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Numerical sensitivity analysis of a rock glacier flow model versus detection of an internal sliding occurrence

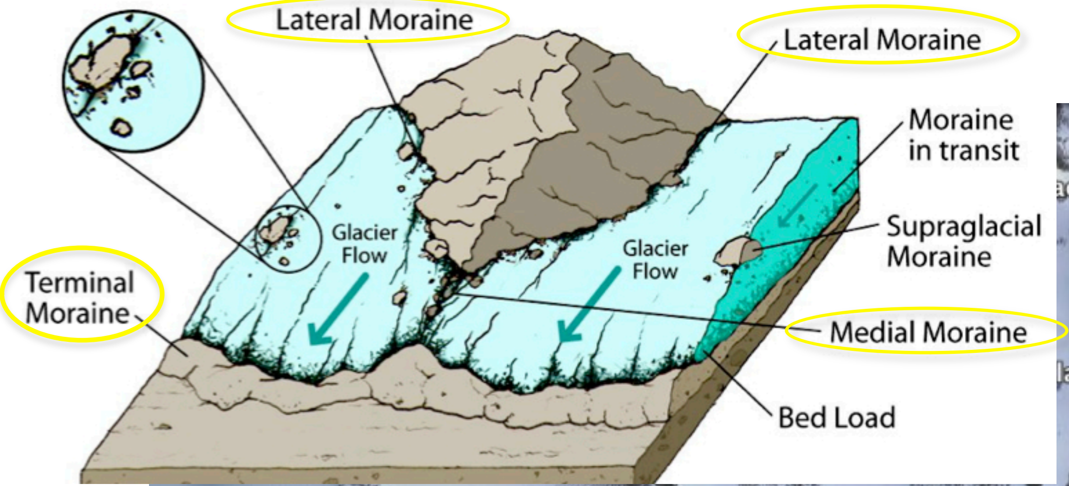
D. Mansutti (1), K. Kannan (2), and K. R. Rajagopal (3)

(1) Ist. per le Applicaz. del Calcolo, CNR, Roma (It)
(2) Dept. Mech. Engrng., Indian Institute of Technology, Madras (In)
(3) Dept. Mech. Engrng., University of Texas A & M, College Station (USA)

OUTLINE

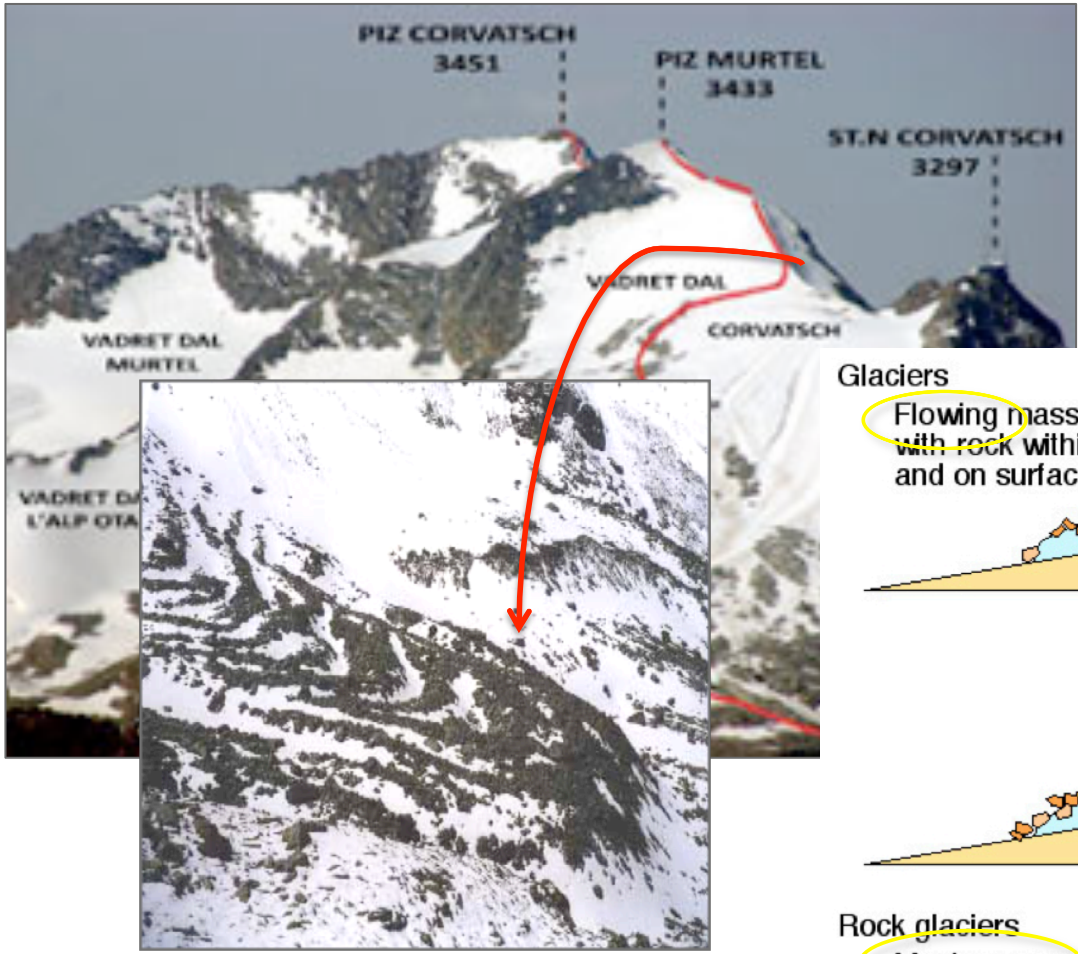
- **mathematical model of a morainic/ rock glacier flow** (slides# 3 – 14)
- **test case: Murtel-Corvatsch alpine rock glacier** (slides# 15 – 16)
- **numerical simulation results** (slides# 17 – 20)
- **sensitivity analysis of the model parameters vs. shear zone detection** (slides# 21 – 29)
- **conclusions** (slides# 30, 31)

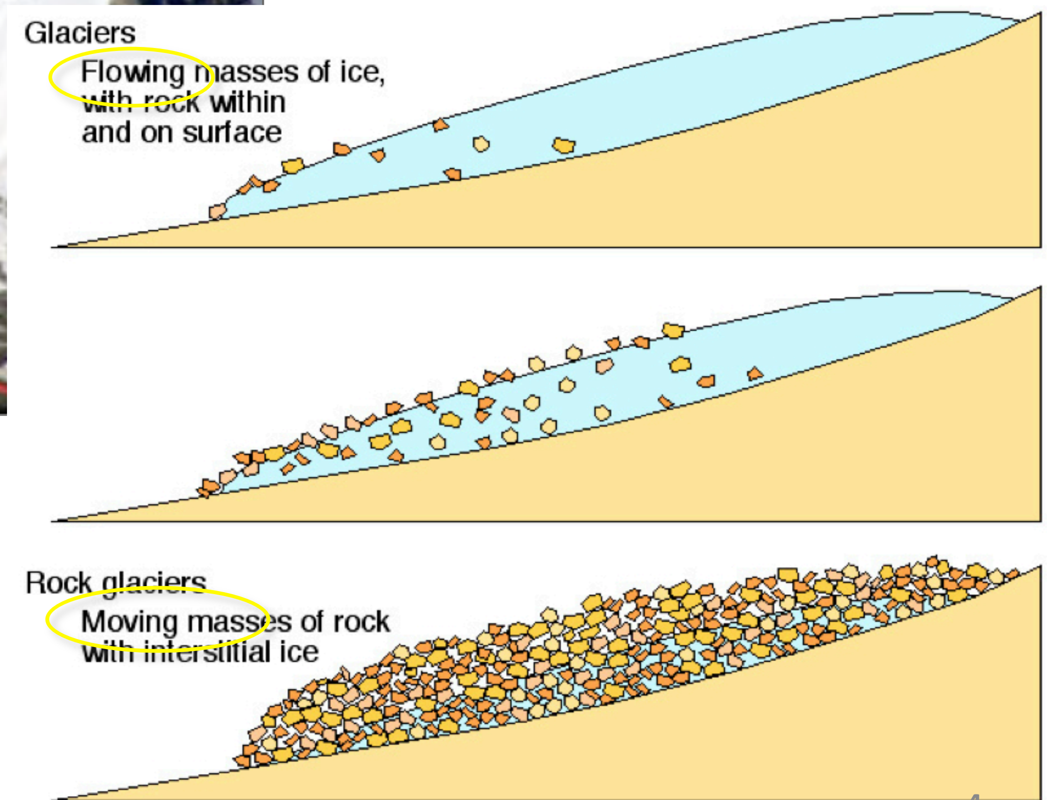
Morainic glaciers and …



project *ENIGMA*
(in progress)

…rock glaciers are multiphase continuum bodies





Governing equations
for multi-phase time dependent
non-isothermal flow problems

from conservation laws (momentum, mass and energy):

$$\rho \frac{d\vec{v}}{dt} = \nabla \cdot \tilde{t}^T + \rho \vec{g}$$

with ρ, density, $\vec{v}$, velocity, $\vec{g}$, gravity

$$\frac{d\rho}{d\mathrm{t}} = -\rho \, \nabla \cdot \vec{v}$$

$\tilde{t}$, Cauchy stress

U, specific internal energy, $\vec{q}$, heat flux

$$\rho \frac{dU}{dt} = -\nabla \cdot \vec{q} + tr\left(\tilde{t} \cdot \left(\nabla \vec{v}\right)^T\right) + \rho r$$

r, specific rate of supplied radiant heating

holding **in each phase** with appropriate
constitutive equations, jump conditions at the interfaces
and **initial** and **boundary conditions**

