Estimating IDF-Relations consistently using a duration-dependent GEV with spatial covariates

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Topic

Intensity-Duration-Frequency (IDF) curves are a popular tool in Hydrology for estimating the properties of extreme precipitation events. They describe the relationship between rainfall intensity and duration for a given non-exceedance probability (or frequency).

Method

We use a duration dependent GEV with spatial covariates, to model the distribution of annual precipitation maxima simultaneously for a range of durations and locations.

Why?

This way, we can obtain return level maps for various durations, as well as IDF curves at all locations. Further advantages are parameter reduction and more efficient use of the available data.

Does it work?

We use the Quantile Skill Score to investigate under which conditions this method leads to an improved estimate compared to using the GEV separately for each duration at every station.

- Case study in **Wupper-Catchment**:
  - → 92 gauge stations in 75 locations (see figure 1)
  - → different measuring periods (see figure 2)
  - → varying length of time series

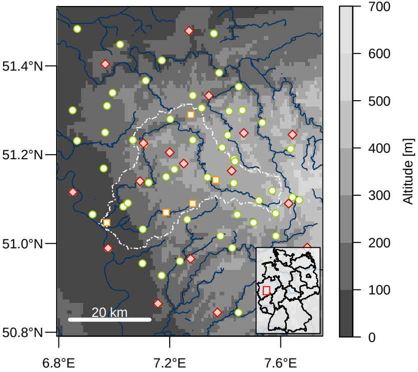


Figure 1: Study area: dashed line shows catchment area. Altitude data: [1], River data: [2]

- Provided by the German Weather Service (DWD) and the Wupperverband (WV)
- **Annual precipitation maxima**
  - → for durations 1, 4, 8, 16, 32 minutes, 1, 2, 3, 8, 16 hours and 1, 2, 3, 4, 5 days

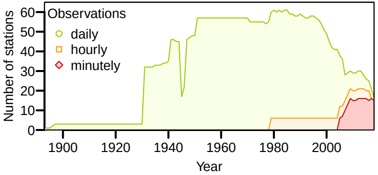


Figure 2: Number of active gauge stations in the study area per year for the different measuring periods.

Provider	Number of stations	Measuring period	Length of time series
DWD	69	1 day	9-121 years
DWD	17	1 min	5-14 years
WV	6	1 hour	38 years

Model

- Modeling precipitation block maxima z with the GEV [3]: $$\boxed{G(z;\mu,\sigma,\xi) = \exp\left\{-\left[1+\xi\left(\frac{z-\mu}{\sigma}\right)\right]^{-1/\xi}\right\}}$$
- Simultaneously model maxima over different **durations** and **locations** using

Duration dependent GEV (d-GEV)

- Assumptions for the dependency of the GEV-parameters on duration d following [4]:

scale: $\sigma(d) = \sigma_0 \cdot (d+\theta)^{-\eta}$

location: $\mu(d) = \tilde{\mu} \cdot \sigma(d)$

shape: $\xi(d) = \xi = \text{const.}$

- Resulting in d-GEV:

$$G(\mu(d),\sigma(d),\xi) = G(\underbrace{\tilde{\mu},\sigma_0,\xi,\theta,\eta}_{\text{5 parameters}})$$

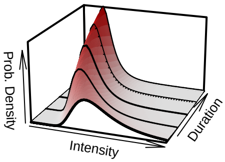


Figure 3: Probability density surface of the d-GEV

with spatial covariates

- Allow d-GEV parameters to vary in space:

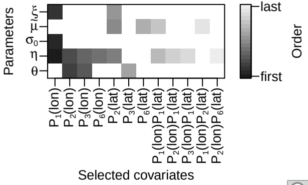
$$G(\ \tilde{\mu}(\vec{r}),\ \sigma_0(\vec{r}),\ \xi(\vec{r}),\ \theta(\vec{r}),\ \eta(\vec{r})\ )$$

- Using orthogonal polynomials of longitude and latitude [5] for all parameters $\Phi \in \{\tilde{\mu},\sigma_0,\xi,\theta,\eta\}$, with maximum orders $J=K=6$:

$$\phi = \phi_0 + \sum_{j=1}^{J}\beta_j^{\phi}\,P_j(\text{lon}) + \sum_{k=1}^{K}\gamma_k^{\phi}\,P_k(\text{lat}) + \sum_{j=1}^{J}\sum_{k=1}^{K}\delta_{j,k}^{\phi}\,P_j(\text{lon})P_k(\text{lat})$$

- Model selection: avoid overfitting, by allowing certain coefficients to remain zero

Figure 4: Model selection result: added covariates colored according to order of their selection. White means parameter remains zero.



