Assessment of Ku- and Ka-Band Dual-Frequency Radar for Retrieval of Snow Properties

Liang Liao\textsuperscript{1}, Robert Meneghini\textsuperscript{2}, Ali Tokay\textsuperscript{3}, and Hyokyung Kim\textsuperscript{1}

\textsuperscript{1}Goddard Earth Sciences Technology & Research/MSU, Maryland
\textsuperscript{2}NASA Goddard Space Flight Center, Maryland
\textsuperscript{3}UMBC/JCET, Maryland
Issues Related to Snow Retrieval

- Snow microphysical models include
  - PSD model, particularly snow mass spectra
    - Because direct/reliable measurements of mass or $D_{eq}(\text{water})$ are usually not available, the snow mass spectrum (liquid-equivalent PSD) is usually obtained by converting the distribution of max dimension of particles (L) to mass or $D_{eq}(\text{water})$ by an empirical mass-size relation ($m-L$ or $m-D$)
    - There are, however, different ways of doing this which results in some ambiguities in the analysis.
  - Snow particle models (shape, orientation and composition)
    - Highly variable in nature but critical for computing higher frequency radar parameters.
- Electric scattering properties of snowflakes
  - These depend on single scattering models that account for shape, orientation and structure [as well as numerical methods for computations].
Objectives

- To characterize the errors (bias and variance) in estimates of liquid-equivalent PSD parameters ($D_m \& N_w$) and bulk parameters (SWC and R) from dual-wavelength radar techniques in association with
  - gamma PSD model (with various $\mu$ values)
  - m-D relationship (sensitivity of m-D to assessment procedures)
  - scattering database (on which the retrievals depend)
- Find an appropriate (or best) PSD model to estimate the PSD and snow bulk properties of interest.
Approach

Flowchart for illustrating the procedures of the retrieval error assessment. The left column enclosed by dashed-line rectangle represents the forward computations, which start from the measured PSD, when coupling when coupling with a mass-size (m-D) relation, to obtain ‘true’ snow liquid-equivalent parameters ($D_m$, $N_w$, SWC and R) as well as radar parameters such as $Z(Ku)$, $Z(Ka)$ and DFR when coupling with the single scattering database. The dashed-line enclosed right column shows the retrieval steps that take on the radar reflectivities computed from the forward model (as shown in the left column) as the input to the integral retrieval look-up table (algorithm), which is calculated from an assumed PSD model and the same single scattering database as used in the forward model. Comparisons between the estimated and ‘true’ snow parameters are made for assessment of retrieval errors.
Single Scattering Database

- Two databases are tested
  - NASA/GSFC scattering database
    Pristine crystals and aggregate snowflakes from a 3-D growth model
  - Florida State Univ. (FSU) database
    Aggregates comprised of 6-branch bullet rosette crystals
Backscattering coefficients of snow obtained from the GSFC (green dot) and FSU (red filled-circle) scattering databases as a function of liquid equivalent diameter at Ku- (left column) and Ka-band (right column). Also plotted are the mean (solid blue curve) and 2-time standard deviation (vertical blue bar) of the GSFC database. For reference, the scattering results computed from the randomly oriented oblate-spheroidal models with an aspect ratio of 0.5 are provided for the densities of 0.05, 0.1 and 0.2 g/cm$^3$, denoted by dotted, dashed and dotted-dashed curves, respectively.
**Dual-λ Retrieval Technique**

Assuming liquid-equivalent PSD:

\[ N(D) = N_w f(\mu) \left( \frac{D}{D_m} \right)^\mu \exp(-\Lambda D), \]

Radar reflectivity factor:

\[ Z_r = \frac{\lambda^4}{\pi^3 |K_w|^2} \int_0^\infty N(D)\sigma_v(D,\lambda)dD, \]

Differential frequency ratio (DFR):

\[ DFR = 10 \log(Z_{Ku}/Z_{Ka}) \]

Snow water content:

\[ SWC = \int_0^\infty N(D)m(D)dD \]

Liquid-equivalent snow rate:

\[ R = \frac{36 \times 10^{-4}}{\rho_w} \int_0^\infty N(D)m(D)V(D)dD \]

Thus, for fixed \( \mu \) and known \( V(D) \)

\[ SWC/Z_{Ku} = f_1(DFR), \quad R/Z_{Ku} = f_2(DFR), \quad D_m = f_3(DFR), \quad N_w/Z_{Ku} = f_4(DFR) \]
Snow retrieval look-up tables in which SWC (top-left), R (top-right) and Nw (bottom-right), which are normalized by the Ku-band radar reflectivity $Z_{Ku}$, as well as $D_m$ (bottom-left) are expressed as a function of DFR ($=\log_{10}(Z_{Ku}/Z_{Ka})$). For these computations, the gamma PSD model with a fixed $\mu$ of 0 is assumed. The results from the GSFC and FSU scattering databases are denoted by the heavy blue and red solid curves, respectively. Also provided are the results from the spheroidal model with the snow densities ranging from 0.05 to 0.5 g/cm$^3$ (thin black curves) for reference.
Example of PSD Measurements

Example of a segment of the PSD data, obtained by averaging the measurements over 1-minute integration time, in time series taken from 8 snow events during winter of 2014 at the NASA Wallops Flight Facility using the SVI/PIP. The particle size spectra (mm$^{-1}$ m$^{-3}$), shown in the color scale, are given in the top panel while equivalent snow fall rate and median mass diameter are displayed in the middle and bottom panels, respectively.

