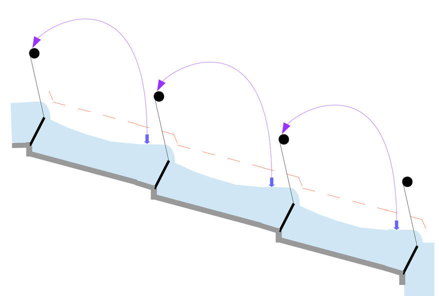
A test of controllers derived from stability rules for a series of identical canals

R. R. P. van Nooijen and A. G. Kolechkina (Delft University of Technology, Netherlands)

1. Introduction

Climate change and economic growth place increasing demands on the management of regional and national waterways. These serve both as part of the drainage network of the catchment and as transport route for raw materials and finished goods. These waterways are often impounded rivers where the management of the weirs must serve both shipping and flood protection. The figure shows reaches of a river separated by weirs.



When tributaries or drainage canals join the river, this disturbs the state of the system. These disturbances can only be compensated for by changing the settings of the weirs. The weirs are usually adjusted at given intervals. This has implications for the design of the controller. If the interval between control actions is relatively long, then the discrete time character of the controller needs to be explicitly taken into account.

2. Model of the system

To allow rapid analysis and simulation of the system a simplified model is constructed. The weirs are modelled by

$$q_{\mathrm{w}}(h_{\mathrm{up}}, h_{\mathrm{cr}}) = b c_{\mathrm{w}} \sqrt{\mathfrak{g}} \cdot \max\left(0, \frac{2}{3}(h_{\mathrm{up}} - h_{\mathrm{cr}})\right)^{3/2}$$

with crest level h_{cr}, crest width b, and c_{w} a weir dependent constant; $\mathfrak{g} = 9.8\mathrm{m/s^2}$. We use the Integrator Delay (ID) model [1, 2, 3] for the reaches, so we approximate each reach by a pure delay τ_j followed by a reservoir with area a_j

$$\frac{dh_1(t)}{dt} = \frac{q_{\mathrm{in}}(t-\tau_1) - q_{\mathrm{w},1}(h_1(t), w_1(t)) + q_{\mathrm{tr},1}(t)}{a_1}$$

$$\frac{dh_2(t)}{dt} = \frac{q_{\mathrm{w},1}(h_1(t-\tau_2), w_1(t-\tau_2)) - q_{\mathrm{w},2}(h_2(t), w_2(t)) + q_{\mathrm{tr},2}(t)}{a_2}$$

$$\frac{dh_3(t)}{dt} = \frac{q_{\mathrm{w},2}(h_2(t-\tau_3), w_2(t-\tau_3)) - q_{\mathrm{w},3}(h_3(t), w_3(t)) + q_{\mathrm{tr},3}(t)}{a_3}$$

with initial condition

$$h_j(0) = h_{0,j}, j = 1,2,3$$

where h_j is the tail end water level in reach j; $q_{\mathrm{in}}(t)$ is inflow to reach 1; $w_j(t)$ is the crest level of weir j located at the tail end of reach j; $q_{\mathrm{tr},j}$ is the flow from tributary j into a reach. To describe the controller we introduce the tail end level set-points h_j^* and the weir crest levels w_j^* for the design flow rate. The control time step τ_{st} is the time between two calculations of new weir settings. A simple discrete local linear proportional controller can now be defined as follows. For $k \in \mathbb{N}$

$$w_j(k+1) = w_j^* + c_{P,j}\left(h_j(t-\tau_{\mathrm{de}}) - h_j^*\right)$$

where τ_{de} is the delay between measurement and control action. We link this to the weir settings by taking

$$\vec{w}(t) = \vec{w}_{\mathrm{discrete}}\left(\left\lfloor \frac{t}{\tau_{\mathrm{st}}} \right\rfloor\right)$$

