

A breakup of a droplet falling into a miscible solution

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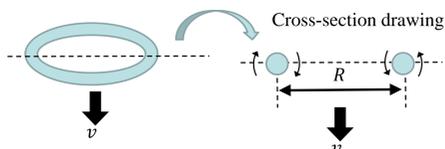
References
 [1] Michiko Shimokawa *et al.*, Phys. Rev. E **93**, 062214, 2016.
 [2] Michiko Shimokawa and Hidetsugu Sakaguchi. Phys. Rev. Fluids **4**, 013603, 2019.

Introduction

● Vortex in our life: Bubble ring, vortex of cigarette, etc...



http://blogs.yahoo.co.jp/ssm_121/63201340.html

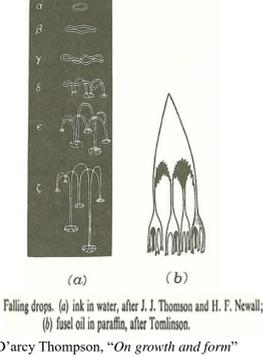
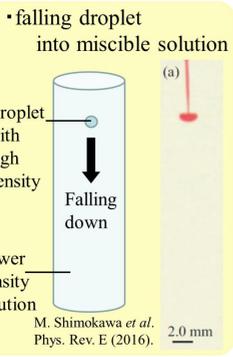
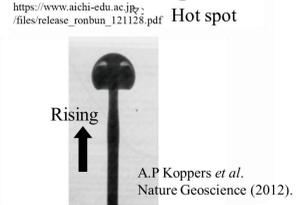
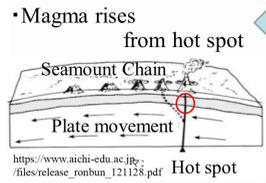


Vortex radius $R(t)$

$$R(t)^2 - R(0)^2 = \frac{gV\Delta\rho}{\pi\rho\Gamma} t$$

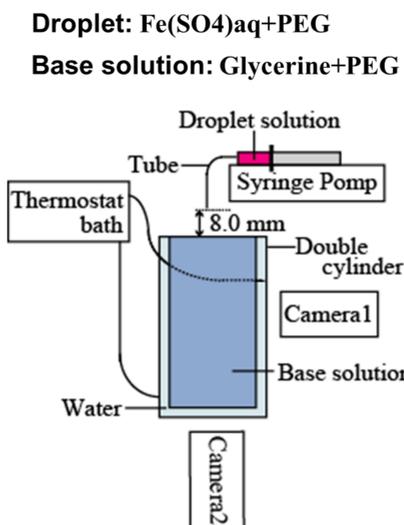
Γ : circulation
 V : vortex volume
 $\Delta\rho$: density difference

● Simple experiments to mimic nature provide to understand the nature.



Experimental Method

- (1) A droplet is put on the surface of the base solution in the double cylinder.
- (2) The process of the droplet is captured from lateral and bottom sides of the cylinder using cameras.

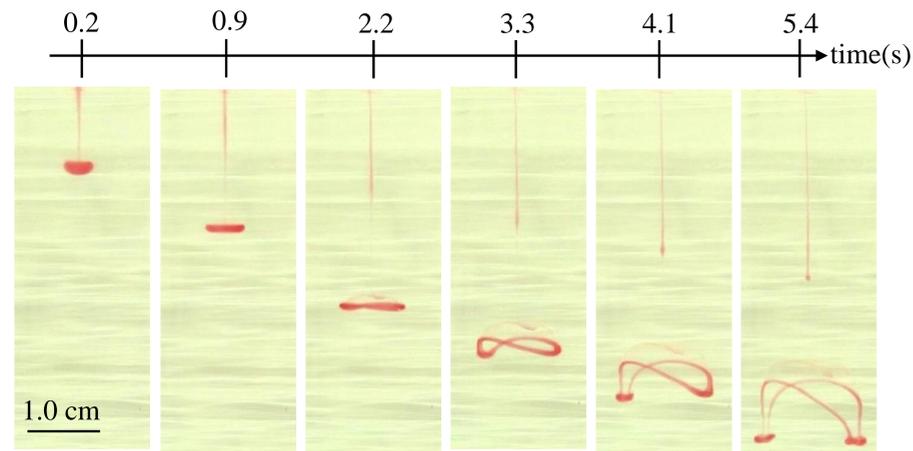


- A size of droplet is controlled by using a syringe pump.
- Viscosities of solutions are controlled by amount of PEG in the solutions. (The value of the droplet viscosity keeps a similar value to that of base solution in our experiments.)
- Density differences between two solutions are controlled by amount of Fe(SO₄)aq in the droplet solution.
- The breakup process is captured from lateral side and bottom of a cylinder.
- Temperature is controlled by thermostat bath for keeping a viscosity constantly.

