

# Dynamic Seepage Meter Theory with Application Examples

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## Abstract

Most single-point methods of measuring seepage fluxes across the surface water-groundwater interface in lakes, streams, and estuaries (e.g., volumetric, head-based, and thermal) have one trait in common: they produce seepage rate values, averaged over substantial periods of time, thereby limiting resolution of the intra-day dynamics. Recently, Solomon et al. (2020) presented a new instrument and modification to a previously tested concept (Solder et al., 2016). This instrument has an open-bottom permeameter (OBP) design, which is commonly used for investigating hydraulic conductivity of the interface with falling or rising head tests, but historically not used for flux estimates. The novel dynamic seepage meter (DSM) evaluates the transient water level in the OBP-based instrument with submillimeter accuracy, exceeding the performance of traditional pressure transducers. The initial dynamics of the water level response over fractions of an hour holds the necessary information to infer the natural seepage rate in both gaining and losing conditions. The tests can be repeated frequently in an automatic regime. If a single test lasts long enough, hydraulic conductivity, in addition to the seepage rate can also be accurately determined. Here, a detailed hydrodynamic theory of the flow systems inside and outside the OBP is presented and the accuracy of measured water fluxes is investigated with emphasis on interpretation of the data with ambient noise. The results of this study will facilitate rapid, accurate, and massive data collection in diverse field conditions.

## Importance and goal

Solomon et al. (2020), also Solder et al. (2016) proposed fundamentally new approach to estimating the seepage across the surface water - groundwater interface. Instead of collecting water volume in a designated closed compartment, the dynamics of the water level in the OBP is used in a dynamic regime. This approach eliminates assumption of steady-state conditions; it applies to both gaining and losing regimes of surface water (streams, fir brevity), and provides previously unavailable temporal resolution of seepage rates. This idea offers substantial advantages of this dynamic seepage meter (DSM) over other known approaches. Used in practice ad hoc analysis needs detailed hydrodynamic foundations.

## Principle and implementation of DSM

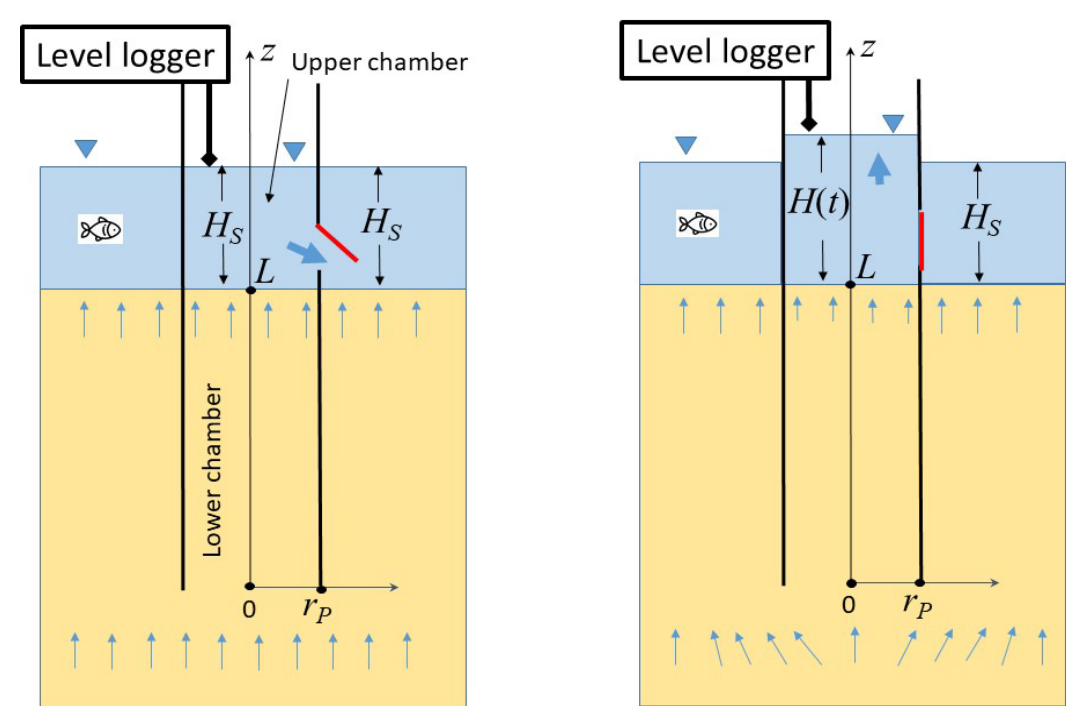


Fig 1: Principle of DSM

Left: Pre-test conditions. The valve (red) is opened and collected natural seepage discharges to the stream; streamlines (blue) are vertical in subsurface. For illustration, case of gaining stream is shown.

