A Dynamic Flexible State Model for Rainfall Nowcasting

Venkat Roy, Marc Schleiss

Dept. of Geoscience and Remote Sensing, Delft University of Technology, Delft, The Netherlands
Acknowledgement: Royal Netherlands Meteorological Society (KNMI)

May 8, 2020

Why nowcasting?

- Prediction up to 6 hours (World meteorological organization)

Applications:

- Short-term weather predictions for air traffic control.
- Early warning systems for flooding
- Outdoor event planning
- Road conditions, traffic management

Principle of radar nowcasting

- Nowcasts are generated by extrapolating rain cells along the principal direction of motion assuming "Lagrangian persistence"
- No temporal evolution except for some random noise

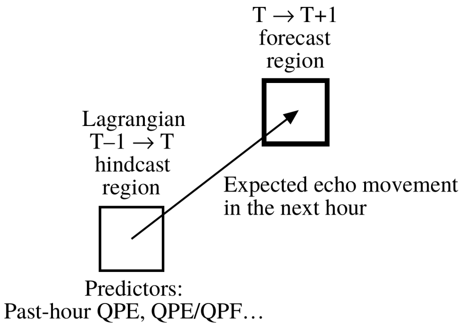


Fig: Radar nowcasting principle [Fabry *et al.*, 2009]

Nowcasting using radar: pros and cons

*A*dvantages:

- Computationally efficient
- High spatial and temporal accuracy (e.g. $1\ \mathrm{km}$ and 5 min)

*D*isadvantages:

- Radar data can be noisy (clutter, blockages, interference, ...)
- Vertical variability, attenuation, calibration, ...
- Radar does not measure rainfall rate but reflectivity. Z-R relation is sensitive to drop size distribution
- Can only predict what has already been observed. Predictions tend to lag behind true state.

State model formalism

- Target $\mathcal{A}$ and measurement area $\mathcal{A}'$
- Spatio-temporal rainfall field:
  $\mathbf{u}_t = [u_t(\mathbf{x}_1), \ldots, u_t(\mathbf{x}_N)]^{\mathrm{T}}$,
  $[\mathbf{x}_1, \ldots, \mathbf{x}_N] \in \mathcal{A}$.
- $\mathbf{u_t}$ can be radar reflectivity or rainfall rate
- Dynamic model : $\boxed{\mathbf{u}_t = \mathbf{H}_t \mathbf{u}_{t-1} + \mathbf{q}_t}$
  $\mathbf{q}_t$: stochastic process noise.
- Estimation of N^2 parameters :
  computationally expensive for large $\mathcal{A}$.

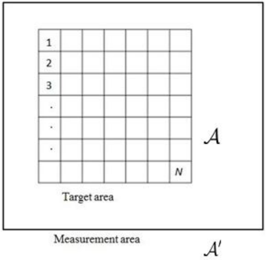


Fig: Target and measurement area

Estimation of $\mathbf{H}_t$

- Estimation of $\mathrm{vec}(\mathbf{H}_t)$ from $\mathbf{u}_t = \underbrace{(\mathbf{u}_{t-1}^{\mathrm{T}} \otimes \mathbf{I}_N)}_{N \times N^2} \mathrm{vec}(\mathbf{H}_t) + \mathbf{q}_t$.
- Simple(iterative) least squares approach:

$$\hat{\mathbf{h}}_t = \arg\min_{\mathbf{h}_t} \|\mathbf{u}_t - \mathbf{X}\mathbf{h}_t\|_2^2, \tag{1}$$

  where $\mathbf{h}_t = \mathrm{vec}(\mathbf{H}_t)$, and $\mathbf{X} = \mathbf{u}_{t-1}^{\mathrm{T}} \otimes \mathbf{I}_N$.
- For $\mathbf{H}_t \approx \mathbf{H}$ for $t = 1, \ldots, T_{\mathrm{s}}$

$$\mathbf{X} = \begin{bmatrix} \mathbf{u}_0^{\mathrm{T}} \otimes \mathbf{I}_N \\ \mathbf{u}_1^{\mathrm{T}} \otimes \mathbf{I}_N \\ \vdots \\ \mathbf{u}_{T_{\mathrm{s}}-1}^{\mathrm{T}} \otimes \mathbf{I}_N \end{bmatrix}_{NT_{\mathrm{s}} \times N^2}, \tag{2}$$

- (1): single snapshot, and (2): multiple snapshot ahead prediction.

