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Temporal variations in ITRF station displacements analyzed with vector spherical harmonics

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Introduction

- Goal: study **global features of time-dependent station coordinate variations** in ITRF solutions
  - **DTRF2014**: non-tidal loading displacements provided (atmosphere and continental water storage)
  - **JTRF2014**: time series approach to TRF determination by Kalman filtering; weekly station positions provided
- New approach: **vector spherical harmonics (VSH)** to estimate global displacements from all three coordinate components
- Comparison to **scalar spherical harmonics** based on the vertical coordinate component (largest signals) and **Helmert transformation parameters**

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Vector spherical harmonics up to degree-2

longitude

$$\begin{aligned}\Delta\lambda\cos\phi = &-T_x\sin\lambda + T_y\cos\lambda\\&-R_1\sin\phi\cos\lambda - R_2\sin\phi\sin\lambda + R_3\cos\phi\\&-a_{2,0}^{M}\sin 2\phi\\&-a_{2,1}^{E,\Re}\sin\phi\sin\lambda - a_{2,1}^{E,\Im}\sin\phi\cos\lambda\\&+a_{2,1}^{M,\Re}\cos 2\phi\cos\lambda - a_{2,1}^{M,\Im}\cos 2\phi\sin\lambda\\&+a_{2,2}^{E,\Re}\cos\phi\sin 2\lambda + a_{2,2}^{E,\Im}\cos\phi\cos 2\lambda\\&+\frac{1}{2}a_{2,2}^{M,\Re}\sin 2\phi\cos 2\lambda - \frac{1}{2}a_{2,2}^{M,\Im}\sin 2\phi\sin 2\lambda\end{aligned}$$

latitude

$$\begin{aligned}\Delta\phi = &-T_x\sin\phi\cos\lambda - T_y\sin\phi\sin\lambda + T_z\cos\phi\\&+R_1\sin\lambda - R_2\cos\lambda\\&-a_{2,0}^{E}\sin 2\phi\\&+a_{2,1}^{E,\Re}\cos 2\phi\cos\lambda - a_{2,1}^{E,\Im}\cos 2\phi\sin\lambda\\&+a_{2,1}^{M,\Re}\sin\phi\sin\lambda + a_{2,1}^{M,\Im}\sin\phi\cos\lambda\\&+\frac{1}{2}a_{2,2}^{E,\Re}\sin 2\phi\cos 2\lambda - \frac{1}{2}a_{2,2}^{E,\Im}\sin 2\phi\sin 2\lambda\\&-a_{2,2}^{M,\Re}\cos\phi\sin 2\lambda - a_{2,2}^{M,\Im}\cos\phi\cos 2\lambda\end{aligned}$$

radial

$$\begin{aligned}\Delta r = &\, D\\&+T_x\cos\phi\cos\lambda + T_y\cos\phi\sin\lambda + T_z\sin\phi\\&-a_{2,0}^{E}\sin^2\phi - \frac{1}{3}\\&+\frac{1}{2}a_{2,1}^{E,\Re}\sin 2\phi\cos\lambda - \frac{1}{2}a_{2,1}^{E,\Im}\sin 2\phi\sin\lambda\\&-a_{2,2}^{E,\Re}\cos^2\phi\cos 2\lambda - a_{2,2}^{E,\Im}\cos^2\phi\sin 2\lambda\end{aligned}$$

Degree 0: scale D

Degree 1: rotations R and dipole deformations T (= translations)

Degree 2: quadrupole deformations a

Vector spherical harmonics vs. Helmert parameters

longitude

$$
\begin{aligned}
\Delta\lambda\cos\phi = &-T_x\sin\lambda + T_y\cos\lambda \\
&- R_1\sin\phi\cos\lambda - R_2\sin\phi\sin\lambda + R_3\cos\phi \\
&- a_{2,0}^{M}\sin 2\phi \\
&- a_{2,1}^{E,\Re}\sin\phi\sin\lambda - a_{2,1}^{E,\Im}\sin\phi\cos\lambda \\
&+ a_{2,1}^{M,\Re}\cos 2\phi\cos\lambda - a_{2,1}^{M,\Im}\cos 2\phi\sin\lambda \\
&+ a_{2,2}^{E,\Re}\cos\phi\sin 2\lambda + a_{2,2}^{E,\Im}\cos\phi\cos 2\lambda \\
&+ \frac{1}{2}a_{2,2}^{M,\Re}\sin 2\phi\cos 2\lambda - \frac{1}{2}a_{2,2}^{M,\Im}\sin 2\phi\sin 2\lambda
\end{aligned}
$$

