

# ***The space-wise approach for cold-atom interferometry geodetic data analysis: the MOCASS study***



**POLITECNICO**  
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# The MOCASS study

- *MOCASS is a study carried out from 2016 to 2018 by POLIMI and other Italian research groups on behalf of the Italian Space Agency (ASI).*
- *The study regarded a new satellite mission proposal for continuously monitoring of the Earth gravity and its changes, based on a satellite-borne interferometer exploiting ultra-cold atom technology.*
- *Goal of the study: to investigate the performance of a cold atom interferometer flying on a low Earth orbiter and its impact on the modeling of different geophysical phenomena, considering:*
  - *the signal requirements from a geophysical point of view,*
  - *the estimation of the global signal characteristics as a result of the geodetic data analysis,*
  - *the technological characteristics of an atom interferometer that delivers the gravity and gradient field at the satellite altitude.*

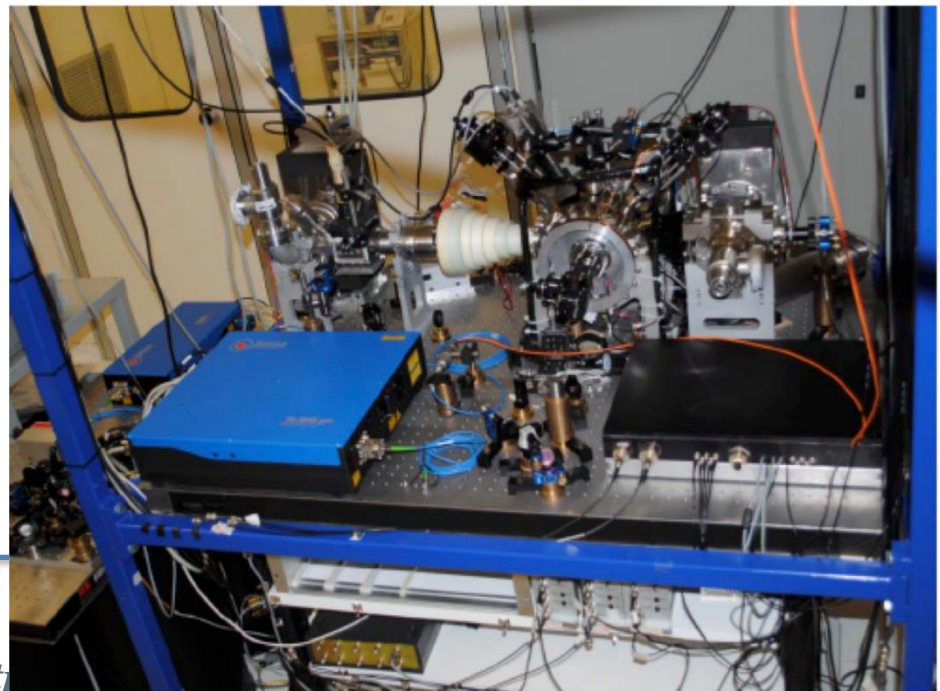
# MOCASS as a GOCE follow-on mission

- *In this study, the basic idea was that of a GOCE follow-on mission, with a unique spacecraft carrying an instrument capable of measuring functionals of the Earth gravitational potential.*
- *Instrument: ultra-cold atom interferometer*  Atom Sensors
- *Study of the signal requirements from a geophysical point of view*  UNIVERSITÀ  
DEGLI STUDI DI TRIESTE
- *Geodetic data analysis by the space-wise approach*  POLITECNICO  
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- *The study was funded by the Italian Space Agency (ASI Contract No. 2016–9-U-0)*  ASI  
Agenzia Spaziale Italiana

# MOCASS as a GOCE follow-on mission

- *In this study, the basic idea was that of a GOCE follow-on mission.*
- *Proposed payload: a spaceborne gradiometer based on laser-cooled atom interferometry, a sensor based on a concept proposed by (Carraz et al., 2014), exploiting the principle of Cold Atom Interferometry.*
- *These instruments operate as inertial sensors and have been used for fundamental physics experiments.*

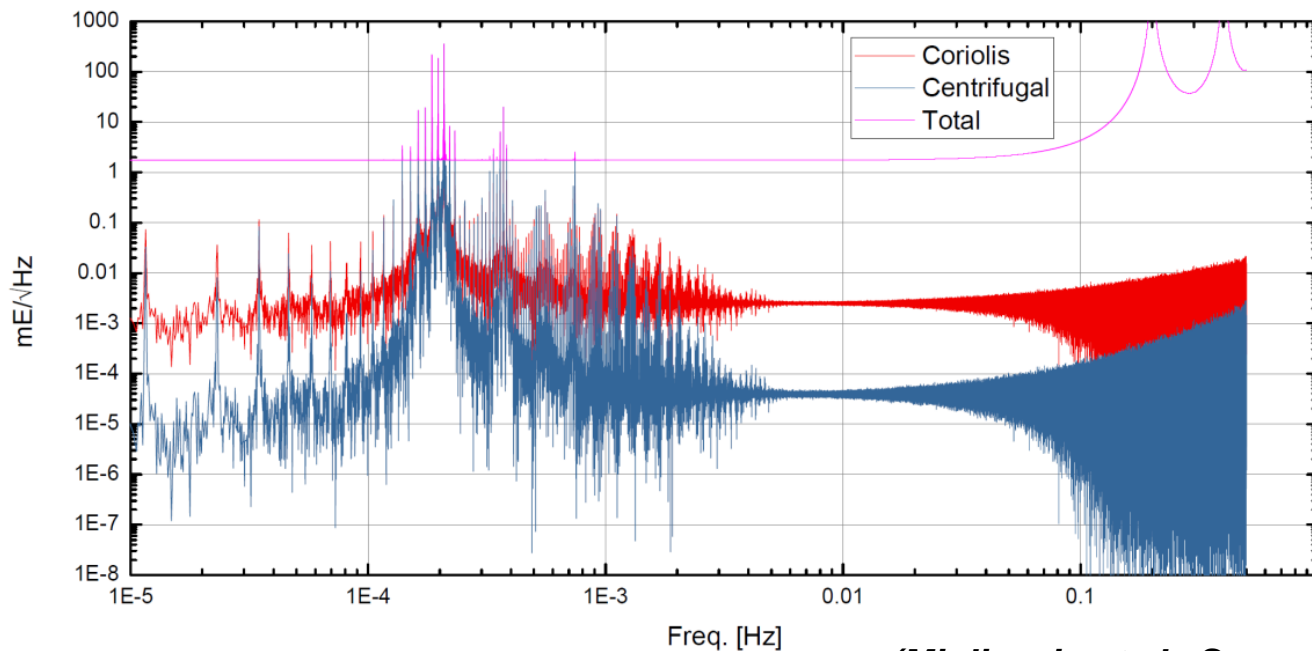
*Complete laser + vacuum system developed in Firenze (Space Optical Clocks Project, 2008), image provided by AtomSensors srl*



# The MOCASS payload

*It has been proved that a sensitivity of  $3.5 \text{ mE}/\sqrt{\text{Hz}}$  ( $1 \text{ E} = 10^{-9} \text{ s}^{-2}$ ) over a wide spectral range can be reached.*