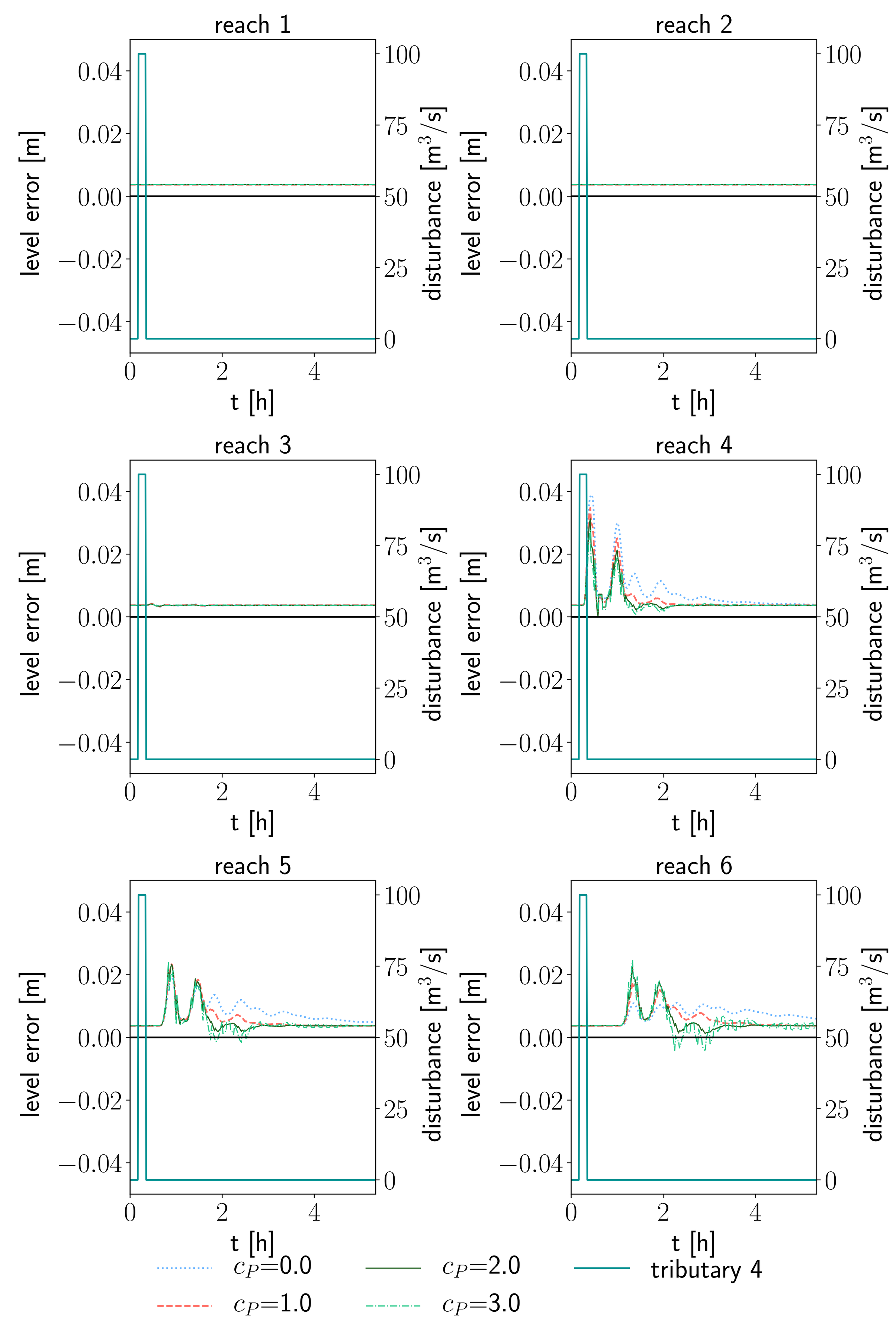


Figure 1: Stable cases ($c_P \leq 3$), time interval around pulse.

