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# Estimating Vertical Land Motion in Northern Adriatic Sea with Coastal Altimetry and In Situ Observations

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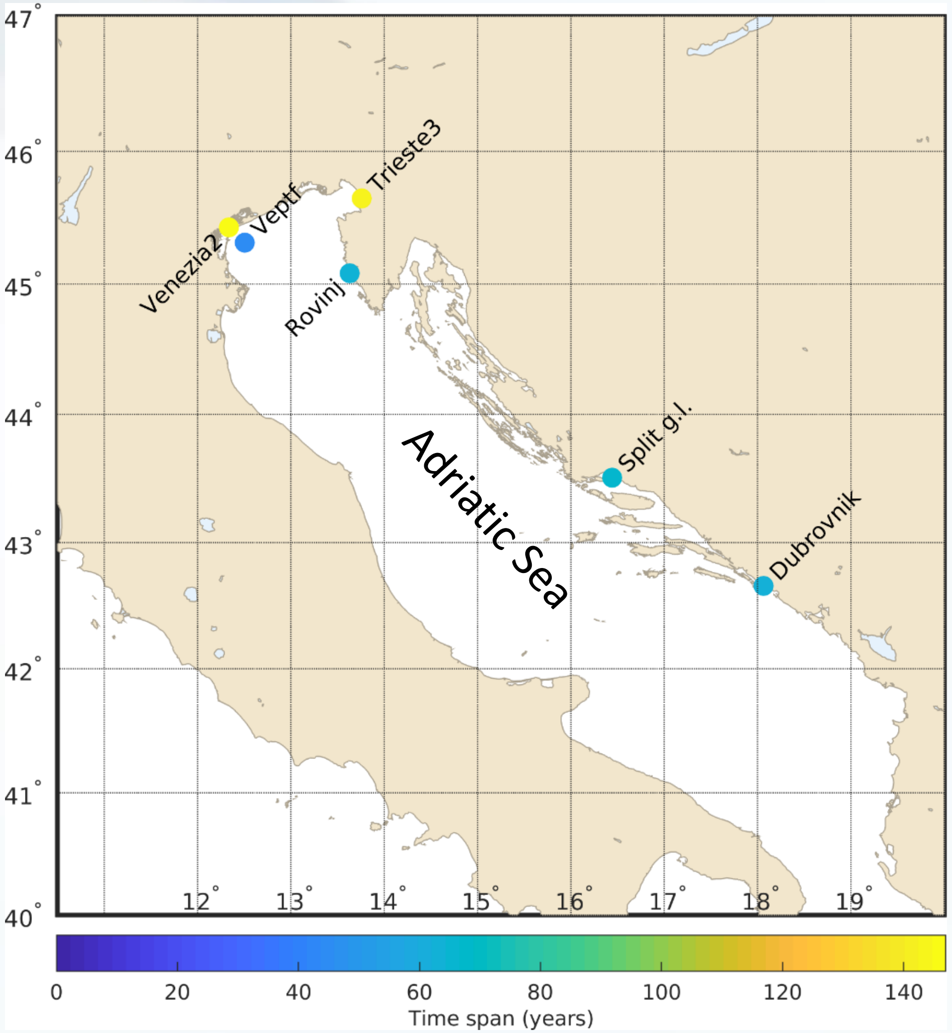
**Objective**: optimal estimation of the vertical land motion (VLM) at some tide gauge location, using two different state-of-the-art altimetry dataset of sea level anomaly (SLA) and tide gauge observations of sea level (SL) and GPS velocities near the tide gauges found on-line

**Why the Adriatic Sea?** Because it is one of the most exposed places in the Mediterranean Sea to the sea level rise and to storm surge related risks, and is thus an ideal place for validating coastal altimetry products




## *OUTLINE*:

- Sea Level Anomaly from altimetry datasets SLCCI and C3S
- Sea Level in situ datasets from tide gauges
- Vertical Land Motion from in situ CGPS
- Methods
- Results
- Summary

# Two altimetry SLA processing chains: ESA and Copernicus C3S

**Gridded monthly means of SLA**[1,2] @¼ degrees **1993-2015** from the ESA Sea Level Climate Change Initiative (**SLCCI**) project:

It is produced by the Climate Change Initiative project on "Sea Level" (SLCCI) of the European Space Agency (ESA). It is an improved set of reprocessed satellite-based sea level products, aimed at being a reference for climate studies

- Multimission: TOPEX/Poseidon, Jason-1, Jason-2, ERS-1, ERS-2, GeoSat Follow-On (GFO), Envisat, SARAL/AltiKa and CryoSat-2
- Processing: editing, cross-calibration, homogeneous corrections, removal of global and regional biases, homogenization of long-spatial-scale errors, monthly optimal interpolation gridding
- This climate data record (CDR) is designed to be the reference for climate/related sea level studies

**Gridded daily means of SLA**[3] @0.125 degrees **1993-2018** from the Copernicus Climate Change Service (**C3S**):

C3S provides this state-of-the-art, climate-oriented dataset of SLA for the Mediterranean Sea at 0.125 deg. resolution grid. Up-to-date altimeter standards are used to estimate the SLA with a mapping algorithm specifically dedicated to the Med Sea. Monthly means were obtained from daily means.

