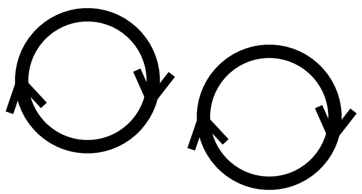
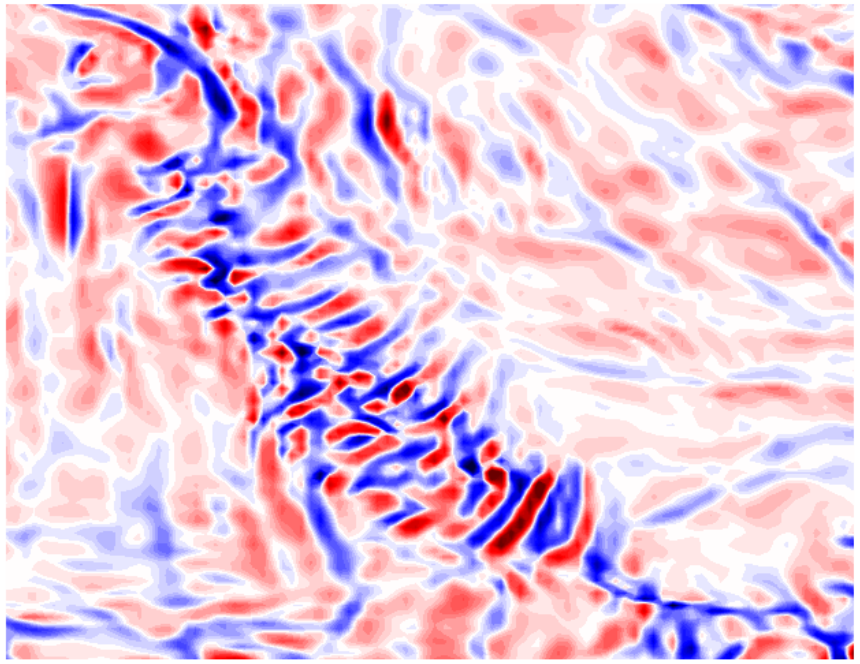
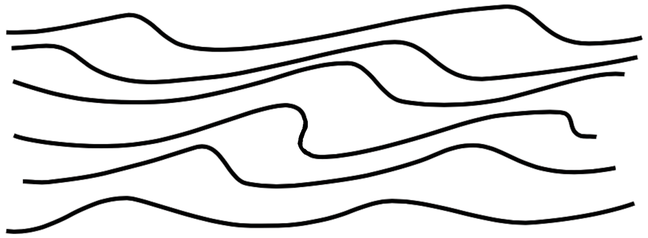
Energy Transfers Between Balanced And Unbalanced Motions In Geophysical Flows



Mesoscale eddies
(balanced)





Internal gravity waves
(unbalanced)

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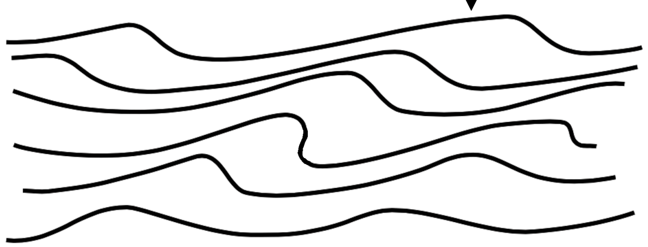
Balanced and unbalanced motions

Geophysical flows

two timescale system



Internal gravity waves
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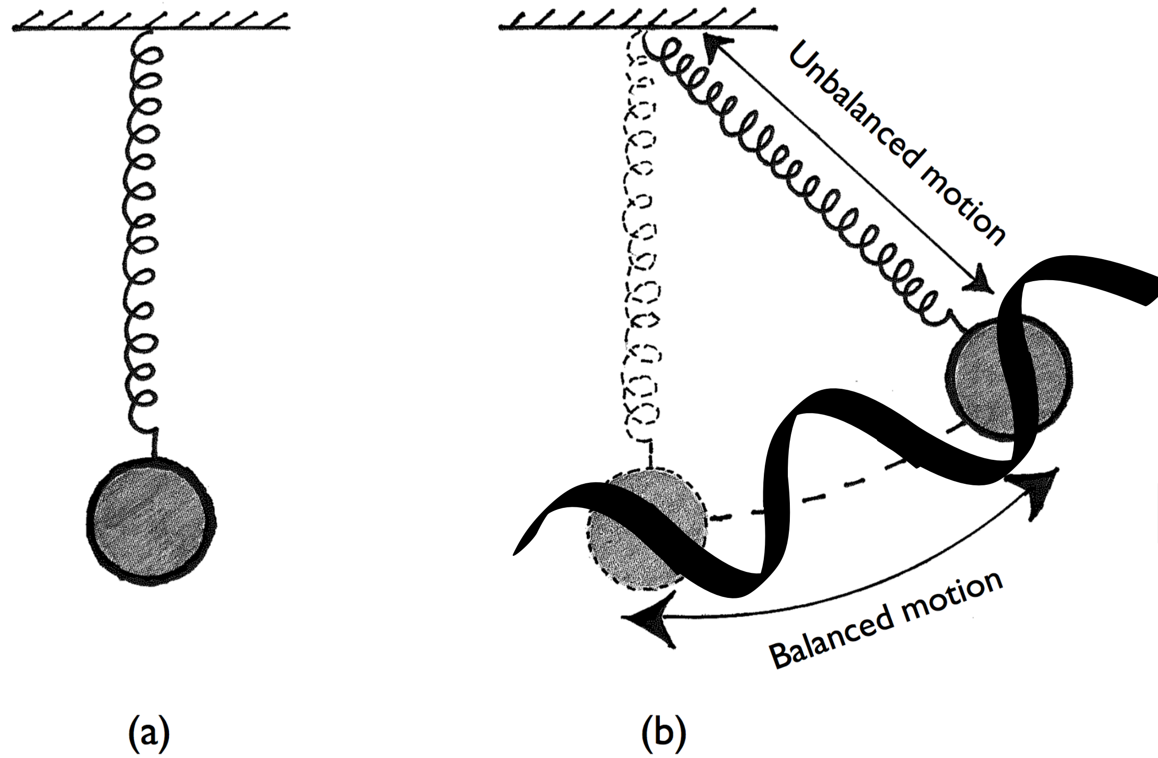
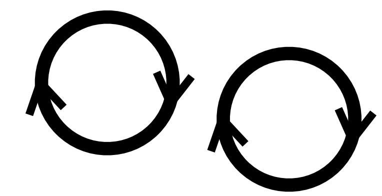




