



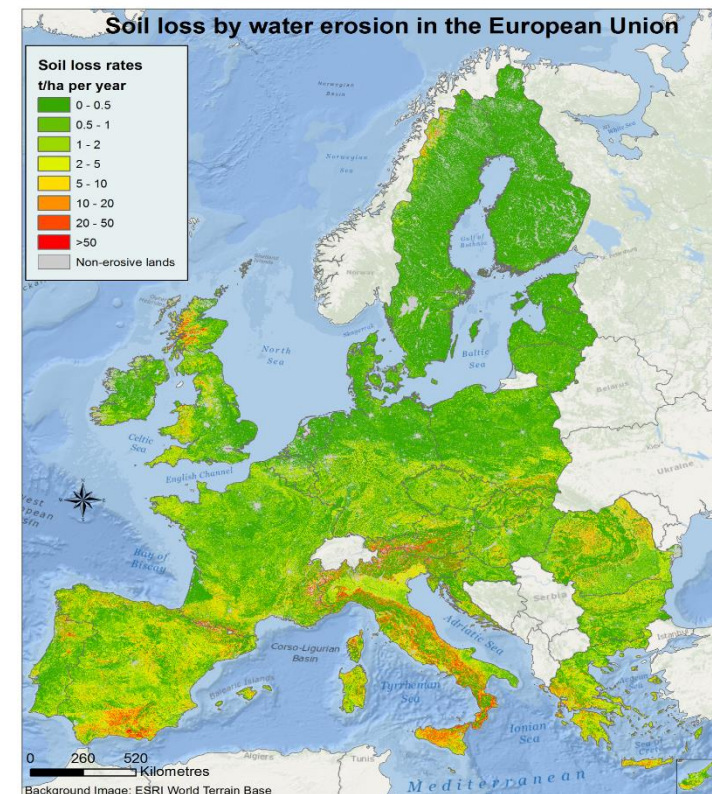
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- Simple and comparable estimates of potential soil loss

The whole dataset (2015) is freely available online, with separate factors and relevant input used in the calculations

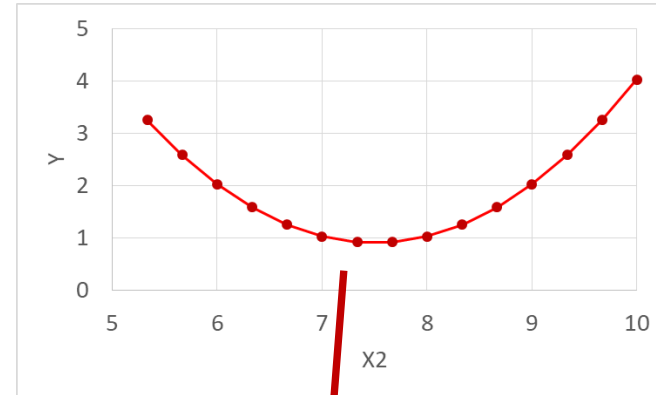
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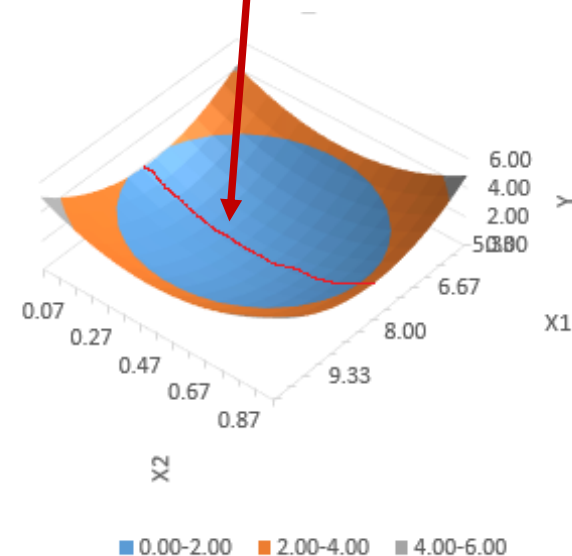


# Relevance of global sensitivity analysis (GSA) for empirical models

- Sensitivity analysis important to assess model uncertainties (data, parameters, structure) and, especially for empirical models, robustness
- Local sensitivity analysis useful to assess individual (not joint) influence of factors with respect to the baseline state of the model
- Global sensitivity analysis assess the behaviour of the model on the whole parameter space of interest
- Choice of the parameter space influences the results of the GSA
- Different possible objectives for the GSA require different sensitivity indices



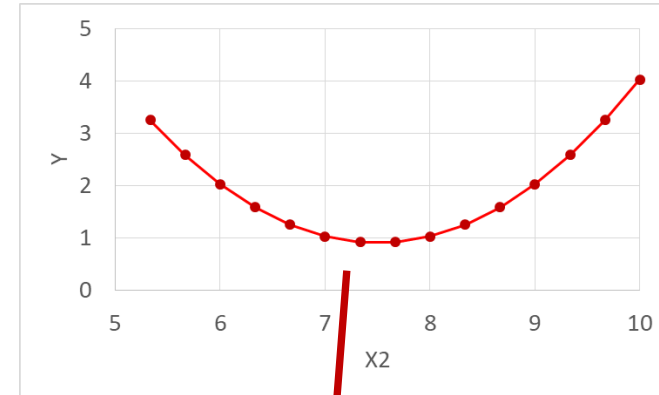
OAT analysis of one variable influence on the output



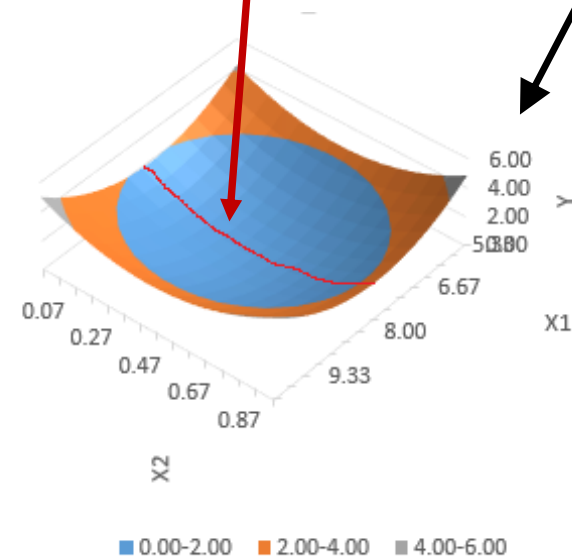


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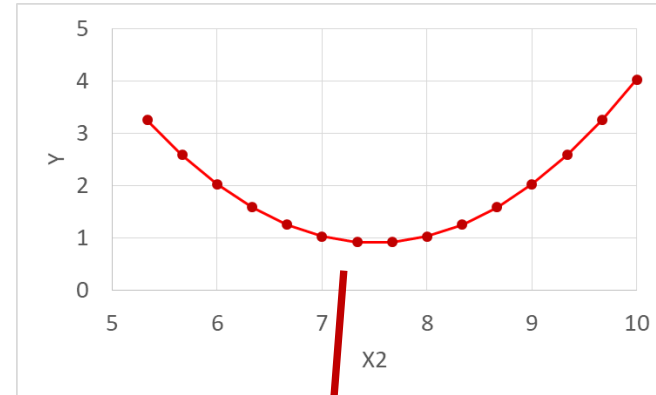