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let's focus on

$$\rho \frac{dU}{dt} = -\nabla \cdot \vec{q} + tr\left(\tilde{t} \cdot (\nabla \vec{v})^T\right) + \rho r$$

U, specific internal energy, $\vec{q}$, heat flux

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holding **in each phase** with appropriate **constitutive equations, jump conditions** at the interfaces and **initial** and **boundary conditions**

Step 1:

Governing equations for time dependent non-isothermal ice flow problems

$$\rho \frac{d\vec{v}}{dt} = \nabla \cdot \tilde{t}^T + \rho \vec{g}$$ with ρ, density, $\vec{v}$, velocity, $\vec{g}$, gravity

$$\frac{d\rho}{dt} = -\rho \nabla \cdot \vec{v}$$ $\tilde{t}$, Cauchy stress

U, specific internal energy, $\vec{q}$, heat flux

$$\rho \frac{dU}{dt} = -\nabla \cdot \vec{q} + tr\left(\tilde{t} \cdot (\nabla \vec{v})^T\right) + \rho r$$ r, specific rate of supplied radiant heating

commonly used in Glaciology, ***Glen's law***:

$$\tilde{t} = -p\tilde{I} + \mu_G \tilde{A}_1$$ with p, pressure, $\tilde{A}_1 = \nabla\vec{v} + (\nabla\vec{v})^T$

$$\mu_G = A\left\{tr\left(\tilde{A}_1^{\,2}\right)\right\}^{1-n/2n}$$ with $n = 1,\ 2,\ 3$ or 4

and $A = A(T, p, \ldots)$ (*v*s. environmental cond's)

a power law newtonian fluid model:

normal stress differences are not supported

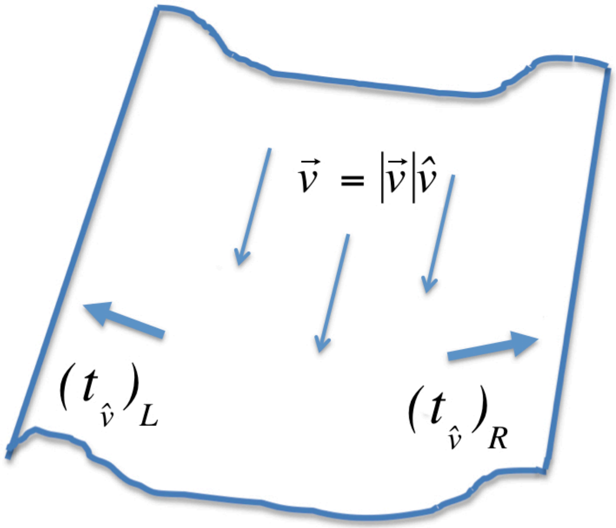
Step 2:

$$\tilde{t} = -p\tilde{I} + \mu_G \tilde{A}_1 + \alpha_1 \tilde{A}_2 + \alpha_2 \tilde{A}_1^2$$

MSOFM, Man & Sun 1987

with p, μ_G and $\tilde{A}_1$ (as for *Glen's law*), and $\tilde{A}_2 = \dfrac{d\tilde{A}_1}{dt} + (\nabla\vec{v})^T \cdot \tilde{A}_1 + \tilde{A}_1 \cdot \nabla\vec{v}$

normal stress differences are supported



with $(t_{\hat{v}})_L \neq (t_{\hat{v}})_R$

free surface depression in channel flow
recovered

in **ice flow**, enhanced by
- *thickness of glacier*,
- *steep slope* and
- *low temperature*

ref:
Man, C.-S., Sun, Q.-S., *On the significance of normal stress effects in the flow of glaciers*, J. Glaciol., 33, 115, pp. 268—273, 1987

Step 3:

Moore 2014: constitutive behaviour of ice-debris mixture (as in rock glaciers) is a "***competition*** *between the* ***role of debris in impeding ice creeping***[1] *and the* ***mitigating effect of unfrozen water***[2] *at debris-ice interface*" (by observation)

[1]as in **'locking' in granular materials**

[2]if a shear zone establishes at a rock glacier bottom, **viscosity can get seven times smaller than for clean ice**: devastating fast shear can occur

ref:

Moore P.L., *Deformation of debris-ice mixtures*, Reviews of Geophysics, 52, pp 435-467, 2014

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for evolving Morainic/Rock Glaciers

$$\mu_{KR}(p,\bar{\phi}) = \mu_G\left[(1-f)\left(1+k_1\sqrt{\frac{p-P_a}{P_a}}\right)+fk_2\frac{p-P_a}{P_a}\right] \quad \text{with f} = \frac{\bar{\phi}}{\bar{\phi}+\mathrm{e}(1-\bar{\phi})}$$

$$\alpha_{KR,i}(\bar{\phi}) = \alpha_i\left[1+k_{i+2}\frac{\bar{\phi}^2}{(1-\bar{\phi})^2}\right] \quad \text{with } \alpha_i \text{ and } k_{i+2} \text{ constant, for } i = 1,2.$$

Kannan & Rajagopal 2013: with pressure and debris volume fraction intrinsic mechanical effects (**multi-phase media treated as a mixture**)

ref:

Kannan K., Rajagopal K.R., *A model for the flow of rock glaciers*, Int. J. Non-lin. Mech., 48, pp. 59—64, 2013

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[2]if a shear zone establishes at a rock glacier bottom, **viscosity can get seven times smaller than for clean ice**: <u>devastating fast shear can occur</u>

for evolving Morainic/Rock Glaciers (inspired to Mills & Snabre, 2009)

$$\mu_{KR}(p,\bar{\phi}) = \mu_G\left[(1-f)\left(1+k_1\sqrt{\frac{p-P_a}{P_a}}\right)+fk_2\frac{p-P_a}{P_a}\right] \quad \text{with } f = \frac{\bar{\phi}}{\bar{\phi}+\mathrm{e}(1-\bar{\phi})}$$

as a **dense suspension** of a **newtonian fluid** and **hard spherical particles**

$\bar{\phi} = \dfrac{\phi}{\phi_{max}}$, *relative* volume fraction of rock and sand grain *trapped* in the ice interstices

e, *extent* of the sliding trend of the *free – to – move* rock particles

f, *equilibrium* solid fraction; $(1 - f)$, *free - to - move* rock particles fraction

Moore 2014: constitutive behaviour of ice-debris mixture (as in rock glaciers) is a "*competition between the role of debris in impeding ice creeping*[1] *and the mitigating effect of unfrozen water*[2] *at debris-ice interface*" (by observation)

[1]as in **'locking' in granular materials**

[2]if a shear zone establishes at a rock glacier bottom, **viscosity can get seven times smaller than for clean ice**: devastating fast shear can occur

for evolving Morainic/Rock Glaciers

$$\mu(p,\bar{\phi}) = \mu_0\left[(1-f)\left(1+k_1\sqrt{\frac{p-P_a}{P_a}}\right)+fk_2\frac{p-P_a}{P_a}\right] \quad \text{with } f = \frac{\bar{\phi}}{\bar{\phi}+e(1-\bar{\phi})}$$

$$\alpha_{KR,i}(\bar{\phi}) = \alpha_i\left[1+k_{i+2}\frac{\bar{\phi}^2}{(1-\bar{\phi})^2}\right] \quad \text{with } \alpha_i \text{ and } k_{i+2} \text{ constant, for } i = 1,2.$$

as a **non-colloidal suspension** of a **non-newtonian fluid** and **hard spherical particles** (inspired to Morris & Boulay, 1999)

$$\bar{\phi} = \frac{\phi}{\phi_{max}}, \qquad \textit{relative} \text{ volume fraction of rock and sand grains } \textit{trapped} \text{ in the ice interstices}$$

Step 4:

Kannan, M. & Rajagopal 2019

with (direct) temperature effects

$$\tilde{t} = -p\tilde{I} + \boldsymbol{\mu}_{KMR}\tilde{A}_1 + \boldsymbol{\alpha}_{KMR,1}\tilde{A}_2 + \boldsymbol{\alpha}_{KMR,2}\tilde{A}_1^{\,2} \qquad (\tilde{A}_1 \text{ and } \tilde{A}_2 \textit{ as above})$$

$$\text{with } \boldsymbol{\mu}_{KMR} = \boldsymbol{\mu}_{KR}(p,\bar{\boldsymbol{\phi}})\ \ exp\left(B\left(\frac{1}{T}-\frac{1}{T_0}\right)\right),$$

$$\boldsymbol{\alpha}_{KMR,i} = \boldsymbol{\alpha}_{KR,i}(\bar{\boldsymbol{\phi}})\ \ exp\left(B_i\left(\frac{1}{T}-\frac{1}{T_0}\right)\right)$$