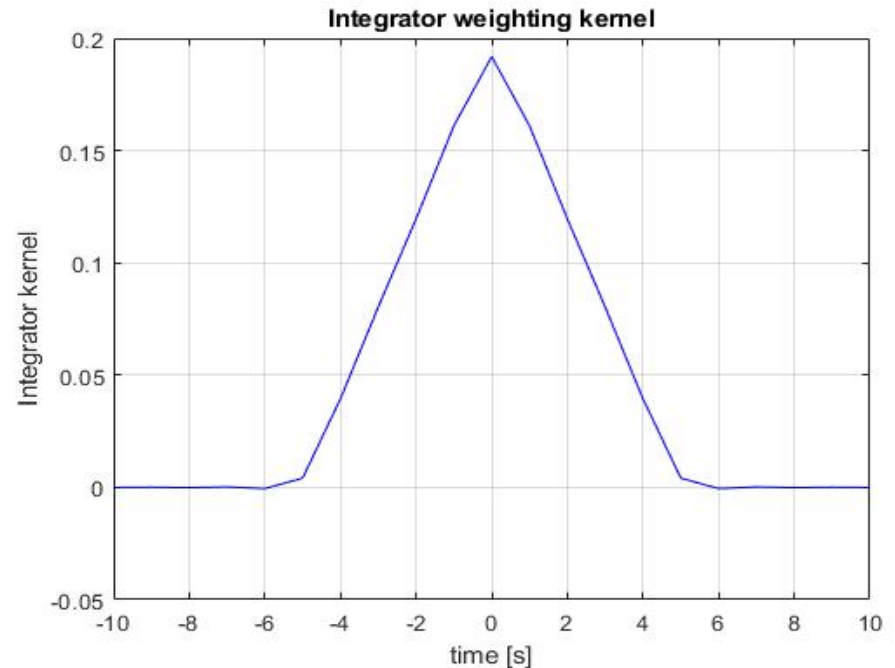
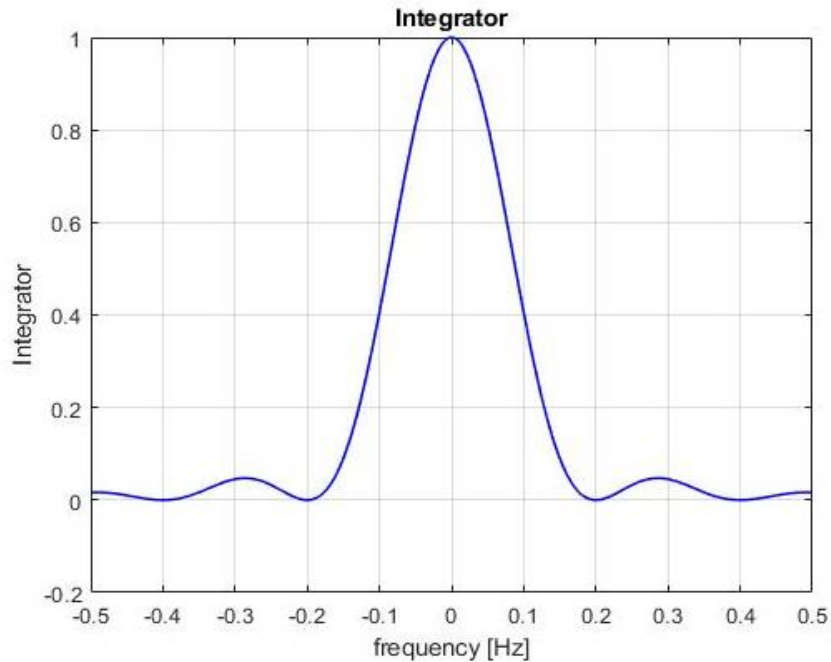
*ASD (Amplitude Spectral Density) of the gravity gradient noise for the cold atom interferometer, assuming fluctuations of  $10 \text{ } \mu\text{m/s}$  RMS for the residual velocity difference between atom clouds, a typical GOCE-like orbit and a compensation of the rotation rate at the level of  $10^{-8} \text{ rad/s}$ . Case: nadir-pointing mode, radial (z) sensitive axis.*



*(Migliaccio et al., Surveys in  
Geophysics 40:1029–1053, 2019)*

# The MOCASS payload

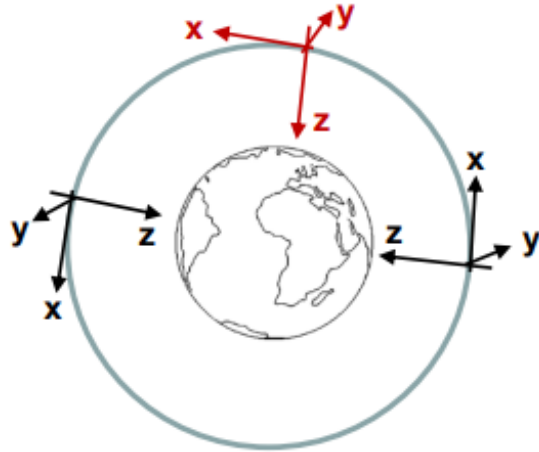
*Transfer function (in the frequency and time domain) of the cold atom interferometer*



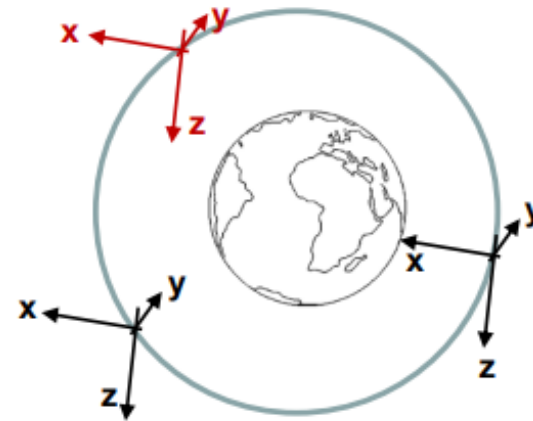
*Noise PSD parameters of the cold atom interferometer*

Interferometer time $T$	Cycle period $T_c$	Number of atoms	Distance $d$
5 s	1 s	$10^6$	0.5 m

# MOCASS: numerical simulations scenarios



***Nadir pointing mode***



***Inertial mode***

- ***High orbit (satellite altitude ~ 259 km)***
- ***Low orbit (satellite altitude ~ 239 km)***
- ***Simulated data:***

***Txx, Tyy, Tzz***

***>> Single-arm gradiometer***

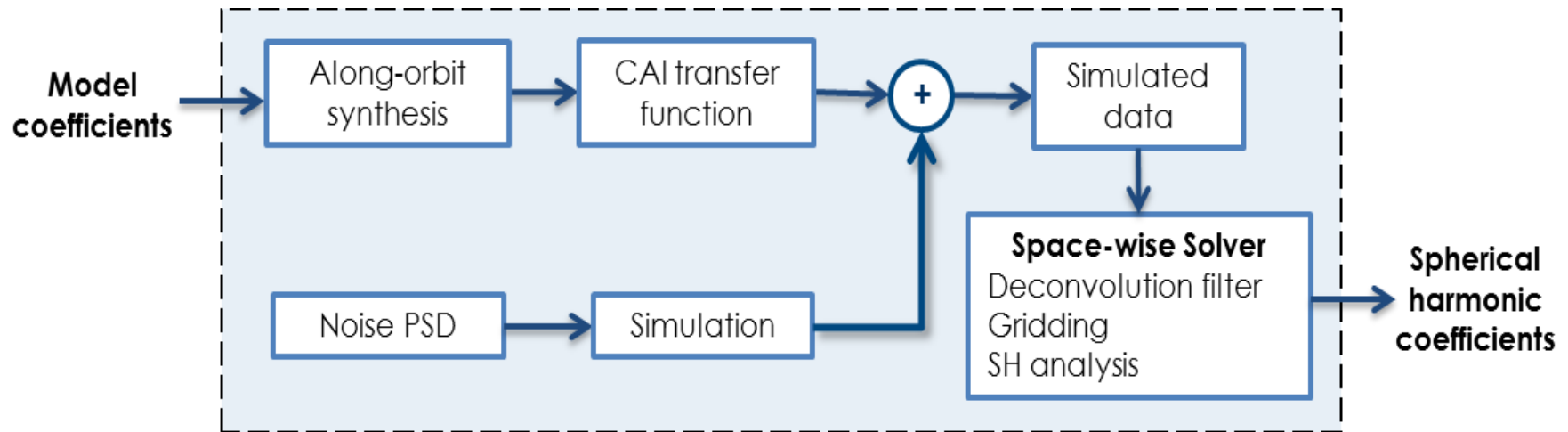
***Txx and Tzz, Tyy and Tzz***

***>> Double-arm gradiometer***



# Geodetic data analysis with the space-wise approach

- *For the geodetic data analysis, numerical simulations were performed at POLIMI applying the space-wise approach for different mission scenarios.*
- *Space-wise simulation and data analysis strategy:*