GSA shows the overall behaviour of the model in the parameter space

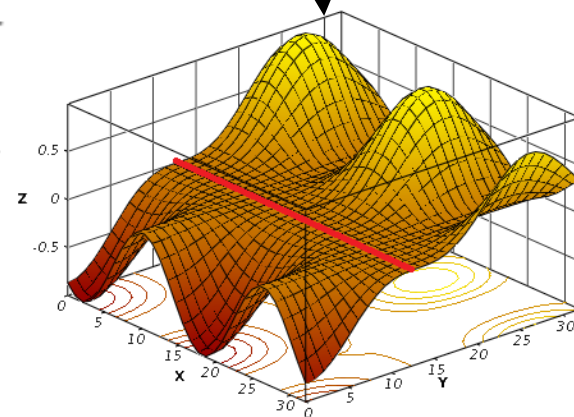
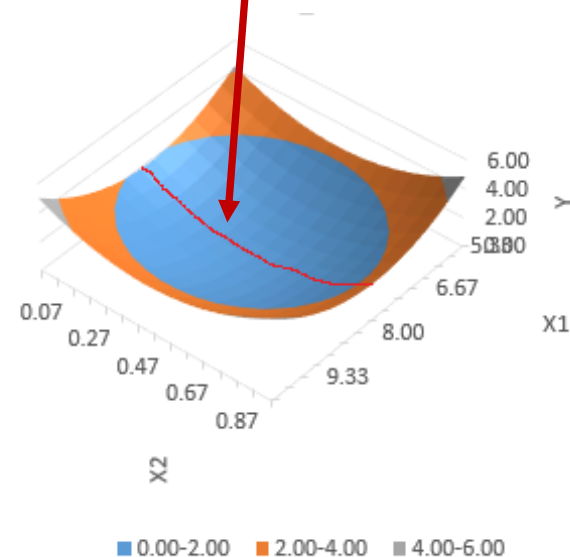


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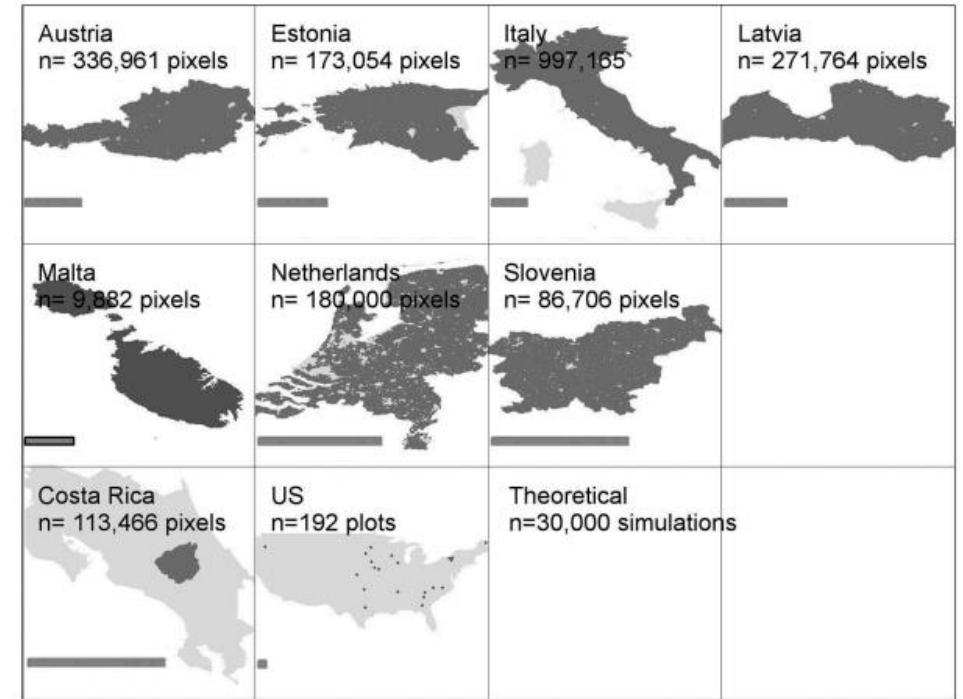


This may give an incomplete representation of the model general behaviour



# Previous sensitivity analysis for the RUSLE model – and what is still missing

- Previous sensitivity analysis performed on RUSLE have been largely local sensitivity analysis
- Estrada-Carmona et al. 2017 performed for the first time a GSA on RUSLE:
  - Objective: factor influence
  - Parameter space: 7 EU states, Costa Rica (2 watersheds), US (192 disconnected plots)
  - No overall assessment on EU parameter space





# Objectives of the present study

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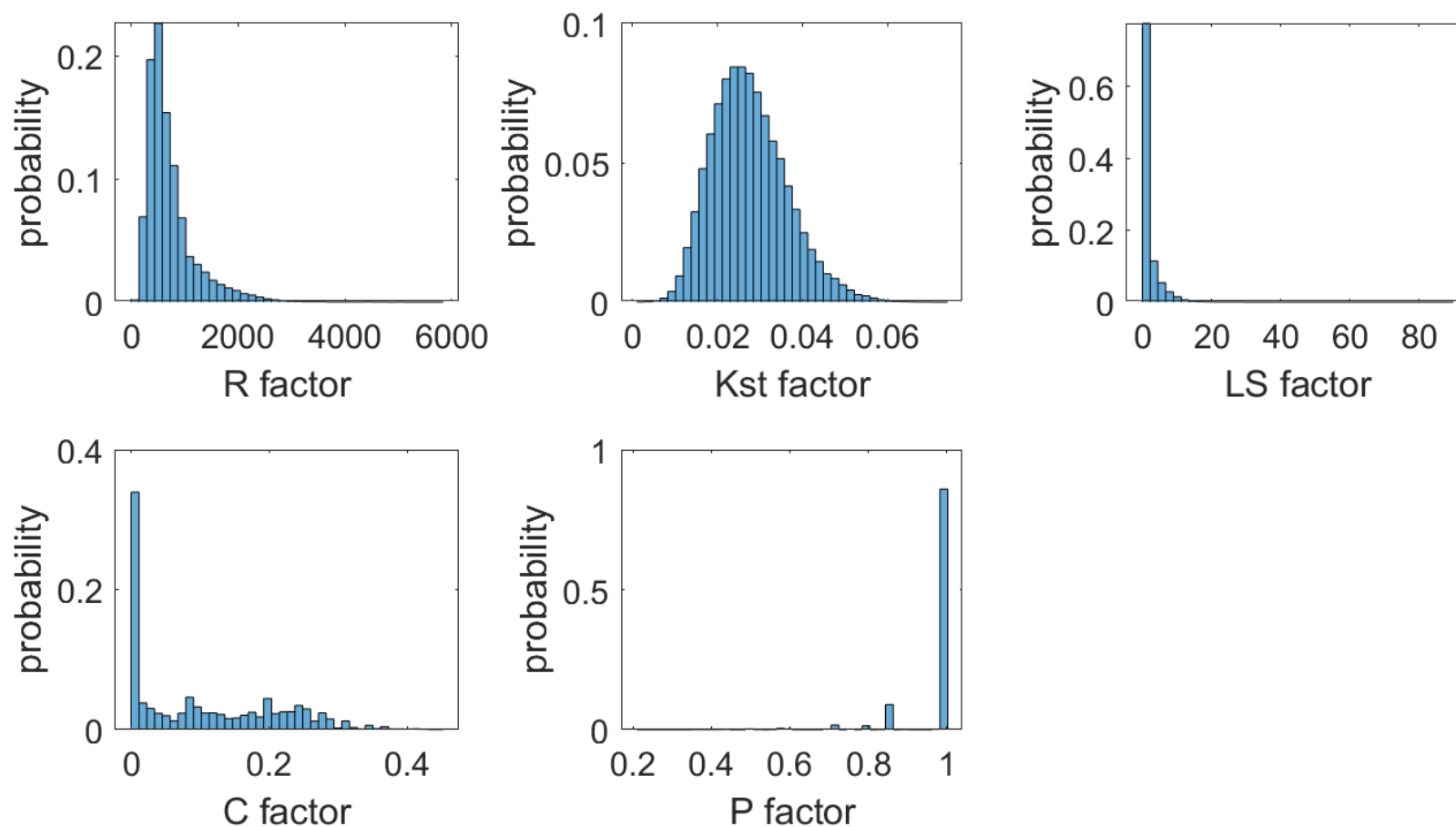
Perform a GSA on the parameter space defined by the JRC RUSLE study available (2015):