Right: Test is initiated by valve closure; seepage moves upward and raises water level in the upper chamber; local velocity field direction and magnitude near the pipe bottom and seepage entry to the lower chamber are perturbed.

However, water level rise at most alluvial systems is very low, and high accuracy of level logger is vital.

Fig 2: View and design

Left: The control unit (white box) sits on top of a thin-walled tube that is inserted into the streambed (typically 30 to 50 cm below the interface). An opening in the side of the tube is fitted with an electric valve. At the beginning of the test the valve is open so that the water level inside the tube is the same as the stream. The valve then closes and the water level changes (up for a gaining stream, down for a losing stream).

Right: The water level is measured by moving the linear actuator with electrode. Water level is recorded by the control unit at the moment when the moving electrode contacts the water level in OBP.

Design was tested in the lab and in the field. Accuracy is on the order 0.05 - 0.1 mm (Solomon et al., 2020).

## Mathematical model: 3D, axisymmetrical, transient

Hydraulic head  $h(r, z, t)$  is a function of time  $t$  and cylindrical coordinates  $(r, z)$ . The streambed has anisotropic hydraulic conductivity ( $K_r, K_z$ ) and specific storage  $S_s$ . Stream stage is  $H_s(t)$ .

$$S_s \frac{\partial h}{\partial t} = \frac{K_r}{r} \frac{\partial}{\partial r} \left( r \frac{\partial h}{\partial r} \right) + K_z \frac{\partial^2 h}{\partial z^2}, \quad 0 < r < \infty, -\infty < z < L$$

Initial and boundary conditions

$$h(r, z, 0) = h_0(z, 0), \quad 0 < r < \infty, -\infty < z < L$$

$$\frac{\partial h(r, z, t)}{\partial z} = -\frac{q_z}{K_z}; \quad r \geq 0, \quad z \rightarrow -\infty; \quad r \rightarrow \infty, \quad -\infty < z < L$$

$$\frac{\partial h(0, z, t)}{\partial r} = 0, \quad -\infty < z < L$$

$$\frac{\partial h(r_p, z, t)}{\partial r} = 0, \quad 0 < z < L$$

OBP mass balance

$$h(r, L, t) = \begin{cases} H(t), & r < r_p \\ H_s(t), & r > r_p \end{cases}$$

$$Q(t) = \pi r_p^2 \frac{dH}{dt} = -2\pi K_z \int_0^{r_p} \frac{\partial h(r, L, t)}{\partial z} r dr, \quad t > 0$$

$$H(0) = H_s(0)$$

Dynamic OBP level  $H(t)$  initially coincides with the stream stage  $H_s(0)$ 

## Reduction to 1D model: competing transient DSM head and seepage in a pipe

Proofs and derivations include application of dimensional analyses, perturbation technique, separation of variables and supporting numerical simulations, resulting in differential equation that accounts for stream dynamics, the OBP geometry, and aquifer properties:

$$\frac{dH}{dt} = -\frac{H - H_s}{t_L} + q_z, \quad H(0) = H_s(0), \quad t_L = \frac{L(1+F)}{K_z}, \quad F \approx \frac{\pi}{5.5} \frac{r_p}{L} \sqrt{\frac{K_z}{K_r}}$$

The main results of water level in the upper chamber, obtained for noiseless record of stream level are as follows:

• For steady stream level ( $H_s(t) = H_s(0) = H_s$ ), and seepage rate ( $q_z(t) = q_z(0) = q_z$ ), result is consistent with Solomon et al. (2019) solution:

$$H(t) = H_s + \Delta H_{\max} \left[ 1 - \exp\left(-\frac{t}{t_L}\right) \right], \quad \Delta H_{\max} = q_z t_L$$

• For short duration of measurements (compared to  $t_L$ ), initial segment of response can be presented as linear function of time with accuracy about 5% and independent of aquifer properties:  $H(t) \approx H_s + q_z t, \quad t < t_{\text{end}} < 0.1 t_L$