- Obtained using a stable two-satellite constellation of altimeters and homogeneous corrections and standards in time
- Processing: editing, cross-calibration, homogeneous corrections, removal of global and regional biases, homogenization of long-spatial-scale errors, optimal interpolation gridding
- This climate data record (CDR) is designed to be the up-to-date extension of the SLCCI SLA dataset to nowadays

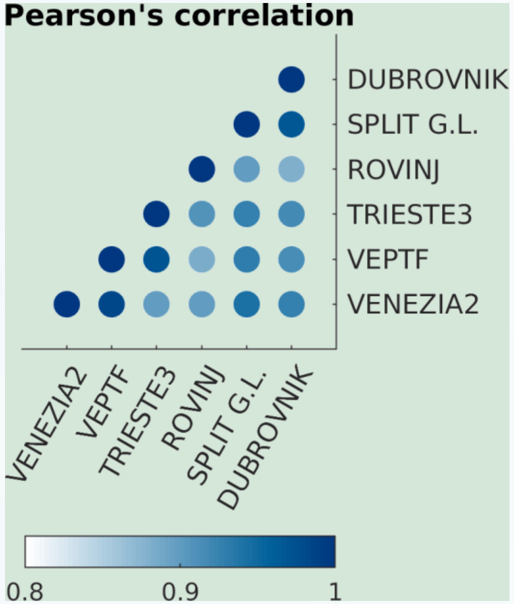
- The SLCCI project has developed consistent altimeter corrections in order to produce a homogeneous and stable global sea level product. The operational production of the climate-oriented global sea level product has now been taken over by the C3S. The main difference is that all available satellites have been included in the SLCCI product, whereas a stable number of two altimeters is used for the C3S product: this contributes to increase the stability of the sea level record, especially on a regional scale[4]
- Dynamic Atmospheric Correction (DAC) from CNES AVISO+ was re-added to both SLA datasets in order to obtain a sea level comparable to TG monthly means observations
- TOPEX-A drift in 1990-1998 was corrected neither in the SLCCI product nor in the C3S product[2,3]
- Satellite altimetry sea level observations are referenced to the ellipsoid, which is an absolute reference system

(1) *DOI: 10.5270/esa-sea_level_cci-MSLA-1993_2015-v_2.0-201612*
(2) *Legeais et al.: DOI: 10.5194/essd-10-281-2018, 2018*
(3) *http://datastore.copernicus-climate.eu/documents/satellite-sea-level/D3.SL.1-v1.2_PUGS_of_v1DT2018_SeaLevel_products_v2.4.pdf*
(4) *Legeais, J.F., personal communication*

# Six tide gauge SL from PSMSL(1,2) and other authorities(3,4)

The tide gauge data was retrieved from the Permanent Service for Mean Sea Level, the Tide Forecast and Early Warning Center of the Venice Municipality and from the Trieste section of the CNR-ISMAR Institute.
The mutual Person's linear correlation coefficient is always > 8.5

| TG name | Lat | Lon | Data (%) | Time span | Record length (Year) |
|---|---|---|---|---|---|
| VENEZIA2 | 45.431 | 12.336 | 97 | 1872 – 2018 | 148 |
| VEPTF | 45.314 | 12.508 | 100 | 1974 – 2018 | 46 |
| TRIESTE3 | 45.647 | 13.760 | 89 | 1875 – 2018 | 145 |
| ROVINJ | 45.083 | 13.628 | 99 | 1955 – 2018 | 65 |
| SPLIT G.L. | 43.507 | 16.442 | 100 | 1952 – 2018 | 68 |
| DUBROVNIK | 42.658 | 18.063 | 99 | 1956 – 2018 | 64 |




- TG SL observations are measured with respect to relative references: usually a benchmark in the TG cabin or nearby
- Processing: X0-filtering for VENEZIA2 and VEPTF
- Trends errors are calculated taking into account serial correlation, and are given with 95% confidence interval.

*(1) Holgate et al. (2013), Journal of Coastal Research, 29, 3, 493 – 504, doi:10.2112/JCOASTRES-D-12-00175.1*
*(2) Permanent Service for Mean Sea Level (PSMSL), 2020, data retrieved 08 Apr 2020 from http://www.psmsl.org/data/obtaining/.*
*(3) VENEZIA2 and VEPTF TG data kindly provided by the Tide Forecast and Early Warning Center of the Venice Municipality*
*(4) TRIESTE3 kindly provided by CNR-ISMAR section of Trieste*