Generalized optimization problem to estimate $\mathbf{H}_t$

- Underdetermined system of equations $\mathbf{u}_t = \underbrace{(\mathbf{u}_{t-1}^{\mathrm{T}} \otimes \mathbf{I}_N)}_{N \times N^2} \mathrm{vec}(\mathbf{H}_t) + \mathbf{q}_t$.
- Regularization using prior spatial information regarding $\mathbf{h}_t = \mathrm{vec}(\mathbf{H}_t)$, given by $f_p(\mathbf{h}_t)$. (e.g. sparsity, covariance structure)

$$\hat{\mathbf{h}}_t = \arg\min_{\mathbf{h}_t} [\underbrace{\|\mathbf{u}_t - \mathbf{X}\mathbf{h}_t\|_2^2}_{\mathrm{Data}} + \lambda_{\mathrm{s}} f_p(\mathbf{h}_t)], \tag{3}$$

- Can also use predictions from a numerical weather prediction model):

$$\hat{\mathbf{h}}_t = \arg\min_{\mathbf{h}_t} [\underbrace{\|\mathbf{u}_t - \mathbf{X}\mathbf{h}_t\|_2^2}_{\mathrm{Data}} + \underbrace{\lambda_{\mathrm{m}} \|\tilde{\mathbf{u}}_t - \mathbf{Y}\mathbf{h}_t\|_2^2}_{\mathrm{NWP}} + \lambda_{\mathrm{s}} f_p(\mathbf{h}_t)], \tag{4}$$

where $\mathbf{Y} = \tilde{\mathbf{u}}_{t-1}^{\mathrm{T}} \otimes \mathbf{I}_N$.

- Weights λ_{s}, λ_{m} tuned based on the accuracy of NWP and prior.

Modelling rainfall dynamics using a scaled affine transform

- Assuming an affine transformation followed by scaling 6 (transformation) + 1 (scaling) parameters.
- Transform:
  $u_t(\tilde{\mathbf{x}}_j) = \alpha_t u_{t-1}(\mathbf{x}_j),\ \alpha_t > 0,\ \tilde{\mathbf{x}}_j \in \mathcal{A}$, where

$$\begin{bmatrix} \tilde{x}_i \\ \tilde{y}_i \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix} = \mathbf{M}_t \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}. \tag{5}$$

- Estimating the best $\alpha_t, \mathbf{M}_t$ using consecutive snapshots.

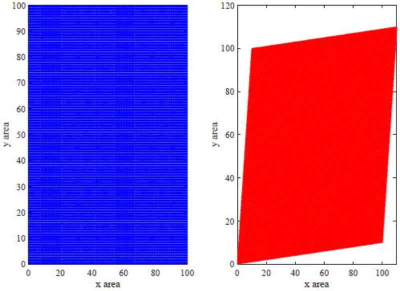
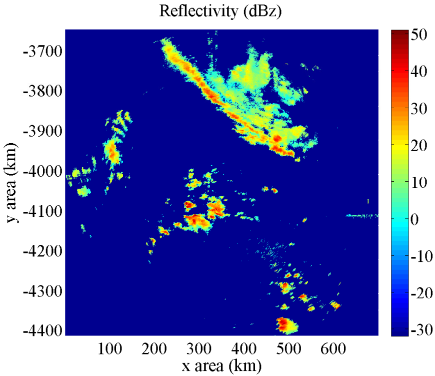


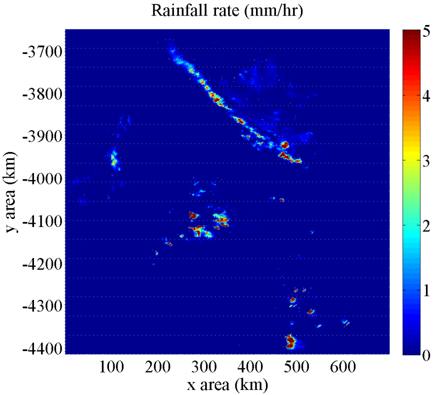
Fig: Affine coordinate transform

Radar reflectivity to rainfall

Rainfall Event on 03:30 a.m., 12.07.2019:

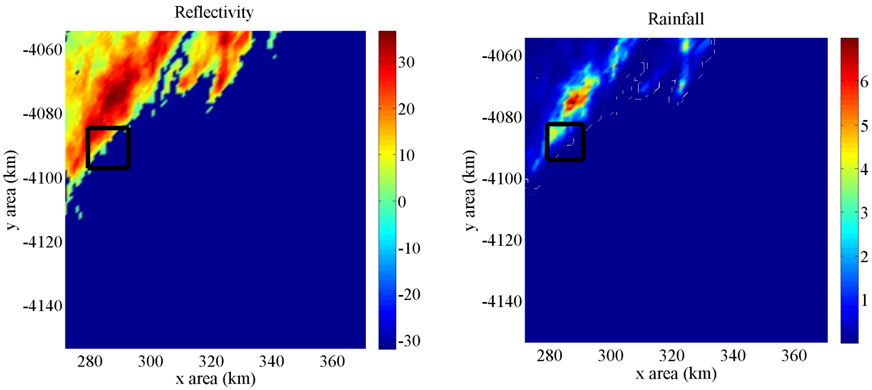
Total area : 700 $\times$ 765 pixels with spatial resolution 1 km^2.





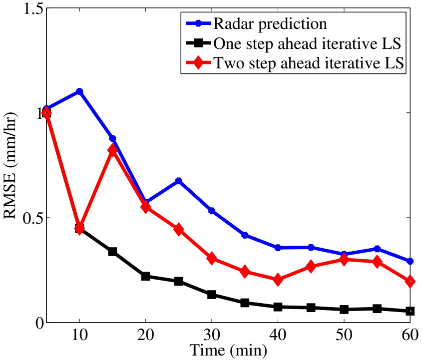
Selected measurement area and target area

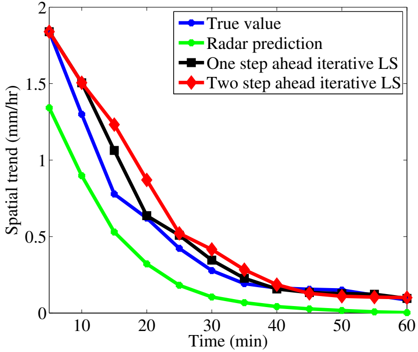
Measurement area : 100×100, Target area: 15×15.



Performance analysis

Used data: Rainfall Event from 03:30 - 04-30 a.m., 12.07.2019





Example of tracking the dynamics using affine transform (simulated field)

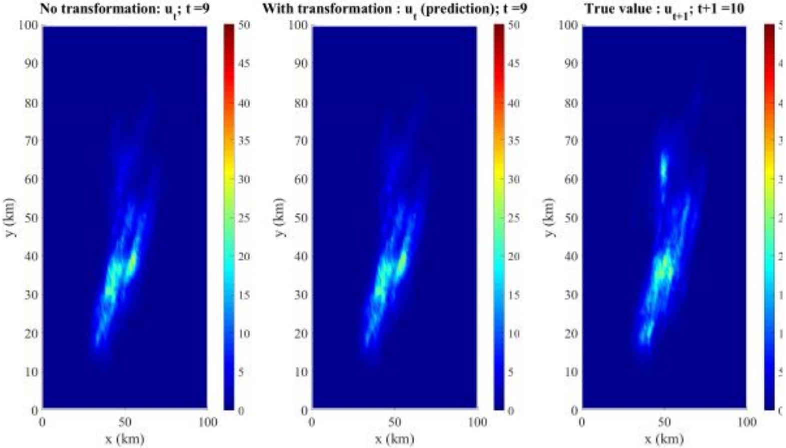


Fig: One step ahead prediction using the scaled affine transform model; No. of pixels: 100×100 can be predicted by only 7 parameters

Summary

- The regularized (iterative) least squares method outperforms Lagrangian persistence for single step ahead prediction. However, performance decreases for multiple step ahead predictions.
- Computational cost quickly grows with size of target area. Scaled affine transformations are less accurate but computationally more efficient.
- External information from NWP can be incorporated into the state model estimation problem using a multi-objective optimization framework.
- The combination of statistical radar extrapolation with physical knowledge from a NWP leads to better multiple step ahead predictions.