Experimental Results

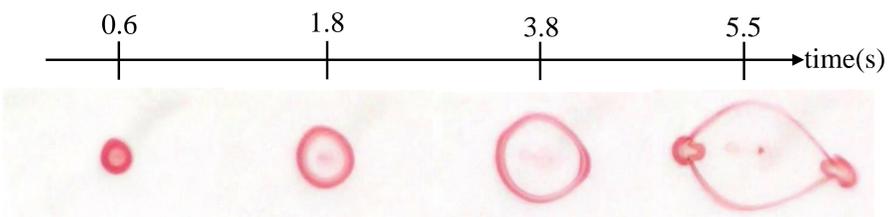
● Time series of breakup and deformation of droplet (droplet size $r=1.3$ mm, mode $m=2$)

★ Side view (Captured from a side of a container)



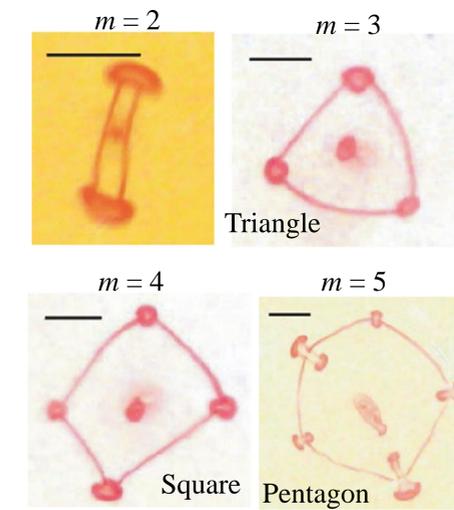
Vortex size increases
 Vertical disturbance becomes large.

★ Horizontal view (Captured from a bottom of a container) [1]



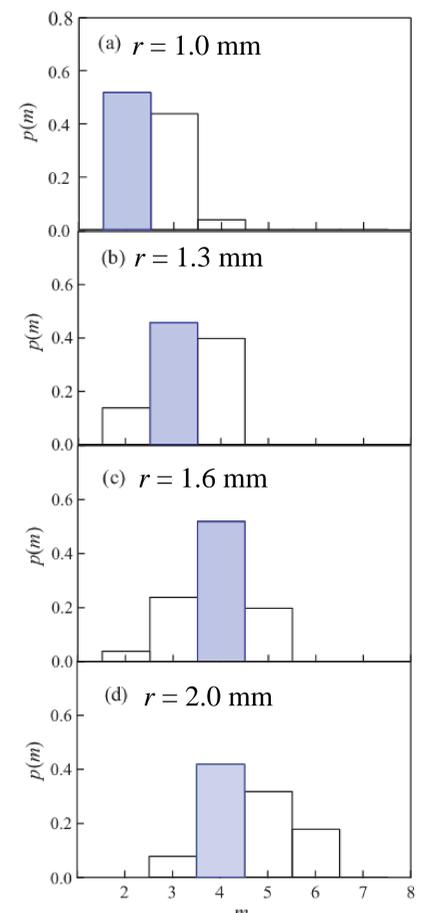
Vortex size increases
 Disturbance becomes large.

● Polygonal deformation of a droplet [1]

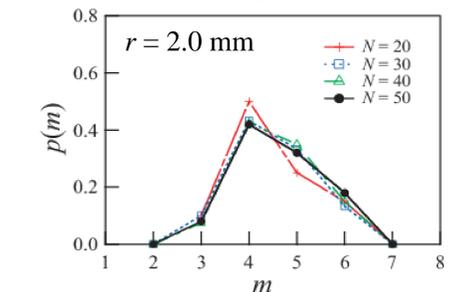


A droplet deforms a polygonal shape. (This is not reported in previous studies.)

