

## Variability in Rainfall and Kinetic Energy across scales of measurement: evaluation using disdrometers in Paris region

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HS 7.6  
Precipitation small scale variability, hydrometeorologic extremes, and land-use feedbacks in the atmospheric water cycle, and beyond

## Aim:

Development of a physically based scale invariant power law relationship between rainfall kinetic energy (KE) and rain rate (R) using the framework of universal multifractals (UM)

$$KE = bR^a$$

Proper representation of KE and R is important in

- predicting soil erosion and
- quantifying leading edge erosion(LEE) in wind turbines

## Data used:

3 Disdrometers in Paris region with 30 s resolution

- 2 OTT Parsivel<sup>2</sup>
- 1 PWS

Campaign's locations :

- ENPC building (part of Fresnel observation platform)
- SIRTA (Site Instrumenté de Recherche par Télédétection Atmosphérique) at Ecole Polytechnique (EP) (part of a intensive campaign over Ile-de-France region)

## Analysis frameworks:

- Theoretical relationship using gamma distribution of DSD
- Universal Multifractals (UM) – based on rainfall events



location	start time	end time
ENPC (1)	18 Jun 2013	10 Nov 2016
EP SIRTA	14 Nov 2016	20 Sep 2017
ENPC (2)	27 Dec 2017	31 Dec 2019

Location of disdrometers and time line of precipitation data used

## Theoretical KE and R:

Rainfall intensity or rate (R) and kinetic energy (KE) from drop size distribution (DSD):

$$R = 3.6 \times 10^{-3} \frac{\pi}{6} \int_0^{\infty} D^3 N_A(D) dD$$

$$KE = 3.6 \times 10^{-6} \frac{\pi \rho}{12} \int_0^{\infty} D^3 v^2(D) N_A(D) dD$$

$N_A(D)$  – DSD at a surface per unit area per unit time, related to volumetric DSD as

$$N_A(D) = v(D) N_v(D)$$

$v(D)$  - terminal fall velocity ( $\text{ms}^{-1}$ ) at equivalent spherical diameter of raindrop  $D(\text{mm})$ .

$$v(D) = cD^\gamma$$

## Gamma DSD:

$$N_v(D) = N_0 D^\mu e^{-(\Lambda D)}$$

Assuming rainfall drop size distribution (DSD) to follow gamma distribution, rainfall rate (R) and kinetic energy (KE) can be derived as

$$R = 6 \times 10^{-4} \pi c N_0 \frac{\Gamma(4 + \gamma + \mu)}{\Lambda^{(4 + \gamma + \mu)}}$$

$$KE = 3.6 \times 10^{-6} \frac{\rho \pi c^3}{12} N_0 \frac{\Gamma(4 + 3\gamma + \mu)}{\Lambda^{(4 + 3\gamma + \mu)}}$$

Which enables expression of theoretical power law:

$$KE = bR^a$$

$$b = 5 \times 10^{-4} \rho c^2 [6\pi c N_0 \times 10^{-4}]^{1-a} \frac{\Gamma(6.01 + \mu)}{(\Gamma(4.67 + \mu))^a}$$

$$a = \frac{6.01 + \mu}{4.67 + \mu}$$

## Universal multifractals (UM):

In UM framework, the variability across scales of geophysical fields is characterized.

- Trace moment analysis (TM) gives quality of scaling:

$$\langle \varepsilon_\lambda^q \rangle \approx \lambda^{K(q)}$$

$\varepsilon_\lambda$  is a normalized conservative field (KE or R)\*,  $K(q)$  is the moment scaling function,  $\lambda$  is the resolution is defined by the ratio between outer scale and observation scale.

- For a conservative field,  $K_c(q)$  is fully characterized with the help of only two exponents:

$$K_c(q) = \frac{C_1}{\alpha - 1} (q^\alpha - q)$$

- multi-fractality index  $\alpha$  and
- mean intermittency codimension  $C_1$

## Power law in UM framework:

If a field is multifractal, then a power law relation of it is also multifractal with known UM parameters.