latitude

$$
\begin{aligned}
\Delta\phi = &-T_x\sin\phi\cos\lambda - T_y\sin\phi\sin\lambda + T_z\cos\phi \\
&+ R_1\sin\lambda - R_2\cos\lambda \\
&- a_{2,0}^{E}\sin 2\phi \\
&+ a_{2,1}^{E,\Re}\cos 2\phi\cos\lambda - a_{2,1}^{E,\Im}\cos 2\phi\sin\lambda \\
&+ a_{2,1}^{M,\Re}\sin\phi\sin\lambda + a_{2,1}^{M,\Im}\sin\phi\cos\lambda \\
&+ \frac{1}{2}a_{2,2}^{E,\Re}\sin 2\phi\cos 2\lambda - \frac{1}{2}a_{2,2}^{E,\Im}\sin 2\phi\sin 2\lambda \\
&- a_{2,2}^{M,\Re}\cos\phi\sin 2\lambda - a_{2,2}^{M,\Im}\cos\phi\cos 2\lambda
\end{aligned}
$$

radial

$$
\begin{aligned}
\Delta r = &\, D \\
&+ T_x\cos\phi\cos\lambda + T_y\cos\phi\sin\lambda + T_z\sin\phi \\
&- a_{2,0}^{E}\sin^2\phi - \frac{1}{3} \\
&+ \frac{1}{2}a_{2,1}^{E,\Re}\sin 2\phi\cos\lambda - \frac{1}{2}a_{2,1}^{E,\Im}\sin 2\phi\sin\lambda \\
&- a_{2,2}^{E,\Re}\cos^2\phi\cos 2\lambda - a_{2,2}^{E,\Im}\cos^2\phi\sin 2\lambda
\end{aligned}
$$

$$\Delta\vec{X} = D\vec{X} + \vec{T} + \mathbf{R}\vec{X}$$

Degree 0: scale D

Degree 1: rotations R and dipole deformations T (= translations)

Degree 2: quadrupole deformations a

Vector vs. scalar spherical harmonics

longitude

$$\begin{aligned}\Delta\lambda\cos\phi = &-T_x\sin\lambda + T_y\cos\lambda\\ &-R_1\sin\phi\cos\lambda - R_2\sin\phi\sin\lambda + R_3\cos\phi\\ &-a^{M}_{2,0}\sin 2\phi\\ &-a^{E,\Re}_{2,1}\sin\phi\sin\lambda - a^{E,\Im}_{2,1}\sin\phi\cos\lambda\\ &+a^{M,\Re}_{2,1}\cos 2\phi\cos\lambda - a^{M,\Im}_{2,1}\cos 2\phi\sin\lambda\\ &+a^{E,\Re}_{2,2}\cos\phi\sin 2\lambda + a^{E,\Im}_{2,2}\cos\phi\cos 2\lambda\\ &+\frac{1}{2}a^{M,\Re}_{2,2}\sin 2\phi\cos 2\lambda - \frac{1}{2}a^{M,\Im}_{2,2}\sin 2\phi\sin 2\lambda\end{aligned}$$