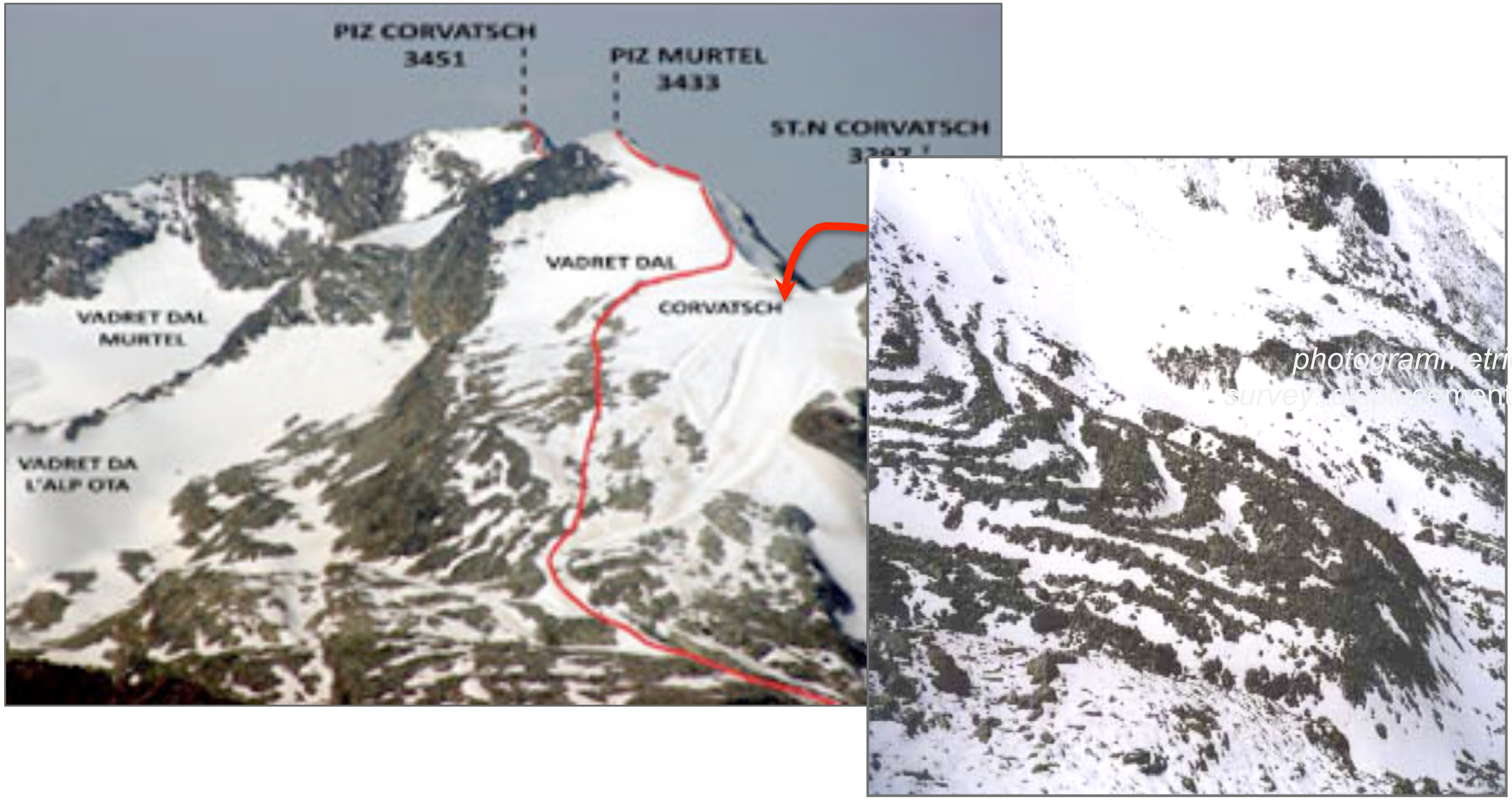
$$\text{for } \ i = 1,\ 2 \text{ and } T_0, \text{reference temperature}$$

<u>*Obs:*</u> Arrhenius type temperature behaviour stems from observation (Cuffey and Paterson 2010)

ref:

Kannan K., M. D., Rajagopal K.R., *Mathematical modeling of rock glacier flow with temperature effects*, in: Mathematical Modelling of Climate Change and its Impacts (Cannarsa P., D.M., Provenzale A., eds.), pp 149-162, Springer-INDAM Series vol.38, 2020

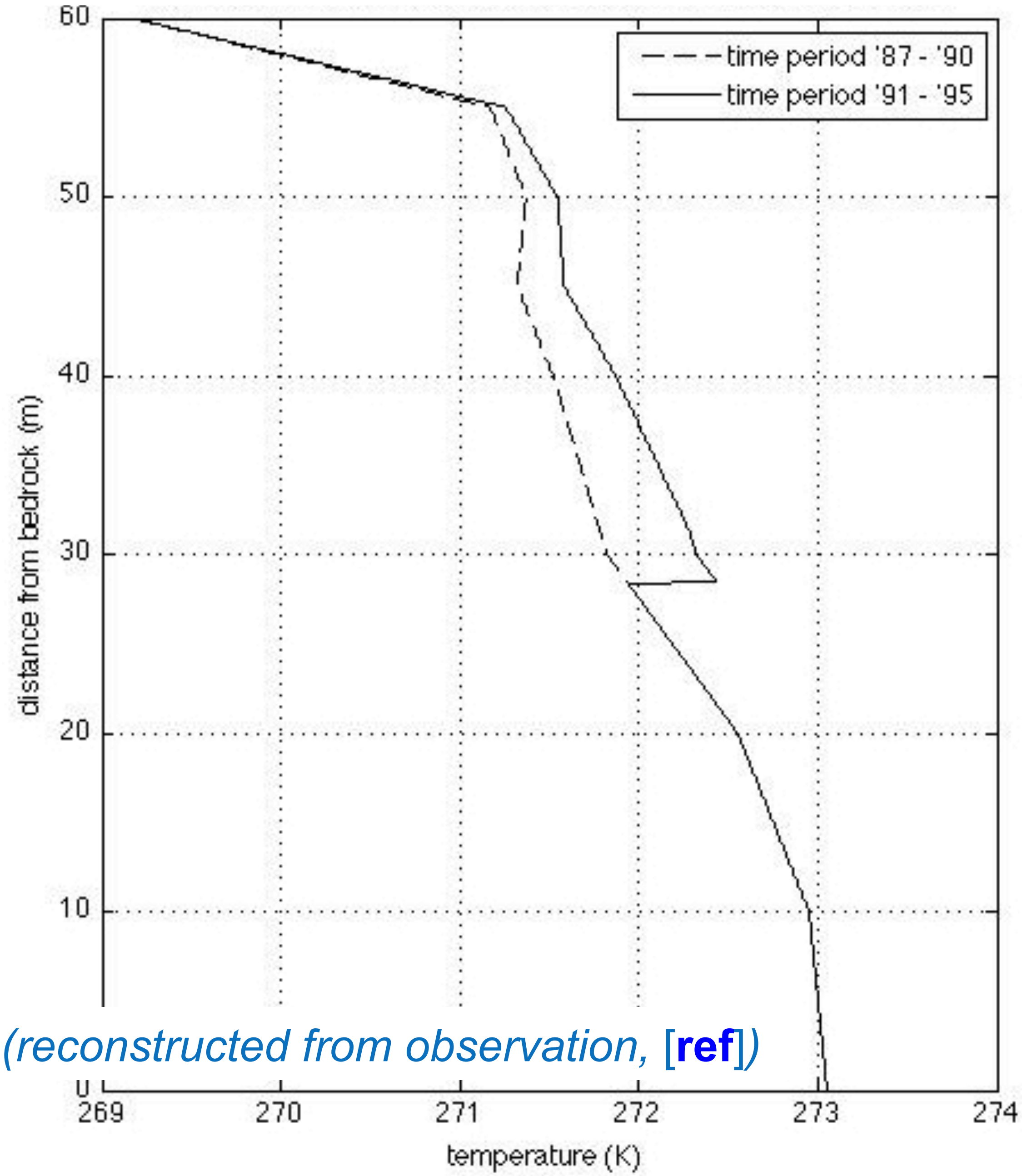
test case: Murtel-Corvatsch alpine rock glacier