KE and R showed multifractal behaviour and power law relationship in UM framework

$$KE = bR^a$$

- where  $a$  is obtained from UM parameters

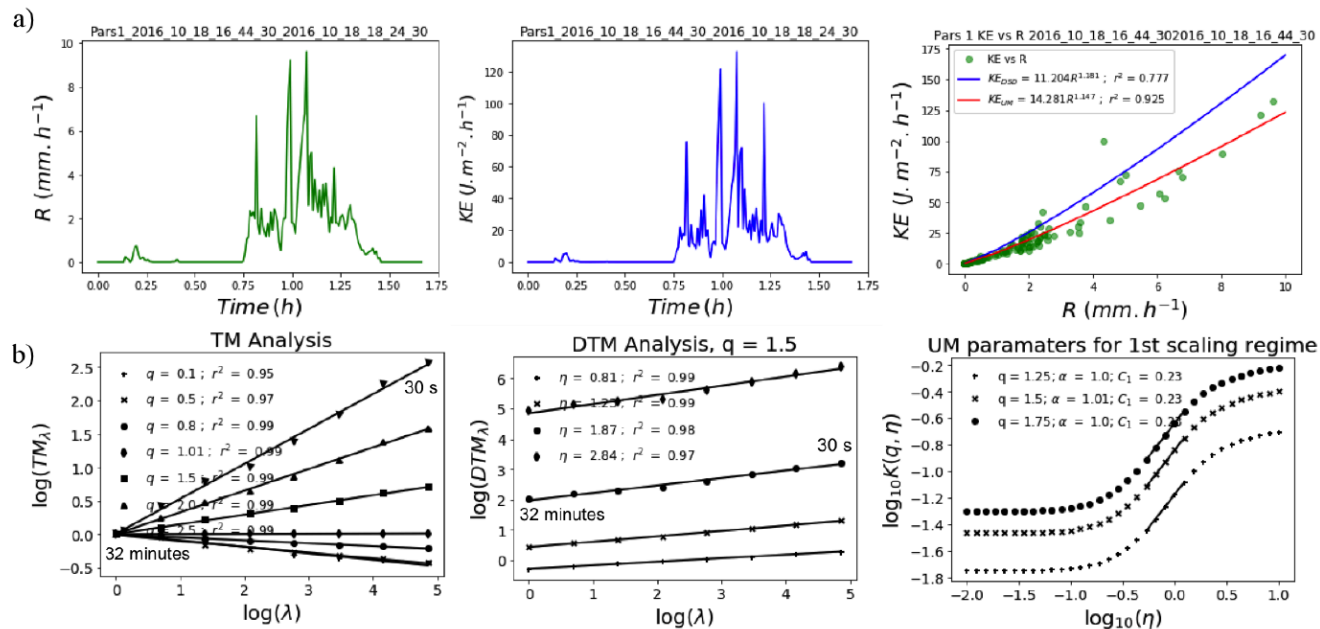
$$\alpha_{KE} \approx \alpha_R$$

$$C_{1,KE} \approx a^\alpha C_{1,R}$$

- $b$  from fitting of KE-R data

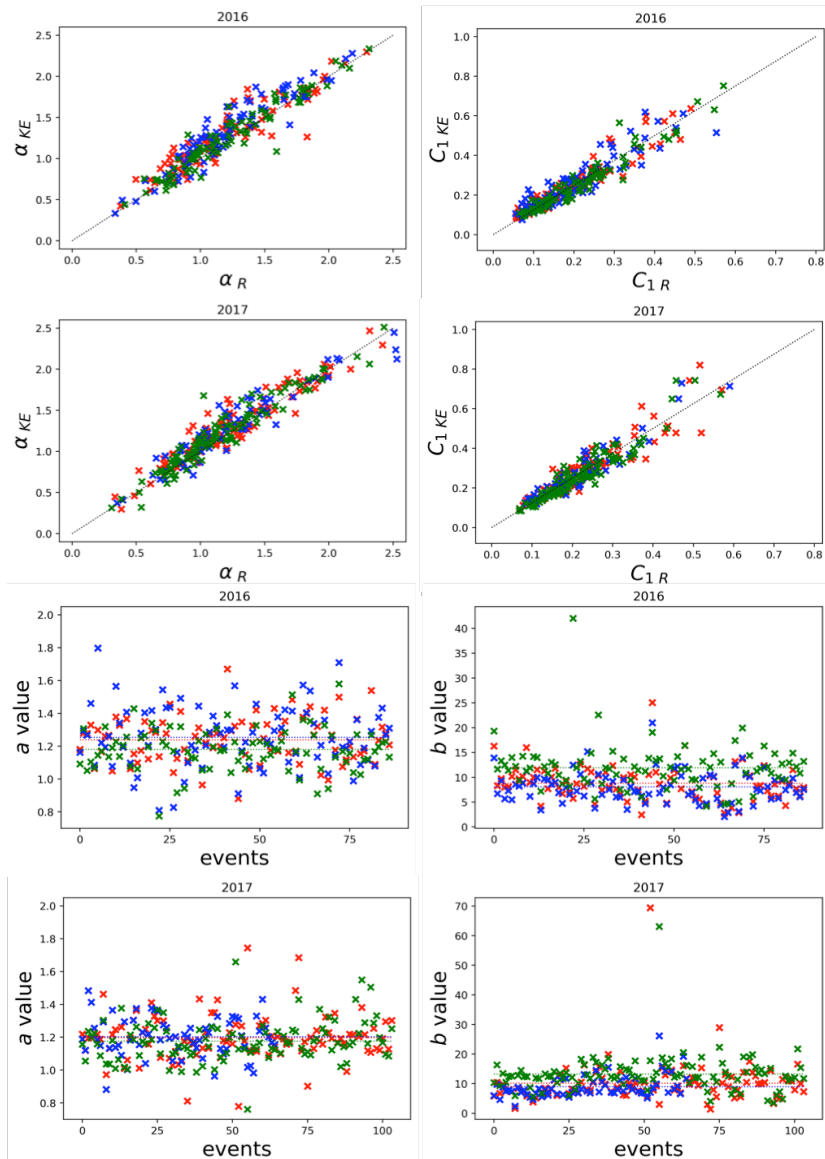
\*since the fields were too smooth, conservative fields were retrieved by taking fluctuations

- Individual rainfall events were identified with following criteria in rain intensity time series - at least 30 consecutive non zero time steps during which the rain rate is not less than  $0.7 \text{ mmh}^{-1}$ .
- a total of 556 rainfall events between 28 Sep 2013 and 31 Dec 2019 of which 493 events were common among all three disdrometers.
- Time series and UM analysis of a sample event below



a) Time series of R and KE and b) MF analysis graphs with KE as field for Pars 1 event 18 October 16:44:30 hr to 18:24:30 hr

## Variation of UM parameters and power law coefficients for the years 2016 and 2017



- Values of UM parameters,  $\alpha$  and  $C_1$  were estimated for all rain events across years 2013 to 2019
- Values of power law coefficients 'a' and 'b' were calculated from UM parameters
- Similar values were computed for whole data, including rainy and non rainy data points (next slide)

year	location	disdrometer	total events		
			# events	avg a	avg b
2016	ENPC	Pars 1	87	1.238	8.802
		Pars 2	87	1.253	8.069
		PWS	87	1.180	11.939
2017	EP-SIRTA*	Pars 1	102	1.197	10.126
		Pars 2	65	1.202	8.944
		PWS	104	1.176	13.238