• For variable stream stage  $H_s(t)$ , fixed head gradient  $q_z/K_z$ , and initial water slug in the upper chamber, the result is as follows:

$$H(t) = H(0) e^{-\frac{t}{t_L}} + \Delta H_{\max} \left[ 1 - e^{-\frac{t}{t_L}} \right] + \frac{1}{t_L} e^{-\frac{t}{t_L}} \int_0^t H_s(t') e^{\frac{t'}{t_L}} dt', \quad H(0) \neq H_s(0)$$

## Noise analysis

Commonly, the field data  $H_F(t_i)$  are collected discretely over time in  $M$  points, each  $i$ -th point having noise. We simulated field data by superimposing the Gaussian noise model with zero mean and standard deviation  $A$ :

$$H_F(t_i) = H_s + \Delta H_{\max} \left[ 1 - \exp\left(-\frac{t_i}{t_L}\right) \right] + A \cdot N(0, 1), \quad \Delta H_{\max} = q_z t_L, \quad 0 \leq t_i = i \cdot \Delta t < t_{\text{end}}, \quad \Delta t = t_{\text{end}} / M$$

Example of synthetic noisy DSM data set below is given for the streambed with  $K_z = 14.4$  m/day,  $q_z = 0.2$  m/day,  $L = 0.30$  m,  $A = 0.2$  mm, resulting in a lag time  $t_L = 1800$  s. To analyze the noise effect, the noiseless model  $H(t)$  is matched to the field data  $H_F(t_i)$  by the root mean square criterion, using Nelder-Mead simplex algorithm in Matlab. The best match produces estimates of  $K_z$  and  $q_z$  values.

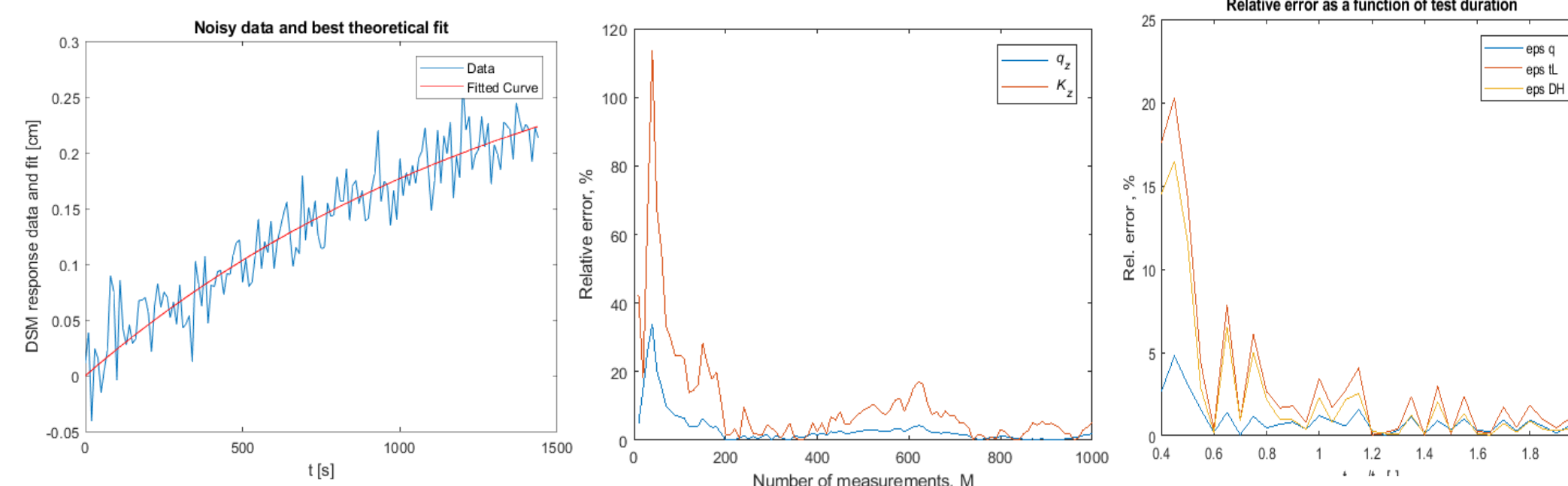


Fig 3: Duration  $t_{\text{end}}$  and number of data points control accuracy of data interpretation at a given noise level. In general  $M \sim 100$  and  $t_{\text{end}} \sim t_L$ , provide accuracy within few percent.