# Geocentric surface velocities from CGPS at four tide gauges from the Nevada Geodetic Laboratory(1), SONEL(2) and ISPRA(3)

| CGPS STATION | LAT | LON | V up NGL (MIDAS) (mm $yr^{-1}$) | Record Length & span (Year) | V up SONEL (mm $yr^{-1}$) | Record Length & span (Year) | V up ISPRA (mm $yr^{-1}$) | Record Length & span (Year) | Vup Pooled mean (mm $yr^{-1}$) |
|---|---|---|---|---|---|---|---|---|---|
| VENEZIA PSAL | 45.431 | 12.337 | -1.70 ± 0.86 | 6 (2014-2020) | - | - | -1.46 ± 0.09 | 5 (2010-2015) | -1.59 ± 0.65 |
| TRIESTE TRIE | 45.710 | 13.764 | -0.52 ± 0.45 | 17 (2003-2020) | 0.20 ± 0.26 | 10 (2003-2013) | - | - | -0.25 ± 0.52 |
| SPLIT SPLT | 45.507 | 16.438 | 0.45 ± 0.68 | 8 (2004-2012) | -0.25 ± 0.34 | 8 (2004-2012) | - | - | 0.10 ± 0.64 |
| DUBROVNIK DUBR | 42.650 | 18.110 | -1.99 ± 0.80 | 12 (2000-2012) | -1.61 ± 0.24 | 12 (2000-2012) | - | - | -1.83 ± 0.70(4) |
| DUBROVNIK DUB2 | 42.650 | 18.110 | -1.94 ± 0.89 | 8 (2012-2020) | - | - | - | - | |

Several solutions are nowadays available on-line for the Continuous GPS monitoring of selected locations, in particular near TGs: SONEL (Université La Rochelle) and Nevada Geodetic Laboratory (University of Nevada). Values of VLM are sometimes very different from centre to centre, and in any case they are often calculated on a limited time-span.
For Venice we report also the solution obtained by the Istituto Superiore per la Protezione e la Ricerca Ambientale (ISPRA), which performs continuous checks on the benchmarks of the geodetic network around the CGPS station of VENEZIA PSAL.

*(1) Blewitt et al. (2018), Eos, 99, [https://doi.org/10.1029/2018EO104623](https://doi.org/10.1029/2018EO104623)*
*(2) On-line: [https://www.sonel.org/-GPS-.html](https://www.sonel.org/-GPS-.html)*
*(3) Baldin G., Crosato F., (2017), ISPRA, Quaderni - Ricerca Marina, 10/2017, Roma*
*(4) This value is the pooled mean of DUBR and DUB2. Pooled mean is defined in «Cochrane Handbook for Systematic Reviews of Interventions», 2° Ed., DOI:10.1002/9781119536604*

# Methods

- The **SLCCI** and **C3S** gridded time-series of **SLA** monthly means, were compared to the TG SL observations, also organized in monthly mean time series
- The altimeter grid point associated with the TG was decided on the base of the proximity and of the maximum correlation
- Another experiment was attempted associating to the TG time series the mean of all the altimeter grid points in the distance range of 10-50 km, and farer than 10 km from coast (to avoid orographic disturbances). No very profound differences were found in this case, and the results have been omitted

We compared the slopes (trends) of the SL time series  derived from the altimetry and SL from the TGs:

- For the altimetry datasets the DAC correction was re-applied to SLA
- For both altimetry and tide gauge time series the annual/inter-annual cyclic variations were subtracted
- In order to get an optimal estimate of the geocentric vertical motion at the TGs, we use the technique developed by Kuo et al.(1), perfected by Wöppelmann and Marcos(2) and based on the solution of the linear inverse problem with constraints (Menke(3))

(1) Kuo, C. Y., C. K. Shum, A. Braun, and J. X. Mitrovica (2004), Geophys. Res. Lett., 31, L01608 DOI: 10.1029/2003GL019106

(2) Wöppelmann, G., and Marcos, M. (2012), J. Geophys. Res., 117, C01007, DOI: 10.1029/2011JC007469

(3) Menke, W. (1989), Geophysical Data Analysis: Discrete Inverse Theory,289 pp., Academic, San Diego, Calif.

# The Linear Inverse Problem with constraints (L.I.P.W.C.) (1)

The rate of absolute vertical land movement at tide gauge *i* is given by the difference between the absolute sea level change rate and the relative sea level change rate at the same place:

**a)** $$\dot{\boldsymbol{u}}_i = \dot{\boldsymbol{g}}_i - \dot{\boldsymbol{S}}_i^{Alt}$$

$\dot{g}_i$, $\dot{S}_i^{Alt}$ = **absolute** and **relative** sea level change **rates** at the tide gauge *i*; dot means time differentiation. In practice, $\dot{g}_i$ is measured by the altimeter, and $\dot{S}_i^{Alt}$ by the tide gauge.

This equation is sufficient to obtain an estimate of the VLM rates at each tide gauge(1). However with often strong uncertainties. Solution: introduce the rate of relative vertical motion between two nearby tide gauges: $r\dot{u}_{ij} = (\dot{g}_i - \dot{S}_i^{Alt})$ - $(\dot{g}_j - \dot{S}_j^{Alt})$ which reduces to:

**b)** $$\boldsymbol{r\dot{u}_{ij}} = \dot{\boldsymbol{S}}_j^{Alt} - \dot{\boldsymbol{S}}_i^{Alt}$$

if $\dot{g}_i = \dot{g}_j$, i.e., if the absolute sea level change rate is the same at the two different locations.

