
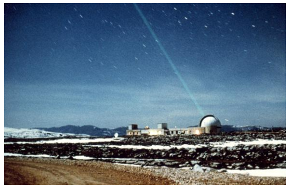
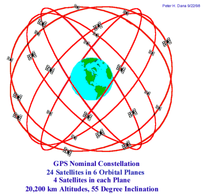


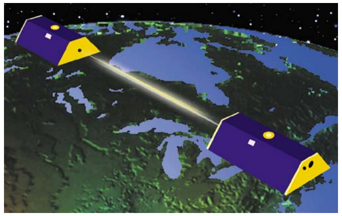
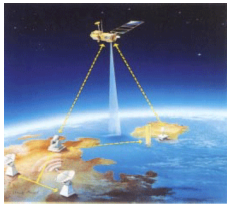

An Advanced Formulation of Kalman Filter Time Series Reference Frame Realization for Geophysical Applications

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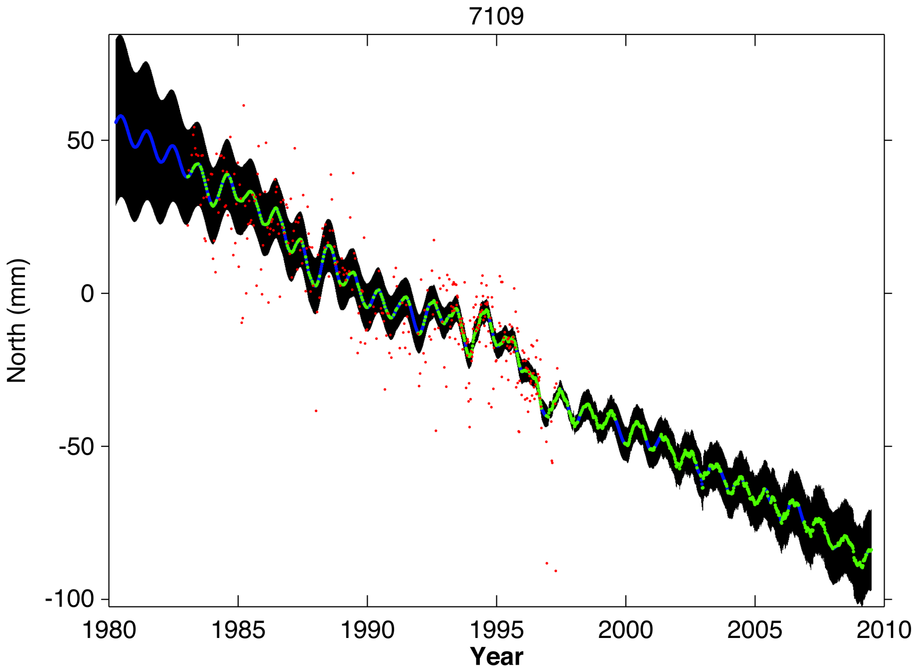
KALREF – Kalman Filter/Smoother Time Series Realization of Terrestrial Reference Frame

- Unify and bridge SLR/VLBI/GNSS/DORIS time series in a frame with origin at nearly instantaneous CM from SLR
- VLBI/GNSS/DORIS series are tied to SLR CM through local ties and co-motion constraints on co-located stations
- Time-variable station coordinates and covariance matrices consistent with time-variable gravity
- Works fine for positioning purposes



State Vector: **Transition Matrix** **Noise**

$$\begin{bmatrix} X_k \\ V_k \\ S_k^{next} \\ S_k^{now} \end{bmatrix} = \begin{bmatrix} 1 & dt & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2e^{-dt/\tau}\cos 2\pi \frac{dt}{T} & -e^{-2dt/\tau} \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X_{k-1} \\ V_{k-1} \\ S_{k-1}^{next} \\ S_{k-1}^{now} \end{bmatrix} + \begin{bmatrix} \varepsilon_x \\ \varepsilon_v \\ \varepsilon_p \\ 0 \end{bmatrix}$$