Mesoscale eddies
(balanced)

Unbalanced flows

» *no balance*
» *high frequency*

fast mode

gravity mode

Gravity manifold

Balanced flows

» *Geostrophic balance*
» *low frequency*

slow mode

vortex/vortical mode

Rossby manifold

Non-linear interactions

Different dynamical regimes

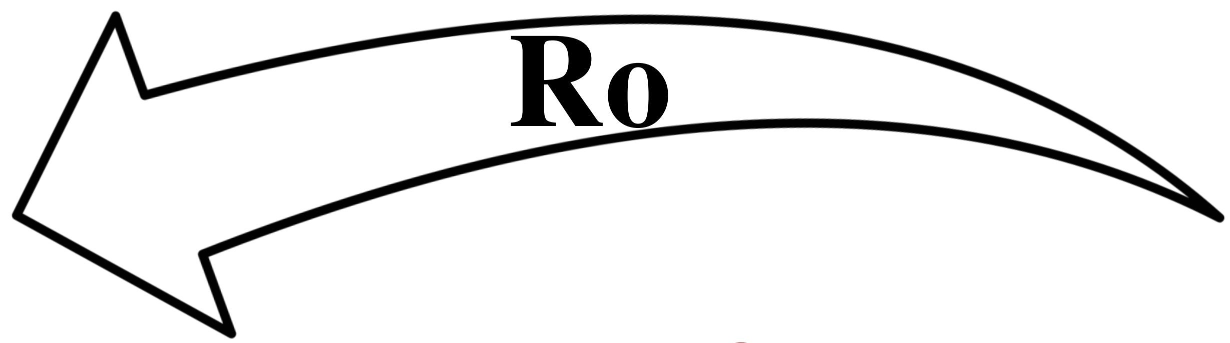
Rossby number

$$Ro = \frac{\text{flow frequency}}{\text{frequency of rotation}}$$

- Weakly stratified
  - e.g. mixed layer

- Strongly stratified
  - e.g. ocean interior

Ro < 1



Ro<< 1

Ageostrophic

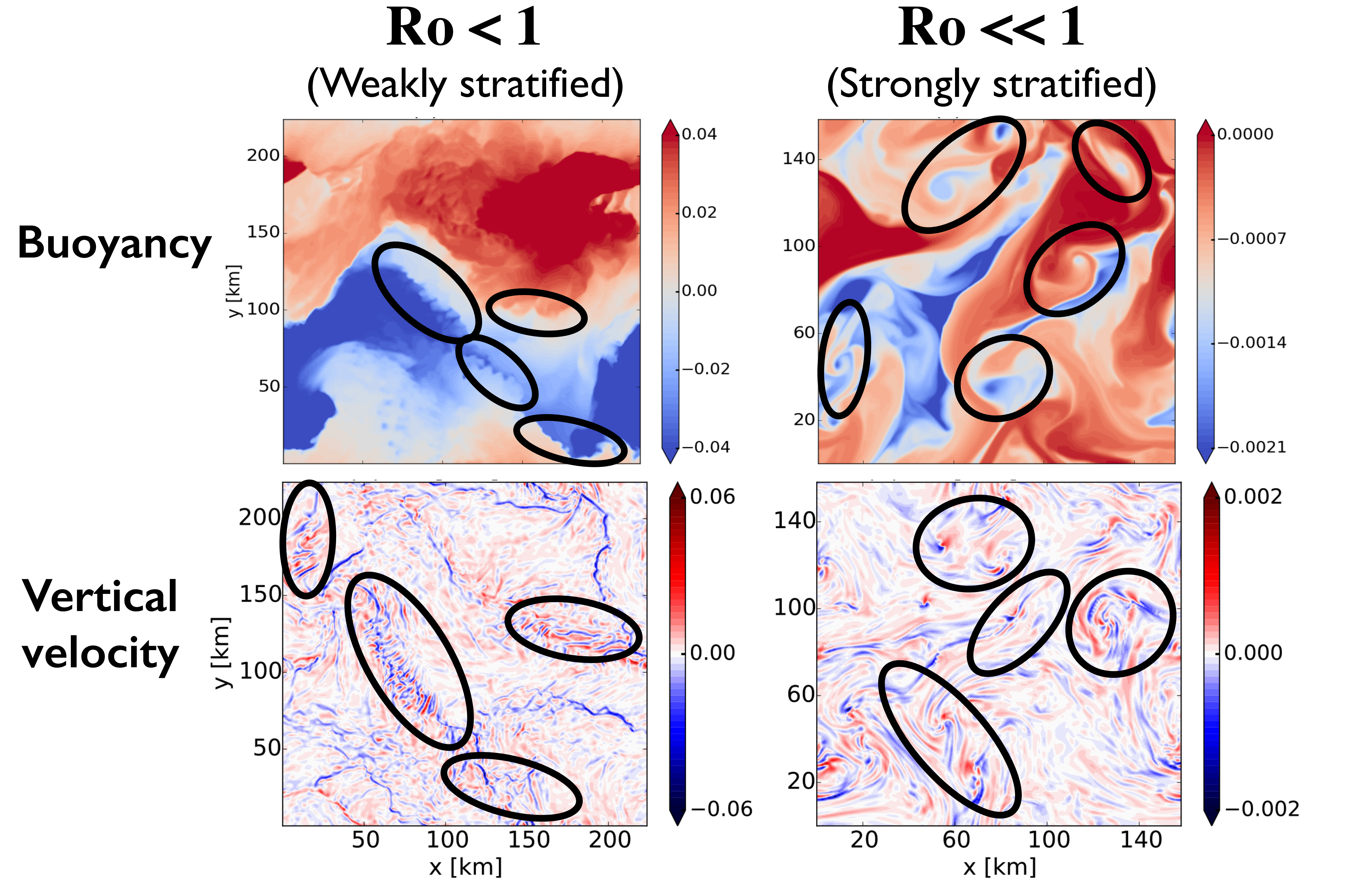
Quasi-geostrophic

Balanced and unbalanced motions are:

- **Strongly coupled**
- Timescale separation is complex

- **Weakly coupled**
- Timescales well-separated

Different dynamical regimes in the Ocean



Flow decomposition methods

Non-linear normal mode initialization (NNMI)

- **Machenhauer (1977):** Initially developed for Numerical Weather Prediction
  - The idea is to initially suppress gravity waves to minimize excitation
  - **Leith (1980):** Quasi-geostrophic balanced state (1st order in Ro), first iteration
- **Warn et. al (1995):** higher oder in Ro (nth order)

Optimal balance

- Masur and Oliver, 2020, JGAFD : Optimal potential vorticity (OPV) balance based on Viúdez and Dritschel (2004) for a quasi-geostrophic balanced state
