Uncertainty Quantification of Borehole Thermal Energy Storage Facilities

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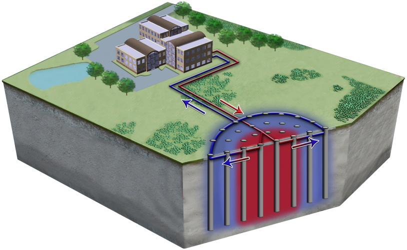
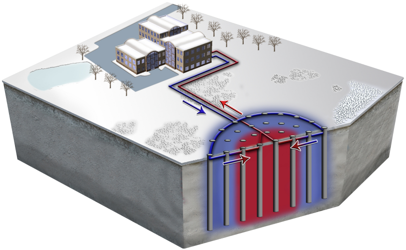
A BTES facility





Motivation for an uncertainty quantification

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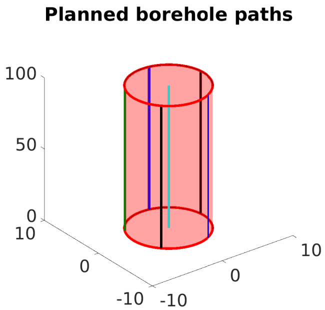
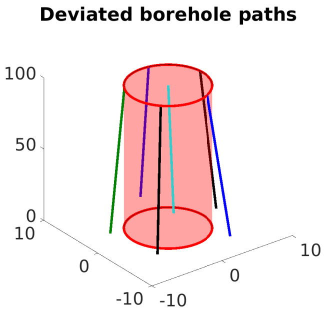




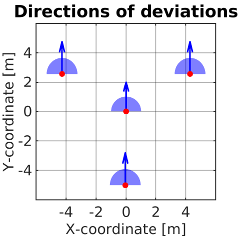
The peak **storage efficiency** of a facility drops from 26.0% for the planned layout to 6.2% for a layout with no thermal interaction.
Question: How much do randomly occurring deviations in the layout geometry affect the performance of a facility statistically?

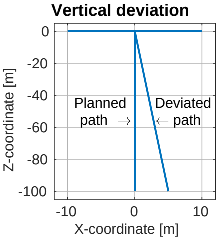
Model

Consider a heat storage facility with several **borehole heat exchangers (BHEs)** perpendicular to the surface, which is operated in an annual extraction and/or storage scenario. This can be modeled as a **partial differential equation (PDE)**.

For each BHE, we consider 2 random variables:

- **horizontal direction of deviation** $X_{Dir} \in [-\alpha_{min}, \alpha_{max}]$
- **vertical angle of deviation** (from the perpendicular course) $X_{Dev} \in [0, \beta_{max}]$





Interpolation of a quantity of interest

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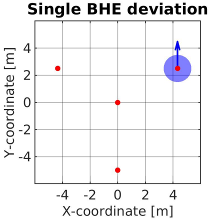
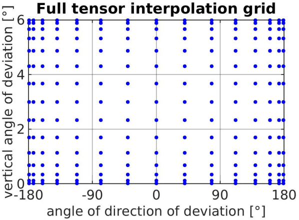
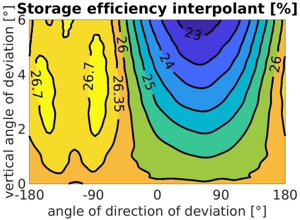
Goal: Calculate an interpolant of a quantity of interest (e.g. the storage efficiency or a mean outlet temperature for several years of operation), which is a scalar function over the space of all random variables ($[-\alpha_{min}, \alpha_{max}] \times [0, \beta_{max}]\,)^{n_{BHE}}$.



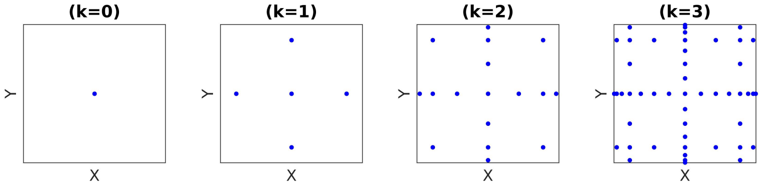




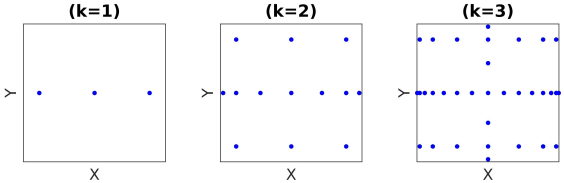
Problem: Each interpolation node requires the solution of the underlying PDE, which is expensive. **'Curse of dimensionality'** further compounds this problem for poorly chosen interpolation rules in multiple dimensions.