Time-Correlation and Geophysical Displacement Covariance Matrices

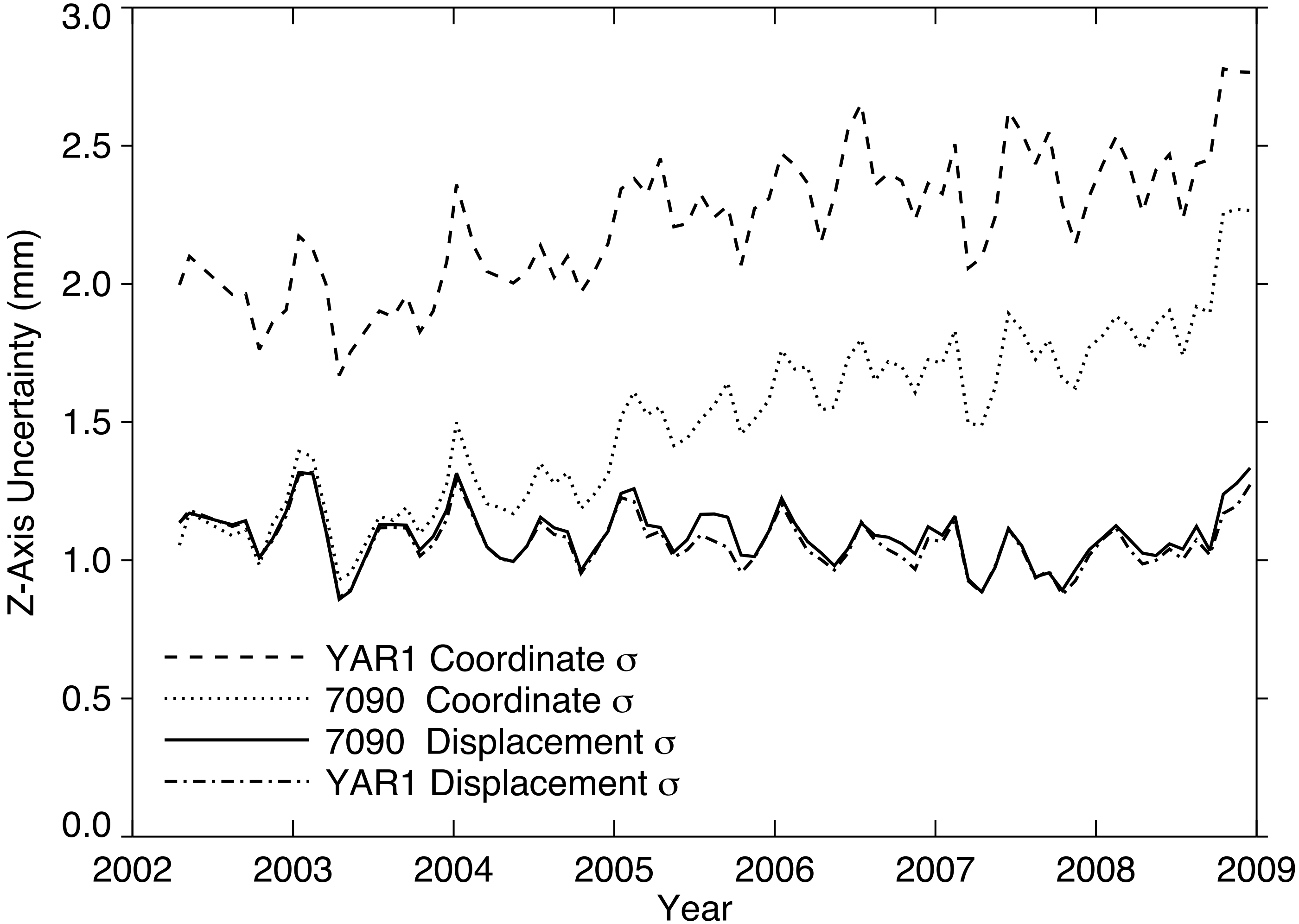
- The unique nature of KALREF results in highly correlated coordinate errors over time. e.g. local tie errors are constant over time
- But Kalman filter cannot keep track of correlations over time
- Ignoring time correlations can result in erroneous covariance matrices for displacement observables in many geophysical applications.
- For example, in KALREF, co-located stations should have the same displacement and uncertainty. But derived from station coordinate covariance matrices through either stacking or differencing without time correlations, the co-located stations have very different displacement covariance matrices.
- Used in subsequent geophysical inversions, these would result in ambiguous and erroneous results. For instance, different co-located stations would result in different geocenter motion results by more than a millimeter.
- Robust geophysical use requires a more advanced KALREF formulation with displacements as explicit state parameters for accurate evaluation of their covariance matrices