latitude

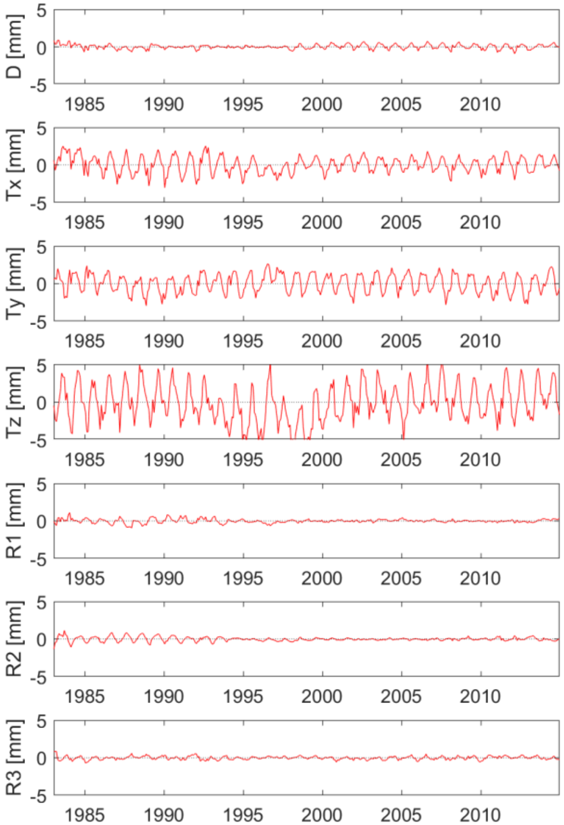
$$\begin{aligned}\Delta\phi = &-T_x\sin\phi\cos\lambda - T_y\sin\phi\sin\lambda + T_z\cos\phi\\ &+R_1\sin\lambda - R_2\cos\lambda\\ &-a^{E}_{2,0}\sin 2\phi\\ &+a^{E,\Re}_{2,1}\cos 2\phi\cos\lambda - a^{E,\Im}_{2,1}\cos 2\phi\sin\lambda\\ &+a^{M,\Re}_{2,1}\sin\phi\sin\lambda + a^{M,\Im}_{2,1}\sin\phi\cos\lambda\\ &+\frac{1}{2}a^{E,\Re}_{2,2}\sin 2\phi\cos 2\lambda - \frac{1}{2}a^{E,\Im}_{2,2}\sin 2\phi\sin 2\lambda\\ &-a^{M,\Re}_{2,2}\cos\phi\sin 2\lambda - a^{M,\Im}_{2,2}\cos\phi\cos 2\lambda\end{aligned}$$

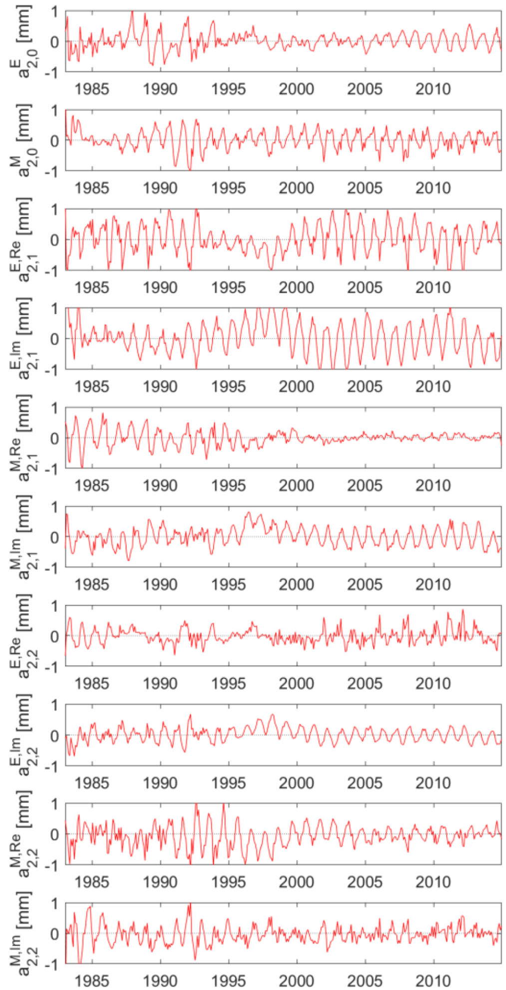
radial

$$\begin{aligned}\Delta r = &D\\ &+T_x\cos\phi\cos\lambda + T_y\cos\phi\sin\lambda + T_z\sin\phi\\ &-a^{E}_{2,0}\sin^2\phi - \frac{1}{3}\\ &+\frac{1}{2}a^{E,\Re}_{2,1}\sin 2\phi\cos\lambda - \frac{1}{2}a^{E,\Im}_{2,1}\sin 2\phi\sin\lambda\\ &-a^{E,\Re}_{2,2}\cos^2\phi\cos 2\lambda - a^{E,\Im}_{2,2}\cos^2\phi\sin 2\lambda\end{aligned}$$

Radial displacements: insensitive to toroidal (“magnetic”) features, such as rotations