## Analysis

Accuracy of test results depends on the magnitude of aquifer parameters, OBP design, and noise level. (compared to the terminal water level change  $\Delta H_{\max}$ ). Accuracy of  $K_z$  values depends on the test duration. Quality of inversion is also determined by the noise amplitude  $A$ . Therefore, relative errors [%] of each parameter were plotted as functions of these parameters, normalized test duration ( $t_{\text{end}}/t_L$ ) that can be controlled and normalized ambient noise level ( $A/\Delta H_{\max}$ ) that cannot be controlled. 2D plots show that  $q_z$  can be estimated accurately in a broader ranger of parameters than  $K_z$ .

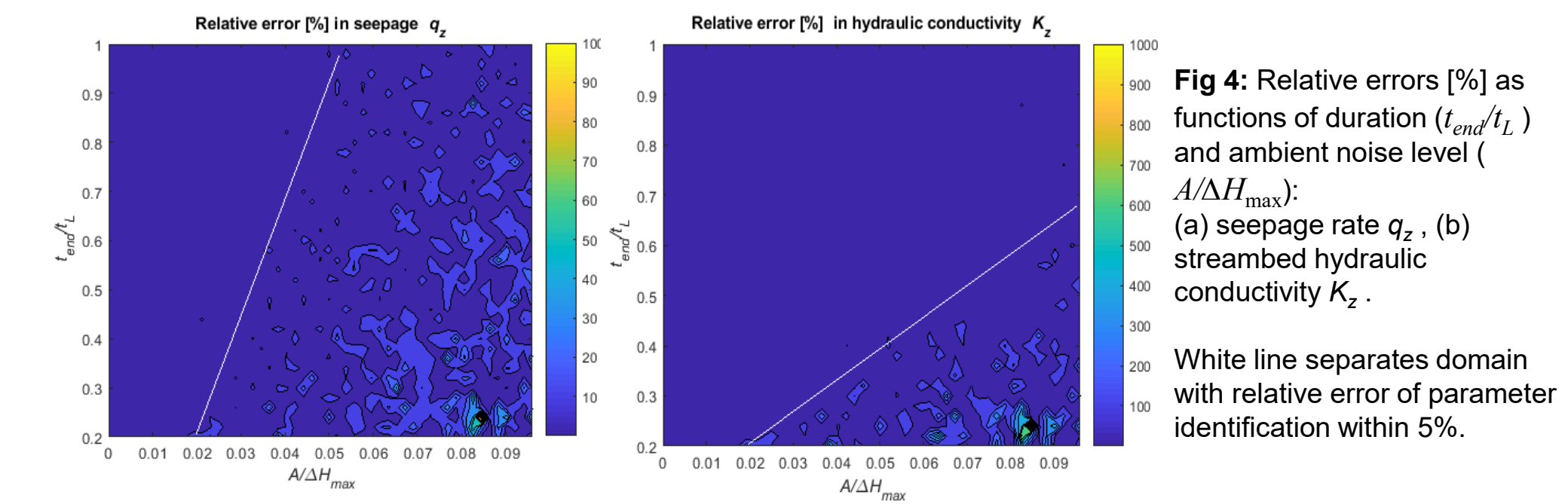
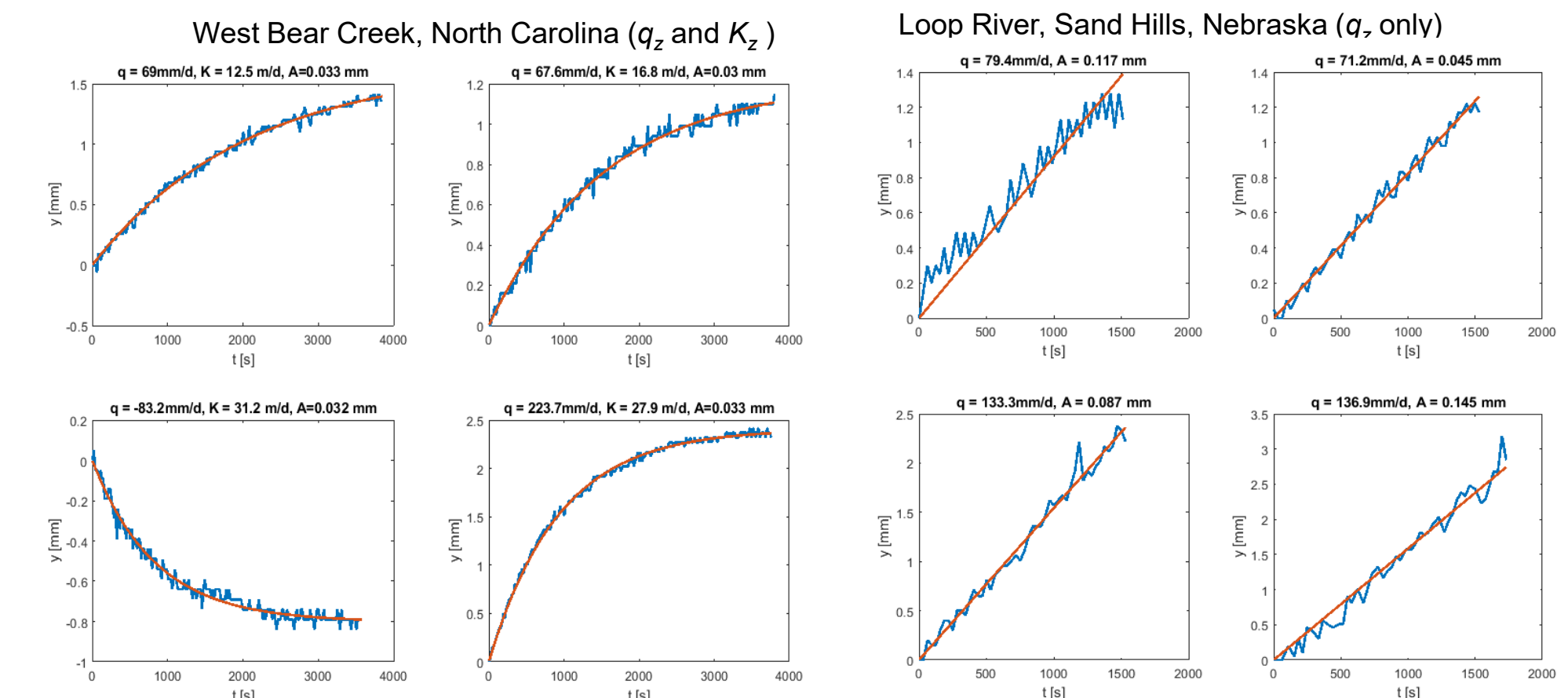