3. Canal data

We will consider local discrete proportional controllers for a series of identical weirs and river reaches. Dimensions: the reach length is $15\mathrm{km}$; each reach has a trapezoidal cross section with bottom width $150\mathrm{m}$ and side slope of 1 in 3; the weir crest width is $300\mathrm{m}$ wide; the setpoint just upstream of the weir is $10\mathrm{m}$ above the river bottom, and a control time step to be chosen from $300\mathrm{s} \leq \tau_{\mathrm{st}} \leq 900\mathrm{s}$. The inflow into the first reach is $500\mathrm{m^3/s}$. We will use $\tau_{\mathrm{st}} = 300\mathrm{s}$ in our tests. After a $24\mathrm{h}$ "warm-up" with the weir crests one meter below the setpoint we switch on the proportional controller. We choose $c_{\mathrm{w}} = 1$. Finally $\tau_{\mathrm{de}} = 0$.

4. A description of the different controllers

When $c_P = 0$, the weirs are fixed in the position corresponding to an approximate weir discharge of $500\mathrm{m^3/s}$ for an upstream water depth of $10\mathrm{m}$. When $c_P > 0$, the deviation from the desired tail end water level determines the increase (or decrease) of the weir crest level at the tail end of that reach

$$w_1(k+1) = w_1^* - c_P(h_1(k\tau_{\mathrm{st}} - \tau_{\mathrm{de}}) - h_1^*)$$
$$w_2(k+1) = w_2^* - c_P(h_2(k\tau_{\mathrm{st}} - \tau_{\mathrm{de}}) - h_2^*)$$
$$w_3(k+1) = w_3^* - c_P(h_3(k\tau_{\mathrm{st}} - \tau_{\mathrm{de}}) - h_3^*)$$