Adaptive Anisotropic Stochastic Collocation

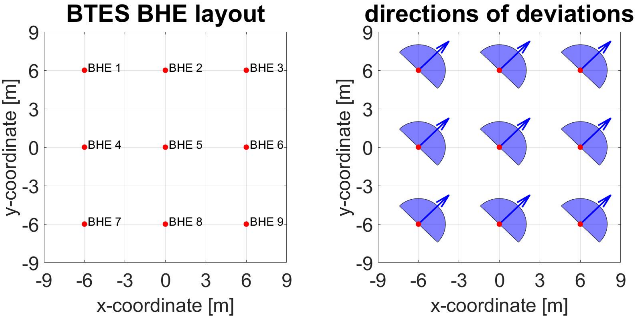
Remedy: Adaptive interpolation on nested Smolyak sparse grids [Smo63].



Improvement: Anisotropic refinement via directional error estimators [GG03].



Practical example - BTES facility layout



The BTES facility is simulated in a heat extraction scenario. The examined quantity of interest dependent on the input uncertainty is the **mean outlet temperature** of the 9 BHEs averaged for 5 years of simulated operation. This quantity is in direct relation to the facility's effectiveness when connected to a heat pump.

Practical example - Uncertainty input

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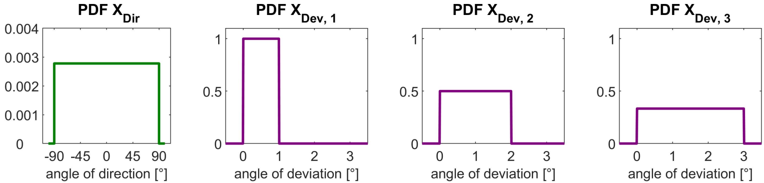
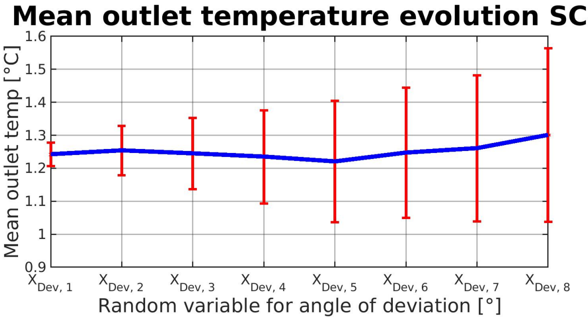
We examine the BTES facility in successive uncertainty quantification scenarios:



We consider uniformly distributed horizontal directions and vertical angles of deviations. The horizontal directions are kept constant, corresponding to X_{Dir}. The range of possible vertical angles is increased by 1° in each successive scenario, up to a maximum of 8°. The corresponding probability density functions for the first 3 scenarios $X_{Dev,1}$, $X_{Dev,2}$ and $X_{Dev,3}$ are pictured above.

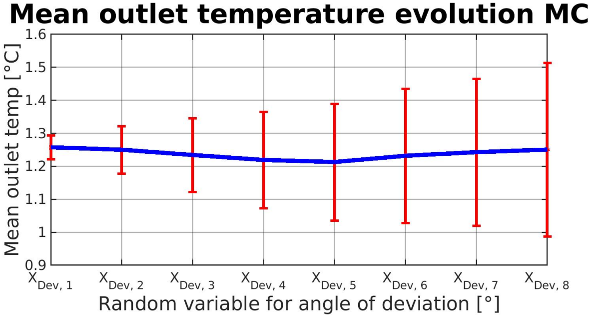
Practical example - results

Using stochastic collocation (SC), we get the mean outlet temperature and its standard deviation (indicated by the red bars) for each scenario. The mean outlet temperature varies very little, but the standard deviation increases with each successive scenario.



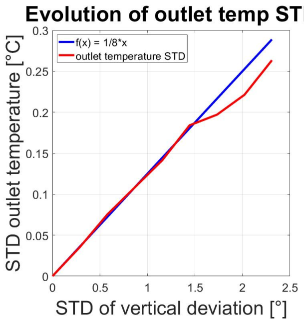
Practical example - results

A Monte Carlo (MC) uncertainty quantification of the same 8 scenarios, with 1000 samples each, provides very similar mean outlet temperatures and standard deviations, validating the results of the stochastic collocation method.