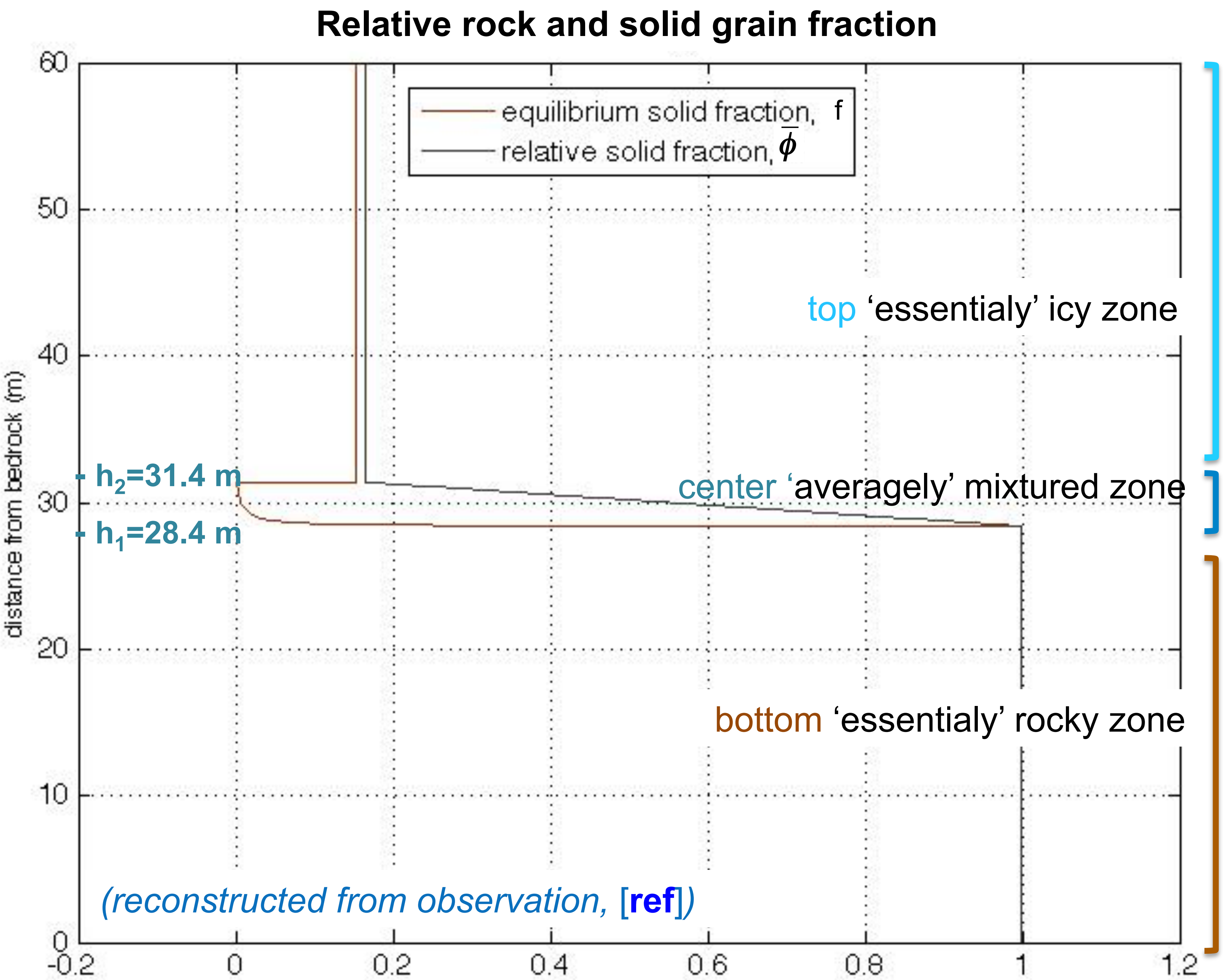
ref:

Arenson L., Hoeltzle M., Springman S., *Borehole deformation measurements and internal structure of some rock glaciers in Switzerland*, Permafrost and periglacial processes, 13, pp. 117-135, 2002

Internal temperature of rock glacier at borehole 2/1987



*(reconstructed from observation, [**ref**])*


Relative rock and solid grain fraction
equilibrium solid fraction, f
relative solid fraction, $\bar{\phi}$
distance from bedrock (m)
60
50
40
30
20
10
0
-0.2
0
0.2
0.4
0.6
0.8
1
1.2
h_2=31.4 m
h_1=28.4 m
top 'essentialy' icy zone
center 'averagely' mixtured zone
bottom 'essentialy' rocky zone
*(reconstructed from observation, [**ref**])*

selected value of numerical parameters:

A=1.5*10^8 (constant multiplying the functional factor of μ_{KMR},
scaled along with Cuffey&Paterson 2010)

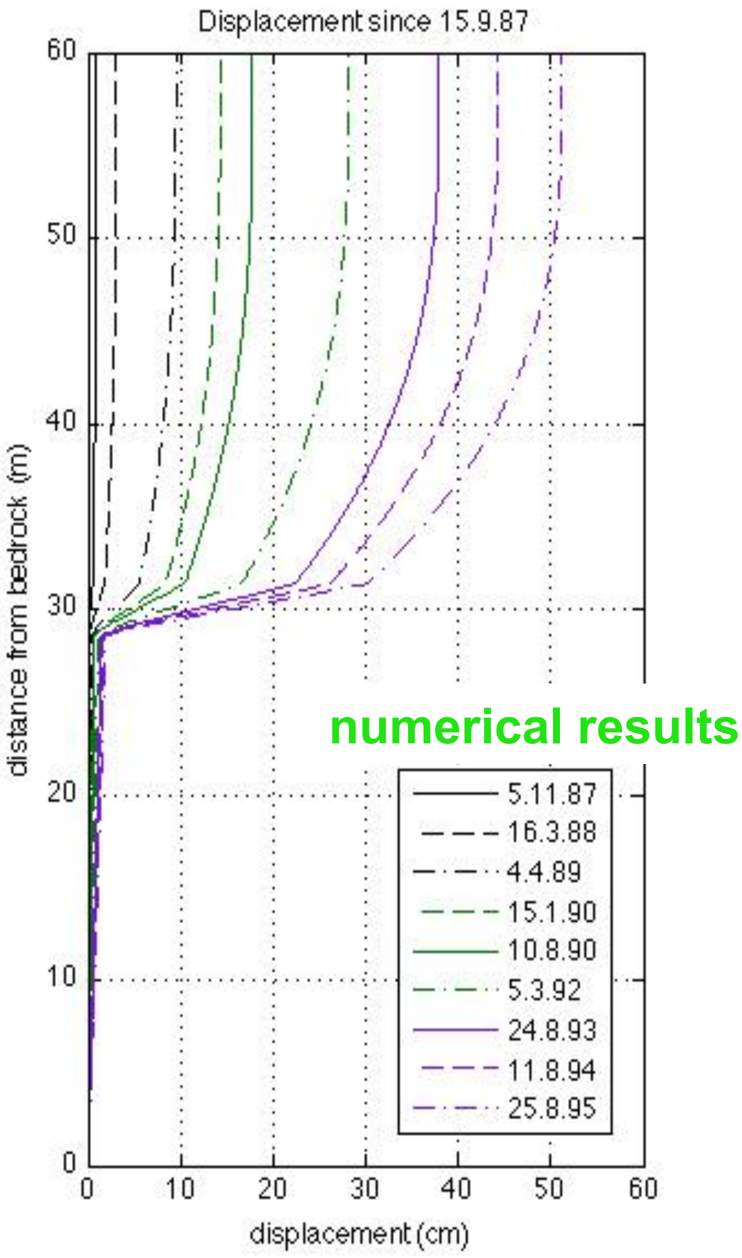
B = **B_1**= **B_2**= 4,610
(exponential constant from Cuffey&Paterson 2010)

T_0= 263 K (reference temperature from Cuffey&Paterson 2010)

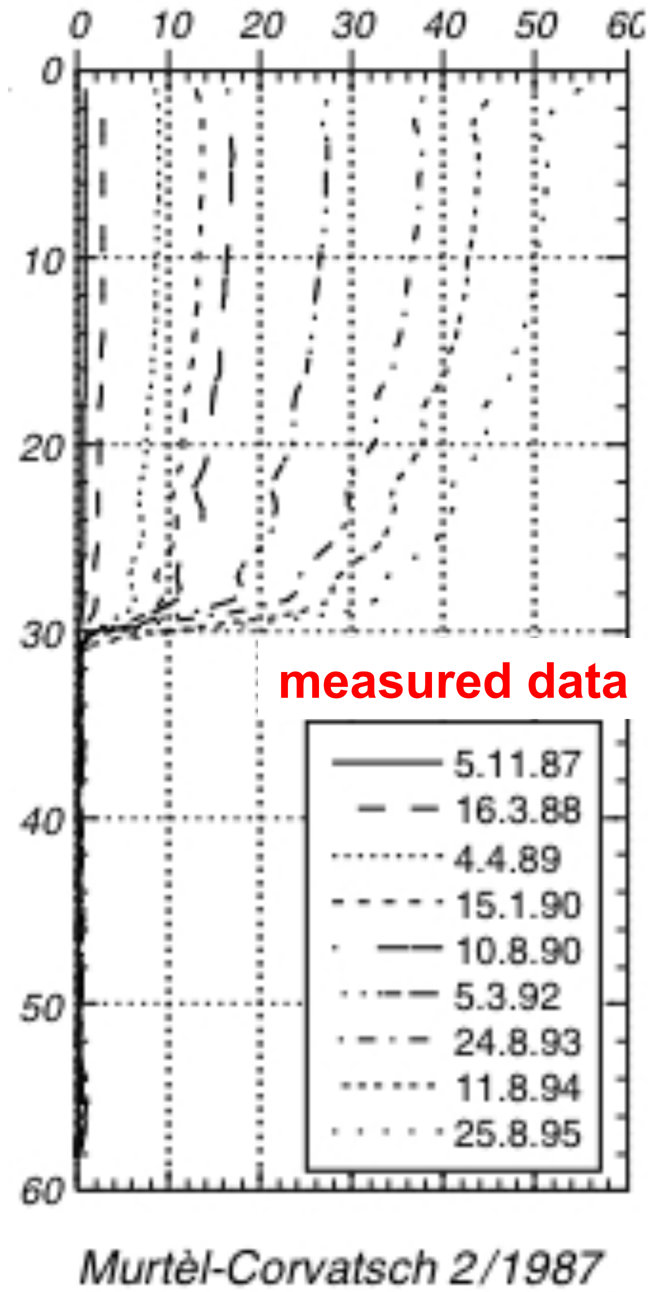
$\alpha_1 = -\alpha_2 = 10^{16}$ (constant multiplying the functional factor of $\alpha_{KMR,1}$)

k_1= 0.015 **k_2**=2. **k_3**=**k_4**=0.02 (constants in μ_{KMR} and $\alpha_{KMR,1/2}$)

e (extent of sliding) = **e_1**=175 at 'essentially' rocky zone (bottom)
e_{12}=175 at ice 'averagely' mixtured
with sand and rocky grains (center)
e_2=1.1 at 'essentially' ice zone (top)



via model KMR [ref]



from figure 9a [ref] *(permission granted)*

further comparison with borehole measured data:

Table 2 Shear velocities.

measured data from [ref]

Site		Depth of shear zone [m]	Data reading time [days]	Velocity in shear zone [cm/year]	Deformation in shear zone/tot. surface def.
Murtèl-Corvatsch	2/1987	28.4–31.4	2891	4.0	59%

numerical results

3.9283	59.23

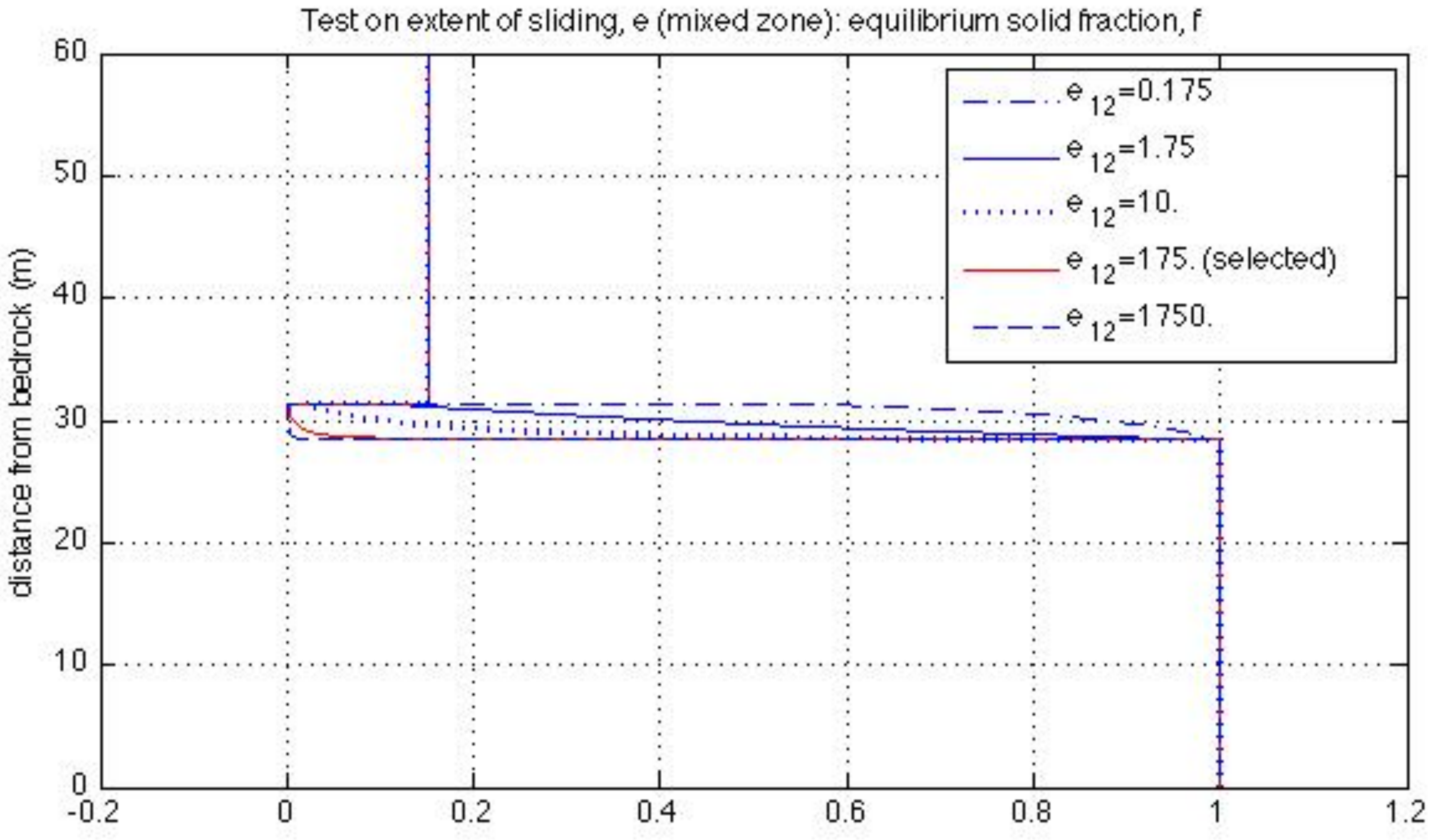
ε_{rel}=1.8% numerical relative error vs observed quantities

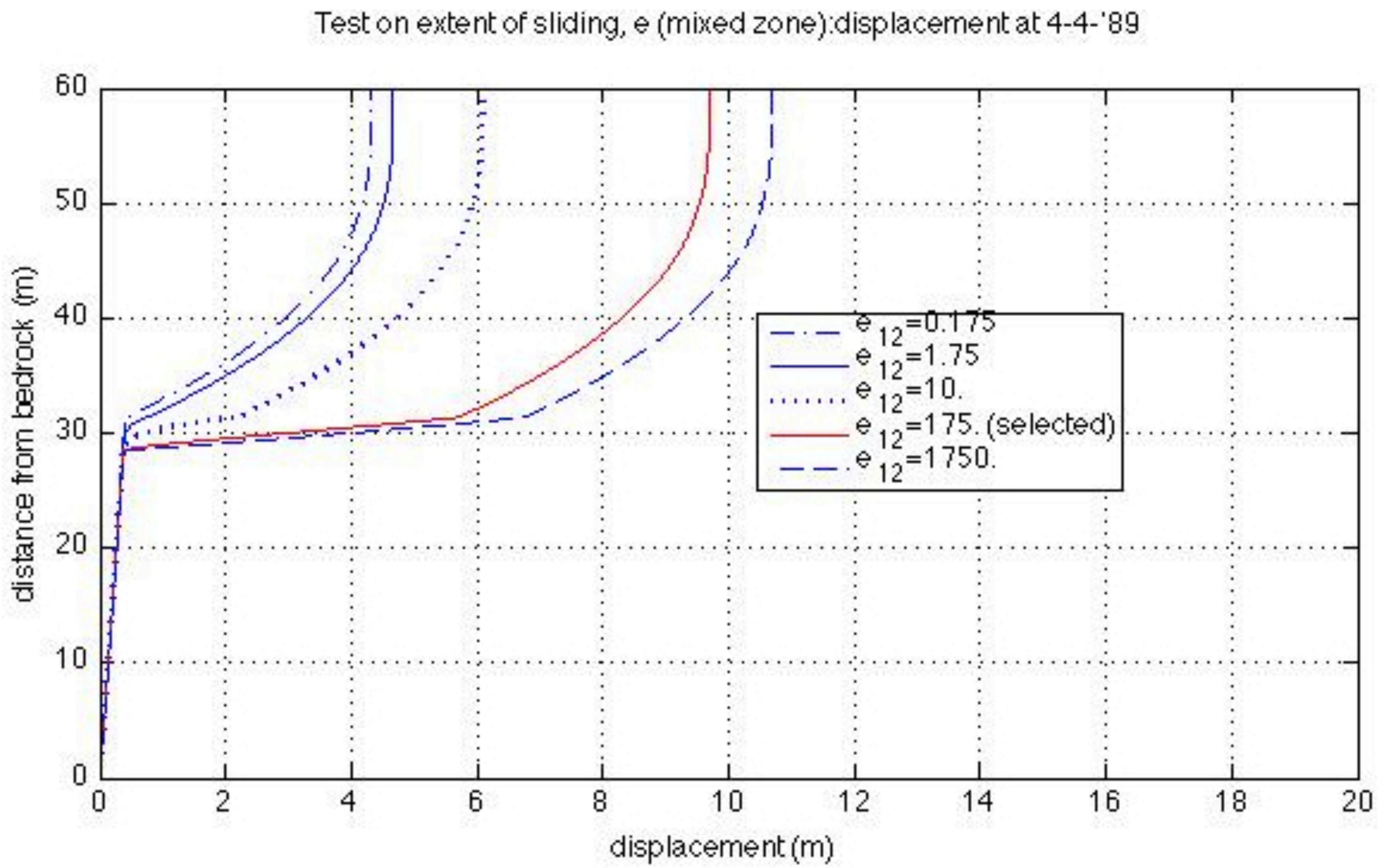
Sensitivity analysis of critical model parameters