As in general the rates $\boldsymbol{r\dot{u}_{ij}}$ have much smaller errors, they can be used to reduce the overall error in the $\dot{\boldsymbol{u}}_i$. This is done by putting the N equations (**a**) in matrix form:

**c)** $$G \cdot \dot{\boldsymbol{u}} = \boldsymbol{d}; \quad \dot{\boldsymbol{u}} = \begin{pmatrix} \dot{u}_1 \\ \vdots \\ \dot{u}_N \end{pmatrix}; \quad \boldsymbol{d} = \begin{pmatrix} \dot{g}_1 - \dot{S}_1^{Alt} \\ \vdots \\ \dot{g}_N - \dot{S}_N^{Alt} \end{pmatrix}; \quad G = Identity$$

and the M<N equations (**b**) as constraints to the linear system:

*d)* $$F \cdot \dot{\boldsymbol{u}} = \boldsymbol{h}; \quad \boldsymbol{h} = -F \cdot \begin{pmatrix} \dot{S}_1^{TG} \\ \vdots \\ \dot{S}_N^{TG} \end{pmatrix}; \quad F = \begin{bmatrix} 1 & -1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & -1 & 0 & \cdots & 0 \\ & \cdots & & & \cdots & \\ 0 & 0 & 0 & \cdots & 1 & -1 \end{bmatrix}$$

The constraints can be chosen arbitrarily, but they have to be linearly independent so that the rank of the matrix F is <=N-1, and that the condition expressed in b) is true $(\dot{g}_i - \dot{g}_j = 0)$.

(1) Cazenave, A., K. Dominh, F. Ponchaut, L. Soudarin, J. F. Crétaux, and C. Le Provost (1999), Geophys. Res. Lett., 26, 2077–2080, doi:10.1029/1999GL900472

# The Linear Inverse Problem with constraints (L.I.P.W.C.) (2)

The linear system **c) + d)** is simoultaneously solved with the use of Lagrange multipliers[(1)]:

$$\begin{bmatrix} G^T \cdot G & F^T \\ F & 0 \end{bmatrix} \begin{pmatrix} \dot{\boldsymbol{u}} \\ \boldsymbol{\lambda} \end{pmatrix} = \begin{pmatrix} \boldsymbol{d} \\ \boldsymbol{h} \end{pmatrix}$$

By using the generalized inverse of the matrix $\begin{bmatrix} G^T \cdot G & F^T \\ F & 0 \end{bmatrix}$.
Errors are calculated with the associated covariance matrix.

Wöppelmann and Marcos[(2)] used a strategy to further reduce the errors on the time series contributing to the rates ***d*** and ***h***: instead of calculating the error on the differences of the rates ($d_i = \dot{g}_i - \dot{S}_i^{Alt}$; $h_{ij} = \dot{S}_j^{TG} - \dot{S}_i^{TG}$ ), they calculated the error on the rates of the differenced time series, as in this way the error results lower.

However, still exists the limitation posed by different absolute sea level rates in forming "homogeneous" relative VLM rates between TG and TG.

**In this study we try to overcome this limitation, in two ways:**

**Bias method 1:** ***introducing "fake" relative sea level rates at each tide gauge, suitably constructed to cancel the local absolute sea level change rates at each TG. They are then removed at the end from the resulting VLM***:

$$\begin{aligned} \dot{S}_i^{Alt} &\rightarrow \dot{S}_i^{Alt} + \dot{g}_i \\ \dot{S}_i^{TG} &\rightarrow \dot{S}_i^{TG} + \dot{g}_i \\ \dot{u}_i &\rightarrow \dot{u}_i - \dot{g}_i \end{aligned}$$

**Bias method 2:** ***adding and subtracting in eq. (a) a constant equal to the local absolute sea level rate, and leaving unaltered the computations, apart from the calculations of the TG-TG time series differences***:

$$\dot{u}_i = \dot{g}_i - \dot{S}_i^{Alt} = \dot{g}_i - \dot{g}_i + \dot{g}_i - \dot{S}_i^{Alt} = \dot{g}_i{}' - \dot{S}_i^{Alt\prime}$$

$$\begin{aligned} \dot{S}_i^{Alt\prime} &= \dot{S}_i^{Alt} - \dot{g}_i \\ \dot{S}_i^{TG\prime} &= \dot{S}_i^{TG} - \dot{g}_i \\ \dot{g}_i{}' &= \dot{g}_i - \dot{g}_i = 0 \end{aligned}$$

We hope also that this work will help to clarify the role of the limitation imposed by eq. b), i.e. the assumption that the relative differences of the TG–TG time series are valid as long as the absolute sea level trends at the TGs are comparable.