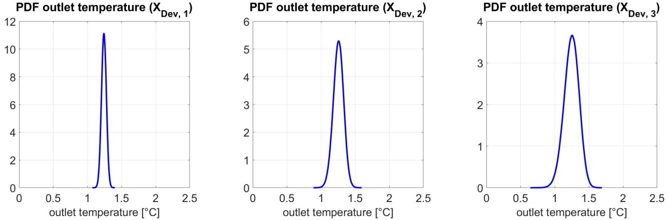
Practical example - results

Plotting the standard deviation of the mean outlet temperature against the standard deviation (STD) of the vertical angles of deviations for each scenario gives a nearly linear relation. Increasing the input uncertainty leads to a proportional increase in the output uncertainty.



Practical example - results

Having access to the interpolant of the mean outlet temperature for each scenario gives a very high amount of statistical information. For example, we can calculate the probability density functions (PDF) of the mean outlet temperature itself. These PDFs answer questions such as 'How likely is the mean outlet temperature going to be above 1°C for a given scenario?'.



Practical example - results

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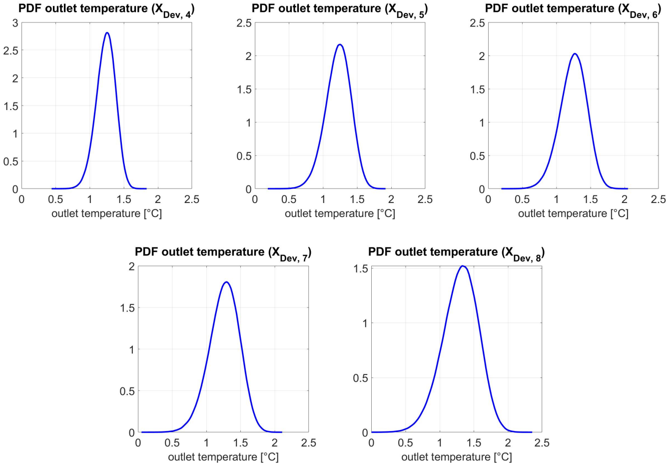

Practical example - results

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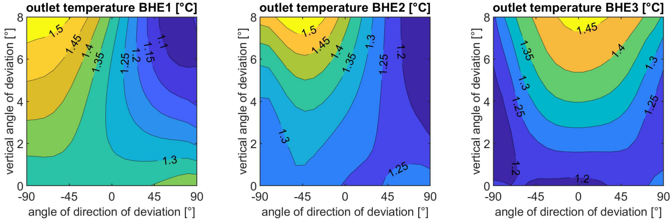
Additionally, we can study the contribution of the deviations of an individual BHE by evaluating the interpolant for its two corresponding random variables while integrating over the remaining ones. This provides results that are in line with our intuitive understanding: The mean outlet temperature rises as the BHE is deviated away from the center of the facility. The plots below belong to the scenario with the strongest considered vertical deviation of up to 8°.



Practical example - results

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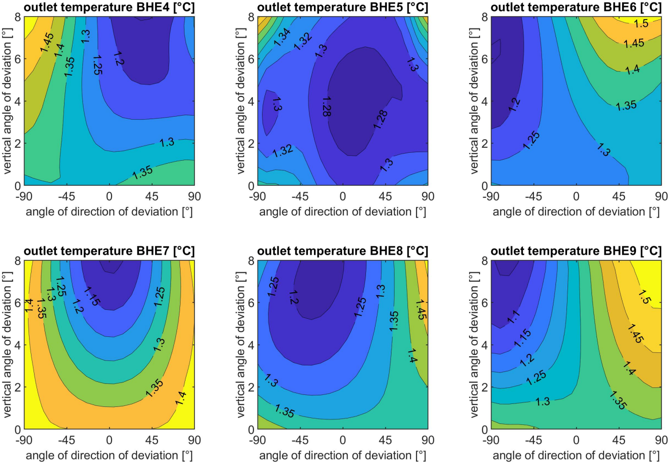

Conclusion

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- Stochastic collocation provides a computationally efficient method to perform uncertainty quantifications in the context of BTES facilities.
- The method's results were validated with Monte Carlo reference uncertainty quantifications.
- The calculated interpolant gives additional statistical information above and beyond a simple Monte Carlo uncertainty quantification.
- For the discussed example, the uncertainty quantification gives a comprehensive study of the effects of deviations in the layout geometry, which provides a tool to assess its viability.

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Thank you for your attention.

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