Advanced Kalman Filter State Parameter Formulation

- Now the state parameter vector includes D_k^{ij} as explicit parameter for station *i*, coordinate axis *j*, and week *k*.
- X_{0k}^{ij} is a constant over time except when there is a position offset
- ε_{Dk}^{ij} is a white noise
- $X_k = X_0 + D_k$ is the total coordinates
- This enabes Kalman filter and smoother to accurately assess the covariance matrices of D_k^{ij}

$$
\begin{bmatrix} X_{0k}^{ij} \\ D_k^{ij} \\ V_k^{ij} \\ \vdots \\ S_{k,next}^{ij,m} \\ S_{k,now}^{ij,m} \\ \vdots \end{bmatrix}
=
\begin{bmatrix}
1 & 0 & 0 & \cdots & 0 & 0 & \cdots \\
0 & 1 & \Delta t & \cdots & 0 & 0 & \cdots \\
0 & 0 & 1 & \cdots & 0 & 0 & \cdots \\
\vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \cdots \\
0 & 0 & 0 & \cdots & 2e^{-\Delta t/\tau_m}\cos(2\pi\Delta t/T_m) & -e^{-2\Delta t/\tau_m} & \cdots \\
0 & 0 & 0 & \cdots & 1 & 0 & \cdots \\
\vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \cdots
\end{bmatrix}
\begin{bmatrix} X_{0k-1}^{ij} \\ D_{k-1}^{ij} \\ V_{k-1}^{ij} \\ \vdots \\ S_{k-1,next}^{ij,m} \\ S_{k-1,now}^{ij,m} \\ \vdots \end{bmatrix}
+
\begin{bmatrix} \varepsilon_{Xk}^{ij} \\ \varepsilon_{Dk}^{ij} \\ \varepsilon_{Vk}^{ij} \\ \vdots \\ \varepsilon_{Sk}^{ij} \\ 0 \\ \vdots \end{bmatrix}
$$

Co-located Australian Station Coordinate and Displacement Uncertainties



- **The coordinate uncertainties differ.**
- **but nearly the same displacement uncertainty from the advanced formulation.**
- **KALREF time series are longer (1980-2009) than the shown time window with coordinate uncertainties bottoming in the middle of the data stream.**

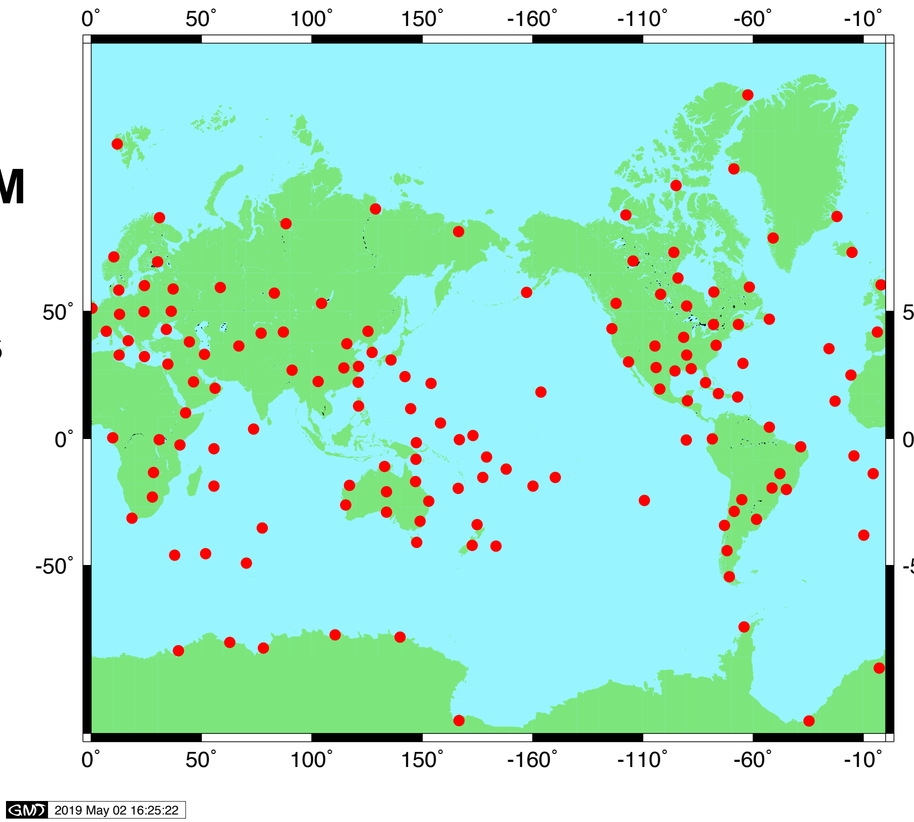
Unified Inversion for Non-Secular Geocenter Motion