Example of PSD Measurements

Wallops: Snow, N(D)

Diameter (mm)

Snowfall rate (mm/h)

$D_{50}$ (mm)

Time sequence (minute)
Impact of $\rho$-$D_L$ relations on retrieval accuracy
To test impact of the m-D relations used for converting measured PSD to snow mass spectra on retrieval, three popular m-D relations are employed, which are documented by


Comparisons between Estimated and True SWC & R ($\mu=0$ & GSFC-LUT)

m-D (Heymsfield 2010)

m-D (Brandes 2007)
Comparisons between Estimated and True $D_m$ & $N_w$ ($\mu=0$ & GSFC-LUT)

m-D (Heymsfield 2010)

m-D (Brandes 2007)
Evaluation of retrieval uncertainties associated with PSD model
Comparisons between Estimated and True SWC & R (Heymsfield m-D & GSFC-LUT)

μ=0

μ=3
Comparisons between Estimated and True $D_m$ & $N_w$ (Heymsfield m-D & GSFC-LUT)

$\mu=0$

$\mu=3$
Relative Errors of Estimates (GSFC-LUT)

For several m-D relations ($\mu=0$)

For several $\mu$ (Heymsfield m-D)
To test how the retrieval accuracy changes if different scattering tables are used, we repeat the same procedure that produced the results shown in Slides 15-17 simply by substituting the GSFC database with the FSU database.
Comparisons between Estimated and True SWC & R ($\mu=0$ & FSU-LUT)

**m-D (Heymsfield 2010)**

![Graphs showing comparisons between estimated and true SWC & R for m-D (Heymsfield 2010).](image1)

**m-D (Brandes 2007)**

![Graphs showing comparisons between estimated and true SWC & R for m-D (Brandes 2007).](image2)
Comparisons between Estimated and True $D_m$ & $N_w$ ($\mu=0 & \text{FSU-LUT}$)

**m-D (Heymsfield 2010)**

- Estimated $D_m$ vs. True $D_m$ (nPoint=3852, $\text{<est>}=0.565$, $\text{<PSD>}=0.651$)
- Estimated $N_w$ vs. True $N_w$ (nPoint=3852, $\text{<est>}=363889$, $\text{<PSD>}=67479$)

**m-D (Brandes 2007)**

- Estimated $D_m$ vs. True $D_m$ (nPoint=3967, $\text{<est>}=0.664$, $\text{<PSD>}=750$)
- Estimated $N_w$ vs. True $N_w$ (nPoint=3967, $\text{<est>}=280002$, $\text{<PSD>}=57994$)
Retrieval error caused by inappropriate LUT

To understand the error caused by the use of unmatched scattering tables, a test is conducted where the GSFC database is used for generating measurements of the radar reflectivities while the FSU scattering database is used for the retrieval, and vice versa.
Comparisons between Estimated and True SWC & R (μ=0 & Heymfield m-D)

GSFC-LUT for $Z_m$ and estimates

GSFC-LUT for $Z_m$ & FSU-LUT for estimates
Relative Errors of Estimates ($\mu=0$)

GSFC $\rightarrow$ $Z_m$ & GSFC $\rightarrow$ SWC, R, $D_m$, $N_w$

GSFC $\rightarrow$ $Z_m$ & FSU $\rightarrow$ SWC, R, $D_m$, $N_w$
To understand and characterize biases and variances of snow parameters (SWC, R, D_m and N_w) derived from dual-frequency radar, we need to evaluate separately uncertainties associated with PSD models and scattering models.

As snow is assumed to obey a gamma distribution, retrieval accuracy has been assessed using measured snowflake size spectra converted to mass spectra by using empirical m-D relations. In the evaluation procedures, the same scattering database is employed to simulate radar reflectivities and to infer snow properties. It is found that:

- Retrieval accuracy is not sensitive to the m-D relation chosen.
- Values of \( \mu \) have various impacts on snow retrieval, e.g., there is less bias in estimates of snowfall rate when \( \mu = 0 \) while better agreement of \( N_w \) with their true values (PSD directly derived) is achieved when \( \mu = 3 \) and 6.
- Less than 10% and 30% negative biases in R estimates when \( \mu = 0 \) and 3, respectively.
- Above findings are not affected by the scattering databases (GSFC/FSU) selected as long as same scattering tables are used for generating radar parameters and for snow retrieval.
It is difficult to assess scattering models without collocated measurements of dual-frequency radar and snow mass spectra or bulk snow properties (SWC and R).

Radar backscattering cross sections from single scattering models of snow in principle depend on shapes, orientations and structures of snow, which are more important at Ka-band than at Ku-band.

GSFC and FSU scattering databases, although both of which nearly depict identical scattering radar cross sections at Ku band, show somewhat differences in scattering properties at Ka-band. This leads to an increase in the bias of snow estimates if one scattering database is used for simulating radar measurements and another for snow retrieval.

The largest snow particles included in both GSFC and FSU databases are up to liquid-equivalent diameters around 3 mm, which, though it covers most of snow particle sizes for light to moderate snowfall rates, may result in truncation errors for relatively heavy snow. This is evidenced by the fact that DFR computed from measured PSD using both databases rarely exceed 8 dB, which is well below measurements from aircraft radar and GPM DPR. Desirable databases should include larger particles up to 5-6 mm in liquid-equivalent diameters.