● Probability distribution $p(m)$ with several droplet sizes r [2]



● Probability distribution $p(m)$ with several sample numbers N [2]



$p(m)$ converges with increasing N .
 → $p(m)$ with several r is investigated in $N=50$.

- Peak values of $p(m)$ increase with r and $\Delta\rho$.
- Peak value decreases in an increasing of μ .

Discussion [2]

● Phenomenological model 1

(1) Navier-Stokes equation (RT instability: Rayleigh-Taylor instability)

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{1}{\rho_2} \nabla P + \nu \nabla^2 \mathbf{u} - \frac{\rho - \rho_2}{\rho_2} g \mathbf{e}_z,$$

(2) Diffusion equation

$$\frac{\partial \rho}{\partial t} + \mathbf{u} \cdot \nabla \rho = D \nabla^2 \rho,$$

Dimensionless $x' = x/r, u' = ur/\nu, t' = tv/r^2, P' = Pr^2/\rho_2\nu^2, \rho' = (\rho - \rho_2)/\rho_2$

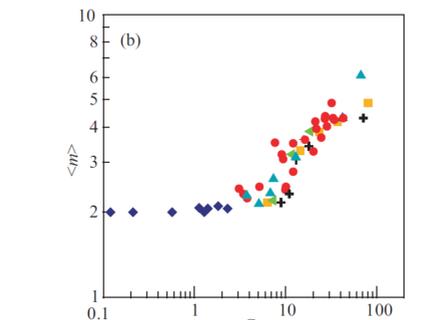
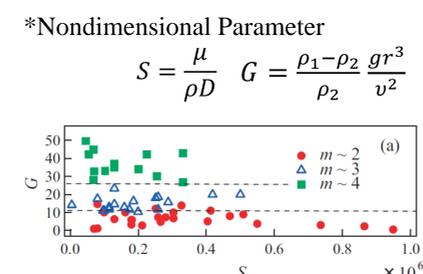
(1') Navier-Stokes equation

$$\frac{\partial \mathbf{u}'}{\partial t'} + \mathbf{u}' \cdot \nabla' \mathbf{u}' = -\nabla' P' + u \nabla'^2 \mathbf{u}' - G \rho' \mathbf{e}_z,$$

(2') Diffusion equation

$$\frac{\partial \rho'}{\partial t'} + \mathbf{u}' \cdot \nabla' \rho' = \frac{1}{S} \nabla'^2 \rho',$$

G is an important factor to determine the mode of the breakup.



We proposed a new phenomenological model

● Navier-Stokes equation

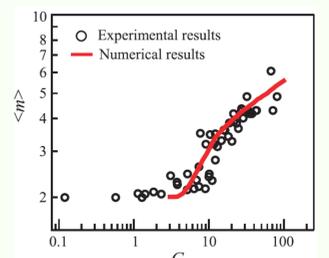
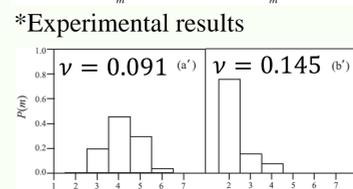
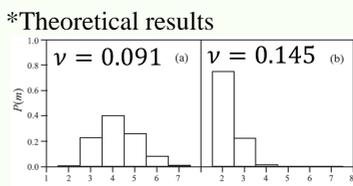
$$\frac{\partial \mathbf{u}'}{\partial t'} + \mathbf{u}' \cdot \nabla' \mathbf{u}' = -\nabla' P' + u \nabla'^2 \mathbf{u}' - G \rho' \mathbf{e}_z,$$

● Expanding of the radius $R(t)$ of the vortex ring

$$R(t) = \sqrt{R(0)^2 + \alpha t},$$

Experimental results provided $\alpha/\nu = (4.3 \times 10^{-2})G - 0.16$

Comparison with experimental results



Our experimental results agree with our theoretical arguments.

Conclusion

- We focused on the breakup of the falling droplet and investigated the numbers of the breakup.
- Our experiments of the probability distributions $p(m)$ agree with those obtained from our phenomenological model.
- (1) RT instability and (2) Expanding of the vortex ring are important factors for the mode selection.