EGU2020 – 6745: Session ERE 6.1

Coupled multiphase flow and geomechanics simulation of hydrate dissociation using FVM-FEM co-located variables arrangement

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7 May 2020

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Introduction

- Gas hydrates are ice-like crystalline solids that form from mixtures of water and light natural gases. They have vast amount of trapped natural gas.

Methane hydrate structure and hydrate samples.

- Methane can be extracted from hydrates by depressurization. The dissociation of solid hydrate produces liquid water and methane gas.
- Dissociation of solid hydrates into fluid constituents weakens the strength bearing capacity of the sediment and can induce subsidence.
- The physical processes involved are hydrate phase change, non-isothermal multiphase flow, change in mechanical properties and strain field, and change in hydraulic properties.
- In this work a THMC coupled numerical solver is developed using node centered Finite volume method (FVM) for flow and Finite element method (FEM) for geomechanics. The state variables are hence co-located.
- The discretized equations are solved using PETSc, an open source suite of solvers.
- The pressure oscillations in numerical solution in the context of the choice of the numerical method - co-located variable arrangement along with unequal order function spaces for flow and displacement variables – and necessity of stabilization method is investigated.
- The performance of iterative coupled approach, where the flow and geomechanics equations are solved separately and sequentially, as against fully-coupled approach is also studied.

Governing Equations

- The mass balance equation for gas, water and hydrate phase ($\alpha = w, g$ and h)

$$\frac{\partial}{\partial t}(\phi\rho_\alpha S_\alpha) + \nabla\cdot(\phi\rho_\alpha S_\alpha \mathbf{v}_{\alpha,t}) = \dot{g}_\alpha$$

where: $\phi S_\alpha \mathbf{v}_{\alpha,t} = \mathbf{v}_\alpha + \phi S_\alpha \mathbf{v}_s, \quad \mathbf{v}_\alpha = -\frac{k k_{r,\alpha}}{\mu_\alpha}(\nabla p_\alpha - \rho_\alpha \mathbf{g}), \quad \mathbf{v}_s = \frac{\partial \mathbf{u}}{\partial t}$

$\mathbf{v}_s$ is the solid velocity and $\mathbf{u}$ is the displacement vector; $\mathbf{v}_h = 0$

- The soil mass balance equation is: $\frac{\partial}{\partial t}[(1-\phi)\rho_s] + \nabla\cdot((1-\phi)\rho_s \mathbf{v}_s) = 0$

- The static equilibrium equation is: $\nabla\cdot\boldsymbol{\sigma} + \rho_{sh}\mathbf{g} = 0$

where ρ_{sh} is the density of the solid phase (soil and hydrate composite), and $\boldsymbol{\sigma}$ is the total stress.

- The stress-strain constitutive equation is:

$$\boldsymbol{\sigma} = 2G_m\boldsymbol{\epsilon} + \left(K_m - \frac{2}{3}G_m\right)(tr\ \boldsymbol{\epsilon})\mathbf{I} - \alpha_b p_p \mathbf{I} - K_m\beta_{sh}(T - T_0)\mathbf{I}.$$

where, K_m , G_m are the bulk and shear modulus of the hydrate bearing medium. β_{sh} and α_b are Biot coefficient and the thermal expansion coefficient.

- The energy equation is:

$$\frac{\partial}{\partial t}\left((1-\phi)\rho_s U_s + \sum_\alpha(\phi\rho_\alpha S_\alpha U_\alpha)\right) + \sum_\alpha \nabla\cdot(\phi\rho_\alpha S_\alpha \mathbf{v}_{\alpha,t} H_\alpha) = \nabla\cdot\left(\lambda_{eff}\nabla T\right) + \dot{Q}^h$$

U_α , H_α and λ_{eff} are internal energy, enthalpy of phase α and effective thermal conductivity.

Numerical Methodology

- The flow and energy equations are discretized using Finite volume method (FVM)
- The stress equilibrium equation is discretized using Finite element method (FEM)
- The discretized equations are solved using PETSc, an open source and parallel suite of routines for solving PDE and ODE.
- Point-centered or node centered FVM is used as it is easier to implement using PETSc.