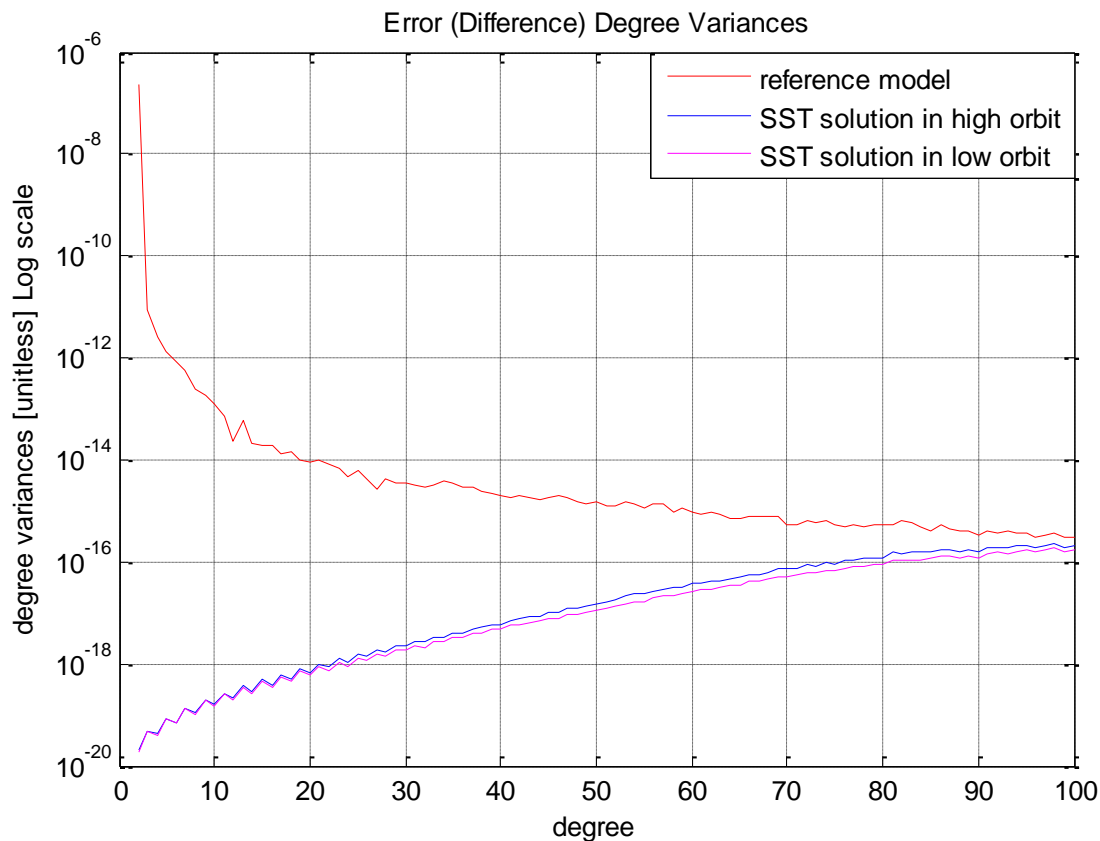
- *Error estimates based on Monte-Carlo simulations.*



# MOCASS: SST solution

***The role of the SST solution (from the on-board GNSS receiver data) is to numerically stabilize the gridding solution, reducing the residual signal amplitude and correlation.***

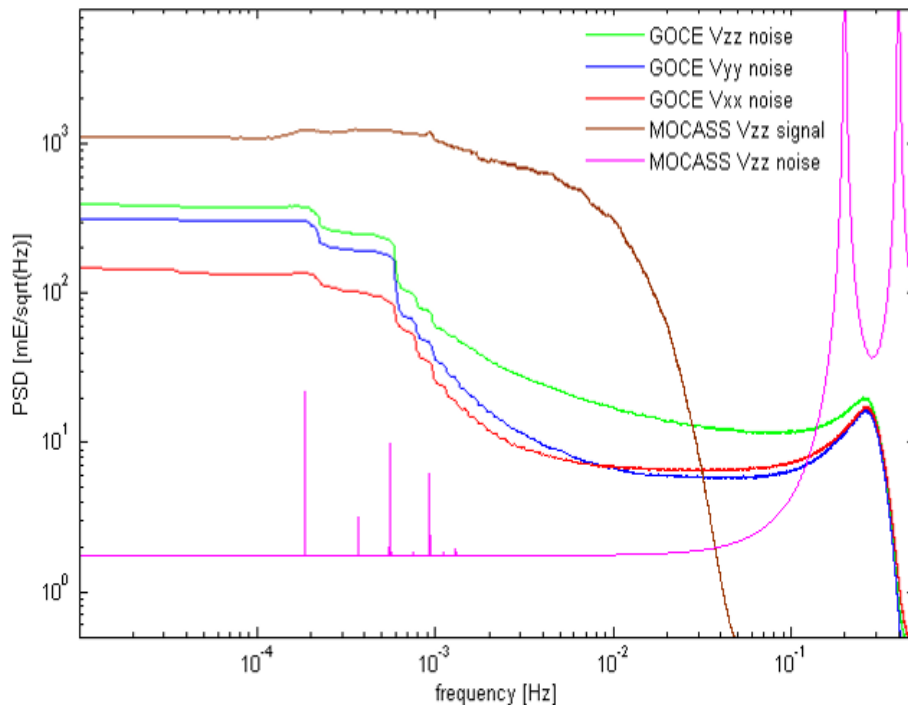
***In the case of GOCE, it also plays a role in the low degree final estimation, which is not the case for MOCASS.***



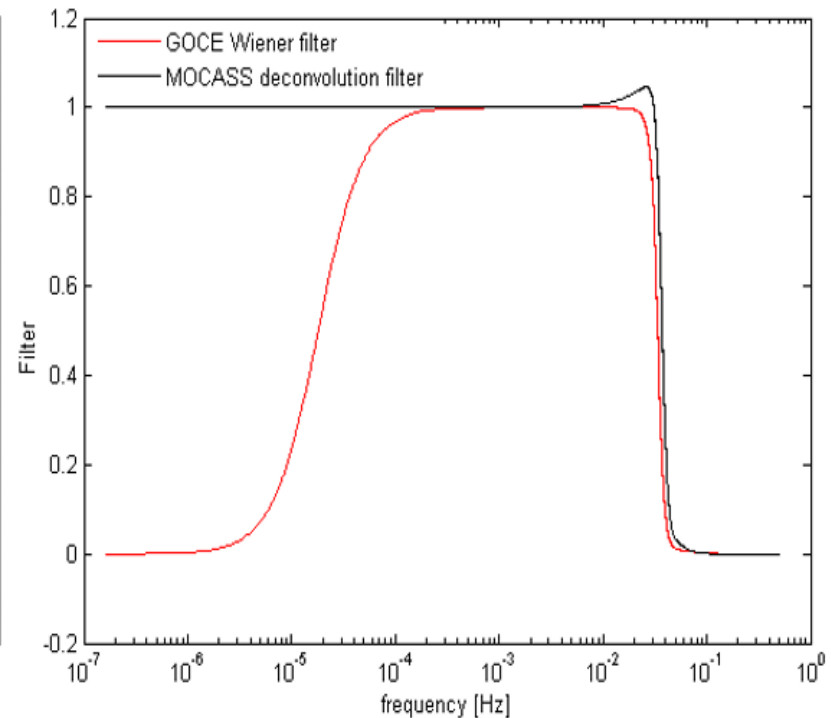
# MOCASS: Wiener deconvolution filter

*Wiener deconvolution filter*

$$W(f) = \frac{H(f) * S_y(f)}{H(f)^2 * S_y(f) + S_v(f)}$$



*Noise ASD of the MOCASS Cold Atom gradiometer compared with the corresponding ASD of the measurement error of the GOCE gradiometer*



*MOCASS Wiener deconvolution filter (black curve) vs. GOCE Wiener filter (red curve)*

