Dispersion upscaling in highly heterogeneous aquifers: The prediction of tracer dispersion at the Macrodispersion Experiment (MADE) site

Alessandro Comolli  
Université Libre de Bruxelles (ULB), Nonlinear Physical Chemistry  
Unit, CP 231, 1050, Bruxelles, Belgium

Marco Dentz and Juan J. Hidalgo  
Spanish National Research Council (IDAEA-CSIC), Barcelona, Spain

Vivien Hakou  
BRGM, University of Montpellier, Montpellier, 34000, France

**Summary**

We derive an upscaled model for the prediction of the plume evolution in highly heterogeneous aquifers based on a stochastic representation in terms of continuous time random walks. Transport is modeled through advective motion of idealized solute particles, which changes their speed at fixed distances. The series of particle speeds is modeled as a stationary Markov chain. The derived model is parameterized by the correlation length, mean and variance of the log-hydraulic conductivity, the mean hydraulic gradient and porosity. Furthermore, it can be conditioned on the conductivity and tracer data at the injection region. The model predicts the non-Fickian evolution of the longitudinal concentration profile observed during the MADE-1 experiment. The mass distribution is characterized by strong localization at the injection region and the correlated motion of particles according to spatially persistent Eulerian flow speeds.

**Upscaled Transport Model**

- Stochastic advective particle motion [1]
  \[
  \frac{d\lambda(x)}{dt} = \frac{1}{v_0} g(x),
  \]
  where \( \gamma \) is advective tortuosity.
  - Distribution of flow speeds \( v_j(x) \)
    \[
    p_j(x) = \frac{g(x)}{\sqrt{2\pi\sigma^2}}
    \]
    where \( p_j(x) \) is distribution of Eulerian flow speed
  - Darcy flow
    \[
    q(x) = -\nabla h(x), \quad \nabla \cdot q(x) = 0.
    \]
  - Eulerian flow speed
    \[
    v_i(x) = v_0 q(x)
    \]
    \[
    v_0 = K_m / \phi
    \]
  - Adiabatic tortuosity is given by \( \chi = (\langle\gamma^2\rangle/\langle\gamma\rangle) \)
  - Propagation of normal speeds \( w(x) \) of particle speeds \( v_j(x) \) through an Ornstein-Uhlenbeck process [1,2].

**Figure 1:** Distribution of \( f = \ln K \) and \( \nu = \ln \lambda \) for a unit hydraulic head gradient and an anisotropy ratio of \( \lambda_2/\lambda_1 = 5 \).

**Prediction of MADE Tracer Data**

**Parameters**

- Hydraulic conductivity is modeled as a multi-Gaussian random field with lognormal point distribution.
- Variance \( \sigma^2 = 5 \mbox{ m}^2 / \mbox{s} \); geometric mean conductivity \( K_m = 5.5 \times 10^{-8} \mbox{ m/s} \).
- Mean hydraulic gradient \( J = 5.5 \times 10^{-3} \mbox{ m/s} \).
- Porosity \( \phi = 0.33 \).

The upscaled model is used for the modeling of the MADE-1 [6] and MADE-2 tracer data based on those transport independent medium and flow characteristics without resorting to fitting parameters.

**MADE-1 Experiment**

**MADE-2 Experiment**

**Figure 2:** Concentration profiles for the MADE1 experiment at \( t = 41 \), 126, 279, 320, 361 days [6].

**Figure 3:** Concentration profiles for the MADE2 experiment at \( t = 27 \), 63 days.

**Conclusion**

The upscaled model predicts the principal macroscopic transport features, the slowly moving peak and the foward tail based on the variability of hydraulic conductivity.

**References:**

[6] Hispanic National Research Council (IDAEA-CSIC), Barcelona, Spain

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