VSH of DTRF2014 non-tidal displacements





Largest signals for translations, in particular Tz

Degree-2 terms almost reach 1 mm

VSH of different DTRF2014 NT displacements

Std. dev. [mm]	NTAL	CWSL	Sum
D	0.27	0.23	0.31
Tx	0.61	0.77	0.9
Ty	1.09	0.98	1.25
Tz	1.68	1.92	2.59
R1	0.15	0.14	0.16
R2	0.14	0.12	0.15
R3	0.25	0.12	0.2
$a^{E}_{2,0}$	0.14	0.16	0.2
$a^{M}_{2,0}$	0.32	0.09	0.26
$a^{E,Re}_{2,1}$	0.31	0.39	0.46
$a^{E,Im}_{2,1}$	0.18	0.53	0.55
$a^{M,Re}_{2,1}$	0.11	0.11	0.13
$a^{M,Im}_{2,1}$	0.19	0.32	0.31
$a^{E,Re}_{2,2}$	0.3	0.17	0.24
$a^{E,Im}_{2,2}$	0.1	0.23	0.22
$a^{M,Re}_{2,2}$	0.2	0.25	0.29
$a^{M,Im}_{2,2}$	0.32	0.12	0.22

After 1995

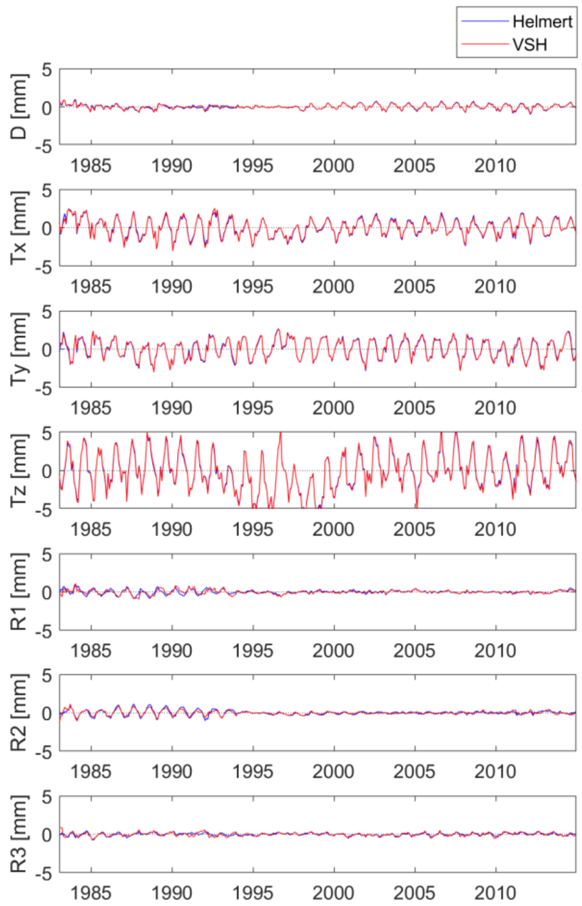
NTAL: weekly

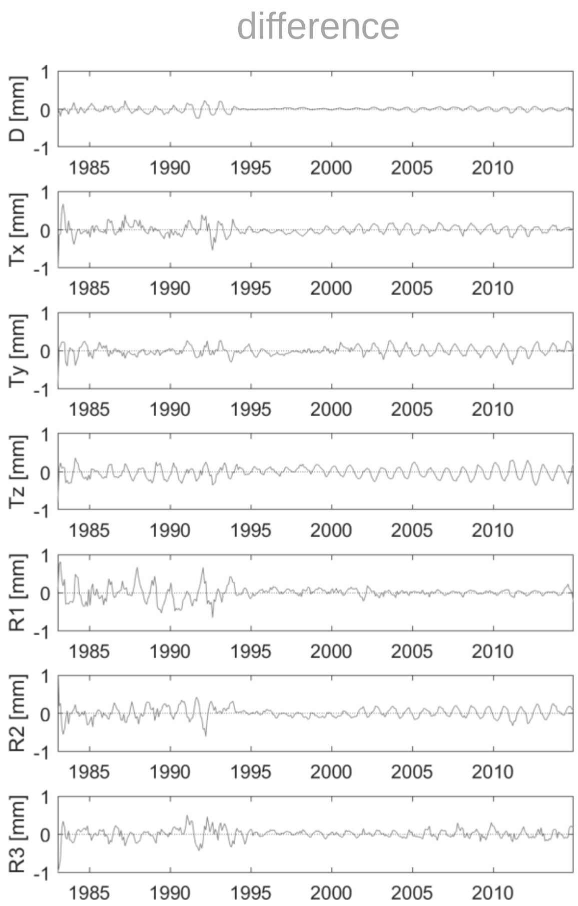
CWSL: monthly

Sum: monthly

Different contributions from atmosphere and continental water storage loading to certain terms