- **Unified inversion of displacements WRT CM + GRACE for geocenter motion**

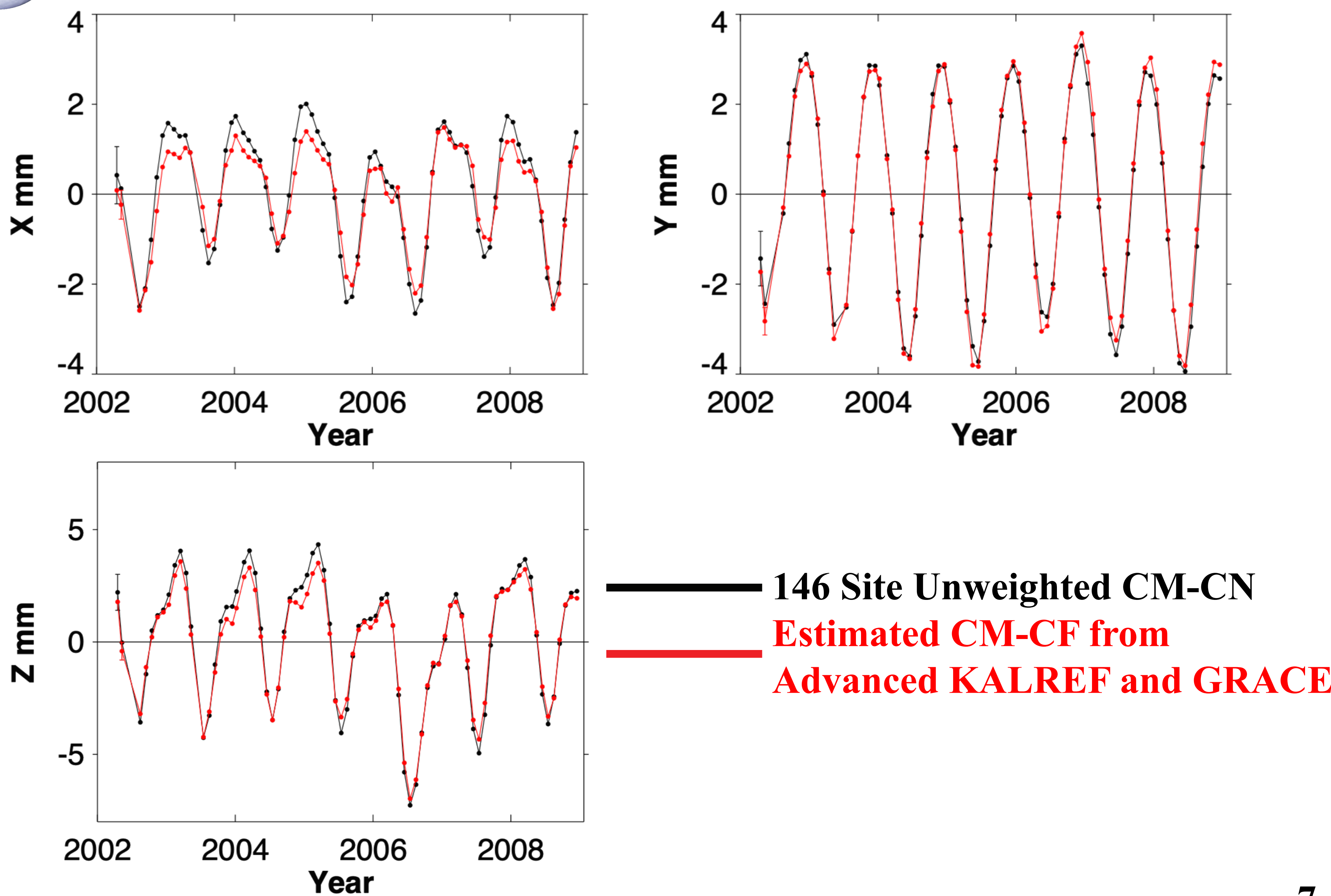
  Wu et al., Geo. J. Int., 2017

  - **Surface Displacements against CM have both (strong) translational and (weak) deformational signatures of n=1 mass variations**
  - **Desire uniform global station coverage**
  - **GRACE+FO data further improve access to CF**
  - **Displacement errors are also correlated in time thus Helmert variance component calibration needed for error propagation**
- **Plan to use geocentric GNSS data with GRACE+FO in the future with rapid progress in GNSS geocenter sensitivity.**