(1) *Menke, W. (1989), Geophysical Data Analysis: Discrete Inverse Theory,289 pp., Academic, San Diego, Calif.*

(2) *Wöppelmann, G., and Marcos, M. (2012), J. Geophys. Res., 117, C01007, DOI: 10.1029/2011JC007469*

# Results 1: the “standard”(1) approach (ALT-TG) and the L.I.P.W.C.

## SLCCI altimetry dataset

| TG name | ALT ($\dot{g}$) | TG$^{Alt}$ ($\dot{S}^{Alt}$) | ALT-TG$^{Alt}$ ($\dot{u}$) | L.I.P.W.C. ($\dot{u}$) | L.I.P.W.C. & DTS(2) ($\dot{u}$) | CGPS ($\dot{u}$) |
|---|---|---|---|---|---|---|
| VENEZIA2 | +4.24±1.95 | +6.08±2.08 | -1.84±0.86 | -1.57±0.40 | -1.87±0.33 | -1.59±0.65 |
| VEPTF | +4.24±1.95 | +6.44±2.07 | -2.19±0.86 | -2.62±0.53 | -1.98±0.43 | - |
| TRIESTE3 | +3.43±1.93 | +4.50±1.98 | -1.07±0.70 | -0.09±0.39 | -0.39±0.32 | -0.25±0.52 |
| ROVINJ | +3.89±1.93 | +1.91±2.04 | +1.98±1.02 | +0.41±0.38 | +0.41±0.33 | - |
| SPLIT G.L. | +3.68±1.49 | +4.15±1.87 | -0.47±0.76 | +0.10±0.36 | +0.09±0.30 | 0.10±0.64 |
| DUBROVNIK | +4.09±1.43 | +4.67±1.70 | -0.58±0.67 | -0.41±0.36 | -0.38±0.30 | -1.83±0.70 |
| *average* | *+3.93±1.78* | *+4.62±1.96* | *-0.70±0.81* | *-0.70±0.40* | *-0.69±0.34* | - |

## C3S altimetry dataset

| TG name | ALT ($\dot{g}$) | TG$^{Alt}$ ($\dot{S}^{Alt}$) | ALT-TG$^{Alt}$ ($\dot{u}$) | L.I.P.W.C. ($\dot{u}$) | L.I.P.W.C. & DTS(2) ($\dot{u}$) | CGPS ($\dot{u}$) |
|---|---|---|---|---|---|---|
| VENEZIA2 | +3.19±1.48 | +5.15±1.73 | -1.96±0.69 | -1.23±0.37 | -1.51±0.31 | -1.59±0.65 |
| VEPTF | +3.16±1.50 | +5.50±1.73 | -2.35±0.71 | -2.28±0.50 | -1.63±0.42 | - |
| TRIESTE3 | +3.63±1.60 | +3.57±1.66 | +0.06±0.63 | +0.26±0.36 | -0.04±0.31 | -0.25±0.52 |
| ROVINJ | +3.15±1.56 | +1.03±1.85 | +2.12±1.07 | +0.75±0.35 | +0.77±0.31 | - |
| SPLIT G.L. | +3.52±1.30 | +2.92±1.65 | +0.60±0.64 | +0.44±0.33 | +0.44±0.29 | 0.10±0.64 |
| DUBROVNIK | +3.19±1.17 | +3.79±1.48 | -0.60±0.58 | -0.07±0.33 | -0.03±0.29 | -1.83±0.70 |
| *average* | *+3.31±1.43* | *+3.66±1.68* | -0.36±0.72 | *-0.36±0.38* | *-0.34±0.32* | - |

- *The SLCCI altimetry dataset reports much variable rates than the C3S dataset in the Adriatic Sea: for the latter the absolute sea level change rates are between 3.15 and 3.52 mm/yr, while the former at the same places reports rates between 3.43 and 4.24 mm/yr*
- *Nonetheless, the VLM calculated from the difference between ALT and TG rates are similar for SLCCI and C3S, apart from Trieste and Split which exhibit significantly different behaviours. This is obviously due to the different characteristics of the two altimetry datasets, which span different tiime periods, have different resolutions, different processing chains and – for C3S – a specific processing for the Mediterranean Sea*
- *The error on the VLM ($\dot{u}$) estimates results lower in the constrained problem with respect to the direct problem: the mean std error passes from 0.81 mm/yr (ALT-TG) to 0.40 in the constrained linear inverse problem, and to 0.34 mm/yr with the DTS(1) method, where std errors are calculated on the differenced time series. Smaller errors are found using the C3S altimetry dataset, maybe for the longer altimetry period with respect to SLCCI*
- *For Rovinj, both datasets (C3S and SLCCI) give a L.I.P.W.C. VLM much lower than the standard (ALT-TG), and unfortunately the nearest CGPS is 16 km far from it, giving no useful information*
- *For Dubrovnik the calculated VLM results markedly different from the CGPS observations: in this case we do suppose that CGPS rates of DUBR and DUB2 (1 km apart from the TG and 400 m above s.l.) do not reflect the effective movement of the TG*
- *Also TRIE CGPS is 6.5 km far from Trieste TG, and at higher level: however the LIPWC results supply for it VLM rates in agreement with CGPS and with the ALT-TG rates calculated with the C3S altimetry dataset*
- *As for the CGPS in Venice, it is not colocated with the TG, but it is just 10 m far from it, and continuous high resolution leveling campaigns conducted by ISPRA keep the TG reference well anchored to the CGPS one. Similar situation, if not better, in Split, where the CGPS is anchored to the TG: in this case the VLM rates obtained by the LIPWC analysis with the SLCCI dataset are very close to the CGPS rate*