- Only factor level, to keep the model simple enough to use a large amount of realizations (basic Monte Carlo can be used)
- Objectives of the GSA: factors influence, policy recommendation
- Variance based methods, established method to get the most robust and comprehensive sensitivity analysis of the model



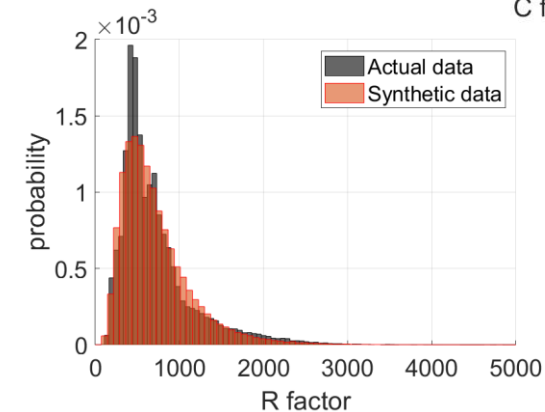
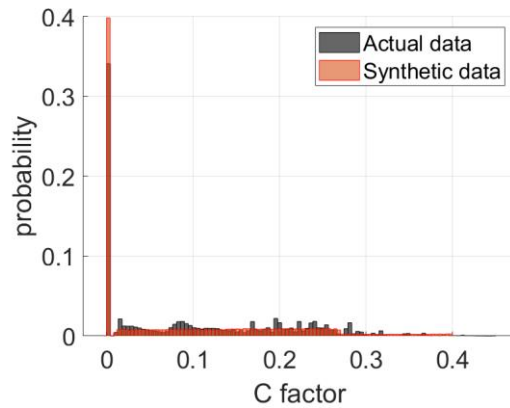
# The JRC dataset for the estimates of soil erosion with RUSLE in the EU

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# Simulation experiment design for the GSA: method 1 - synthetic data sampling

## Method 1 (the robust one)

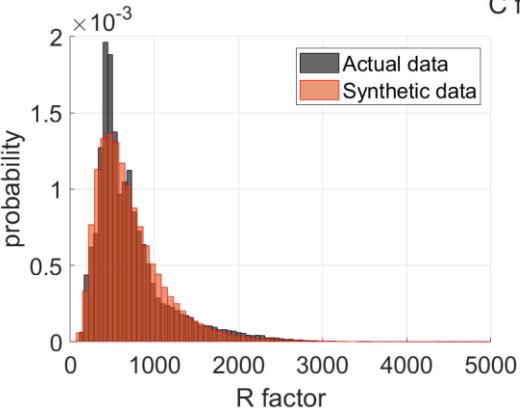
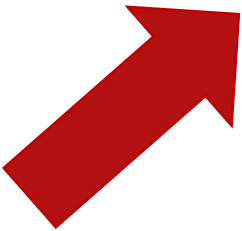
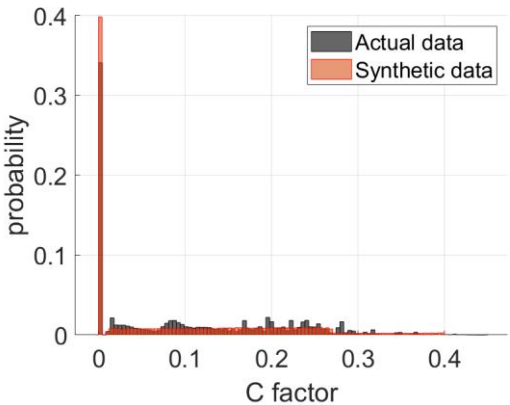


Step 1: fit pdf functions  
to mapped factors

# Simulation experiment design for the GSA: method 1 - synthetic data sampling

## Method 1 (the robust one)

Step 2: extract randomly  $10^4$  realisations for each factor's pdf – creating 5 vectors



Step 1: fit pdf functions to mapped factors

R
6.601E+02
9.481E+02
5.477E+02
1.449E+03
4.194E+02
6.449E+02
5.896E+02
1.343E+03
2.455E+02
6.652E+02

Kst
2.664E-02
2.374E-02
3.714E-02
2.333E-02
3.404E-02
2.271E-02
3.477E-02
3.037E-02
1.274E-02
2.265E-02

LS
6.764E+00
6.262E+00
6.358E+00
6.517E+00
5.929E+00
6.308E+00
6.021E+00
3.781E+00
8.454E+00
5.395E+00

C
1.246E-03
8.890E-04
1.497E-01
1.206E-03
2.885E-01
1.231E-01
8.863E-02
1.178E-03
1.908E-01
9.294E-04

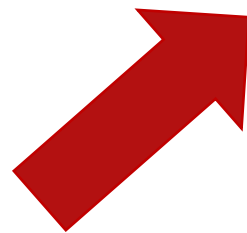
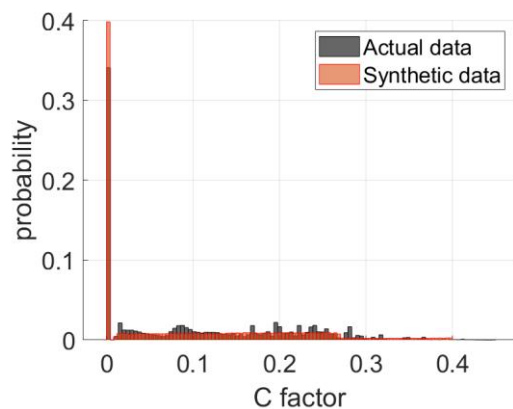
P
1.000E+00
1.000E+00
7.878E-01
1.000E+00
1.000E+00
1.000E+00
1.000E+00
1.000E+00
1.000E+00
1.000E+00



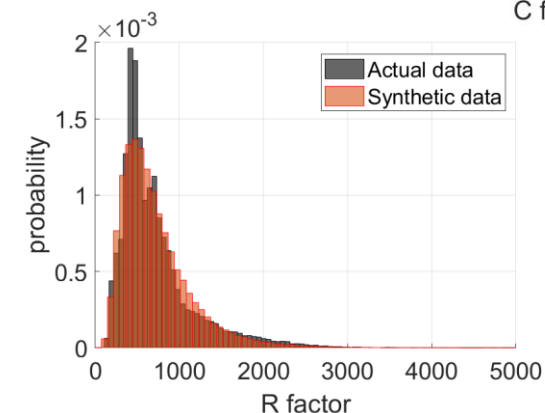
# Simulation experiment design for the GSA: method 1 - synthetic data sampling