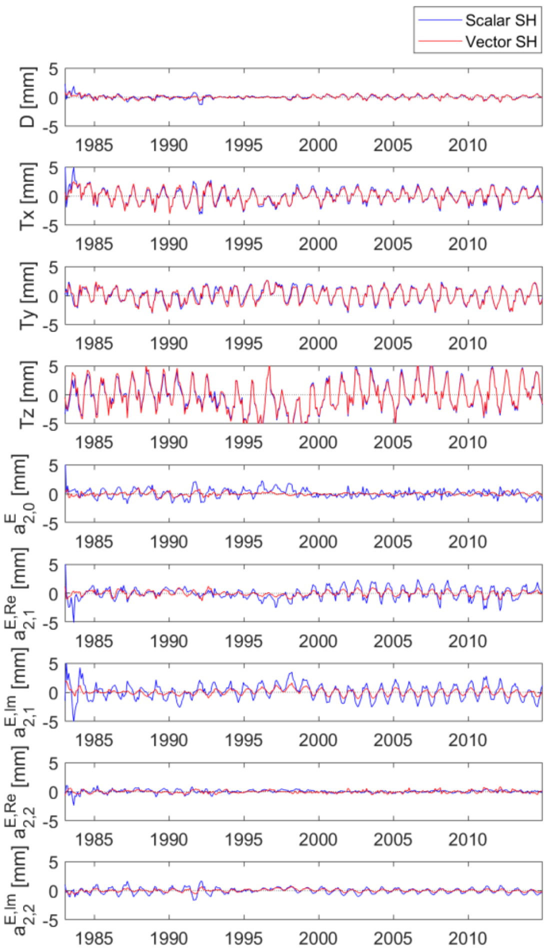
VSH vs. Helmert parameters for DTRF2014

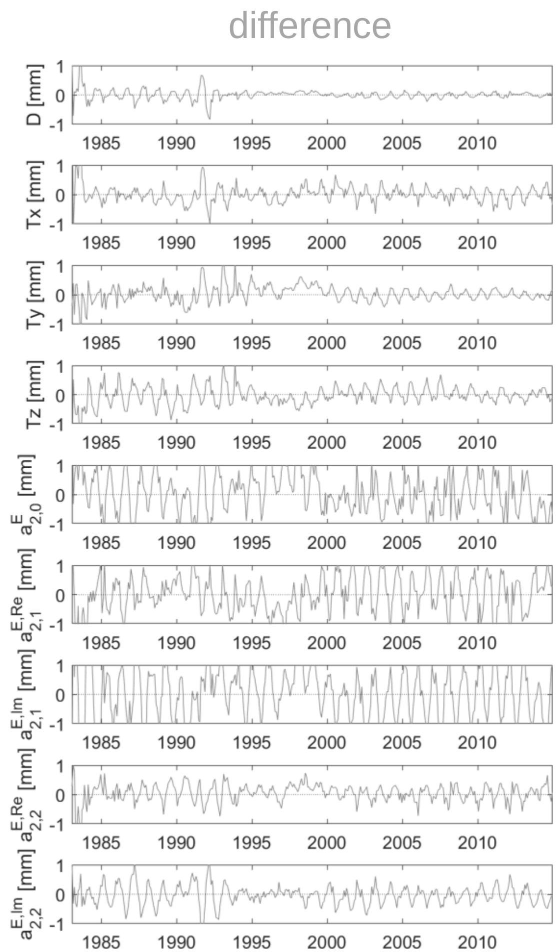




Sub-mm differences

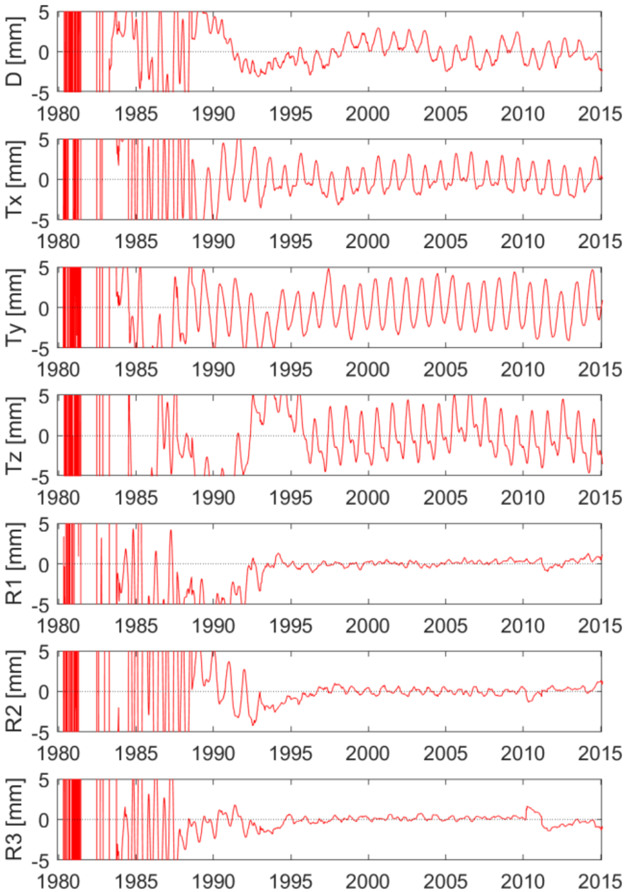
Vector vs. scalar SH for DTRF2014

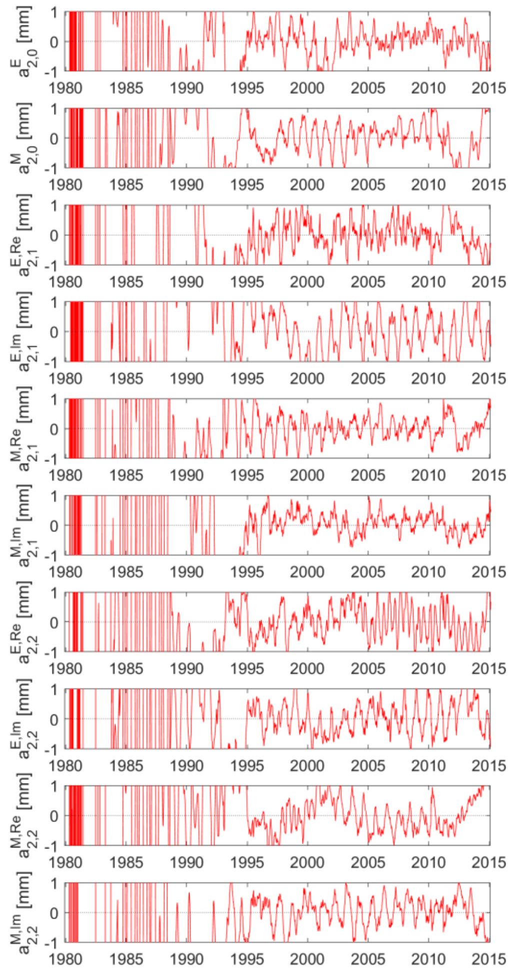




Differences exceed 1 mm

VSH of JTRF2014 coordinate displacements





Displacements defined as coordinate time series minus linear fit

Based on observations → contain more signals (and errors) than NT model of DTRF2014

Before 1995: stronger variations due to smaller networks and worse data quality

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Comparison between DTRF2014 and JTRF2014