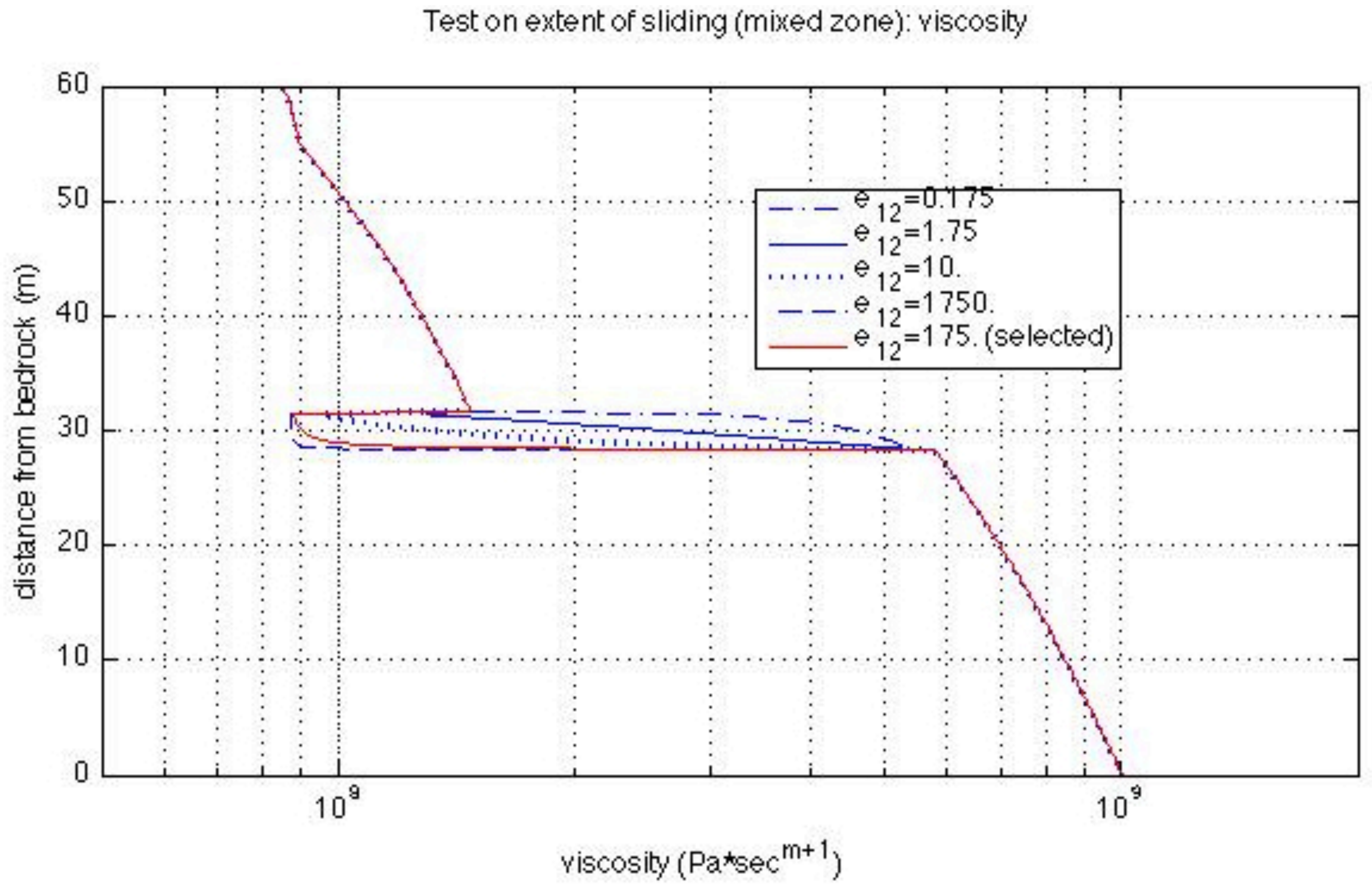
For the calibration of the values of parameters
e_1, e_{12}, e_2 and k_1, k_2,
thorough numerical testing has been conducted.

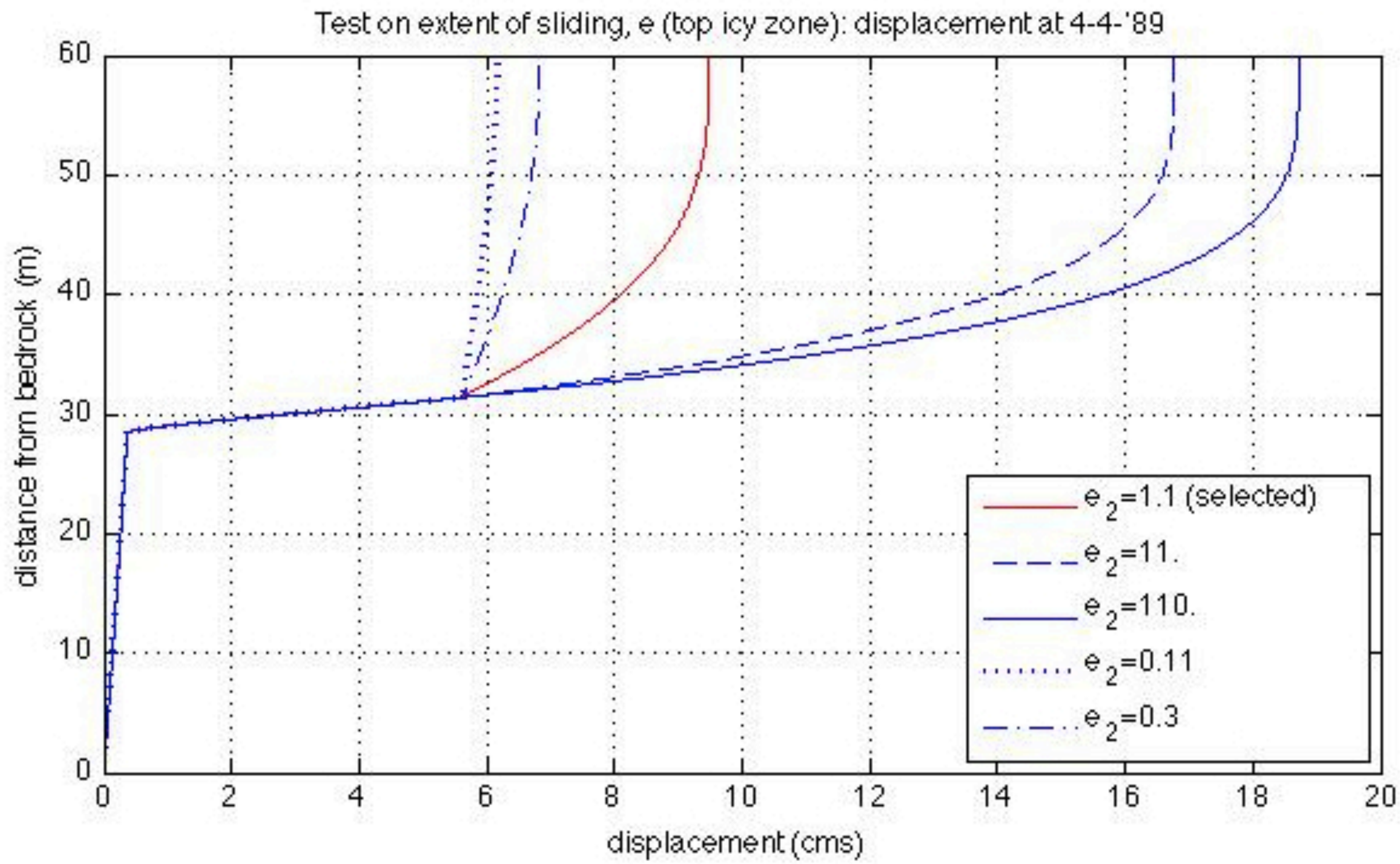
In the following slides,
the plot of the related most significant results are reported
in view to explain the role of each one
in characterizing an Internal Sliding Occurrence (**ISO**),
here studied in Murtel Corvatsch rock glacier flow.