- This choice of grid implies that the state variables for flow and geomechanics are **co-located**.
- The rate of change of volumetric strain term couples the flow and geomechanics equations.
- After calculating strain from the geomechanics solver (finite element mesh), it is interpolated to the finite volume mesh as

$$\varepsilon_{v_{ABCD}} = \frac{1}{4}\left(\frac{\varepsilon_{v_A}V_A + \varepsilon_{v_B}V_B + \varepsilon_{v_C}V_C + \varepsilon_{v_D}V_D}{V_{b_{ABCD}}}\right)$$

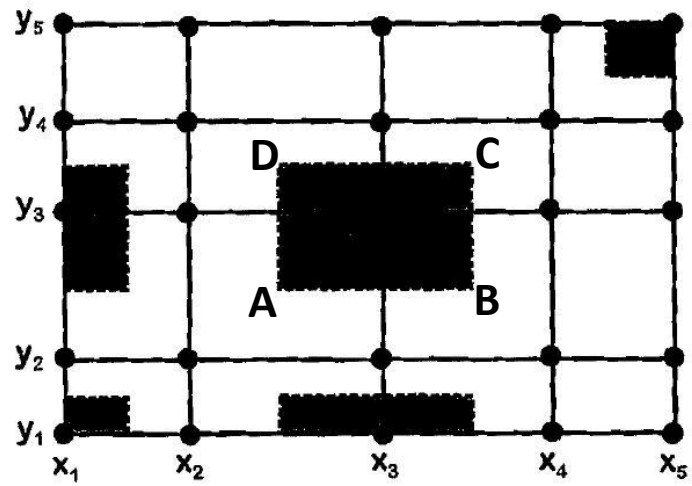


Fig 1: Point centered grid showing the corresponding control volumes (shaded area).

- The iterative coupled approach, where the flow and geomechanics equations are solved separately and sequentially, and fully-coupled approach, where all the equations are solved simultaneously are implemented.
- It is found that **unequal order function spaces** for flow and displacement alone cannot mitigate pressure oscillations for co-located variable arrangement, hence a physical influence scheme (PIS) stabilization method (Honorio et al. 2018) is implemented to mitigate it.

Validation

Flow code validation (Thermo-Hydro-Chemical, THC coupling)

The flow code is validated against experimental results of Tang et al. (2007), a cylindrical pressure vessel insulated and impermeable at one end and depressurized at the other end.

Parameter	Value
Length	50 cm
Cross-sectional area	11.4 cm^2
Porosity	0.308
Absolute Permeability	300 mD
Initial pressure	3.535 MPa
Initial water saturation	0.2961
Initial hydrate saturation	0.2183
Initial Temperature	1.54 ^oC
Boundary Pressure	0.93 MPa
Boundary Temperature	1.54 ^oC

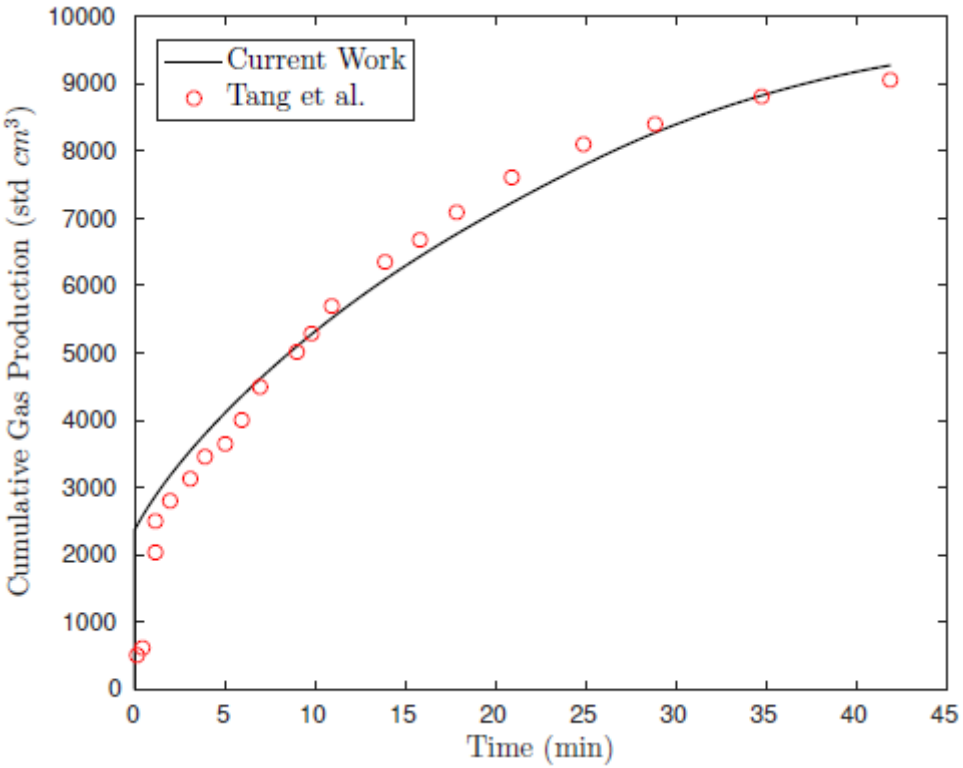


Fig 2: Cumulative gas production rate vs Time.