(1) «standard» means the direct calculation of the VLM from the difference $\dot{g}_i - \dot{S}_i^{Alt}$ as in Cazenave et al. (1999)

(2) DTS: Differenced Time Series as in Wöppelmann & Marcos (2012)

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# Results 2: the L.I.P.W.C. + the bias method 1 (“fake” bias)

## SLCCI altimetry dataset

| TG name | ALT-TG[Alt] ($\dot{u}$) | L.I.P.W.C. ($\dot{u}$) | L.I.P.W.C. & DTS[(1)] ($\dot{u}$) | L.I.P.W.C. + bias 1 ($\dot{u}$) | L.I.P.W.C. + bias 1 & DTS[(1)] ($\dot{u}$) | CGPS ($\dot{u}$) |
|---|---|---|---|---|---|---|
| VENEZIA2 | -1.84±0.86 | -1.57±0.40 | -1.87±0.33 | -1.57±0.42 | -1.87±0.33 | -1.59±0.65 |
| VEPTF | -2.19±0.86 | -2.62±0.53 | -1.98±0.43 | -2.62±0.51 | -1.98±0.43 | - |
| TRIESTE3 | -1.07±0.70 | -0.09±0.39 | -0.39±0.32 | -0.09±0.42 | -0.39±0.32 | -0.25±0.52 |
| ROVINJ | +1.98±1.02 | +0.41±0.38 | +0.41±0.33 | +0.41±0.40 | +0.41±0.33 | - |
| SPLIT G.L. | -0.47±0.76 | +0.10±0.36 | +0.09±0.30 | +0.10±0.39 | +0.09±0.30 | 0.10±0.64 |
| DUBROVNIK | -0.58±0.67 | -0.41±0.36 | -0.38±0.30 | -0.41±0.39 | -0.38±0.30 | - |
| ***average*** | *-0.70±0.81* | *-0.70±0.40* | *-0.69±0.34* | *-0.70±0.42* | *-0.69±0.34* | *-0.58±0.60* |

## C3S altimetry dataset

| TG name | ALT-TG[Alt] ($\dot{u}$) | L.I.P.W.C. ($\dot{u}$) | L.I.P.W.C. & DTS[(1)] ($\dot{u}$) | L.I.P.W.C. + bias 1 ($\dot{u}$) | L.I.P.W.C. + bias 1 & DTS[(1)] ($\dot{u}$) | CGPS ($\dot{u}$) |
|---|---|---|---|---|---|---|
| VENEZIA2 | -1.96±0.69 | -1.23±0.37 | -1.51±0.31 | -1.23±0.37 | -1.51±0.31 | -1.59±0.65 |
| VEPTF | -2.35±0.71 | -2.28±0.50 | -1.63±0.42 | -2.28±0.48 | -1.63±0.42 | - |
| TRIESTE3 | +0.06±0.63 | +0.26±0.36 | -0.04±0.31 | +0.26±0.37 | -0.04±0.31 | -0.25±0.52 |
| ROVINJ | +2.12±1.07 | +0.75±0.35 | +0.77±0.31 | +0.75±0.35 | +0.77±0.31 | - |
| SPLIT G.L. | +0.60±0.64 | +0.44±0.33 | +0.44±0.29 | +0.44±0.34 | +0.44±0.29 | 0.10±0.64 |
| DUBROVNIK | -0.60±0.58 | -0.07±0.33 | -0.03±0.29 | -0.07±0.34 | -0.03±0.29 | - |
| ***average*** | *-0.36±0.72* | *-0.36±0.38* | *-0.34±0.32* | *-0.36±0.38* | *-0.34±0.32* | *-0.58±0.60* |

***In the table we report the results obtained using the “fake” bias (bias method 1; in red and blue) alongside the ALT-TG and L.I.P.W.C.&DTS methods already shown in the previous slide, for comparison***

- ***The errors are generally lower in the DTS counterpart of the calculated rates, as expected.***
- ***Taking as indicators Venice and Split, which have the most adequate locations of the CGPS stations, the SLCCI dataset seems to give the best agreement in terms of VLM rates, with the LIPWC&DTS method, irrespective of the introduction of the fake bias, while the C3S dataset ensures the lowest errors, maybe for the longest time span, 13% longer than SLCCI***
- ***The results of the SLCCI dataset seem to better reflect the rates calculated by the CGPS stations near the VENEZIA2, TRIESTE3 and SPLIT G.L. TG***
- ***The results of the “fake” bias method seem indistinguishable from the corresponding results of L.I.P.W.C. without bias (orange and pale blue); however the bias should have granted the validity of eg. b) irrespective of the choice of the TG-TG couple: one could ask himself which benefit brings the introduction of the “bias” in the calculation. The answer is not easy as the biases have been introduced in order to cancel the dependence of the TG relative sea level rates from the absolute sea level seen at the tide gauge. However, no evidence are seen that this happens. Anyway, the two bias methods are seen to maintain the invariance of the LIPWC method to independent permutations of the TG-TG couples, confirming that the biases are correctly formulated in the calculations***
- ***This holds true as long as the LIPWC method is used without the DTS: the reason is obvious: both errors calculated with the covariance matrix and DTS are non-linear in the rates, and specifically in the biases. The non-linearity of the DTS calculation brings asymmetry, and thus breaks the invariance w.r.t. permutations of the TG-TG couples***