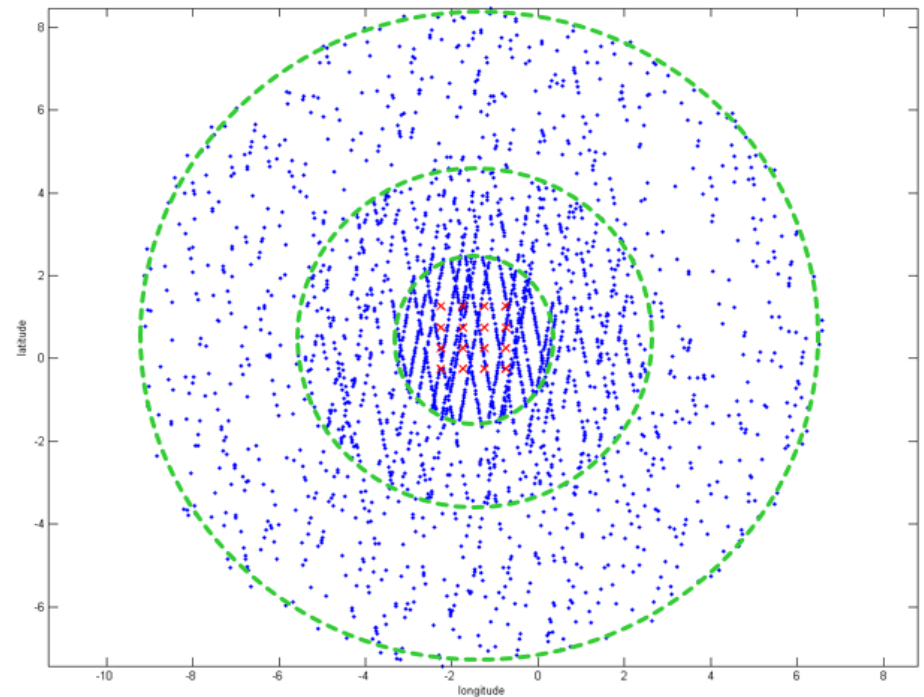
# MOCASS: local collocation gridding

*Local collocation gridding*

$$\hat{z}(\vartheta, \lambda, r)_{r=R} = C_{zy}(C_{yy} + C_{ee})^{-1}\hat{Y}_0(\vartheta, \lambda, r)$$

**$T_{xx}$ ,  $T_{yy}$  and  $T_{zz}$  observations were used to predict grids of  $T$ ,  $T_{rr}$  and  $T_{\lambda\lambda}$  values.**

***A local collocation procedure was applied to the filtered data, using a suitable method for data selection, in order to enhance local information and reducing or possibly avoiding undersampling (Reguzzoni et al., 2014).***

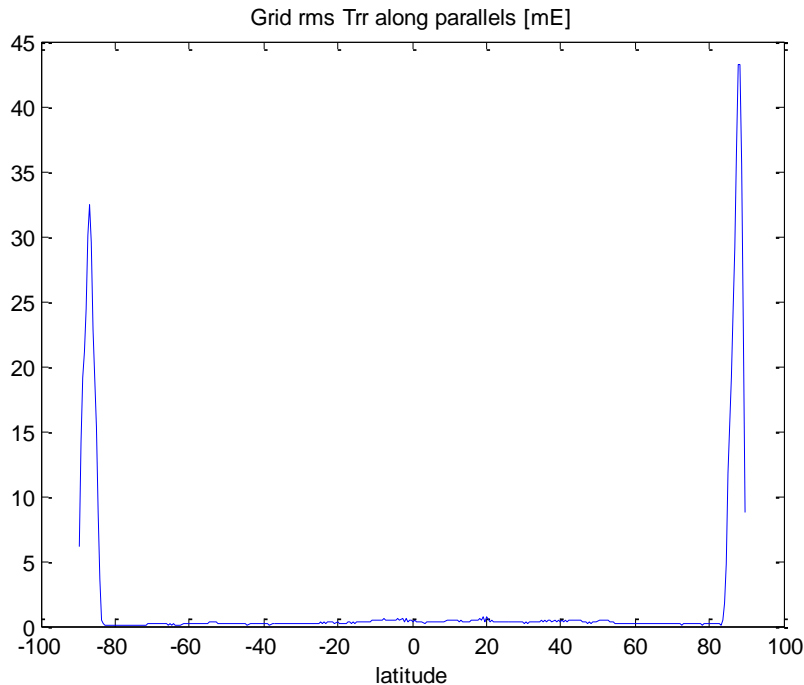


# Error r.m.s of predicted grids of $T_{rr}$ , $T_{\lambda\lambda}$ and $T$ values

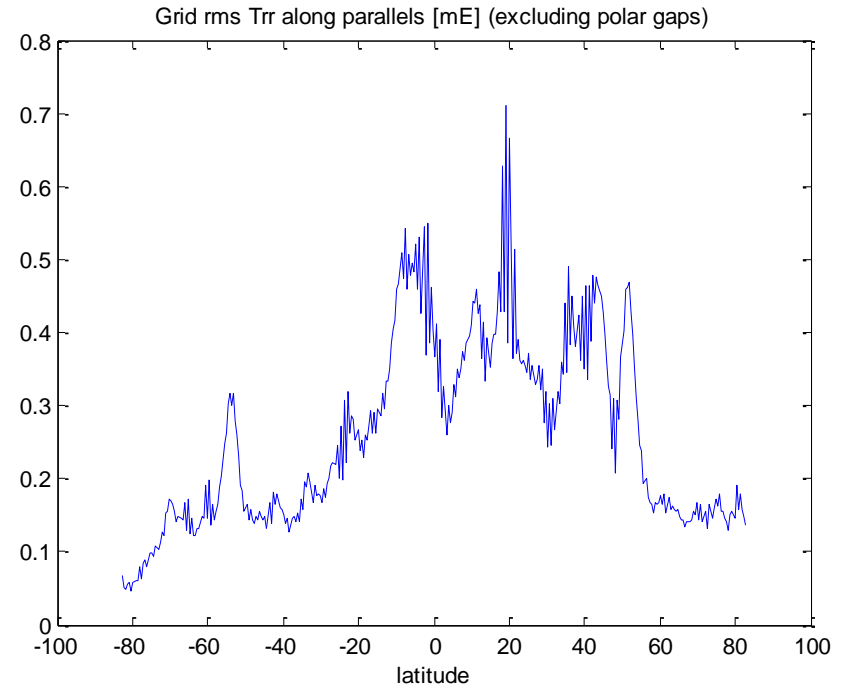
Simulated observation data	Pointing mode	Orbit	Error r.m.s. of predicted $T_{rr}$ grid values [mE]	Error r.m.s. of predicted $T_{\lambda\lambda}$ grid values [m <sup>2</sup> /s <sup>2</sup> ]	Error r.m.s. of predicted $T$ grid values [m <sup>2</sup> /s <sup>2</sup> ]
$T_{xx}$	Nadir	High	1.216	39.193	0.048
		Low	0.915	30.203	0.047
	Inertial	High	4.521	9.834	0.067
		Low	1.499	5.816	0.057
$T_{yy}$	Nadir	High	1.765	7.928	0.051
		Low	1.290	5.202	0.049
	Inertial	High	<b>0.453</b>	<b>6.834</b>	<b>0.041</b>
		Low	<b>0.328</b>	<b>5.283</b>	<b>0.042</b>
$T_{zz}$	Nadir	High	<b>0.358</b>	<b>6.069</b>	<b>0.042</b>
		Low	<b>0.2174</b>	<b>4.879</b>	<b>0.041</b>
	Inertial	High	2.906	21.396	0.070
		Low	2.770	25.717	0.067

Simulated observation data	Pointing mode	Orbit	Error r.m.s. of predicted $T_{rr}$ grid values [mE]	Error r.m.s. of predicted $T_{\lambda\lambda}$ grid values [m <sup>2</sup> /s <sup>2</sup> ]	Error r.m.s. of predicted $T$ grid values [m <sup>2</sup> /s <sup>2</sup> ]
$T_{xx}$ and $T_{zz}$	Nadir	High	0.525	22.742	0.064
		Low	0.307	4.203	0.035
	Inertial	High	0.761	12.920	0.114
		Low	0.403	5.194	0.038
$T_{yy}$ and $T_{zz}$	Nadir	High	<b>0.552</b>	<b>12.102</b>	<b>0.041</b>
		Low	<b>0.338</b>	<b>3.771</b>	<b>0.035</b>
	Inertial	High	<b>0.546</b>	<b>8.249</b>	<b>0.055</b>
		Low	<b>0.341</b>	<b>4.731</b>	<b>0.038</b>

# Grid r.m.s for $T_{rr}$ along parallels



*With polar gaps*



*Excluding polar gaps*

# MOCASS: spherical harmonic analysis

$$\hat{T}_{lm} = \frac{1}{4\pi a} \int_{\Sigma} \hat{z}(\vartheta, \lambda) Y_{lm}(\vartheta, \lambda) d\sigma \approx \frac{1}{4\pi a} \sum_i \sum_j \hat{z}(\vartheta_i, \lambda_j) Y_{lm}(\vartheta_i, \lambda_j) \Delta\sigma_{ij}$$