146 KALREF Sites



CM-CN (JTRF2008-146 sites) and Estimated CM-CF from Unified Inversion

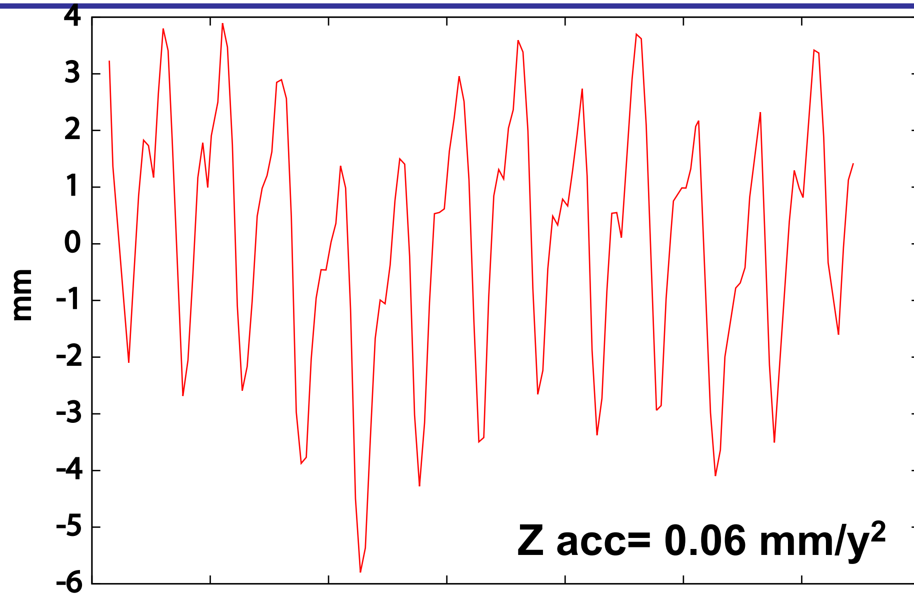


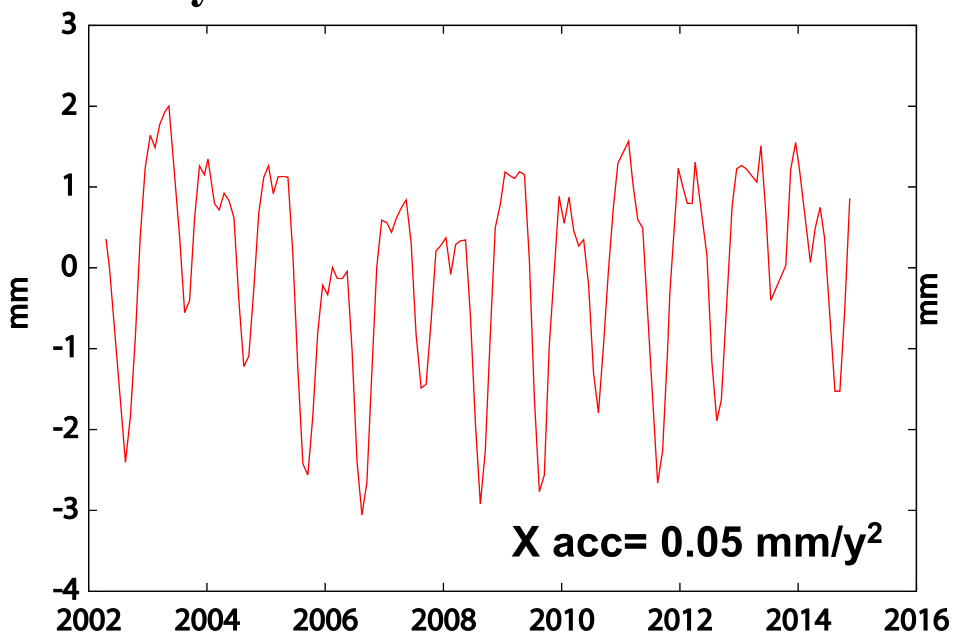
Annual Geocenter Motion Estimates

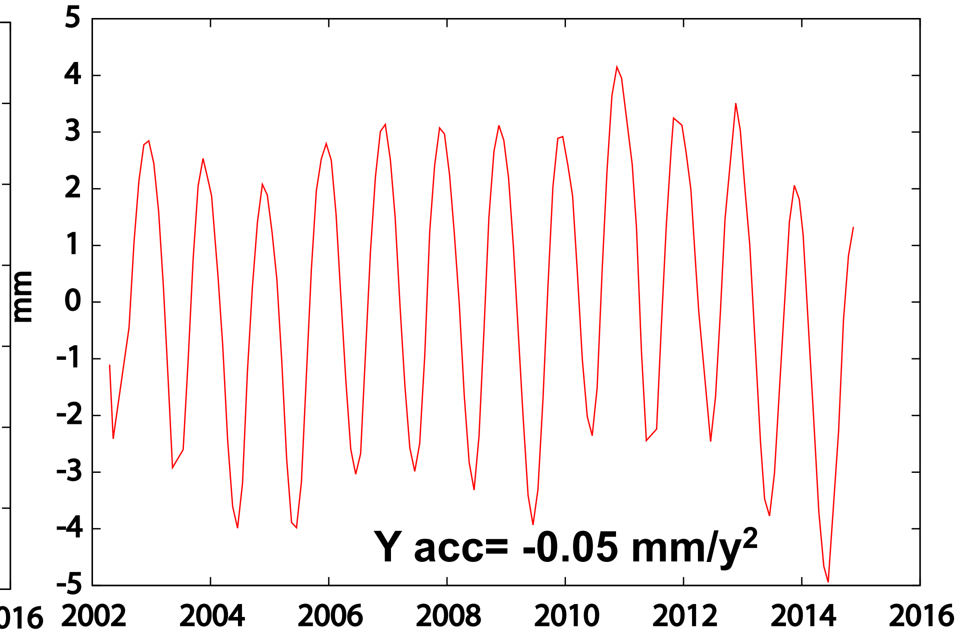
Data	X_g		Y_g		Z_g		Ref
	Amp mm	Phase day	Amp mm	Phase day	Amp mm	Phase day	
SLR (Monthly)	3.2±0.4	33 ± 3	2.6±0.2	306 ± 2	4.3±0.3	31 ± 2	*Cheng 2013 2002-2010*
ILRS (Weekly)	3.0±0.2	55 ± 4	2.7±0.2	328 ± 4	5.4±0.4	23 ± 4	*ILRS 2002-2009*
GNSS GRACE tracking + Acc. data	1.1±	54 ±	2.8±	332 ±	3.6 ±	45 ±	*Kuang 2019 2006-2010*
GPS Deformation +OBP+GRACE	1.9±0.1	48 ± 5	2.9±0.1	325 ± 3	4.3±0.2	30 ± 3	*Wu 2013 2002.3-2009.3*
OBP + GRACE	2.3±0.1	52 ± 3	2.8±0.1	327 ± 2	2.9±0.2	69 ± 4	*Sun 2016 2002.6-2014.5*
Unified Inversion	1.3±0.1	50 ± 4	3.3±0.1	338 ± 2	2.9±0.2	27 ± 3	*This study 2002.2-2009.0*

Unified Inversion for Longer Term Geocenter Motion Estimates

- **JTRF2014 input data (Abbondanza et al., 2017)**
- **Advanced KALREF formulation**
- **Results largely consistent with global inversion of relative GPS + GRACE+ECCO but more precise**
- **Interannual variations rather than steady acceleration**