Std. dev. [mm]	DTRF2014	JTRF2014
D	0.31	1.31
Tx	0.9	1.46
Ty	1.25	2.32
Tz	2.59	2.34
R1	0.16	0.38
R2	0.15	0.47
R3	0.2	0.49
$a^{E}_{2,0}$	0.2	0.47
$a^{M}_{2,0}$	0.26	0.58
$a^{E,Re}_{2,1}$	0.46	0.51
$a^{E,Im}_{2,1}$	0.55	0.74
$a^{M,Re}_{2,1}$	0.13	0.41
$a^{M,Im}_{2,1}$	0.31	0.36
$a^{E,Re}_{2,2}$	0.24	0.58
$a^{E,Im}_{2,2}$	0.22	0.53
$a^{M,Re}_{2,2}$	0.29	0.55
$a^{M,Im}_{2,2}$	0.22	0.49

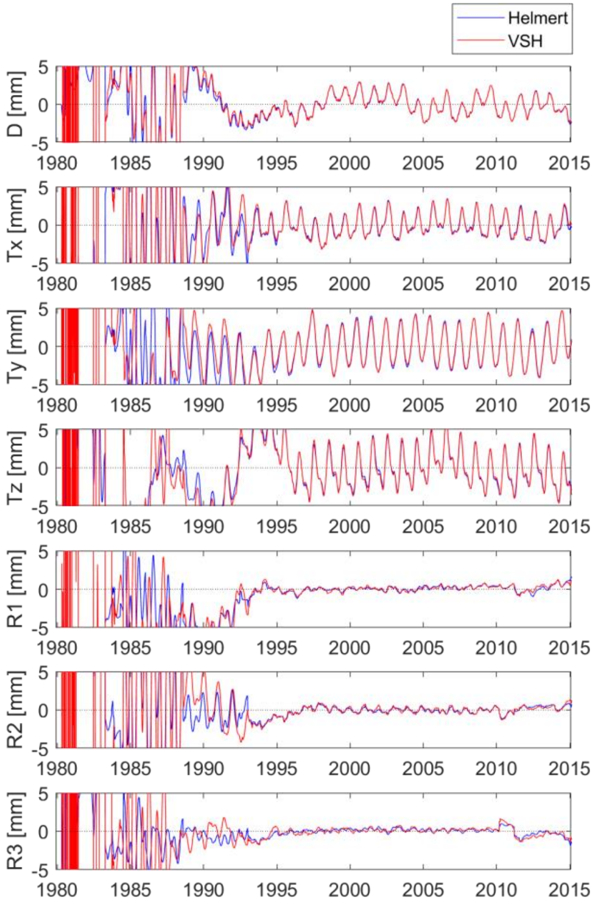
After 1995

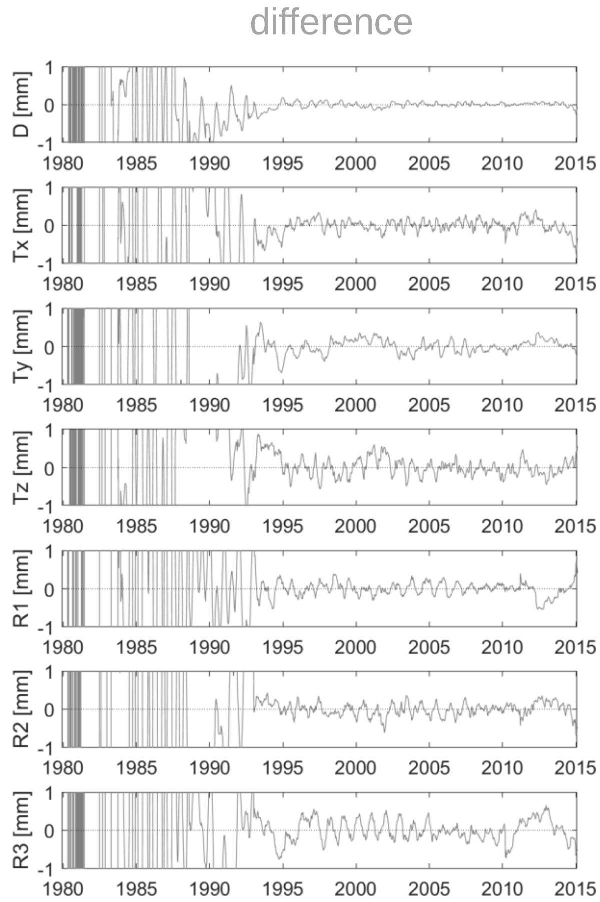
DTRF2014:

NTAL+CWSL (monthly)