## Method 1 (the robust one)

Step 3: calculate the sensitivity indices on all possible combinations of the vectors



Step 1: fit pdf functions to mapped factors



R	Kst	LS	C	P
6.601E+02	2.664E-02	6.764E+00	1.246E-03	1.000E+00
9.481E+02	2.374E-02	6.262E+00	8.890E-04	1.000E+00
5.477E+02	3.714E-02	6.358E+00	1.497E-01	7.878E-01
1.449E+03	2.333E-02	6.517E+00	1.206E-03	1.000E+00
4.194E+02	3.404E-02	5.929E+00	2.885E-01	1.000E+00
6.449E+02	2.271E-02	6.308E+00	1.231E-01	1.000E+00
5.896E+02	3.477E-02	6.021E+00	8.863E-02	1.000E+00
1.343E+03	3.037E-02	3.781E+00	1.178E-03	1.000E+00
2.455E+02	1.274E-02	8.454E+00	1.908E-01	1.000E+00
6.652E+02	2.265E-02	5.395E+00	9.294E-04	1.000E+00










# Simulation experiment design for the GSA: method 1 - synthetic data sampling

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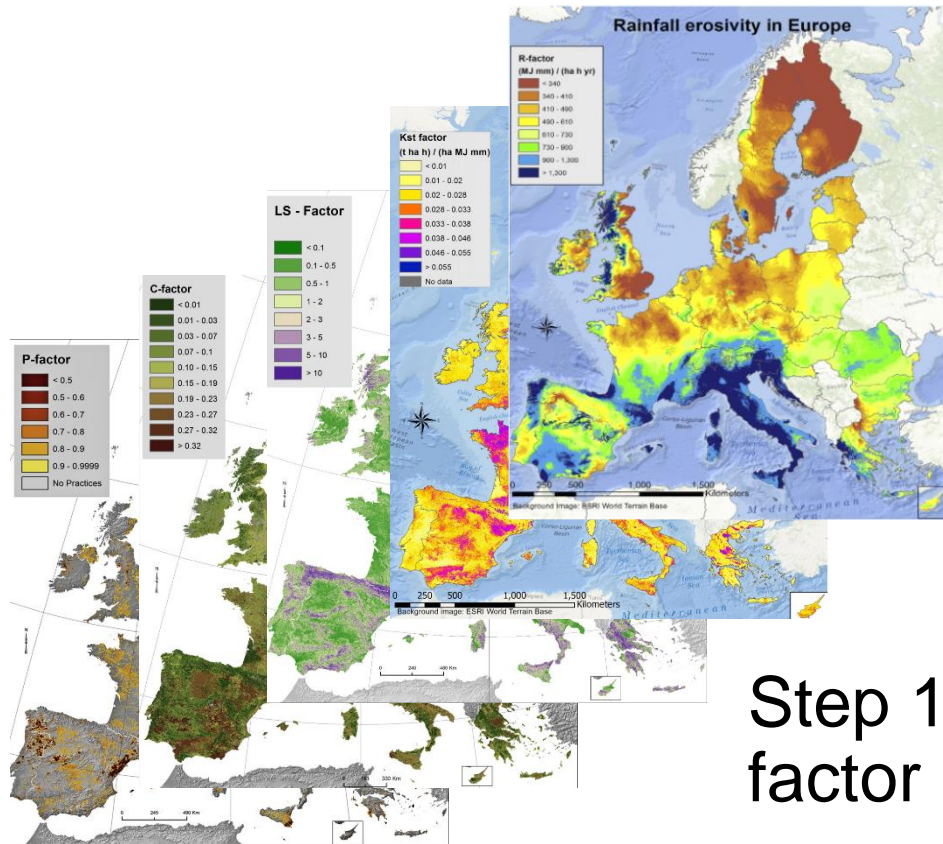
## Method 1 (the robust one)

RUSLE factors distributions fitted pdfs for random realizations of datasets:

-  independent variables assumption verified
-  explores all the possible factors interactions
-  explores the whole parameter space (not just local points)
-  Robust, unbiased statistical analysis
-  does not give proper representation of real correlation between factors

# Simulation experiment design for the GSA: method 2 – actual map data sampling

## Method 2 (focus on factors correlation)

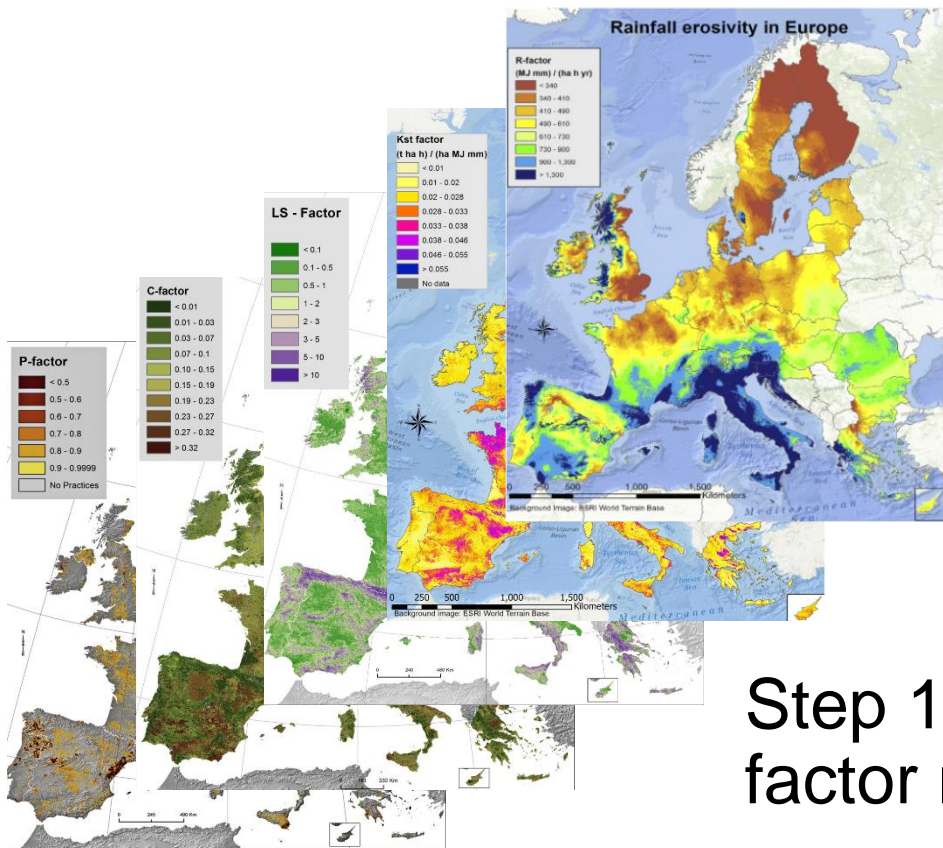


Step 1: superimpose  
factor maps pixel by pixel

# Simulation experiment design for the GSA: method 2 – actual map data sampling

## Method 2 (focus on factors correlation)

Step 2: create a matrix with geo-referenced superimposed maps – factors ordered by geographical location