- ***Spherical harmonic coefficients were computed by numerical integration (Migliaccio et al., 2004), and then combined according to MC solution errors to obtain a final estimate of the global geopotential model.***
- ***The MOCASS results were compared with GOCE and GRACE solutions based on the same amount of data (two-month solutions).***

# MOCASS: results of spherical harmonic analysis

*In the next slides:*

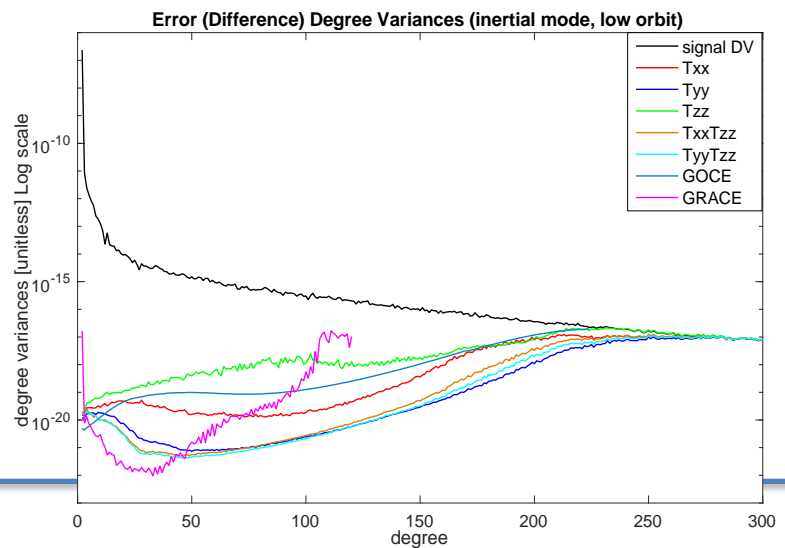
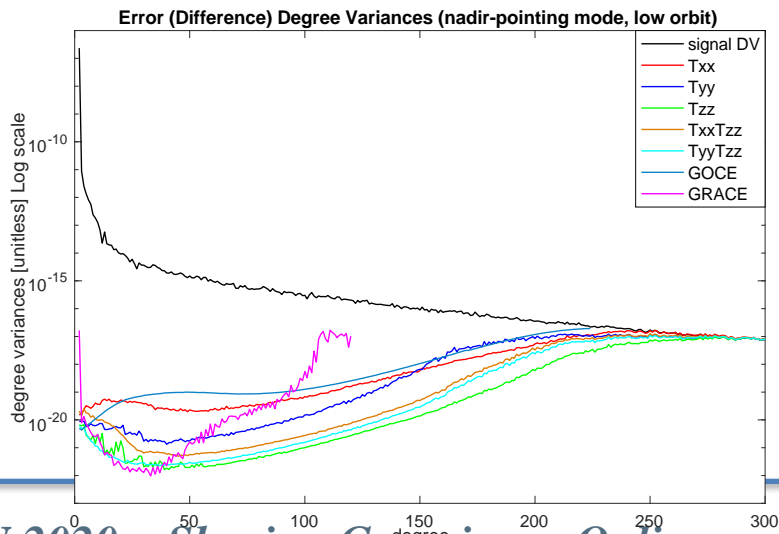
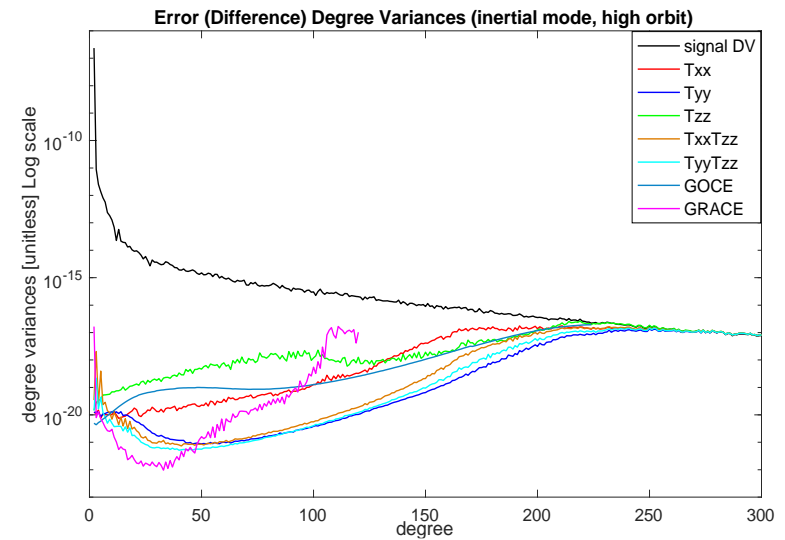
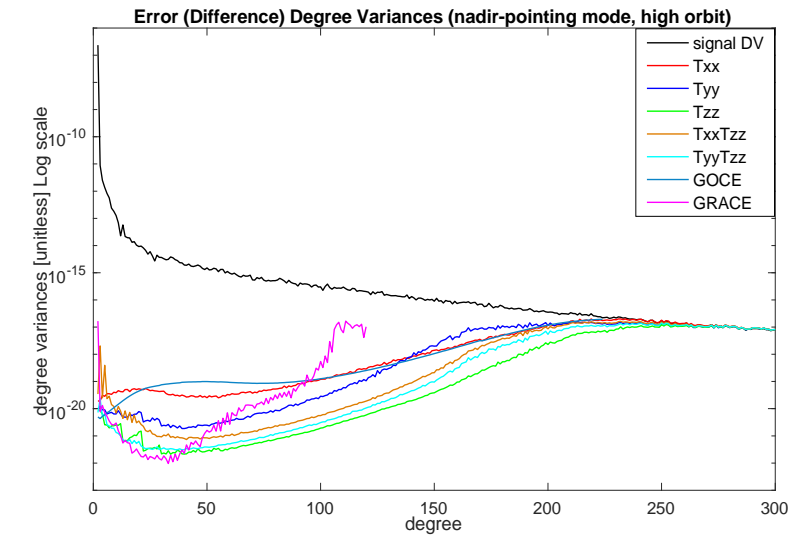
- *Error degree variances for reconstruction of a global model (2-month data).*
- *Cumulative error for  $\Delta g$  at ground level (2-month data).*
- *Cumulative error for  $T_{rr}$  at satellite altitude (2-month data).*
- *Cumulative error for  $\Delta g$  at ground level and  $T_{rr}$  at satellite altitude, for 1-year, 2-year and 5-year mission.*
- *Cumulative errors of  $\Delta g$  and  $T_{rr}$ : linear trend of monthly solutions.*

*Curves are plotted for the cases      nadir / inertial pointing configuration;  
high / low orbit;  
one-arm / double-arm gradiometer.*