  - Iterative procedure
  - Ramp time to match a target potential vorticity

Modal decomposition

Single layer model (scaled): $\partial_t \boldsymbol{u} + \underset{\neg}{\boldsymbol{u}} + \boldsymbol{\nabla} h = -Ro\ \boldsymbol{u} \cdot \boldsymbol{\nabla} \boldsymbol{u}$ $\quad \partial_t h + c^2 \boldsymbol{\nabla} \cdot \boldsymbol{u} = -Ro\ \boldsymbol{\nabla} \cdot h\boldsymbol{u}$

Fourier space:

vector $\hat{z}(\boldsymbol{k}) = (\hat{u}, \hat{v}, \hat{h})^T$

$$\partial_t \hat{z} - \underbrace{i\boldsymbol{A} \cdot \hat{z}}_{\text{Linear}} = \underbrace{Ro\ \hat{\boldsymbol{n}}}_{\text{Non-linear}}$$

$$\boldsymbol{A} = \begin{pmatrix} 0 & -i & -k_x \\ i & 0 & -k_y \\ -c^2 k_x & -c^2 k_y & 0 \end{pmatrix}$$

Modal decomposition

Single layer model (scaled): $\partial_t \boldsymbol{u} + \underset{\neg}{\boldsymbol{u}} + \nabla h = -Ro\ \boldsymbol{u} \cdot \nabla \boldsymbol{u}$ $\quad \partial_t h + c^2 \nabla \cdot \boldsymbol{u} = -Ro\ \nabla \cdot h\boldsymbol{u}$

Fourier space: vector $\hat{z}(\boldsymbol{k}) = (\hat{u}, \hat{v}, \hat{h})^T$

$$\partial_t \hat{z} - \underbrace{i\boldsymbol{A} \cdot \hat{z}}_{\text{Linear}} = \underbrace{Ro\ \hat{\boldsymbol{n}}}_{\text{Non-linear}}$$

$$\boldsymbol{A} = \begin{pmatrix} 0 & -i & -k_x \\ i & 0 & -k_y \\ -c^2 k_x & -c^2 k_y & 0 \end{pmatrix}$$