Average value of power law coefficients, years 2016 & 2017



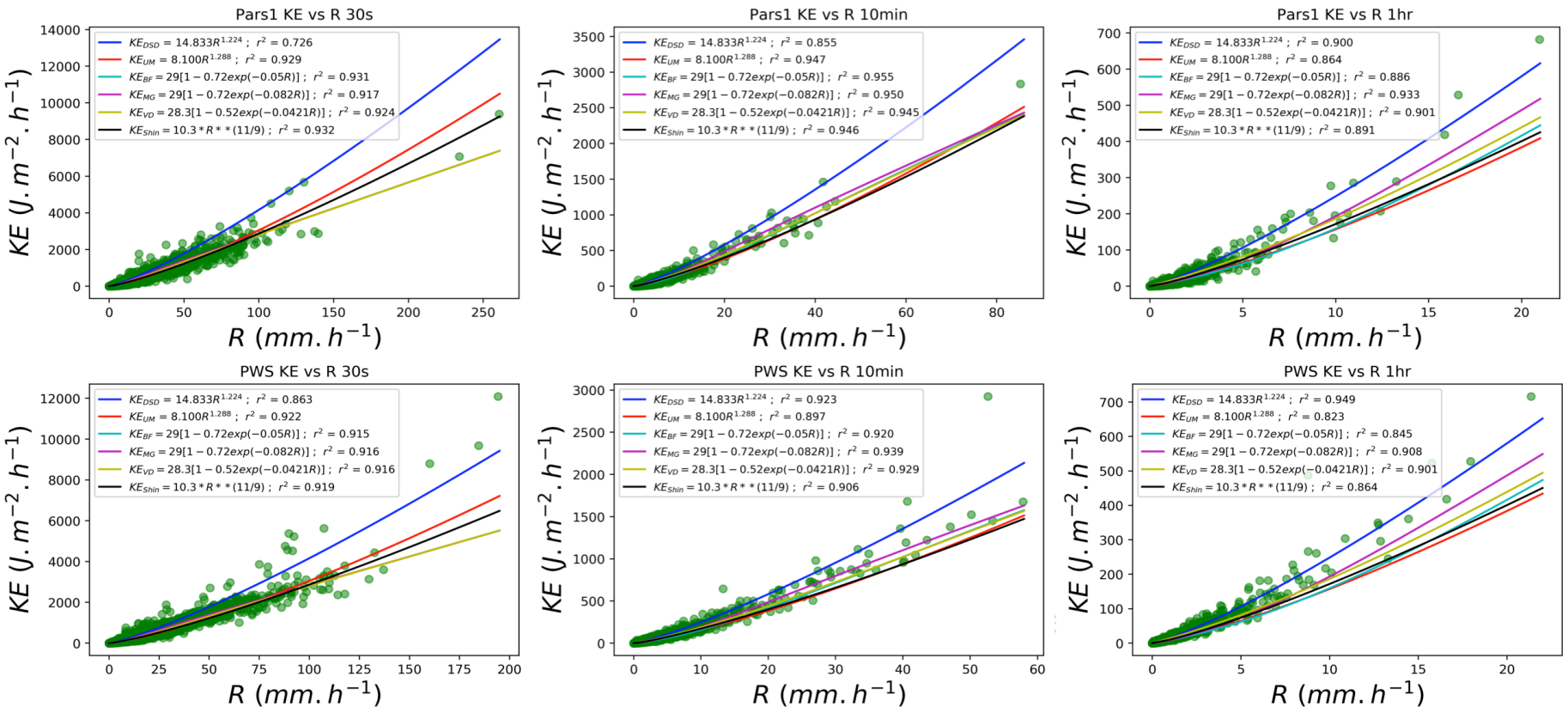
“a” and “b” rather stable despite strong variability in UM parameters among events

- Values of 'a' and 'b' for proposed power law relationship between KE and R were estimated using event based analysis. Stable estimates are found despite strong variations in UM parameters of single events.
- UM provides consistent results of 'a' with good correspondence to theoretical values.
- Obtained KE-R power law works well for different resolutions along side existing relations.

**30 seconds**

**10 minutes**

**1 hour**



KE-R relationships compared for Parsivel<sup>2</sup> and PWS at different resolutions



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