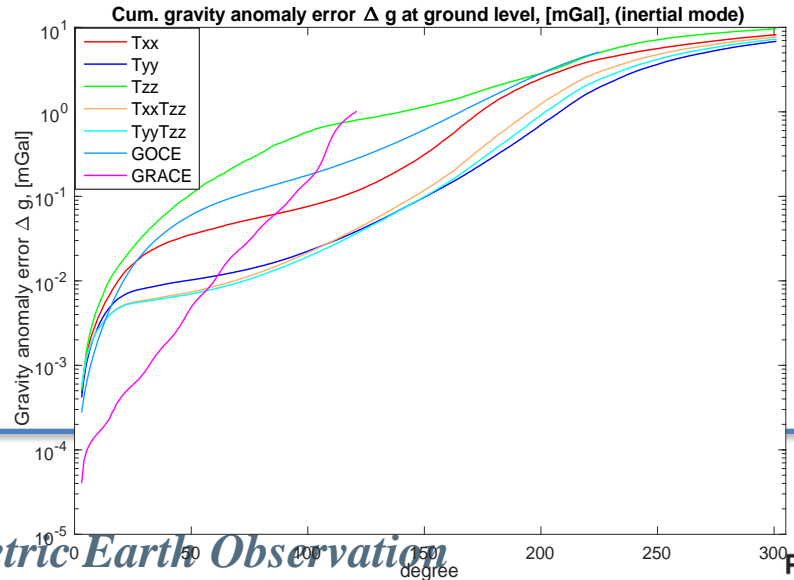
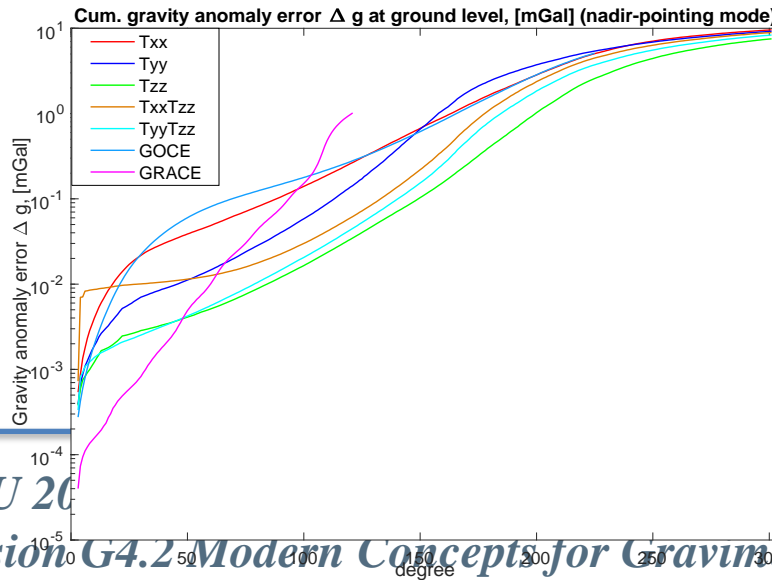
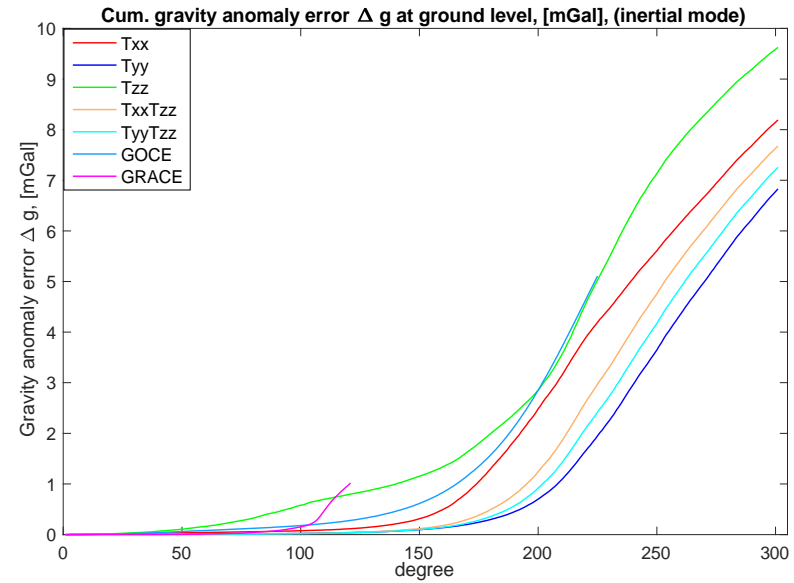
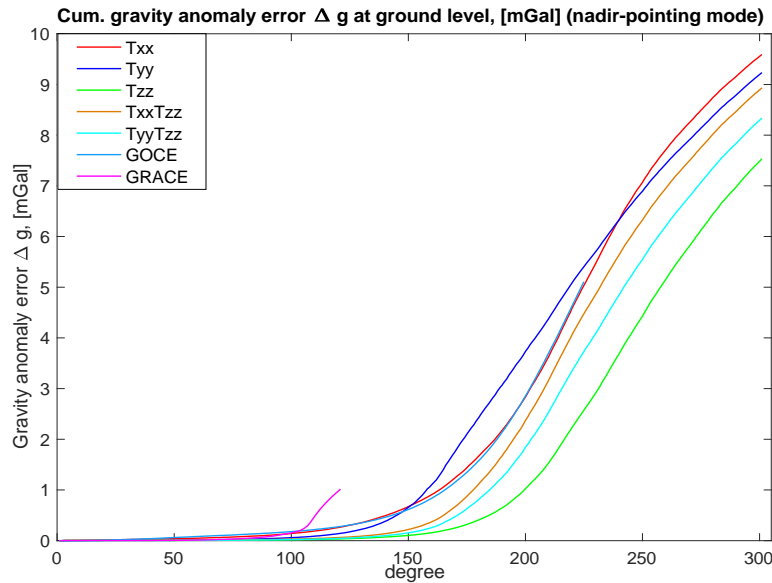
*A comparison with corresponding curves for GRACE and GOCE is shown.  
For all curves low-order coefficients affected by polar gap degradation are disregarded.*



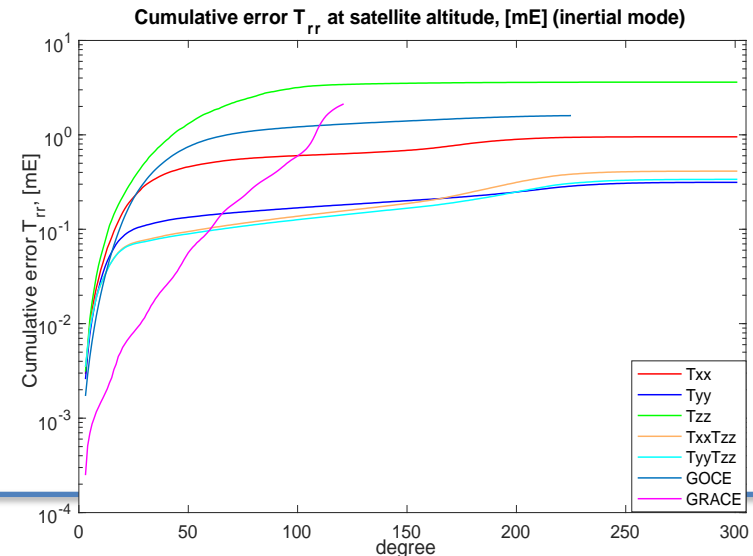
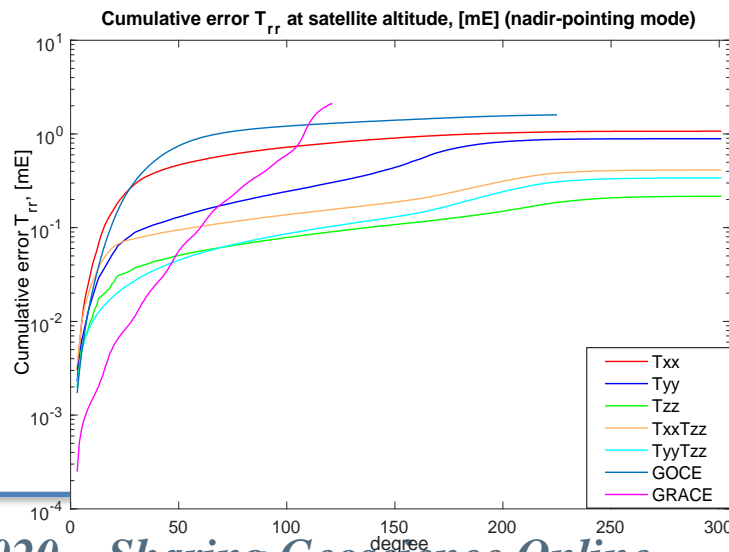
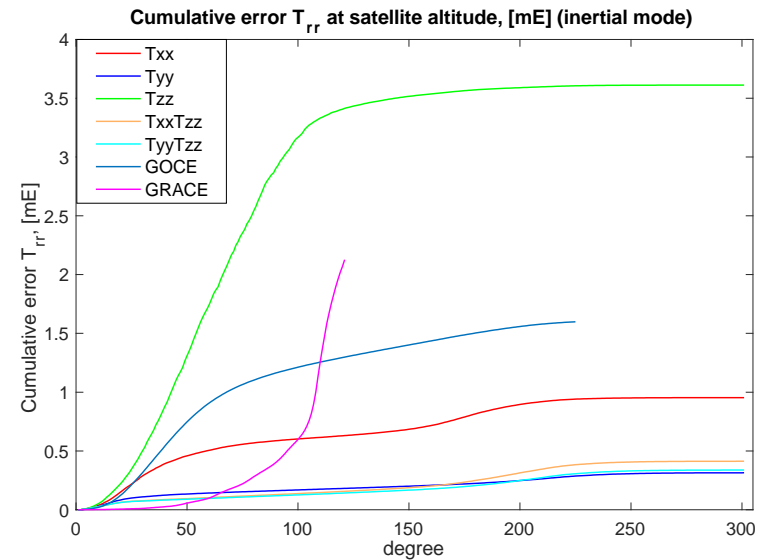
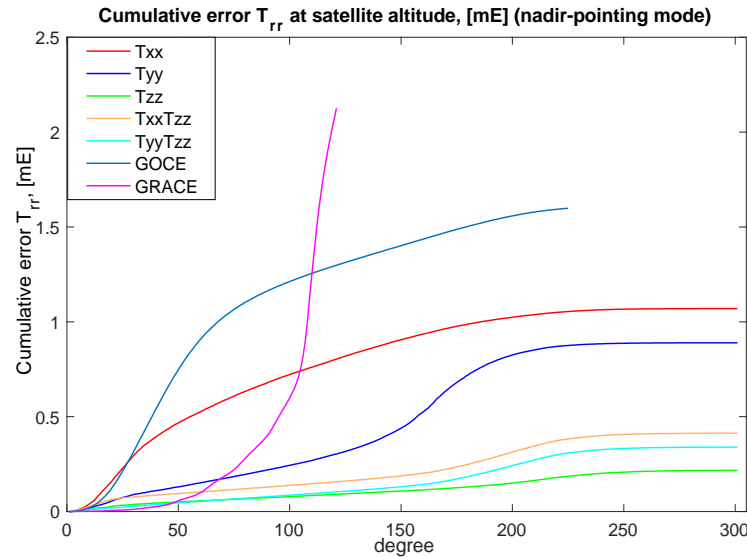
# Estimated global gravity model: error degree variances