R	Kst	LS	C	P	A
7.302E+02	1.338E-02	1.575E+00	8.872E-02	1.000E+00	1.365E+00
3.896E+02	1.894E-02	1.805E-01	8.401E-04	1.000E+00	1.119E-03
3.161E+02	3.260E-02	2.361E-01	1.284E-03	1.000E+00	3.124E-03
7.413E+02	1.589E-02	1.004E+00	8.964E-02	8.535E-01	9.050E-01
1.534E+03	3.416E-02	1.153E+00	1.457E-01	1.000E+00	8.797E+00
3.811E+02	2.353E-02	9.945E-01	6.884E-02	1.000E+00	6.141E-01
1.177E+03	2.073E-02	7.541E-01	3.575E-02	1.000E+00	6.577E-01
3.454E+02	2.153E-02	4.091E-01	8.198E-04	1.000E+00	2.494E-03
6.685E+02	2.605E-02	1.230E+00	1.640E-03	1.000E+00	3.512E-02
8.318E+02	3.185E-02	1.646E+00	1.953E-01	7.886E-01	6.715E+00

Step 1: superimpose factor maps pixel by pixel



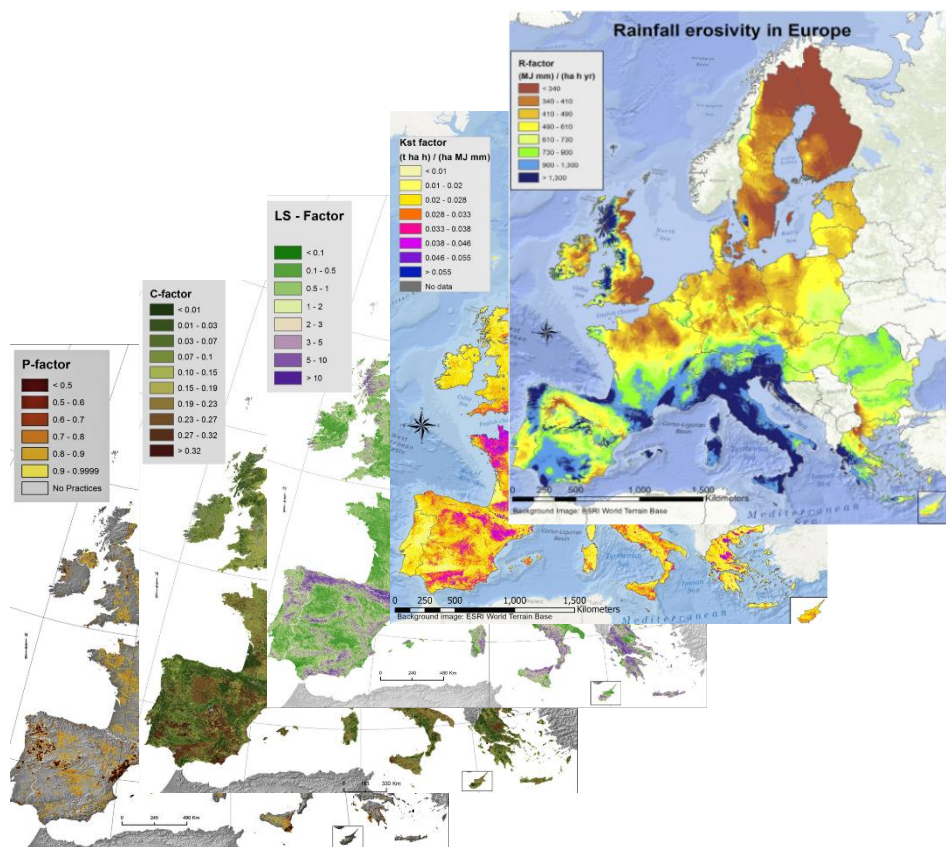


# Simulation experiment design for the GSA: method 2 – actual map data sampling

## Method 2 (focus on factors correlation)

Step 3: randomly extract  $10^6$  rows for the sensitivity analysis

Step 4: bootstrap the procedure to cross validate results







R	Kst	LS	C	P	A
7.302E+02	1.338E-02	1.575E+00	8.872E-02	1.000E+00	1.365E+00
3.896E+02	1.894E-02	1.805E-01	8.401E-04	1.000E+00	1.119E-03
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3.811E+02	2.353E-02	9.945E-01	6.884E-02	1.000E+00	6.141E-01
1.177E+03	2.073E-02	7.541E-01	3.575E-02	1.000E+00	6.577E-01
3.454E+02	2.153E-02	4.091E-01	8.198E-04	1.000E+00	2.494E-03
6.685E+02	2.605E-02	1.230E+00	1.640E-03	1.000E+00	3.512E-02
8.318E+02	3.185E-02	1.646E+00	1.953E-01	7.886E-01	6.715E+00

# Simulation experiment design for the GSA: method 2 – actual map data sampling

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## Method 2 (focus on factors correlation)

RUSLE factors from superimposed maps:

-  factors correlation is realistic
-  explores the joint effects observed in reality
-  Independence assumption not respected: less robust sensitivity indices
-  Limited exploration of the parameter space – many holes in the model estimates





# How conditional variance based methods for GSA work

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**First order variance** sensitivity index gives the influence of a factor on the model output, without considering factor interaction but on the whole parameter space

$$S_{Z_i} = \frac{V_{Z_i} (E_{Z_{\sim i}} (Y|Z_i))}{V(Y)}$$

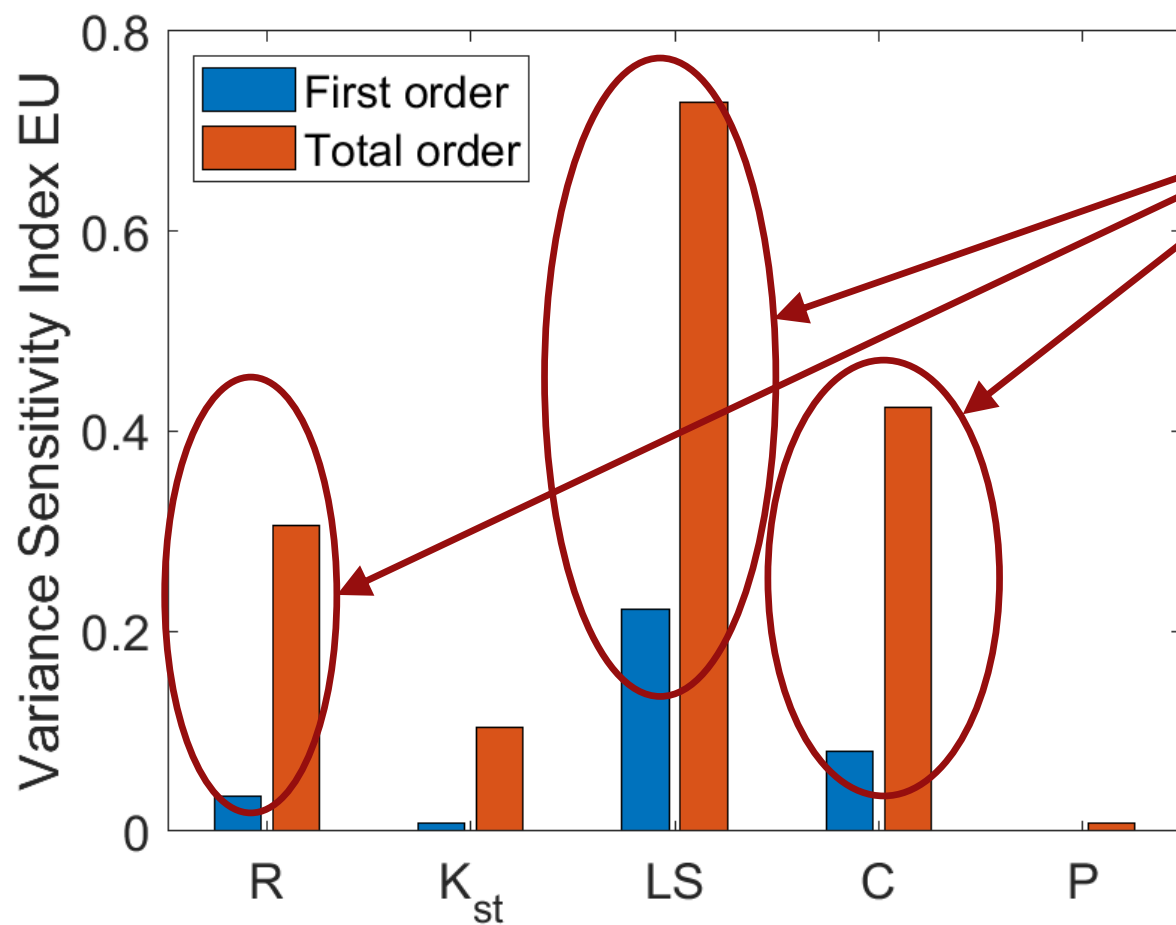
➡  $1 - \sum_{i=1}^n S_{Z_i}$  gives a measure of overall factor interaction