	Balanced mode	**Unbalanced mode**
Eigenvalues:	$\omega^0 = 0$	$\omega^\pm = \pm\sqrt{1 + c^2 k^2}$
Eigenvectors:	$\boldsymbol{q}^0, \boldsymbol{p}^0$	$\boldsymbol{q}^\pm, \boldsymbol{p}^\pm$

from C-grid discrete operators

Projection: mode amplitude $g^s = \boldsymbol{p}^s \cdot \hat{z}$ with $s = 0, \pm$

$$\partial_t g^s - i\omega^s g^s = Ro\ \boldsymbol{p}^s \cdot \hat{\boldsymbol{n}} = -iRo\ I^s(g^0, g^\pm)$$

$$I^s(g^0, g^\pm) = I^s(g^0, 0) + I^s(0, g^\pm) + K^s(g^0, g^\pm)$$

Modal decomposition

» **Modal representation:** $\partial_t g^s - i\omega^s g^s = Ro\, \boldsymbol{p}^s \cdot \hat{\boldsymbol{n}} = -iRo\, I^s(g^0, g^\pm)$

$$(Ro\,\partial_T + \partial_{t*})\, g^s - i\omega^s g^s = -iRo\left(I^s(g^0,0) + I^s(0,g^\pm) + K^s(g^0,g^\pm)\right)$$

» **Weak interaction assumption:** weakly growing waves $g^\pm = Ro\, f_1^\pm + Ro^2\, f_2^\pm + \ldots$

» expansion in Ro as e.g. in Warn (1996), Kafiabad and Bartello (2017)

» introduce fast and slow time scale with $T = Ro\; t^*$ and $\partial_t = Ro\; \partial_T + \partial_{t^*}$

» slow mode g^0 varies on T only, while fast mode $g^\pm$ has two time scales t^* and T

Modal decomposition

» **Modal representation:** $\partial_t g^s - i\omega^s g^s = Ro\, \boldsymbol{p}^s \cdot \hat{\boldsymbol{n}} = -iRo\, I^s(g^0, g^\pm)$

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» slow mode g^0 varies on T only, while fast mode $g^\pm$ has two time scales t^* and T

for increasing order in Ro:

» **SLOW MODE s=0**

$$\partial_T g^0 = -iI^s(g^0, 0)$$
$$\partial_T g^0 = -iI^s(g^0, f_1^\pm) + iI^s(0, f_1^\pm)$$
$$\partial_T g^0 = -iI^s(g^0, f_2^\pm) + iI^s(0, f_2^\pm) - iI^0(0, f_1^\pm)$$

» **FAST MODE s=±**

$$\partial_{t^*} f_1^\pm - i\omega^\pm f_1^\pm = -iI^\pm(g^0, 0)$$
$$\partial_T f_1^\pm + \partial_{t^*} f_2^\pm - i\omega^\pm f_2^\pm = -iK^\pm(g^0, f_1^\pm)$$
$$\partial_T f_2^\pm + \partial_{t^*} f_3^\pm - i\omega^\pm f_3^\pm = -iI^\pm(0, f_1^\pm) - iK^\pm(g^0, f_2^\pm)$$

» suppress any wave generation by $\partial_{t^*} f_n^\pm = 0 \rightarrow$ 'slaved' modes f_n

» **Machenhauer(1977)**

» **QG balanced state**

$$\boxed{f_1^\pm = I^\pm(g^0, 0)/\omega^\pm}, \quad f_2^\pm = \left(K^\pm(g^0, f_1^\pm) - i\partial_T f_1^\pm\right)/\omega^\pm, \quad \ldots$$

» **first order slaved mode**

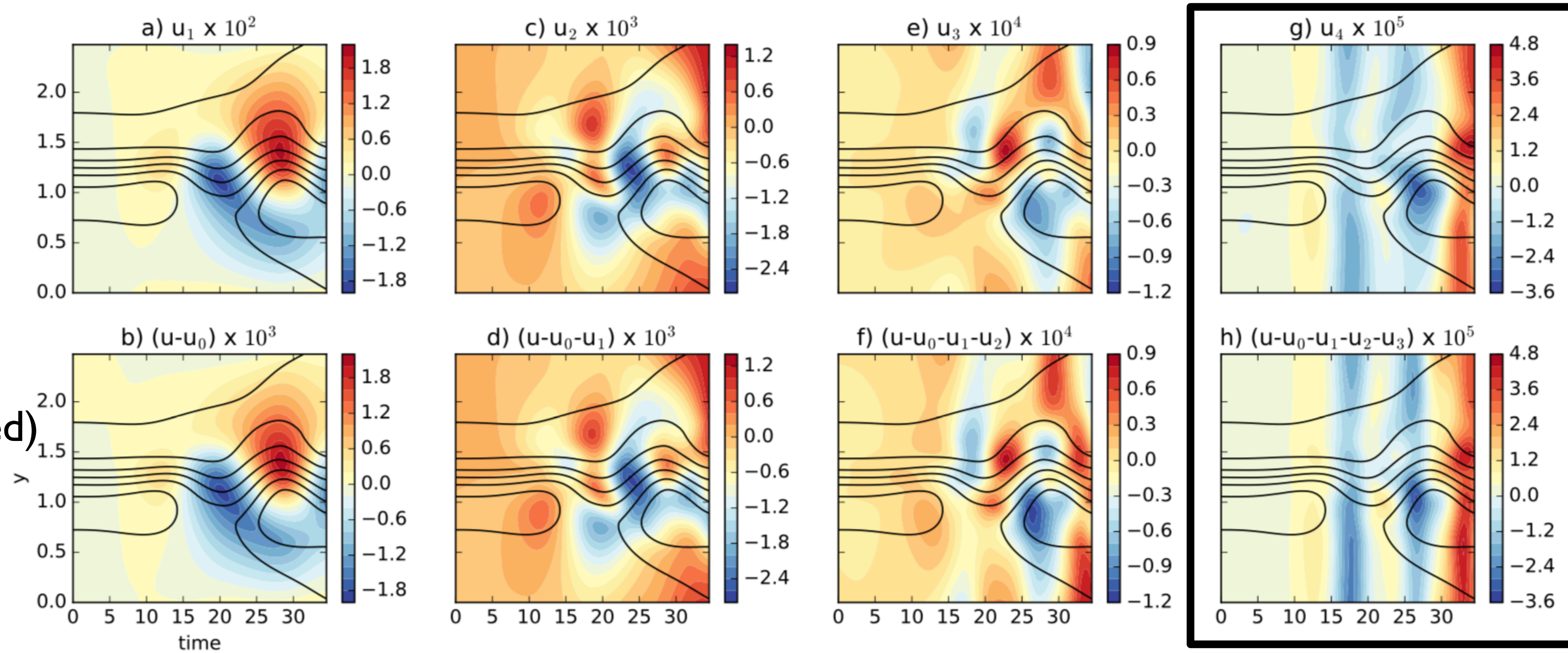
Wave emission at higher orders: single layer model