Validation

Geomechanics validation (Hydro-Mechanics, HM coupling)

The geomechanics code is validated against the analytical solution of the Mandel's problem.

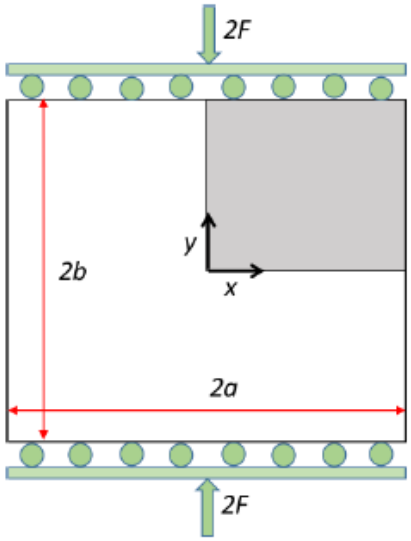
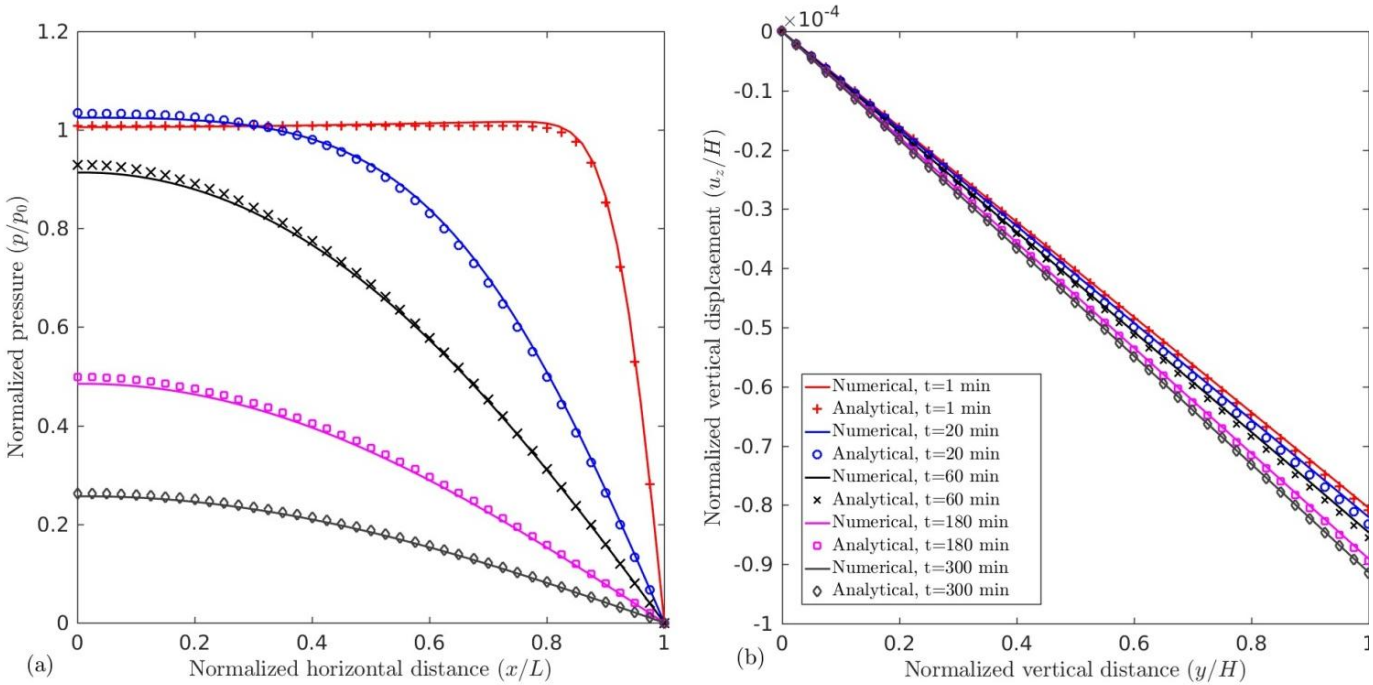