JTRF2014 (weekly) with larger variations, except for Tz

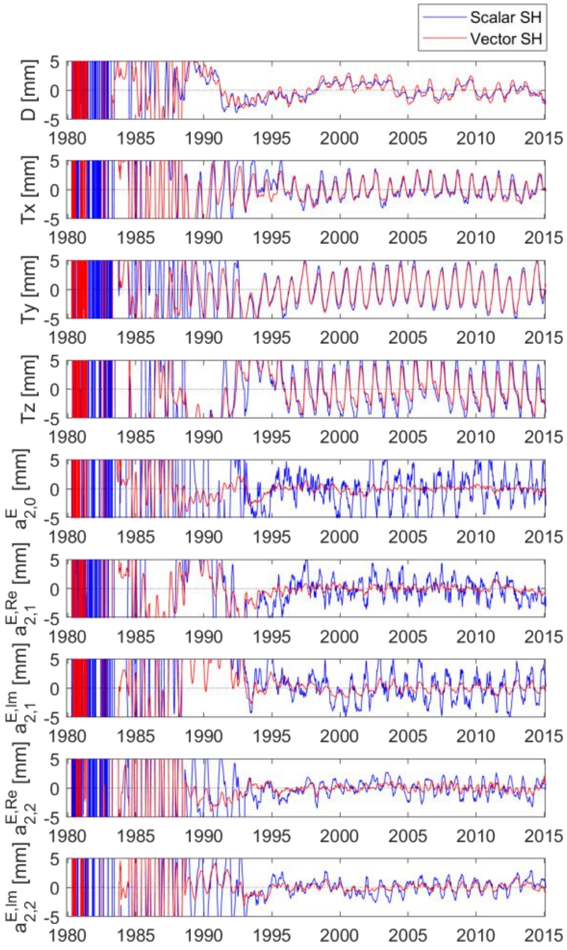
VSH vs. Helmert parameters for JTRF2014



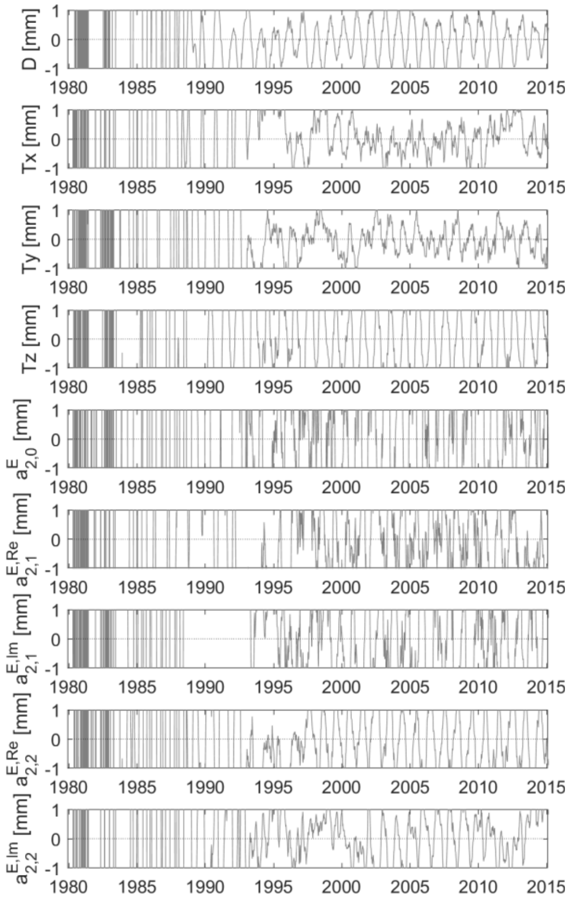


Sub-mm differences after 1995

Vector vs. scalar SH for JTRF2014



difference



Degree-2 terms much more variable for scaler spherical harmonics, exceeding 5 mm for $a_{2,0}$

Conclusions

- Applied **vector spherical harmonics** to study time series of coordinate displacements
- Fundamentally different results for DTRF2014 and JTRF2014 due to model- vs. observation-based approach
- Global features at the mm-level for certain degree-2 terms
- Differences between Helmert transformation parameters and VSH below 1 mm
- Differences between scalar and vector SH larger than 5 mm for certain terms

Thank you very much!

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