Fig 4: Relative errors [%] as functions of duration ( $t_{\text{end}}/t_L$ ) and ambient noise level ( $A/\Delta H_{\max}$ ): (a) seepage rate  $q_z$ , (b) streambed hydraulic conductivity  $K_z$ .

White line separates domain with relative error of parameter identification within 5%.

## Applications

$$y[\text{mm}] = H(t) - H_s - \text{rise or fall of water level in OBP-DSM}$$



## Conclusions

- Theory shows that 3D transient interpretation of axisymmetric flow can be reduced to 1D flow in the OBP pipe
- The exponential response reaches asymptote at a time scale ( $\sim t_L$ ), defined by streambed and instrument parameters
- For short times (when curvature of response is undetectable), linear response is adequate for determining just  $q_z$ .
- For larger times (comparable to characteristic time  $t_L$ ) curvature allows for determining  $K_z$  in addition to  $q_z$ .
- Test duration (geometry and hydraulic properties) and noise level determine accuracy of  $q_z$  and  $K_z$  in OBP-DSM tests
- Increasing test duration of the test improves accuracy of  $K_z$  estimation more than of  $q_z$  for a given noise level
- Water level measurements accuracy of 0.1 mm may allow to determine  $q_z$  seepage rates with magnitude of 10 mm.
- The theory allows to account for dynamic changes of both surface water head and seepage rates.
- Performance of the test dramatically accelerates seepage data collection and interpretation

## References

- Solder et al. (2019) A tube seepage meter for in situ measurement of seepage rate and groundwater sampling, *Groundwater*, 54(4), 588-595.
- Solomon et al. (2020). An automated seepage meter for streams and lakes. *Water Resour. Res.*, 56, e2019WR026983. <https://doi.org/10.1029/2019WR026983>

## Acknowledgements

Research was supported by the NSF grant EAR – 1744719. We acknowledge contributions of G. Ledder, Univ. Nebraska-Lincoln and J.P. Cao, Sun Yat-Sen Univ. at early stages of the study