(1) *DTS: Differenced Time Series as in Wöppelmann & Marcos (2012)*

# Results 3: the L.I.P.W.C. + the bias method 2

## SLCCI altimetry dataset

| TG name | ALT-TG$^{Alt}$ ($\dot{u}$) | L.I.P.W.C. ($\dot{u}$) | L.I.P.W.C. & DTS[1] ($\dot{u}$) | L.I.P.W.C. + bias 2 ($\dot{u}$) | L.I.P.W.C. + bias 2 & DTS[1] ($\dot{u}$) | CGPS ($\dot{u}$) |
|---|---|---|---|---|---|---|
| VENEZIA2 | -1.84±0.86 | -1.57±0.40 | -1.87±0.33 | -1.25±0.42 | -1.55±0.33 | -1.59±0.65 |
| VEPTF | -2.19±0.86 | -2.62±0.53 | -1.98±0.43 | -2.30±0.54 | -1.67±0.43 | - |
| TRIESTE3 | -1.07±0.70 | -0.09±0.39 | -0.39±0.32 | -0.59±0.41 | -0.90±0.32 | -0.25±0.52 |
| ROVINJ | +1.98±1.02 | +0.41±0.38 | +0.41±0.33 | +0.38±0.40 | +0.38±0.33 | - |
| SPLIT G.L. | -0.47±0.76 | +0.10±0.36 | +0.09±0.30 | -0.15±0.39 | -0.17±0.30 | 0.10±0.64 |
| DUBROVNIK | -0.58±0.67 | -0.41±0.36 | -0.38±0.30 | -0.25±0.39 | -0.23±0.30 | - |
| ***average*** | *-0.70±0.81* | *-0.70±0.40* | *-0.69±0.34* | *-0.70±0.42* | *-0.69±0.34* | *-0.58±0.60* |

## C3S altimetry dataset

| TG name | ALT-TG$^{Alt}$ ($\dot{u}$) | L.I.P.W.C. ($\dot{u}$) | L.I.P.W.C. & DTS[1] ($\dot{u}$) | L.I.P.W.C. + bias 2 ($\dot{u}$) | L.I.P.W.C. + bias 2 & DTS[1] ($\dot{u}$) | CGPS ($\dot{u}$) |
|---|---|---|---|---|---|---|
| VENEZIA2 | -1.96±0.69 | -1.23±0.37 | -1.51±0.31 | -1.35±0.38 | -1.63±0.31 | -1.59±0.65 |
| VEPTF | -2.35±0.71 | -2.28±0.50 | -1.63±0.42 | -2.43±0.52 | -1.78±0.42 | - |
| TRIESTE3 | +0.06±0.63 | +0.26±0.36 | -0.04±0.31 | +0.58±0.38 | +0.28±0.31 | -0.25±0.52 |
| ROVINJ | +2.12±1.07 | +0.75±0.35 | +0.77±0.31 | +0.60±0.36 | +0.61±0.31 | - |
| SPLIT G.L. | +0.60±0.64 | +0.44±0.33 | +0.44±0.29 | +0.65±0.35 | +0.65±0.29 | 0.10±0.64 |
| DUBROVNIK | -0.60±0.58 | -0.07±0.33 | -0.03±0.29 | -0.19±0.35 | -0.14±0.29 | - |
| ***average*** | *-0.36±0.72* | *-0.36±0.38* | *-0.34±0.32* | *-0.36±0.39* | *-0.34±0.32* | *-0.58±0.60* |

- *The table shows the results obtained using the second bias method (bias method 2; in red and blue) alongside the ALT-TG and L.I.P.W.C.&DTS methods already shown in the previous slides, for comparison. The results of the bias 2 method, in contrast with the bias 1 method, are different from the corresponding results of L.I.P.W.C. without bias (orange and pale blue), but still, the invariance w.r.t. independent permutations of the TG in the constraints is maintained. Errors instead change magnitude, due to the non-linearity in the std error of the covariance matrix*
- *The bias 2 method provides VLM rates less similar to the CGPS observations (VENEZIA2, TRIESTE3 and SPLIT G.L.), even if the averages (last row) are identical to those of the bias 1 method*

(1) DTS: Differenced Time Series as in Wöppelmann & Marcos (2012)