Toward Higher Precision and Low-Latency Geocenter Motion Determination

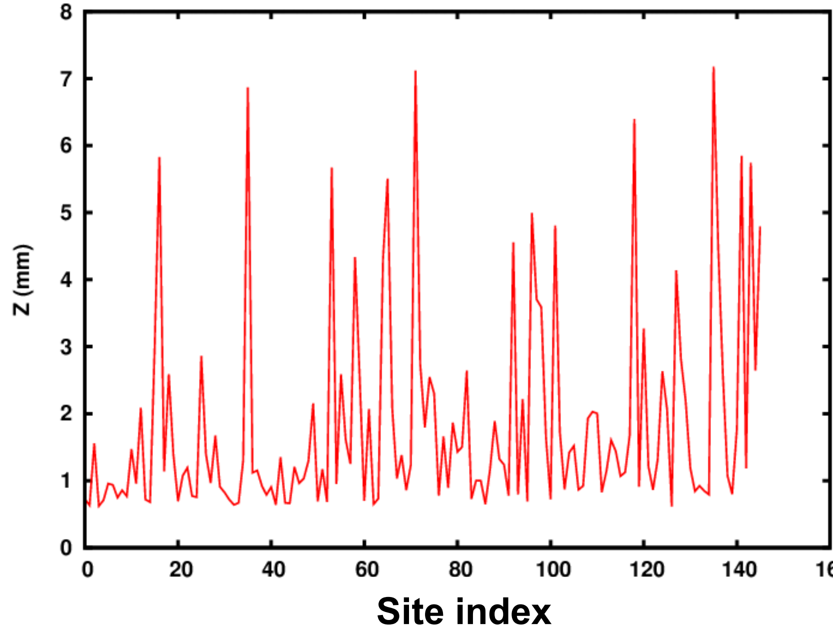
- **KALREF-type Displacements have complex and heterogenous error structure**

- **TRF products have latencies of > 5 years**

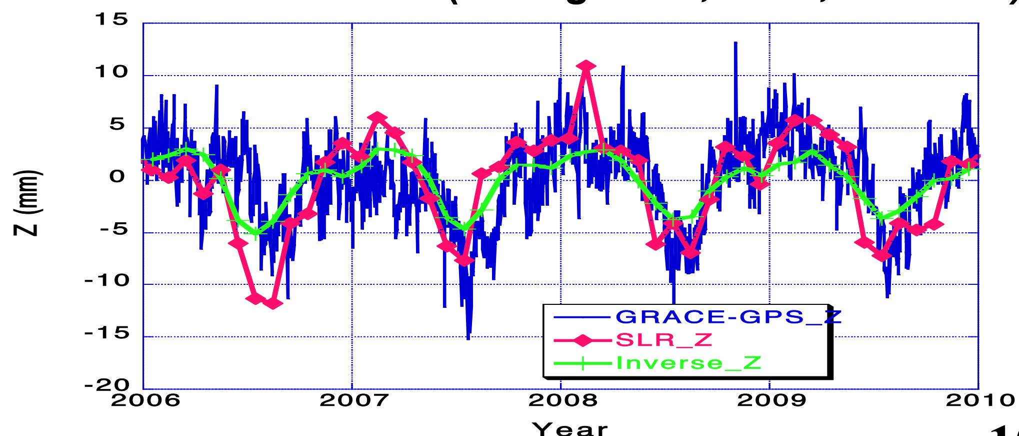
- **Recent developments in JPL demonstrate GNSS's sensitivity to CM (*Haines et al., 2015, Kuang et al., 2019*)**

- **Orbit tracking with point-positioning for larger-network displacements WRT GNSS's CM + GRACE+FO gravity for unified inversion with better homogeneity and latency**

Displacement Uncertainties for 1 month



CM-Flinn CN from GNSS tracking to GRACE+ accelerometer data (*Kuang et al., 2019, J. Geod.*)



Summary

- For positioning purposes, the standard KALREF formulation in time-variable coordinates are fine.

- For geophysical investigations relying on displacements, an advanced KALREF formulation with displacements as explicit parameters is required.

- The advanced formulation results in accurate displacement covariance matrices to be used in unified inversion with GRACE data for non-linear geocenter motion.

- Wider and more even ground networks and GRACE gravity data reduce biases and enhance access to CF

See *Wu et al., JGR., 2020.*

- Plan to apply the unified approach to linear trend inversion and use improved geocentric GNSS data with GRACE to improve latency