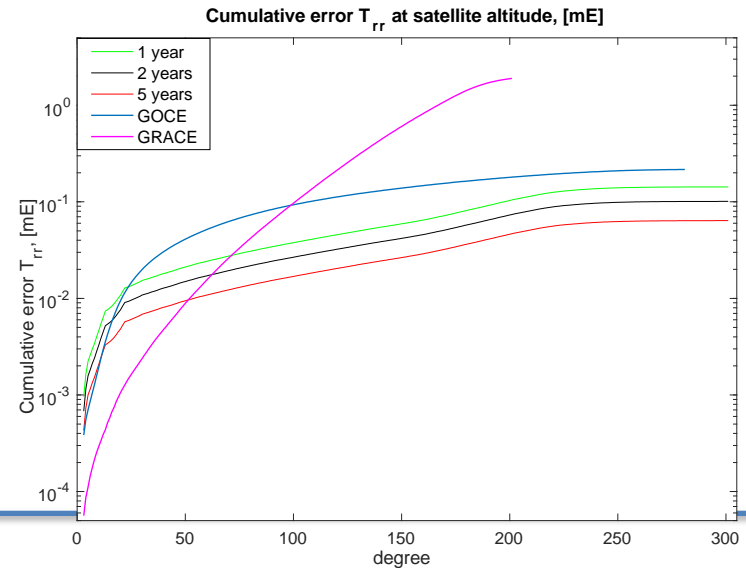
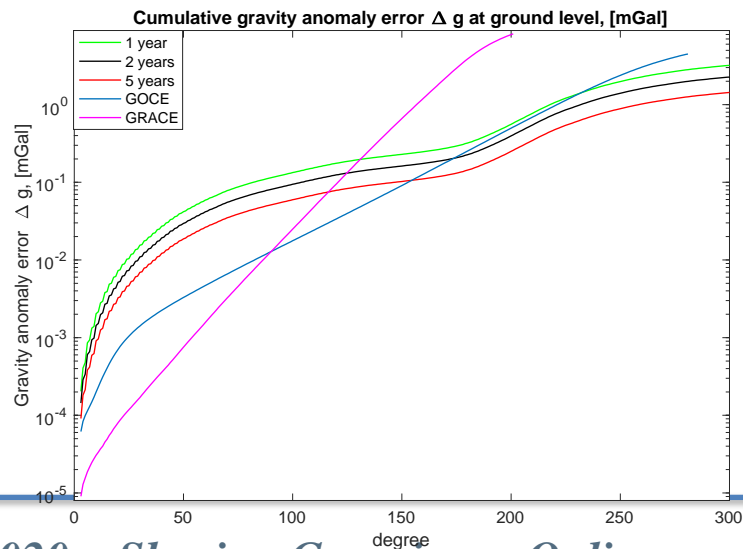
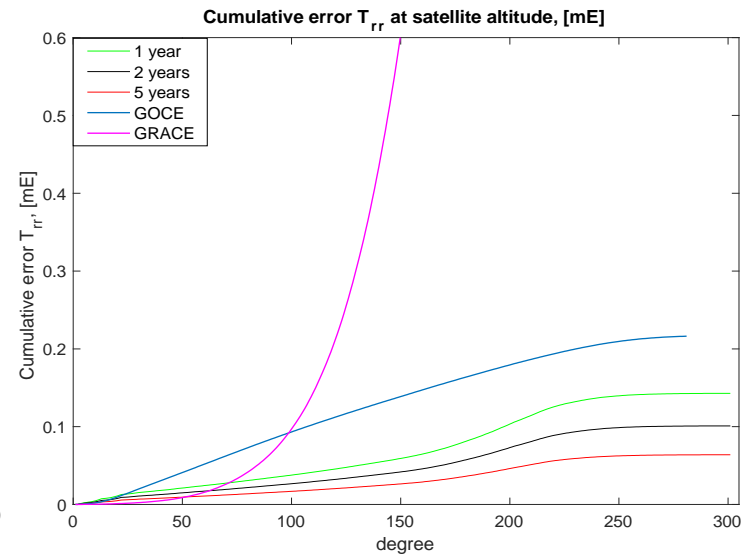
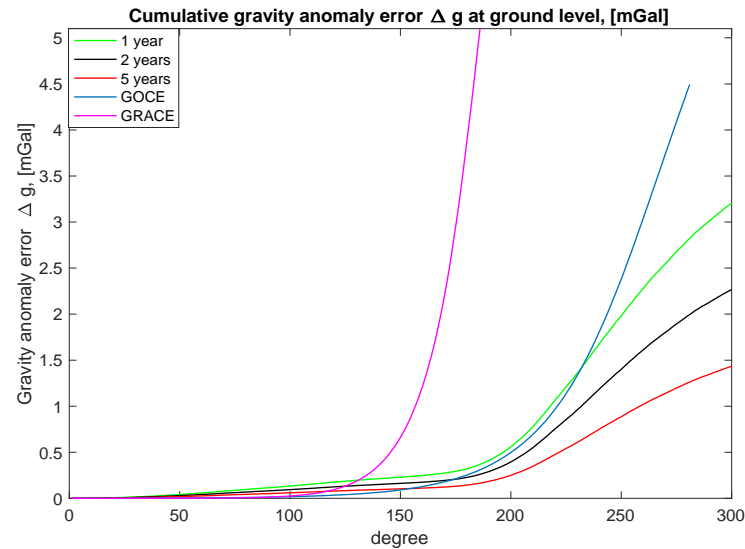
# Cumulative error for $\Delta g$ at ground level