# Results 4: the L.I.P.W.C. and the bias method 1&2: a metrics

## SLCCI altimetry dataset

| METHOD «X» | RMSD(X,ALT-TG$^{Alt}$) $\delta_1$ | RMSD(X,CGPS) $\delta_2$ | <σ(X)> $\delta_3$ | $\Delta=\sqrt[2]{\sum_{i=1}^{3}\delta_i^2}$ |
|---|---|---|---|---|
| **LIPWC** | 0.82 | 0.10 | 0.40 | **0.92** |
| **LIPWC DTS** | 0.74 | 0.18 | 0.34 | **0.83** |
| **LIPWC b1** | 0.82 | 0.10 | 0.42 | **0.93** |
| **LIPWC b1 DTS** | 0.74 | 0.18 | 0.34 | **0.83** |
| **LIPWC b2** | 0.75 | 0.31 | 0.42 | **0.91** |
| **LIPWC b2 DTS** | 0.73 | 0.40 | 0.34 | **0.90** |

## C3S altimetry dataset

| METHOD «X» | RMSD(X,ALT-TG$^{Alt}$) $\delta_1$ | RMSD(X,CGPS) $\delta_2$ | <σ(X)> $\delta_3$ | $\Delta=\sqrt[2]{\sum_{i=1}^{3}\delta_i^2}$ |
|---|---|---|---|---|
| **LIPWC** | 0.68 | 0.41 | 0.38 | **0.88** |
| **LIPWC DTS** | 0.70 | 0.24 | 0.32 | **0.81** |
| **LIPWC b1** | 0.68 | 0.41 | 0.38 | **0.88** |
| **LIPWC b1 DTS** | 0.70 | 0.24 | 0.32 | **0.81** |
| **LIPWC b2** | 0.72 | 0.59 | 0.39 | **1.01** |
| **LIPWC b2 DTS** | 0.70 | 0.44 | 0.32 | **0.89** |

*To understand which analysis method is better, and which altimetry dataset provides more coherent rates, we introduce a metric based on three indexes $\delta_i$:*

1. *$\delta_1$: the RMSD of the ALT-TG rates and the "METHOD" rates, where "METHOD" is one of the six analyzed in this study, namely L.I.P.W.C., L.I.P.W.C. DTS, …*
2. *$\delta_2$: the RMSD of the CGPS rates and the "METHOD" rates, limited to the three CGPS stations which are coherent with the ALT-TG rates (VENEZIA PSAL, TRIESTE TRIE, SPLIT)*
3. *$\delta_3$: the mean of the std deviations associated with the METHOD (<σ(METHOD)>)*

***From these three indexes the lowest square root of the sum of the squares is taken as the "overall" index Δ. The red squares identify, for each altimetry dataset, the analysis methods with lowest Δ. The formulation consisting in the solution of the linear inverse problem with constraints (LIPWC) where TG-TG and ALT-TG rates are calculated by differentiation of the time series of SL higth (DTS) with and without bias (method 1 – fake bias) show the best score.***

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# Summary

*We have estimated **trends and errors** at six locations **in the Adriatic Sea**; we have used **three different measuring systems** (tide gauges, radar altimetry, continuous gps), **integrating the information** coming from each of the three system, in order **to maximize the knowledge, qualitatively and quantitatively**. We have **assessed** two different **altimetry** products (ESA **SLCCI** and Copernicus **C3S**) specifically processed for climate studies. We have compared the results with the **direct method** (subtracting the relative from the absolute sea level rates) and as a **constrained linear inverse problem**, which permits to **simoultaneously solve for the rates of all TGs**. We also tested the **robustness of the constrained linear inverse problem with respect to a known limitation**.*

***We found that:***

- *The two altimetry products, SLCCI and C3S, supply very similar results: errors on the calculated VLM rates are slightly lower for the C3S dataset, which cover a period 13% longer w.r.t. SLCCI*
- *The simoultaneous solution of the VLM rates with the constrained linear inverse problem, where the TG-TG and the ALT-TG rates were calculated by differencing the time series of sea level, had the best performance, both in the simple formulation and using a «fake» bias (bias method 1). The errors on the VLM rates are of the order of 0.3-0.4 mm yr$^{-1}$*
- *The use of biases does not bring any improvement in the derived rates and errors: this could suggest that the LIPWC method is robust enough not to suffer from deviation from the rule* $\dot{g}_i - \dot{g}_j = 0$
- *Overall, for the Adriatic Sea we obtain a consistent representation of the absolute and relative sea level change rates , from altimetry and tide gauges, but with a little difference which can be explained by the vertical motion of TGs, moving with mean rate of 0.35-0.70 mm yr$^{-1}$, but with different rates and signs from place to place. These rates are confirmed by the derived mean rates from 3 CGPS stations*

***To be considered:***

- *The SLCCI and C3S datasets cover slightly different periods*
- *The SLCCI and C3S products are generated from different processing chains, have different spatial and temporal resolutions, and C3S relies on a* mapping algorithm specifically dedicated to the Mediterranean Sea
- GPS data span very different time periods, but always shorter than altimetry and TGs time series; sometimes it is difficult to understand if CGPS stations do effectively reflect the tide gauge movement

***Open questions:***

- *Open question 1: can we use this strategy to analyze sea level rates in other regions of the Mediterranean Sea or elsewhere?*
- *Open question 2: how can we maximize the exploitation of the existing CGPS stations in this context, and improve the integration of the available measurement systems?*