Ro = 0.1

1st 2nd 3rd 4th

Balanced velocity

Residual (unbalanced) velocity



- » double periodic domain 10x5 (dimensionless)
- » initialized with an unstable zonal jet
- » zonal jet meanders and dissolves into eddies
- » range of Ro = 0.02 to 0.3, i.e. from mesoscale to sub-mesoscale conditions

- » wave signal seen: is not related to spontaneous emission by shear instability of the balanced flow
- » rather, the wave signal could be related to local Rossby numbers with Ro_{local} >1: symmetric instability becomes possible

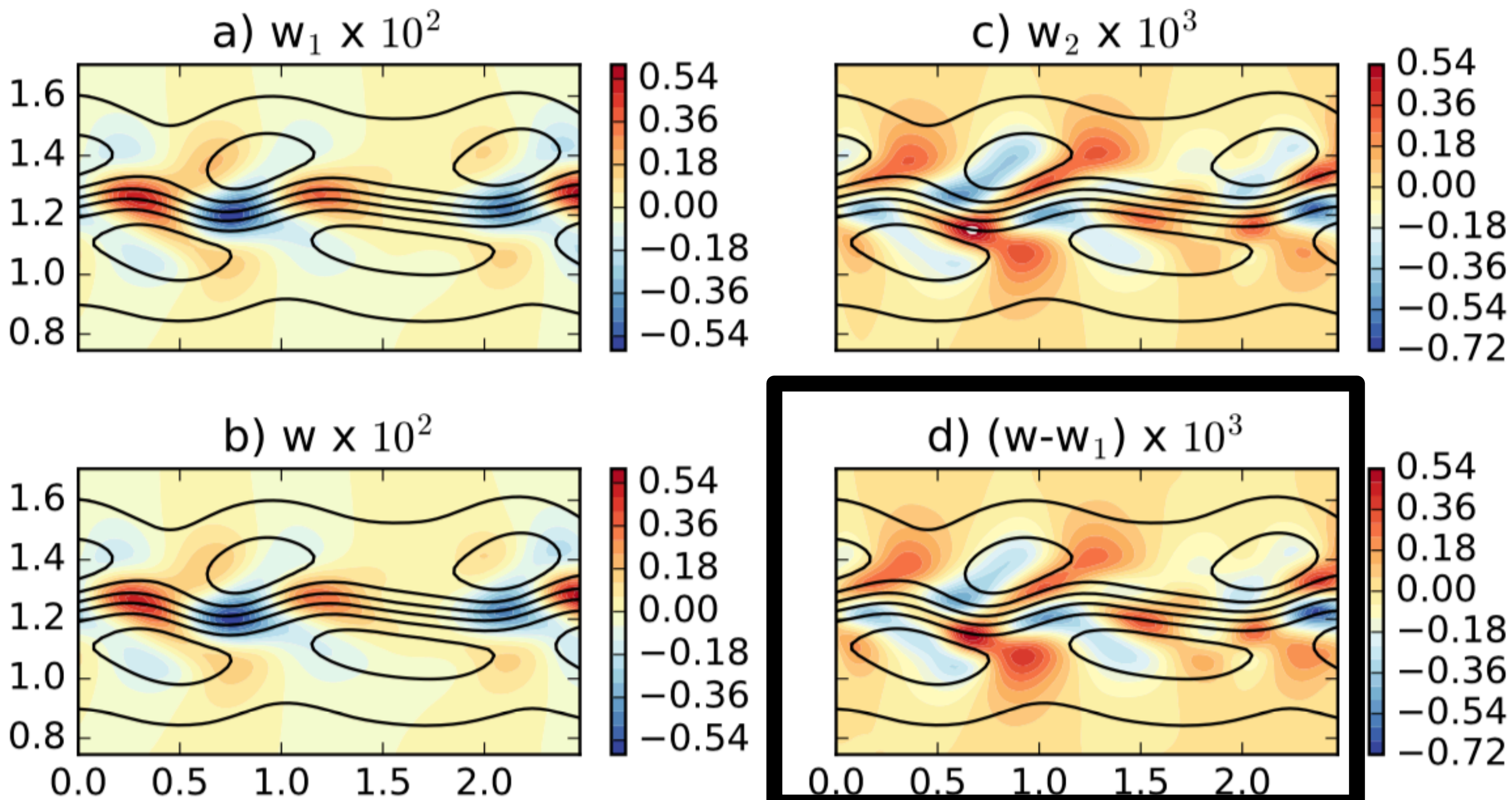
from Eden, Chouksey, and Olbers, JPO, 2019: Gravity wave emission by shear instability