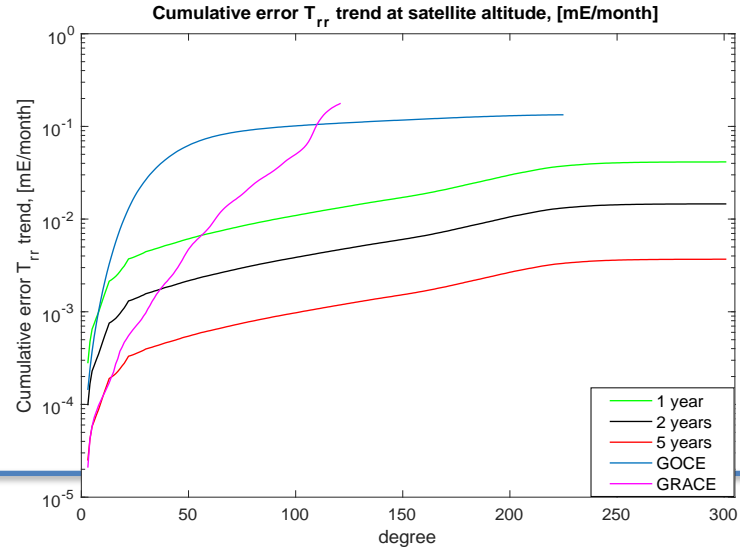
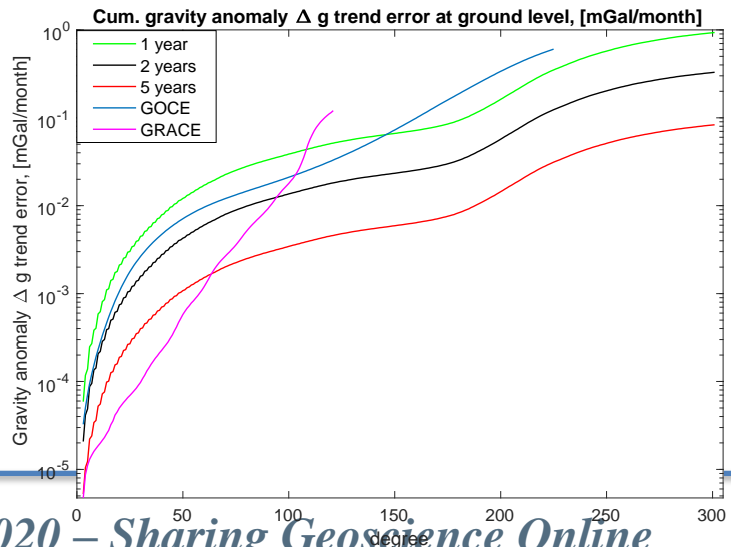
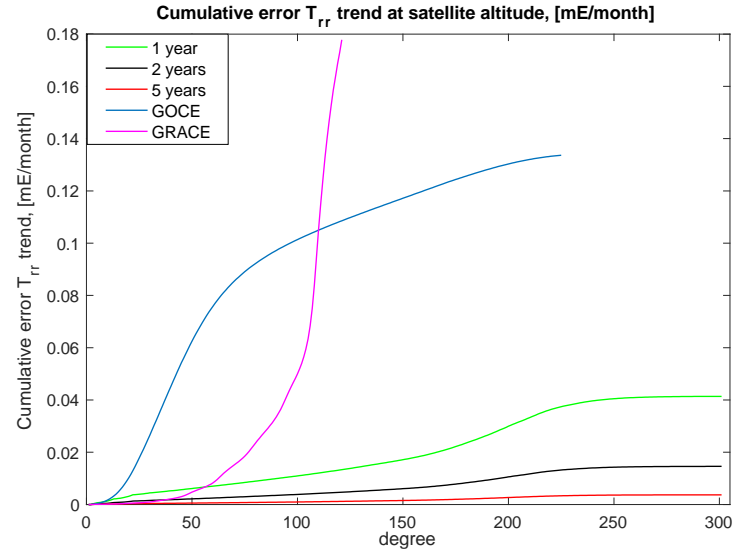
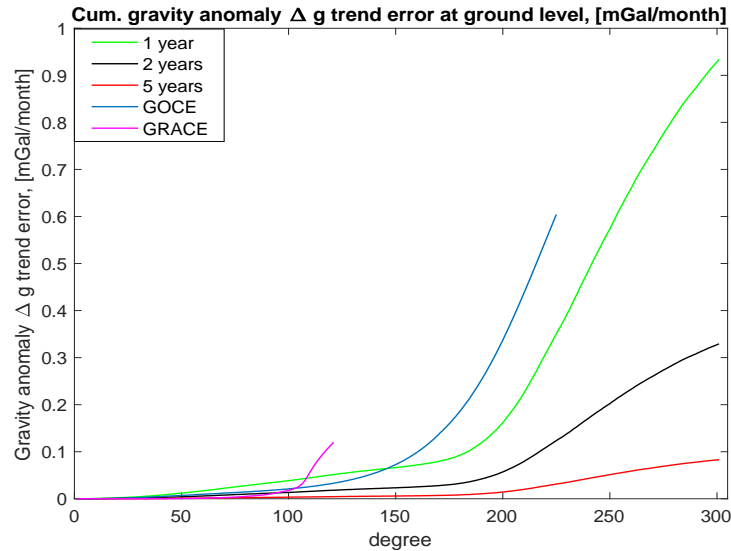
# Cumulative error for $T_{rr}$ at satellite altitude (2-month data)



# Cumulative error for 1-year, 2-year and 5-year mission, for $\Delta g$ at ground level (left) and $T_{rr}$ at satellite altitude (right)



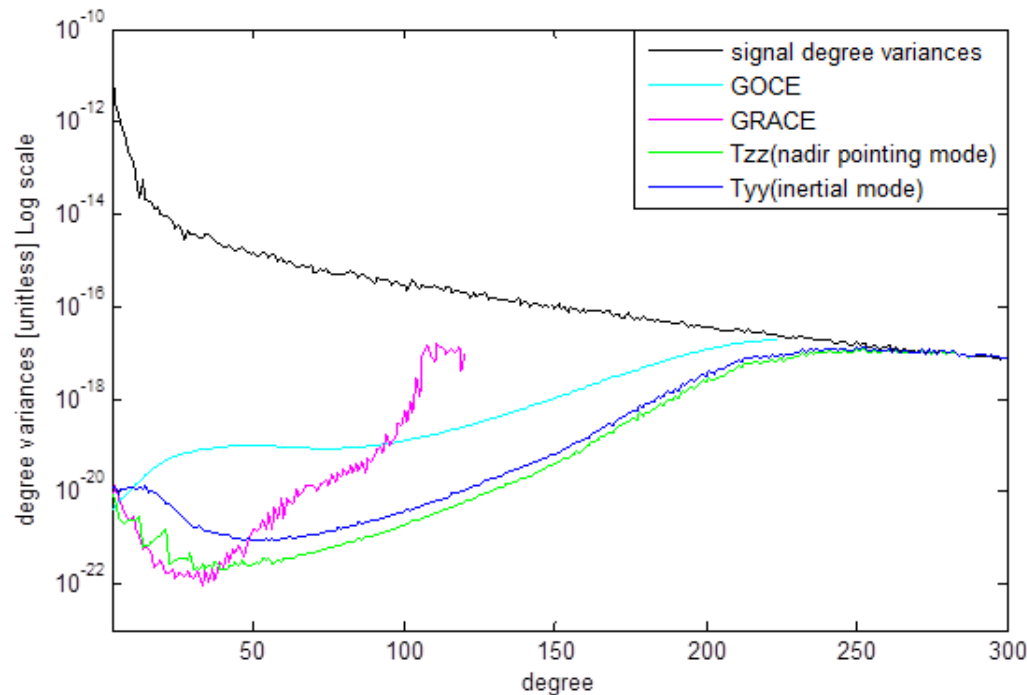
# Cumulative errors of $\Delta g$ (left) and $T_{rr}$ (right): linear trend of monthly solutions



# MOCASS: results of spherical harmonic analysis

**Summarizing, the best solutions are obtained:**

- **with data along the zz direction, for the nadir-pointing configuration;**
- **with data along the yy direction, for the inertial-pointing configuration.**



**Note: in the case of double-arm gradiometer there is no improvement because the atom cloud is not duplicated but it is split into two directions.**

# Results

- *Regarding the recovery of the static gravity field, the MOCASS error estimates can improve the performances of GOCE over all the harmonic spectrum, especially at high degrees, with a commission error of about of 1.4 mGal at degree 300 (0.9 mGal at degree 250) for a 5-year mission.*
- *For the time-variable gravity field, the MOCASS error estimates are promising (the 2-month solution is better than the GRACE one already above degree 40, while in the case of GOCE this occurs above degree 90). However, the accuracy at very low degrees seems to be still insufficient for this type of applications.*



# The MOCAST+ study

- *A new study called MOCAST+ has been started in February 2020 proposing an enhanced cold atom interferometer which can deliver gravity gradients and time measurements.*
- *The study will investigate whether this could give the possibility of improving the estimation of gravity models even at low harmonic degrees, with inherent advantages in the modeling of mass transport and its global variations: this would represent fundamental information, e.g. in the study of variations in the hydrological cycle and relative mass exchange between atmosphere, oceans, cryosphere and solid Earth.*