Quantile estimation

Using the d-GEV with spatial covariates, we can obtain
return level maps for various durations (figure 5) and **IDF-curves at all locations** (figure 6).

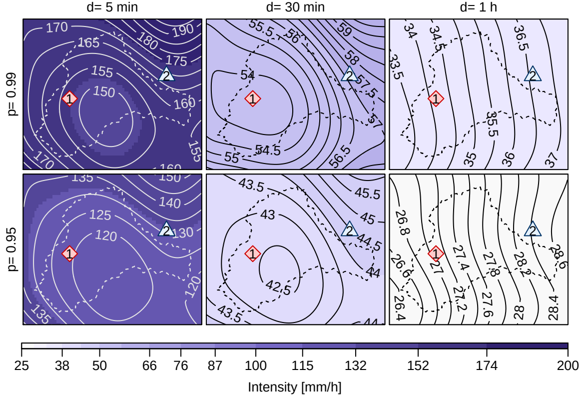


Figure 5: 100-year (upper panel) and 20-year (lower panel) return level maps for durations: 5 minutes, 30 minutes and 1 hour. Diamond and Triangle mark the locations for the IDF-curves in figure 6

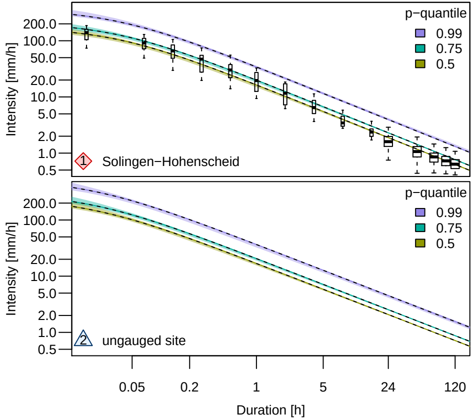


Figure 6: IDF-curves for a station with minutely precipitation measurements and an ungauged location. 95% confidence Intervals were obtained by the bootstrap percentile method.

Verification

- How does the d-GEV perform in the study area?
  1. When using an individual model at each station
  2. When using one spatial model

- Investigation of model performance using the Quantile Score QS [6] corresponding to weighted mean difference between observations $o_{s,d}$ at certain station s, for certain duration d and modeled p-quantile $q_{s,d}(p)$
- Mean Quantile Score for probability p and duration d, averaged over stations:

$$\overline{\mathrm{QS}}_d(p) = \frac{1}{S_d} \sum_{s=1}^{S_d} \frac{1}{N_{s,d}} \sum_{n=1}^{N_{s,d}} \rho_p(o_{s,d,n} - q_{s,d}(p)), \qquad \rho_p(u) = \begin{cases} pu & , u \geq 0 \\ (p-1)u & , u < 0. \end{cases}$$

- Compare model score with reference using the Quantile Skill Score QSS
- Reference = individual GEV model for each duration at every station

$$\overline{\mathrm{QSS}}(p)_d = 1 - \frac{\overline{\mathrm{QS}}_d^{\text{d-GEV}}(p)}{\overline{\mathrm{QS}}_d^{\text{GEV}}(p)}$$

- **Results** (see figure 7):
  - $\rightarrow$ Both approaches: **improved modeling of rare events**
  - $\rightarrow$ $d \geq 24$ h (right of dashed line): more data availability, pooling information over durations is less important
    $\rightarrow$ **Improvement for stations with short observation time series**
  - $\rightarrow$ Short durations $d \leq 0.5$ h: loss in skill for both approaches (dashed circle)
    $\rightarrow$ d-GEV might not be flexible enough to model durations $d \leq 0.5$ h properly

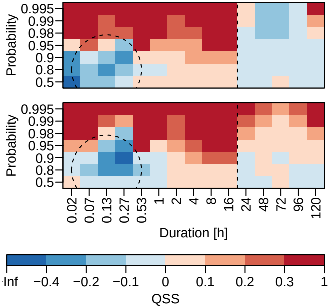


Figure 7: Mean Quantile Skill Score $\overline{\mathrm{QSS}}_d(p)$ for station-wise d-GEV (upper panel) and spatial d-GEV (lower panel). Positive values (red) indicate improvement compared to the reference.

References

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