(apparent) Wave emission at higher orders: primitive equation model

1st

2nd

Apparent spontaneous emission or 1st or 2nd order slaved modes?



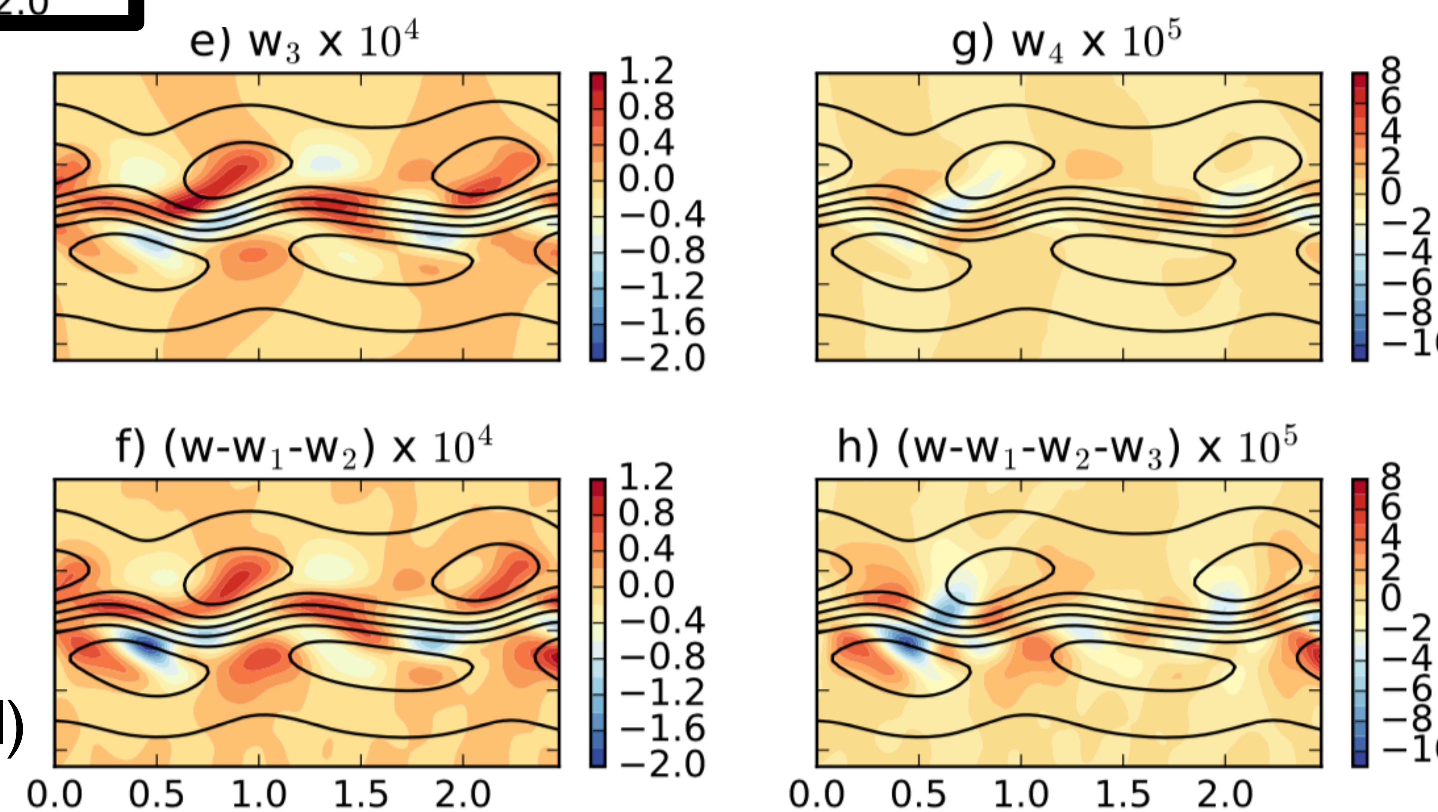
Balanced velocity

Residual (unbalanced) velocity

3rd

4th

Balanced velocity

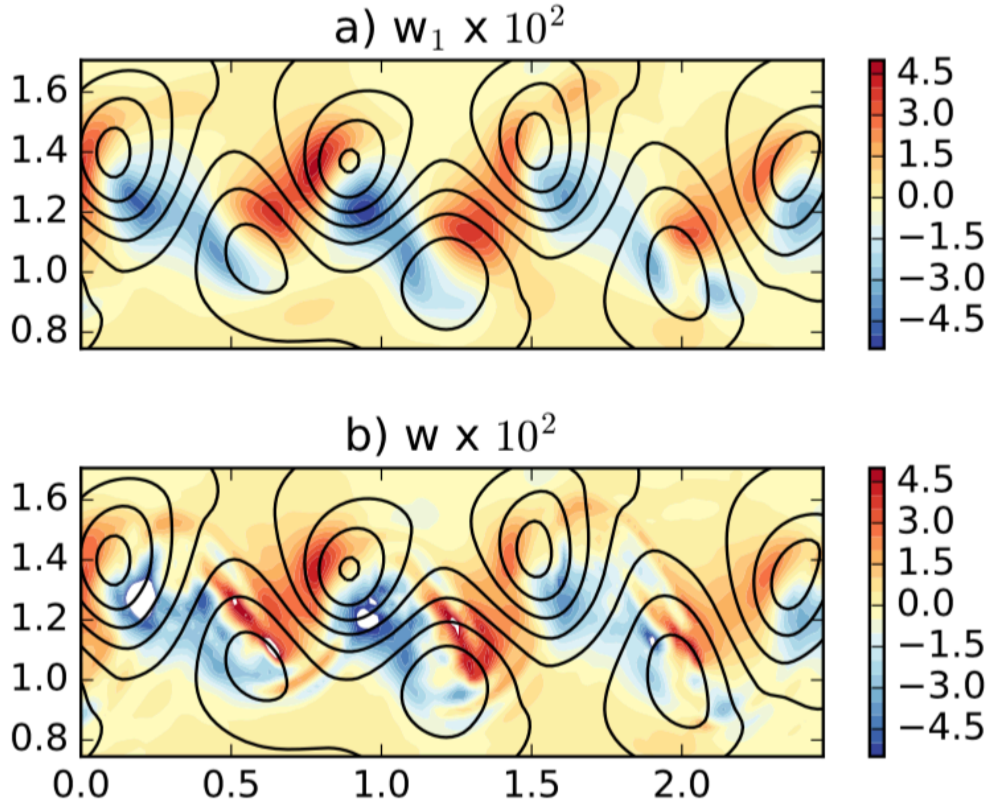