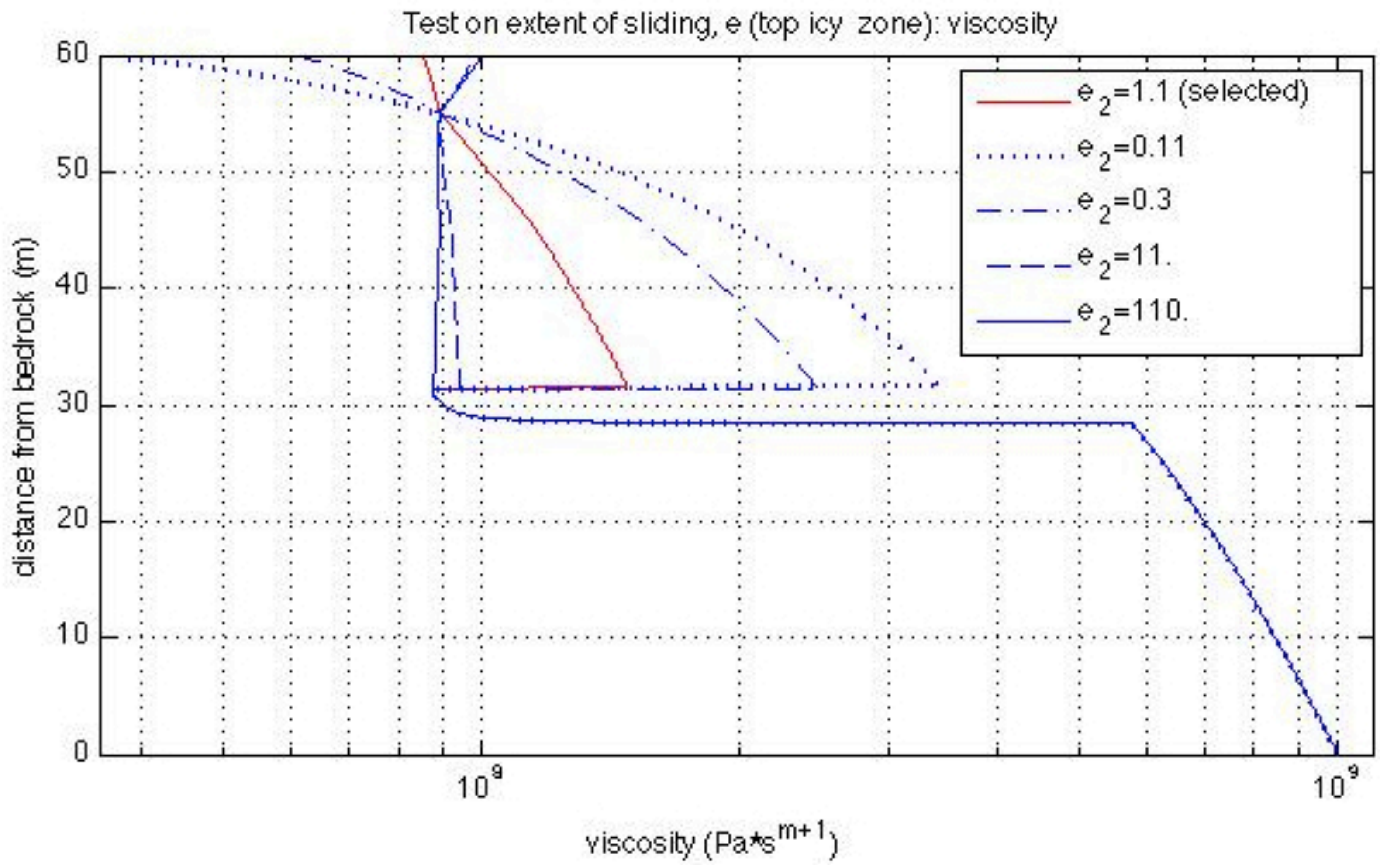
Numerical testing has revealed that,
at least in Murtel Corvatsch rock glacier conditions,
remaining parameters have negligible impact on **ISO**. So
they have been kept fixed to values either inspired
to Cuffey and Paterson (2010) (e.g. A, B, T_0), or
they have been taken from [ref] where temperature effects
are not included (e.g. α_1, α_2, k_3 and k_4).

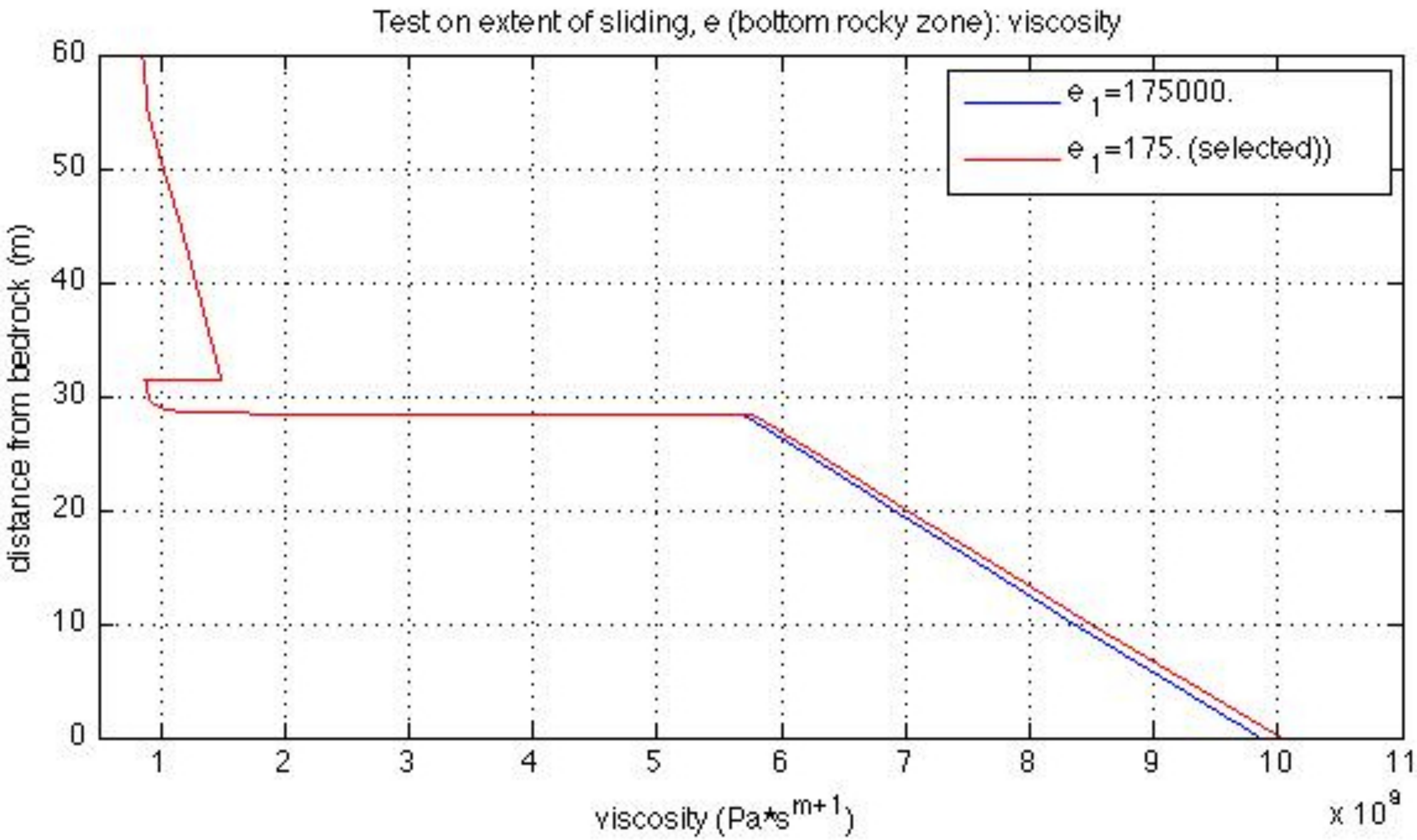

Test on extent of sliding, e (mixed zone): equilibrium solid fraction, f
distance from bedrock (m)
e 12=0.175
e 12=1.75
e 12=10.
e 12=175. (selected)
e 12=1750.
60
50
40
30
20
10
0
-0.2
0
0.2
0.4
0.6
0.8
1
1.2


Test on extent of sliding, e (mixed zone):displacement at 4-4-'89
60
50
40
30
20
10
0
distance from bedrock (m)
0
2
4
6
8
10
12
14
16
18
20
displacement (m)
e 12=0.175
e 12=1.75
e 12=10.
e 12=175. (selected)
e 12=1750.


Test on extent of sliding (mixed zone): viscosity
distance from bedrock (m)
60
50
40
30
20
10
0
10^9
10^9
viscosity (Pa*sec^m+1)
e 12=0.175
e 12=1.75
e 12=10.
e 12=1750.
e 12=175. (selected)