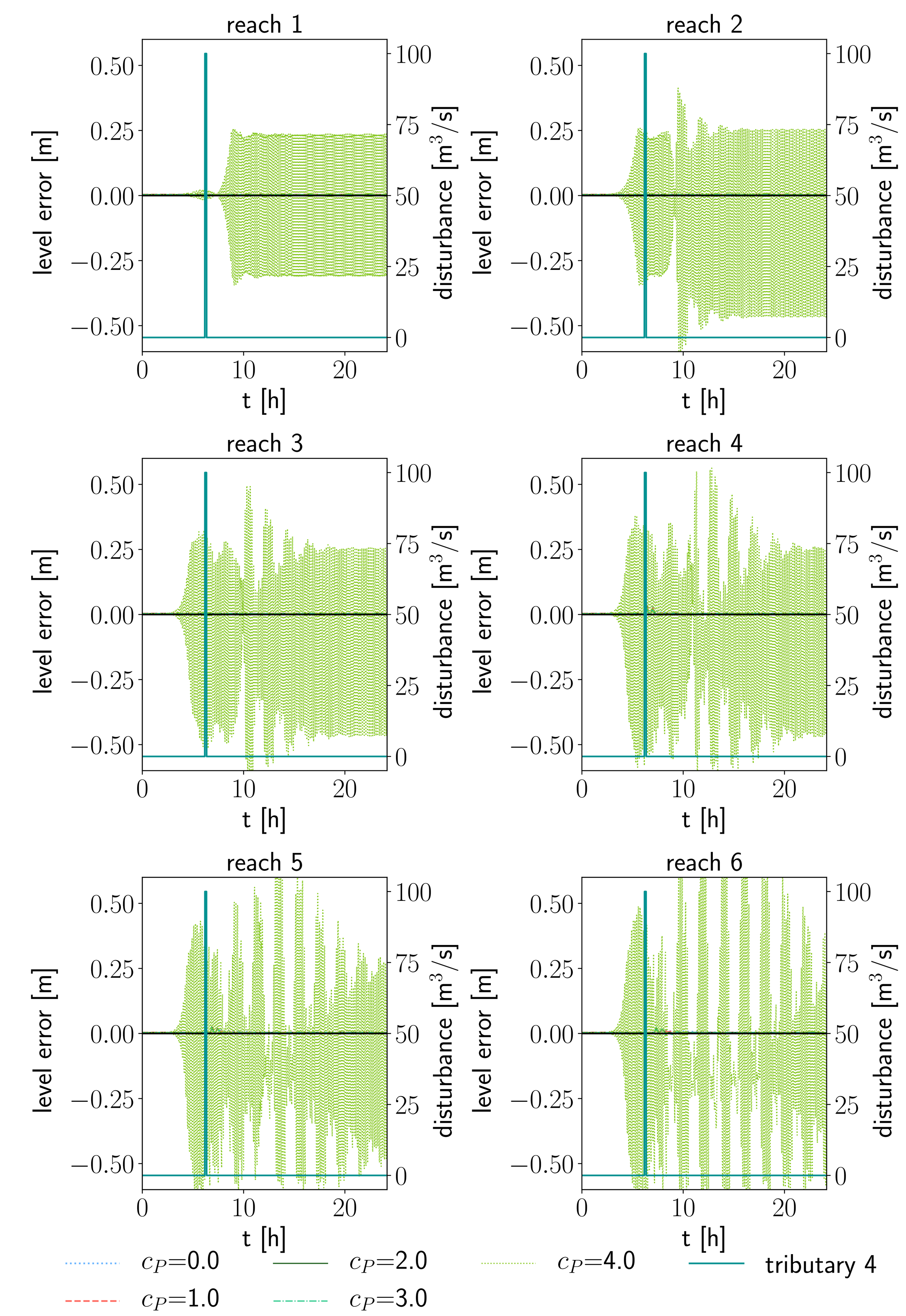


Figure 2: Add unstable case $c_P = 4$, note the different vertical scale, time interval starts when controller is switched on

6. Simulation results

For fixed weir settings, the additional inflow into the river leads to a rise in water level. This in turn leads to a rise in weir discharge. If the inflow is not in the last reach, then it leads to a rise in water level in the downstream reaches later on. In the simulation a temporary non-zero inflow into reach 4 simulates such an additional inflow.

Adding identical local proportional controllers speeds up the return to setpoint after the inflow from the tributary stops, but c_P should be chosen with care.

If c_P is too large, even the small deviation from setpoint at the time when the controller is switched on is sufficient to initiate oscillations that do not damp out. The simulations were performed using the Sobek 1D flow simulation software [4], which is a full 1D hydraulic flow simulation.

7. Discussion

Today careful use of water is of great importance as demand for water goes up while the availability decreases. This study shows that even for a relatively simple system and for relatively simple controllers, careless controller design can lead to unstable systems

During initial experiments it seemed that wave reflection might play a role under some circumstances. That would imply that an Integrator Delay Zero (IDZ) model [2] is needed instead of the ID model.

6. Future plans

A combination of the theory provided by [2] and [5] will be applied to predict the boundary between stable and unstable control systems.

Integraton of the IDZ model into this is also foreseen.

References

[1] X. Litrico and V. Fromion. Simplified modeling of irrigation canals for controller design. *Journal of Irrigation and Drainage Engineering*, 130(5):373–383, 2004.

[2] X. Litrico and V. Fromion. *Modeling and Control of Hydrosystems.* Springer, 2009.

[3] J. Schuurmans. *Control of water levels in open-channels.* PhD thesis, Faculty of Civil Engineering, Delft University of Technology, Stevinweg 1, 2628 CN, Delft, the Netherlands, 1997.

[4] Deltares. D-Flow 1D in Delta Shell: User manual. Technical report, Deltares, Mar. 2019. Released for SOBEK Suite 3.7.

[5] A. N. Michel, L. Hou, and D. Liu. *Stability of dynamical systems.* Birkhäuser/Springer, second edition, 2015.

TUDelft