# Dispersion upscaling in highly heterogeneous aquifers: The prediction of tracer dispersion at the Macrodispersion Experiment (MADE) site

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## Summary

We derive an upscaled model for the prediction of the plume evolution in highly heterogeneous aquifers based a stochastic transport representation in terms of continuous time random walks. Transport is modeled through advective motion of idealized solute particles, which changes their speed at fixed distances. The series of particles speeds is modeled as a stationary Markov chain. The derived model is parameterized by the correlation length, mean and variance of the log-hydraulic conductivity, the mean hydraulic gradient and porosity. Furthermore, it can be conditioned on the conductivity and tracer data at the injection region. The model predicts the non-Fickian evolution of the longitudinal concentration profile observed during the MADE-1 experiment. The mass distribution is characterized by strong localization at the injection region and a strong forward tail. These features are explained by conductivity heterogeneity at the injection region, and the correlated motion of particles according to spatially persistent Eulerian flow speeds.

## Upscaled Transport Model

- Stochastic advective particle motion [1]

$$\frac{dx(s)}{ds} = \chi^{-1}, \qquad \frac{dt(s)}{ds} = \frac{1}{v_\ell(s)},$$

  where $\chi$ is advective tortuosity.
- Distribution of flow speeds $v_\ell(s)$

$$p_v(v) = \frac{v p_e(v)}{\langle v_e \rangle},$$

  where $p_e(v)$ distribution of Eulerian flow speed
- Darcy flow

$$\mathbf{q}(\mathbf{x}) = -K(\mathbf{x})\nabla h(\mathbf{x}), \qquad \nabla \cdot \mathbf{q}(\mathbf{x}) = 0,$$

- Eulerian flow speed

$$v_e(\mathbf{x}) = v_0|\mathbf{q}(\mathbf{x})| \qquad v_0 = K_g J/\phi\theta,$$

  where $K_g$ is the geometric mean conductivity, $J$ the mean hydraulic graident, $\phi$ is porosity and $\theta$ retardation coefficient.
- Advective tortuosity is given by $\chi = \langle |q| \rangle / \langle q_1 \rangle$
- Propagation of normal scores $w(s)$ of particle speeds $v_\ell(s)$ through an Ornstein-Uhlenbeck process [1,2].