**Total order variance** sensitivity index gives the influence of a factor accounting also for interaction with other factors

$$S_{T_i} = \frac{E (V (Y|Z_{\sim i}))}{V(Y)}$$

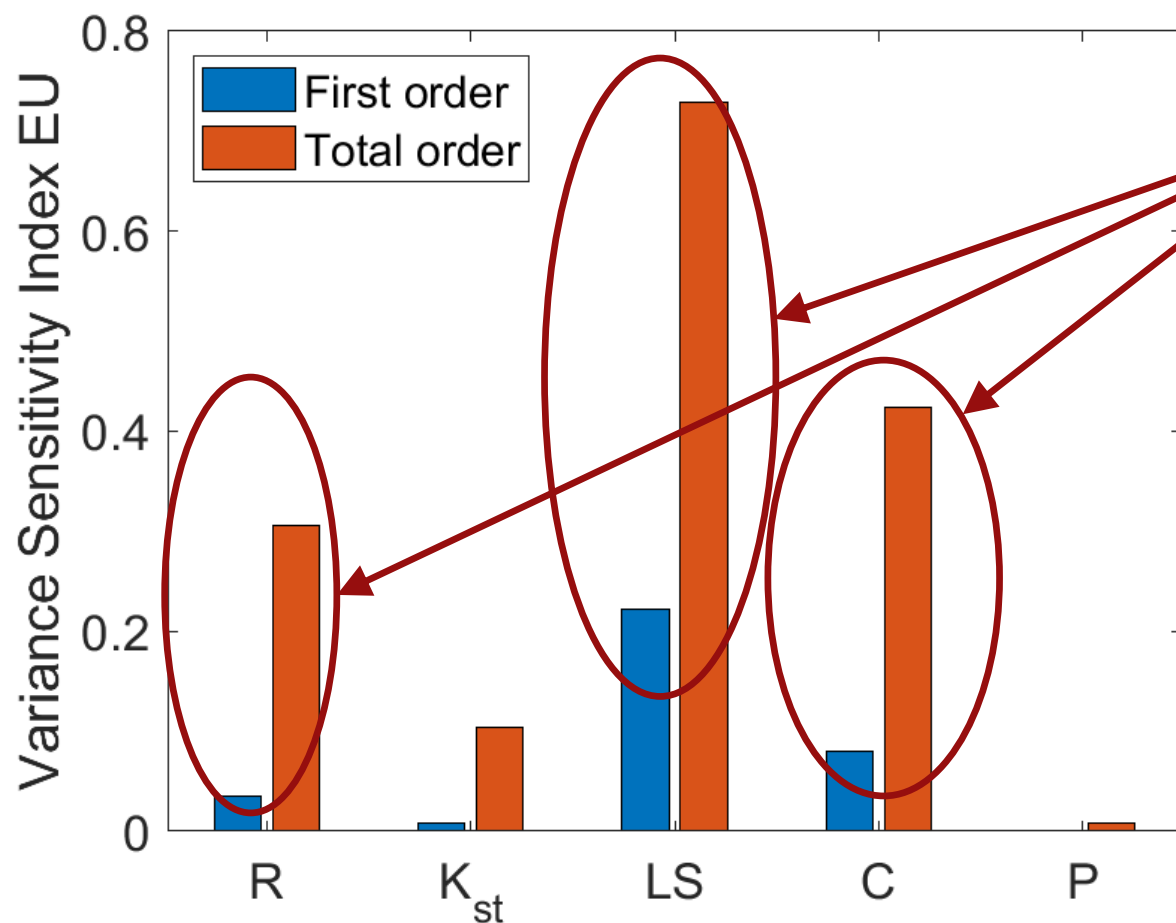


# First and total order variance on **method 1** shows *C* and *R* most influential parameters



*LS* most influential factor,  
followed by *C* and *R*

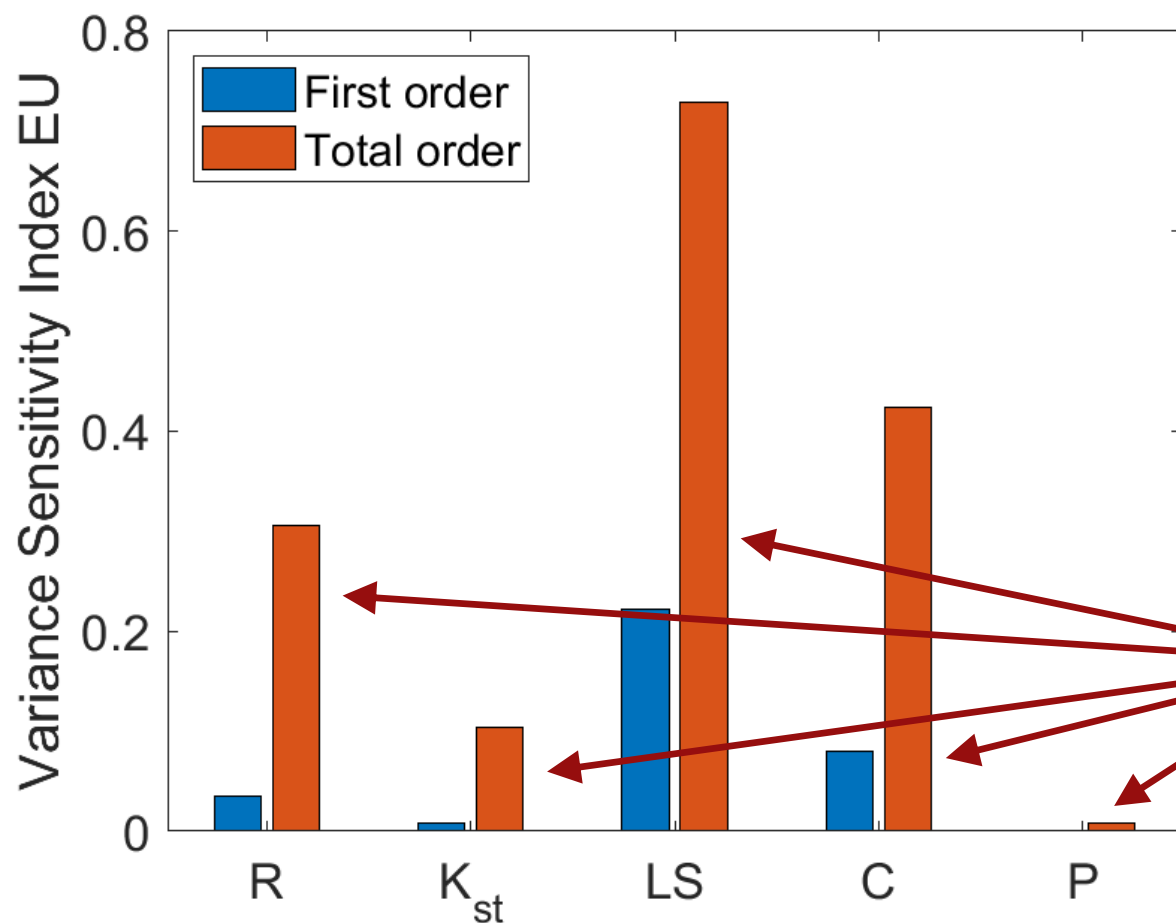
# First and total order variance on **method 1** shows C and R most influential parameters