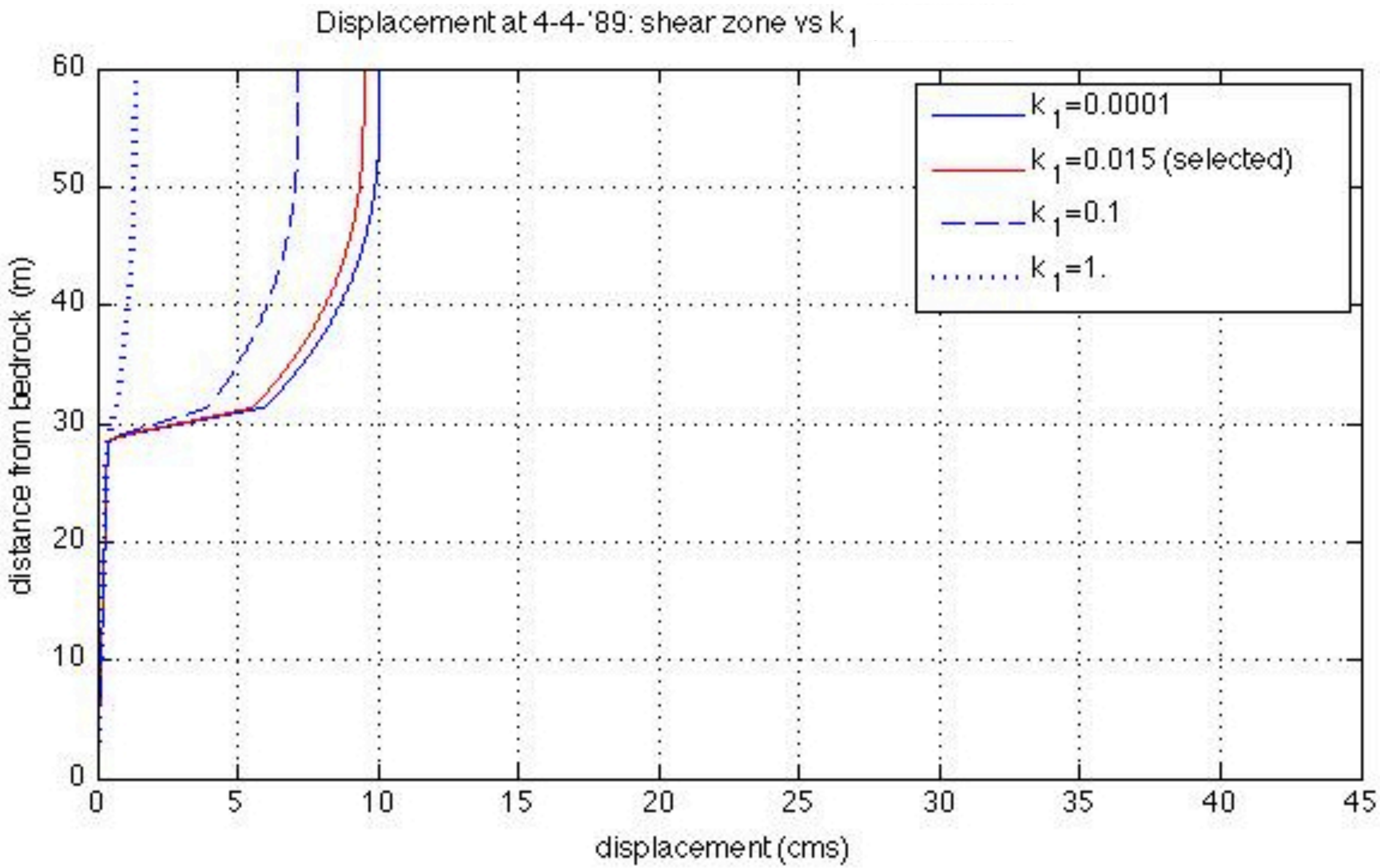

Test on extent of sliding, e (top icy zone): displacement at 4-4-'89
distance from bedrock (m)
displacement (cms)
0
10
20
30
40
50
60
0
2
4
6
8
10
12
14
16
18
20
e 2=1.1 (selected)
e 2=11.
e 2=110.
e 2=0.11
e 2=0.3

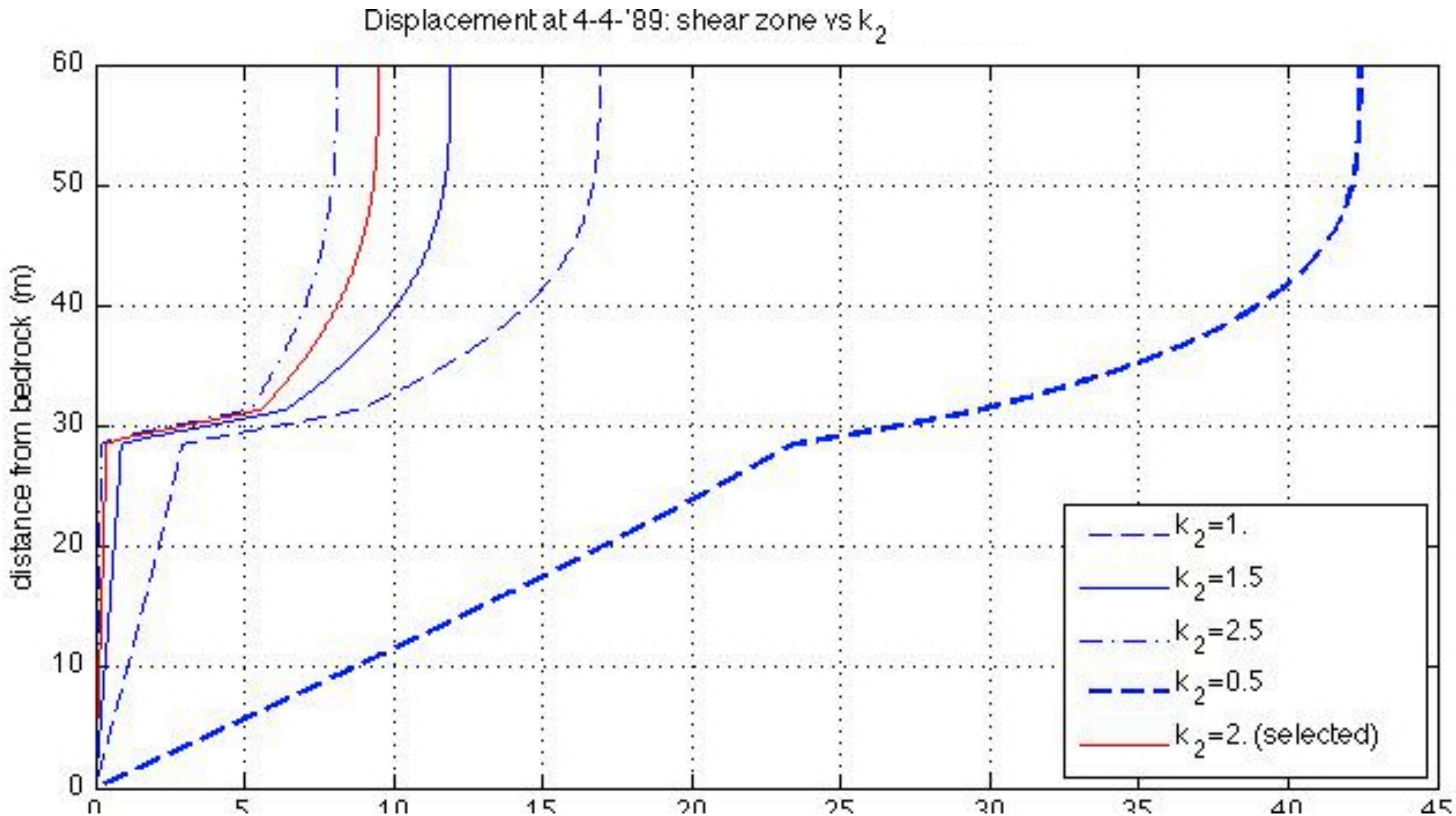

Test on extent of sliding, e (top icy zone): viscosity
60
50
40
30
20
10
0
distance from bedrock (m)
10^9
10^9
viscosity (Pa*s^{m+1})
e 2=1.1 (selected)
e 2=0.11
e 2=0.3
e 2=11.
e 2=110.



velocity u at reference distance from bedrock:

e_1	$u(h_1)$	$u(h_2)$	$u(y_{max})$
175	0.243cm/yr	3.619 cm/yr	6.139 cm/yr
175,000	0.253 cm/yr	3.637 cm/yr	6.139 cm/yr


Displacement at 4-4-'89: shear zone vs k1
distance from bedrock (m)
displacement (cms)
0
10
20
30
40
50
60
0
5
10
15
20
25
30
35
40
45
k1=0.0001
k1=0.015 (selected)
k1=0.1
k1=1.



Displacement at 4-4-'89: shear zone vs k2
distance from bedrock (m)
60
50
40
30
20
10
0
0
5
10
15
20
25
30
35
40
45
k2=1.
k2=1.5
k2=2.5
k2=0.5
k2=2. (selected)

CONCLUSIONS

- f, equilibrium solid fraction, extends the notion of relative solid fraction, ϕ, as it takes into account also the thinning effect on viscosity played by the free-to-move rock particles surrounded by water film formed for particle-to-particle and particle-ice interactions. The parameter e in the expression of f is a measure of the extent of sliding of the free-to-move rock particles (see slides# 12 and 23).

- Internal Sliding occurs in correspondence of the aveargely mixtured ice in layer [h_1,h_2], just due to thinning of ice viscosity which is described via increasing value of e (see slide# 24 and 25).

- for the same mechanism, at top layer ($y>h_2$), increasing values of e correspond to increasing velocity profiles (see slide# 26 and 27)

- at bottom layer, where relative solid fraction is very close to 1 ($y<h_1$) almost whole rock particles are packed together and the effect of e is almost imperceptible (see slide# 28)

CONCLUSIONS/2

➢ **parameters k_1 and k_2 support the effect of pressure on viscosity, in particular k_1 is mostly effective where f << 1 (e.g. icy top zone) and k_2 is mostly effective where f $\cong$1 (bottom rocky zone) (see slide# 12). For increasing value of k_1 and k_2 , viscosity increases and glacier flow slows down (see slides# 29 and 30). Then larger values of k_1 and k_2 counteract the onset of an Internal Sliding Occurrence (ISO).**

> complete numerical validation of the model KMR and further discussion on the mechanisms related to ISO in different physical contexts, which may be deduced via numerical simulation with model KMR, are the content of a paper in preparation

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Thank you very much

for

your attention and comments

... and good luck!