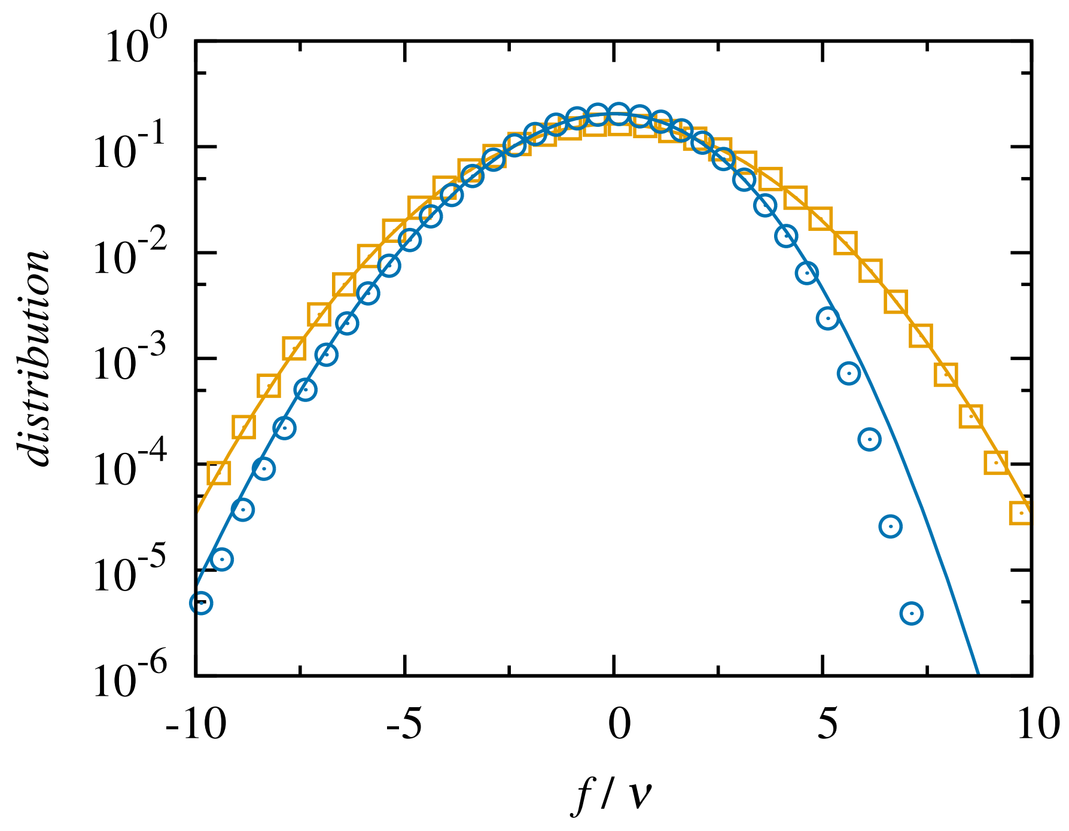


*Figure 1: Distribution of $f = \ln K$ and $\nu = \ln q$ for a unit hydraulic head gradient and and anisotropy ratio of $\lambda_h/\lambda_v = 5$.*

## Prediction of MADE Tracer Data

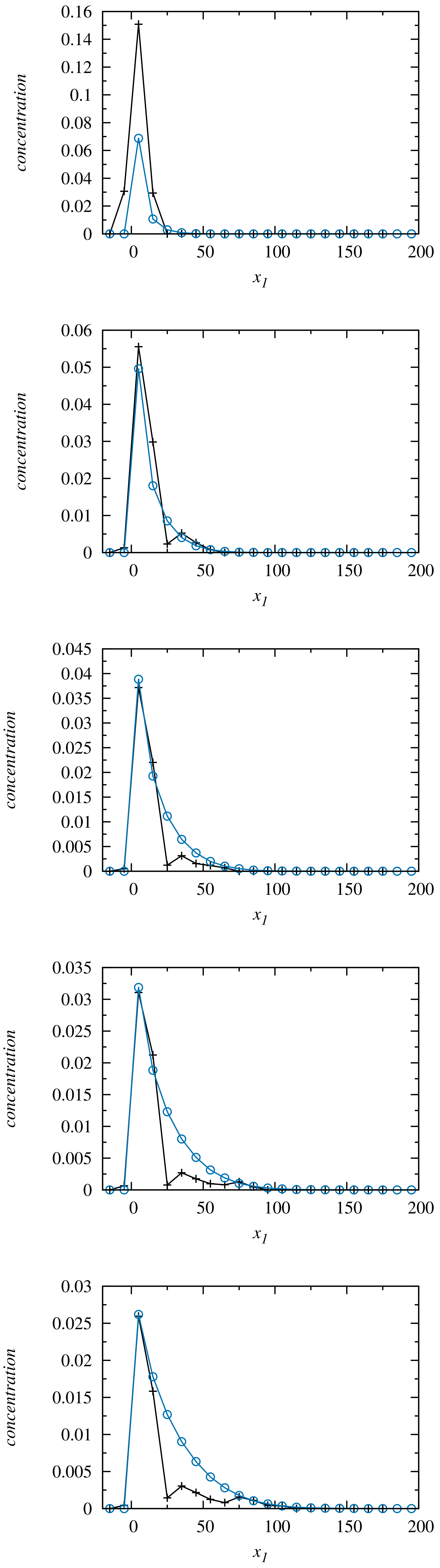
### Parameters

The relevant medium and flow parameters are obtained from [3–5].