Fig 3: Mandel's problem configuration



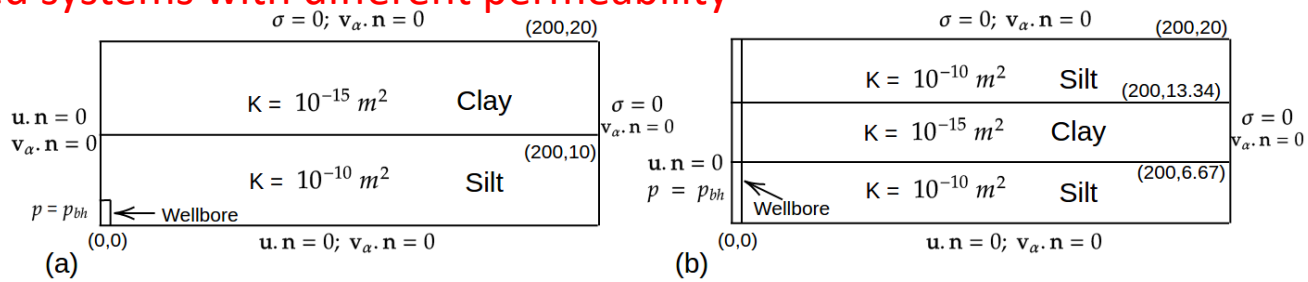
Comparison of numerical and analytical solutions at various times:

(a) Non-dimensional pressure vs non-dimensional horizontal distance.

(b) Non-dimensional vertical displacement vs non-dimensional vertical distance.

Coupled hydrate dissociation and deformation

Hydrate Dissociation in layered systems with different permeability



Parameter	Value
Porosity	0.308
Absolute Permeability	Clay – 10^{-15} m^2 Sand - 10^{-10} m^2
Initial pressure	6.913 MPa
Initial water saturation	0.3
Initial hydrate saturation	0.6
Initial Temperature	283.15 K
Biot Coefficient	0.8
Poisson's Ratio	0.15
Young's Modulus	260 MPa
Well bottom hole pressure	4 MPa

Fig 5: (a) Two layers with well bore penetrating small part of lower layer (b) Three layer system with well bore through out.

- Permeability variation is 5 orders of magnitude.
- PIS (Honorio et al. 2018) stabilization mitigates pressure oscillations.

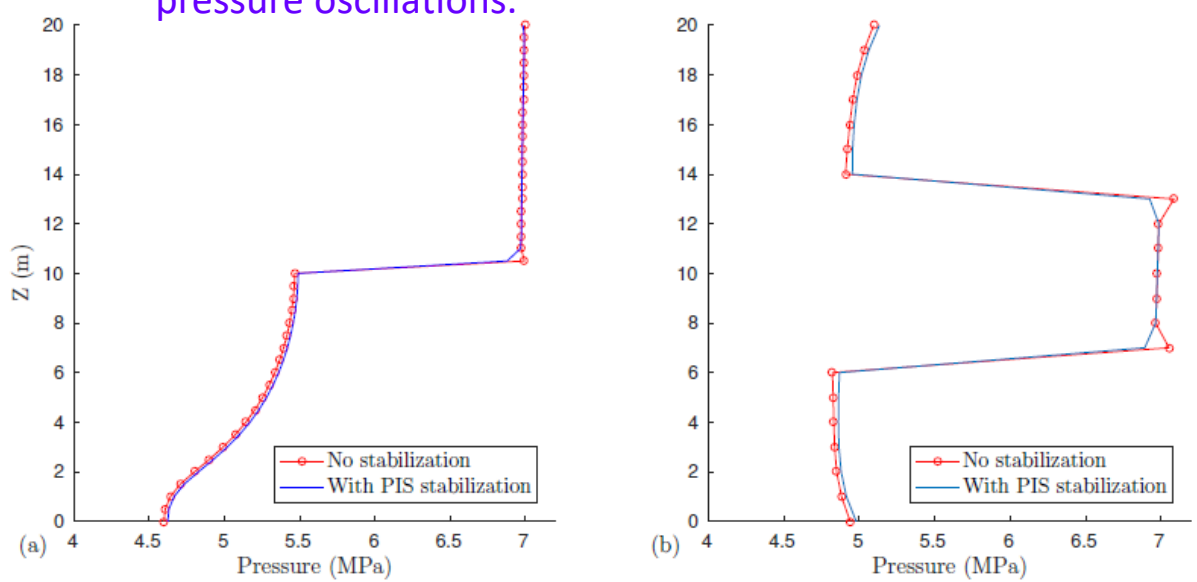


Fig 6: Comparison of pressure profiles near well bore with and without stabilization at t=1 s (Δt=0.1 s) (a) domain with two layers (b) domain with three layers.