*LS* most influential factor, followed by *C* and *R*

$1 - \sum_{i=1}^n S_{Z_i} = 0.35$  high factor interaction

# First and total order variance on **method 1** shows C and R most influential parameters



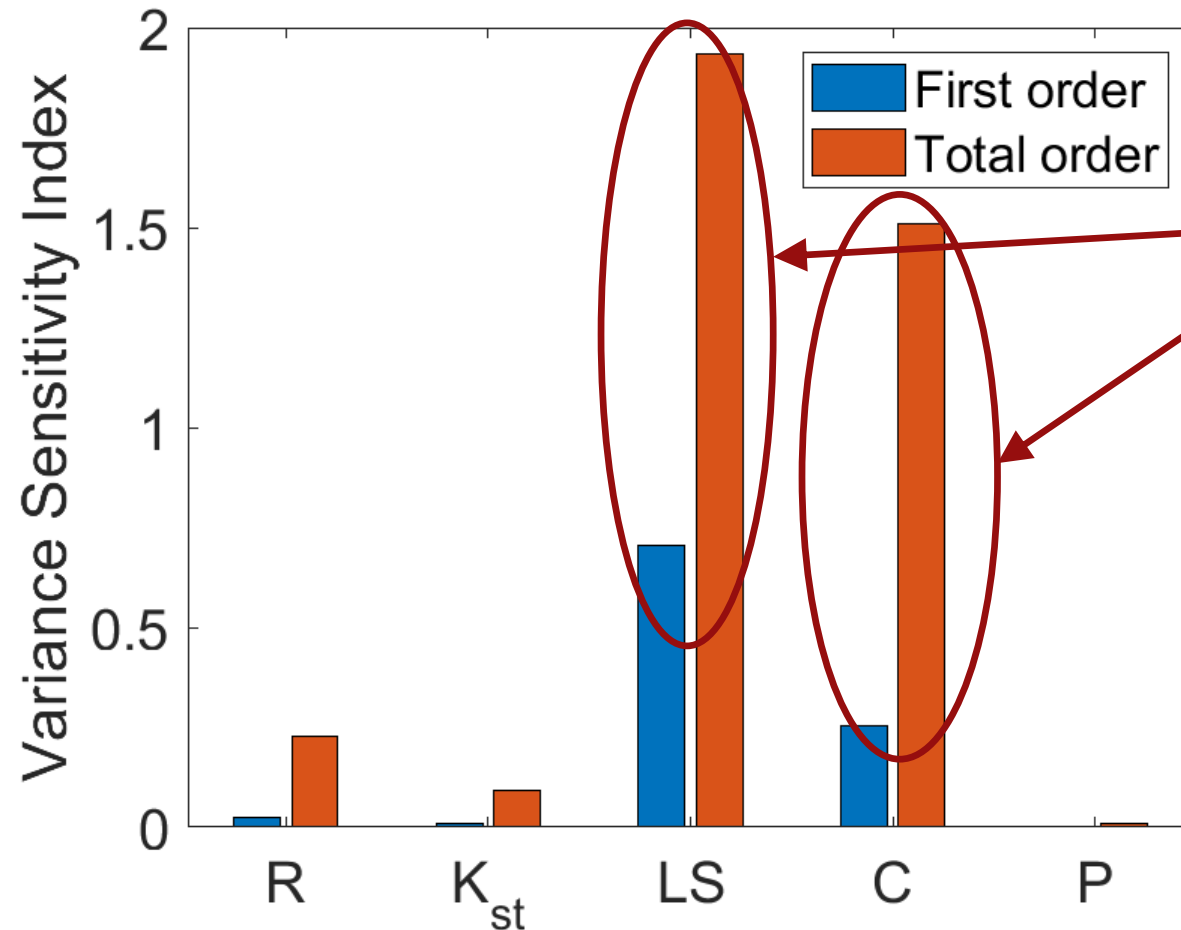
*LS* most influential factor, followed by *C* and *R*

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First order and total order analysis show same factor influence order