- Hydraulic conductivity is modeled as a multi-Gaussian random field with lognormal point distribution.
- Variance $\sigma_f^2 = 5.9$, geometric mean conductivity $K_g = 5.5 \times 10^{-6}$ m/s, $\lambda_h = 9.1$ m, $\lambda_v = 1.8$ m
- Mean hydraulic gradient $J = 3.6 \times 10^{-3}$ m/s
- Porosity $\phi = 0.31$, retardation coefficient for bromide (MADE-1) $\theta = 1.2$ and for tritium (MADE-2) $\theta = 1$.

The upscaled model is used for the modeling of the MADE-1 [6] and MADE-2 tracer data based on these transport independent medium and flow characteristics without resorting to fitting parameters.

### MADE-1 Experiment



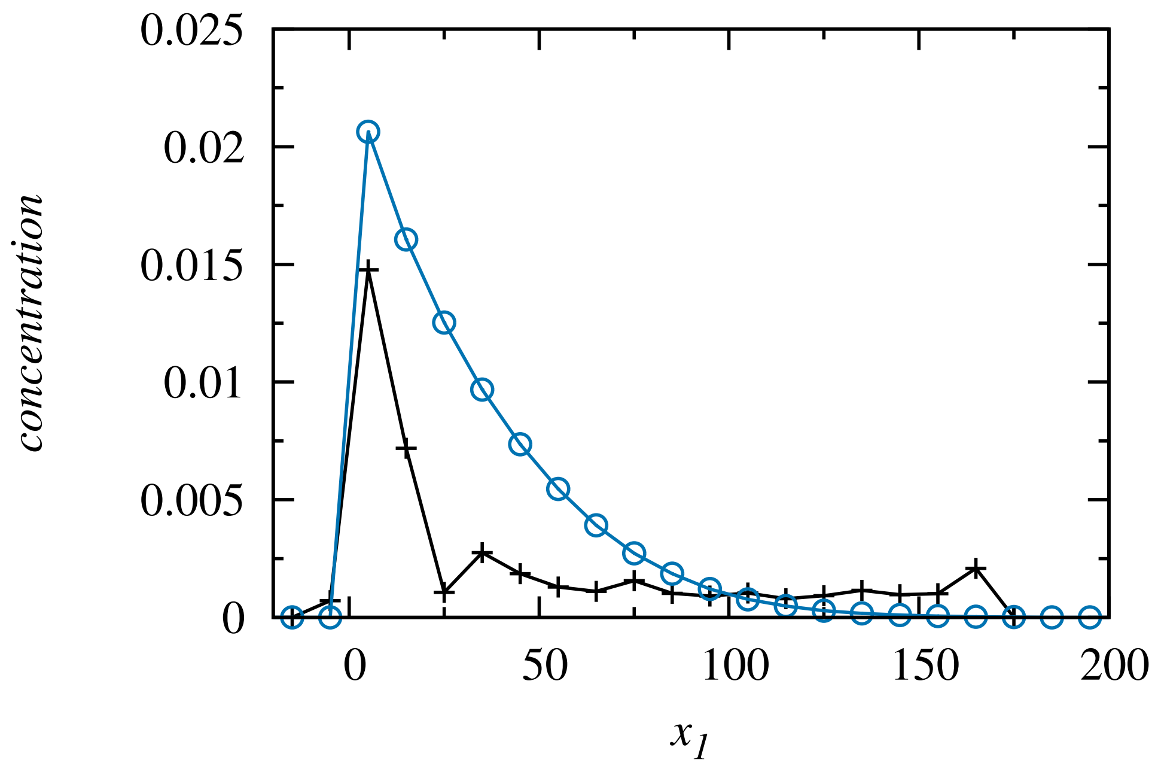


*Figure 2: Concentration profiles for the MADE1 experiment at $t = 49, 126, 279, 370, 503$ days [6].*