Weak wave signal is seen only at 4th order.

Residual (unbalanced) velocity

Convectively generated waves at higher orders: primitive equation model

1st



Balanced velocity

Residual (unbalanced) velocity

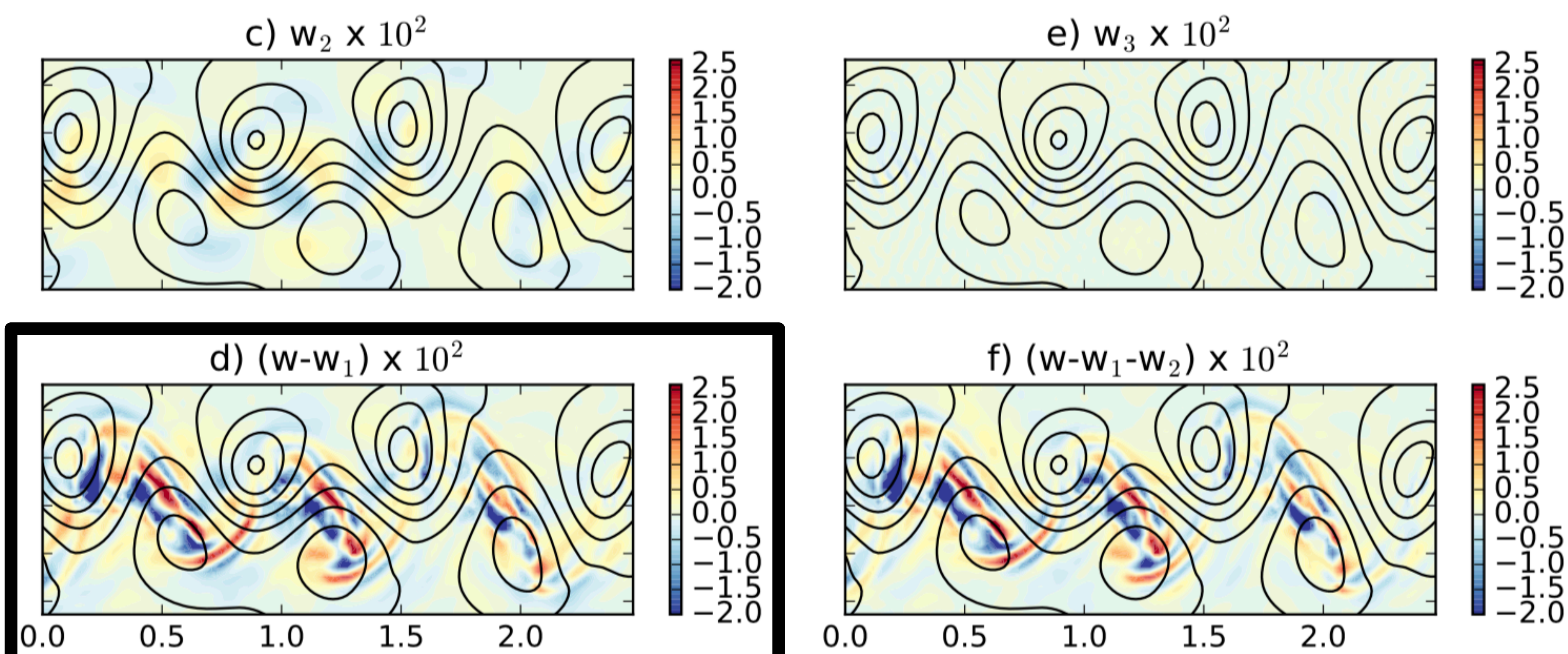
Stronger wave signal seen already at 2nd order.