Coupled hydrate dissociation and deformation

Hydrate Dissociation: Comparison of iterative and fully-coupled solvers.

- Computational time for fully coupled solver is 1539 s and for the iterative solver is 760s for 5 hours of well operation.

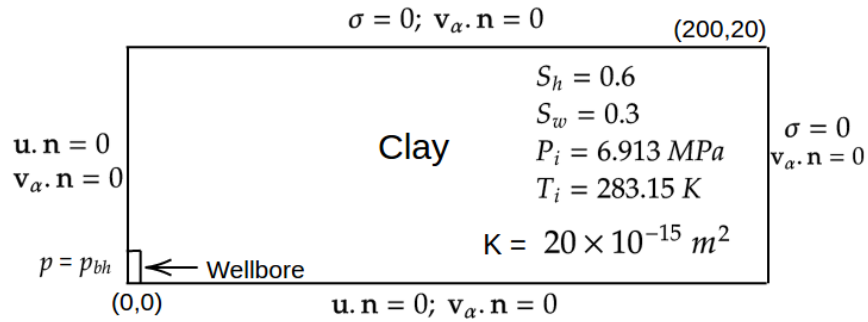


Fig 7: 2D domain for coupled dissociation and deformation problem.

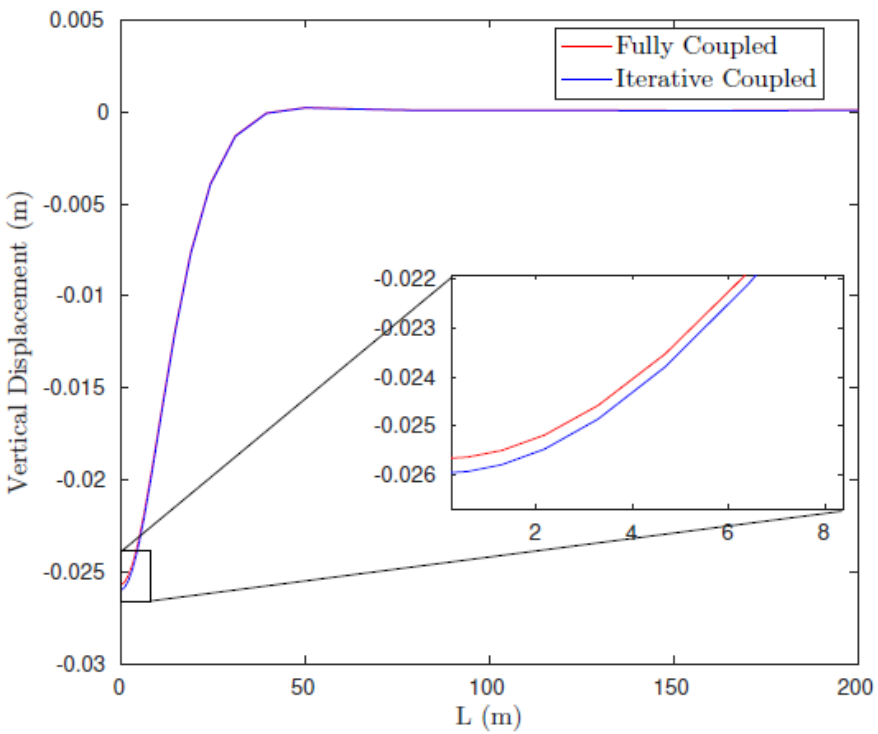


Fig 8: Comparison of iterative and fully-coupled solvers after 5 hours of well operation.

Summary

- A robust and efficient numerical THMC solver using PETSc routines was developed for gas hydrate dissociation with deformation.
- A node centered FVM for flow and FEM for geomechanics, that is, a co-located variable approach has been used.
- Unequal order function spaces for pressure and displacement along with co-located variable arrangement could not mitigate numerical pressure oscillations occurring at early times of simulation.
- PIS based stabilization scheme could mitigate the numerical pressure oscillations.
- The iteratively coupled approach is faster than the fully coupled approach due to smaller matrix size.

References

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