### MADE-2 Experiment

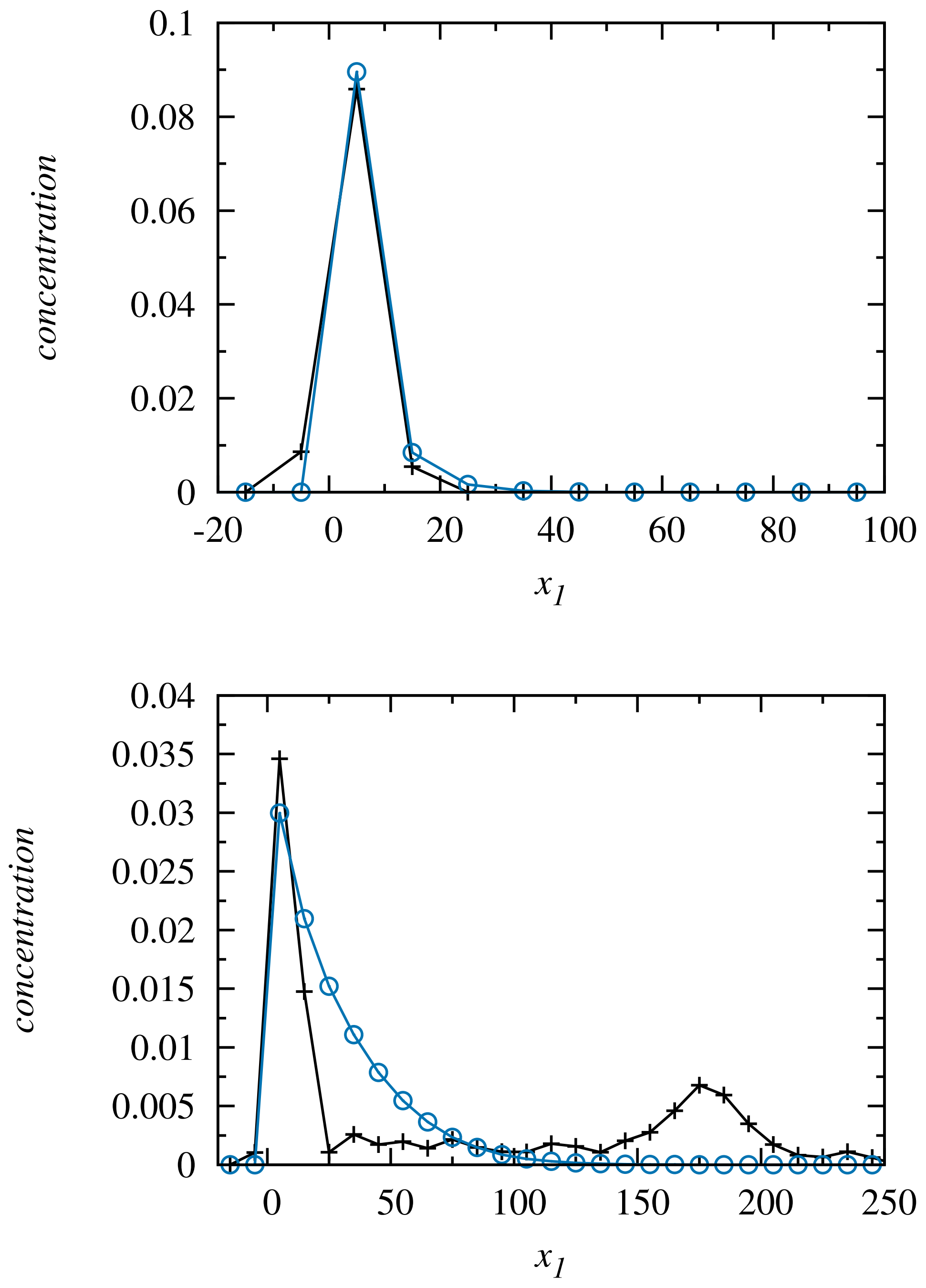


*Figure 3: Concentration profiles for the MADE2 experiment at $t = 27, 328$ days.*

## Conclusion

The upscaled model predicts the principal macroscopic transport features, the slowly moving peak and the foward tail based on the variability of hydraulic conductivity.

**References:**
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