# First and total order variance on **method 2** shows LS relevance and factors correlations

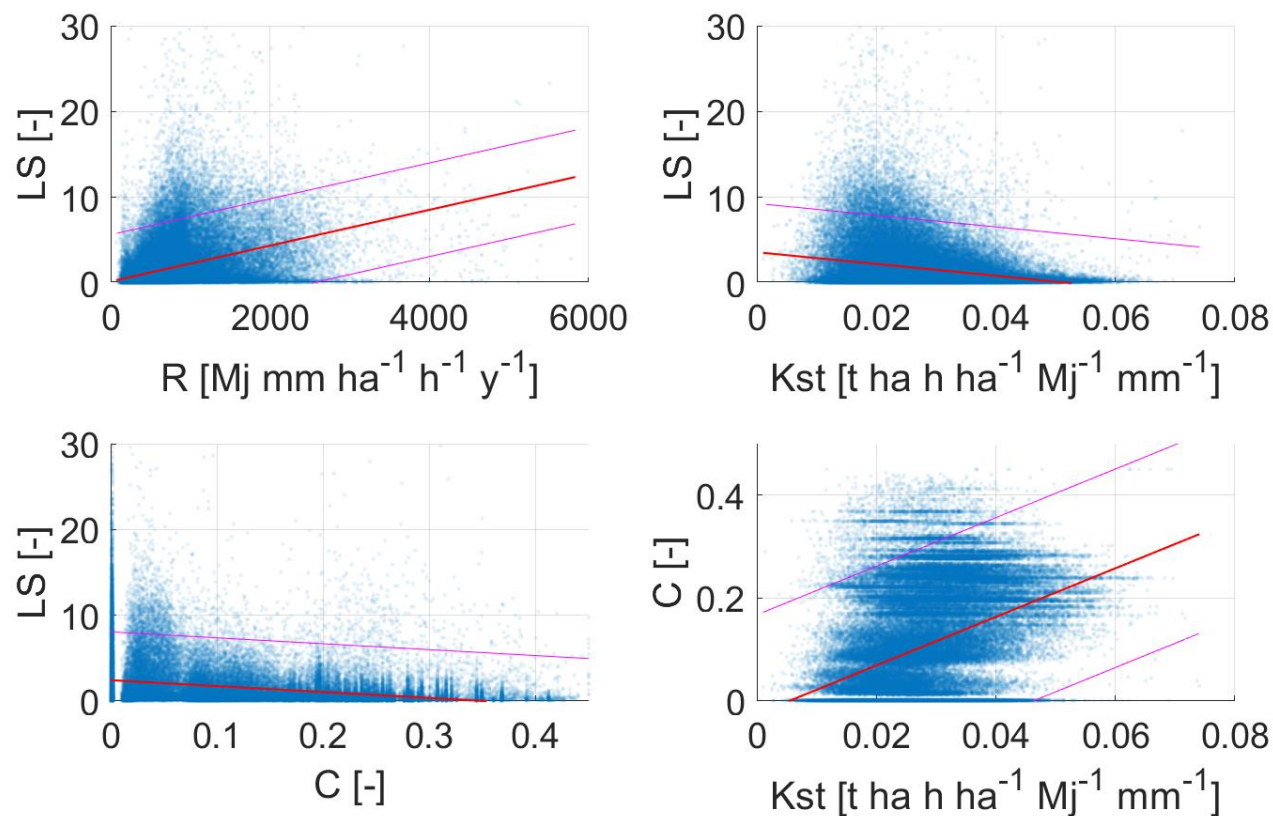


When factors are combined by their geographical location, *LS* relative influence increases, followed by *C*, other factor non-influential

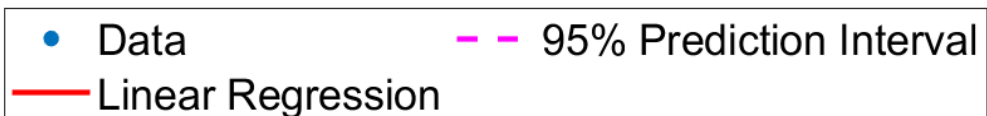
Total order analysis is not robust due to factor dependence; however, it shows the same patterns as the first order



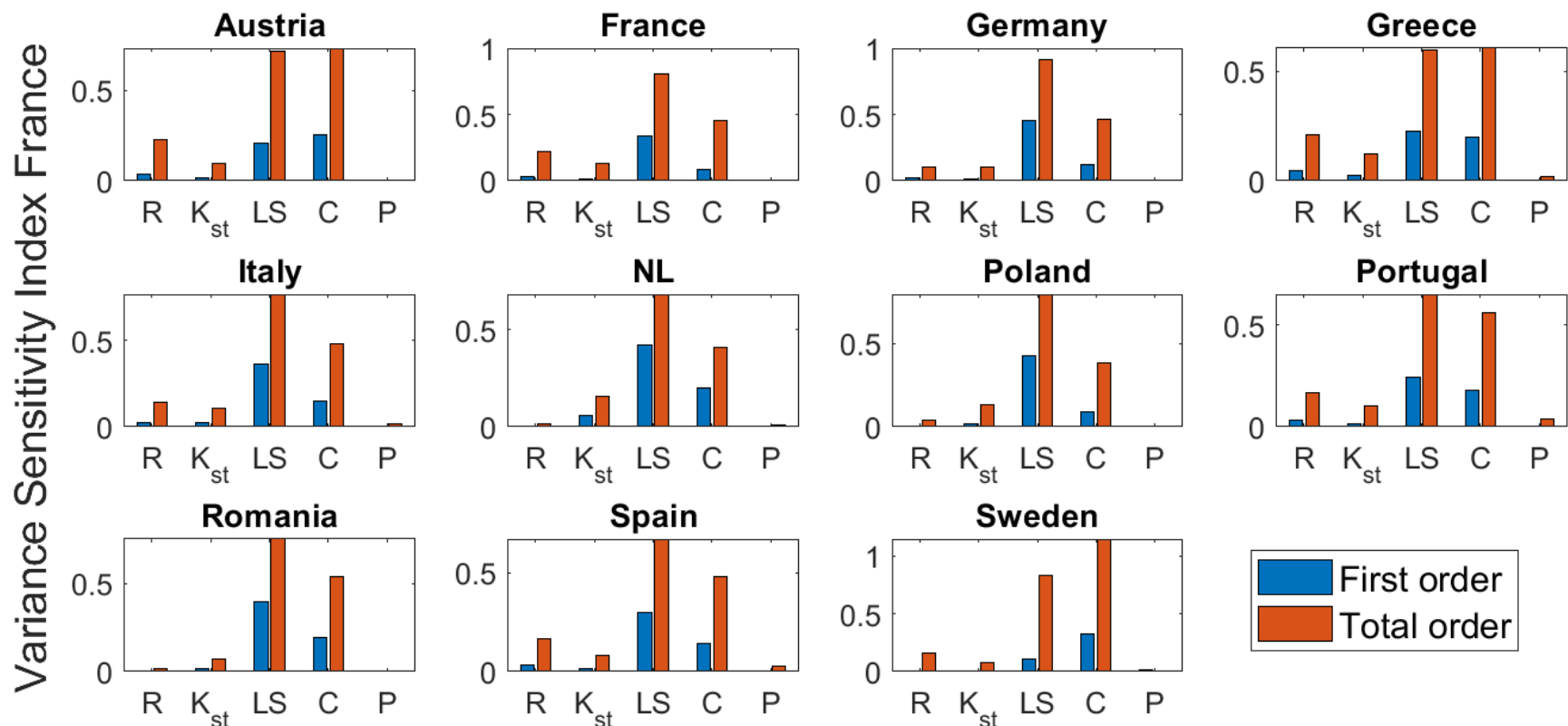
# First and total order variance on **method 2** shows LS relevance and factors correlations



- Correlation analysis shows that factors  $R$ ,  $K_{st}$ , and  $C$  correlates with  $LS$
- $C$  and  $K_{st}$  show also some correlation
- Correlations are not linear, and display large variance



# Preliminary analysis for individual EU countries for a more detailed GSA

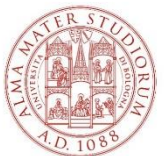


- *LS* remains the most influential factor overall, followed by *C*.
- *C* is the most influential factor in Austria and Sweden, and it is very relevant in Greece and Portugal
- Results are different with respect to those obtained by Estrada Carmona et al. 2017

# Findings and recommendations

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- *LS* factor (controlled by morphology) *C* factor (which can be influenced by humans) and *R* factor (influenced by climate change) are the most influential factors determining RUSLE erosion estimates
- Focus should be on land use change and agricultural practices decreasing *C* factor – *C* factor calculation should also be carefully checked (empirical factor from literature data)
- Relevance of the *R* factor reiterates the importance of tackling climate change
- *P* factor has no significant relevance on RUSLE erosion estimates on EU scale but local scale should be assessed



# Limitations of the study

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- We explored only the factor level – a deeper analysis down to input level is programmed in the future
- Since the GSA results depend on the parameter space analysed, the results are valid only on the scale of the whole EU  
→ the country specific analysis is still preliminary, but shows the robustness of the analysis
- Factor mapping, as an objective for the GSA, would give clear thresholds for the input variables useful to remain under a prescribed (by policy) level of potential soil erosion



**Thank you for your attention!**  
**Questions, comments and suggestions**  
**are welcome!**

Please contact us at [enrico.balugani2@unibo.it](mailto:enrico.balugani2@unibo.it) for more information on our work and supplementary material

We are working on an article on the study, which will be going to country level

Our intention is to extend the study to the main variables making up the different factors