2nd

3rd

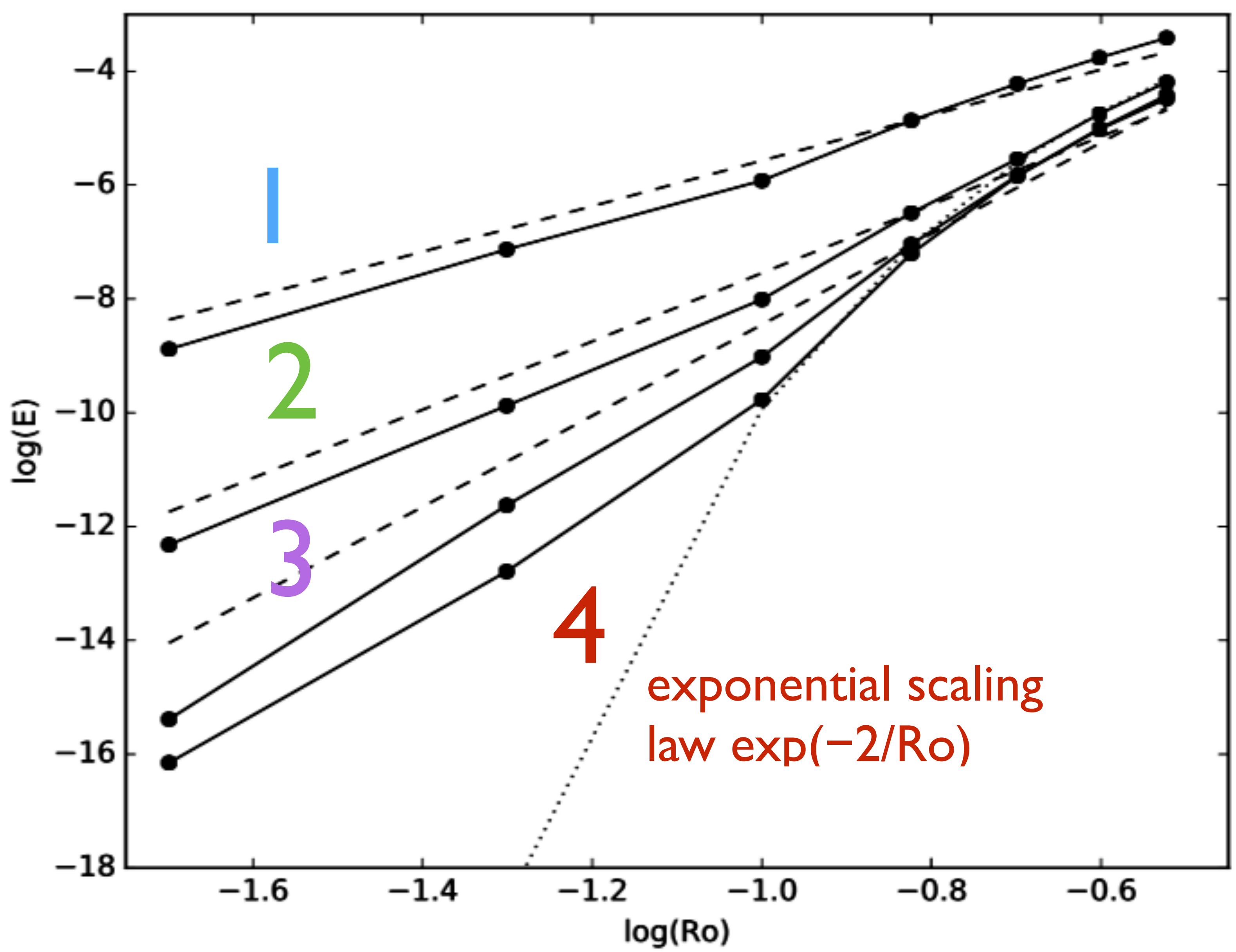
Balanced velocity

Residual (unbalanced) velocity



Wave emission at higher orders: *Ro* scaling

Wave generation scales exponentially at higher orders for large Rossby number, *Ro*.



Total residual wave energy normalized with the total energy integrated over the model domain.

Dashed lines: different power laws

Eden, Chouksey, and Olbers, 2019: Gravity wave emission by shear instability, JPO

Summary

- The non-linear decomposition of balanced and unbalanced motions is achieved and implemented in model in different dynamical regimes up to fourth order.
  - Machenhauer (1977) and Warn et al. (1995)
  - Chouksey et. al (2018) JPO, Eden et. al (2018) JPO, Eden et. al (2019) JPO

- Balanced state is diagnosed in a single layer and primitive equation model using higher order Ro expansion.

- The use of C-grid discrete operators is important for the obtained balanced state.

- **Convective instability generates gravity waves rather than spontaneous emission.**

- Gravity wave generation scales exponentially at higher orders for large *Ro*.

- Spontaneous wave emission by shear instability is negligible.

balanced or unbalanced?!

It’s as much numerics, as it’s the realm of philosophy.