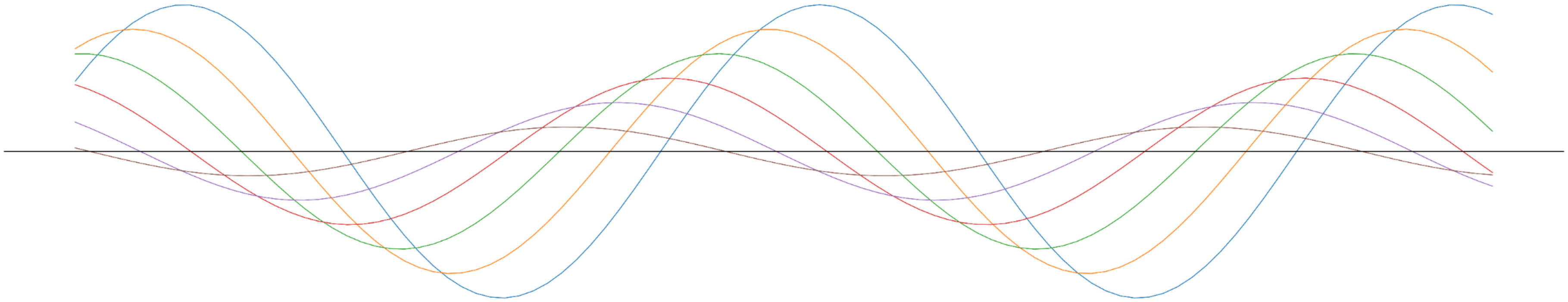
Karlsruher Institut für Technologie

Determining the limits for harmonic constituent Amplitude and Phase ESTimation (APES) from time series measurements using least-squares

Daniel Schweizer, Vincent Ried, Gabriel Rau, Jonthan Tuck and Peter Stoica

INSTITUT FÜR ANGEWANDTE GEOWISSENSCHAFTEN, ABTEILUNG INGENIEURGEOLOGIE

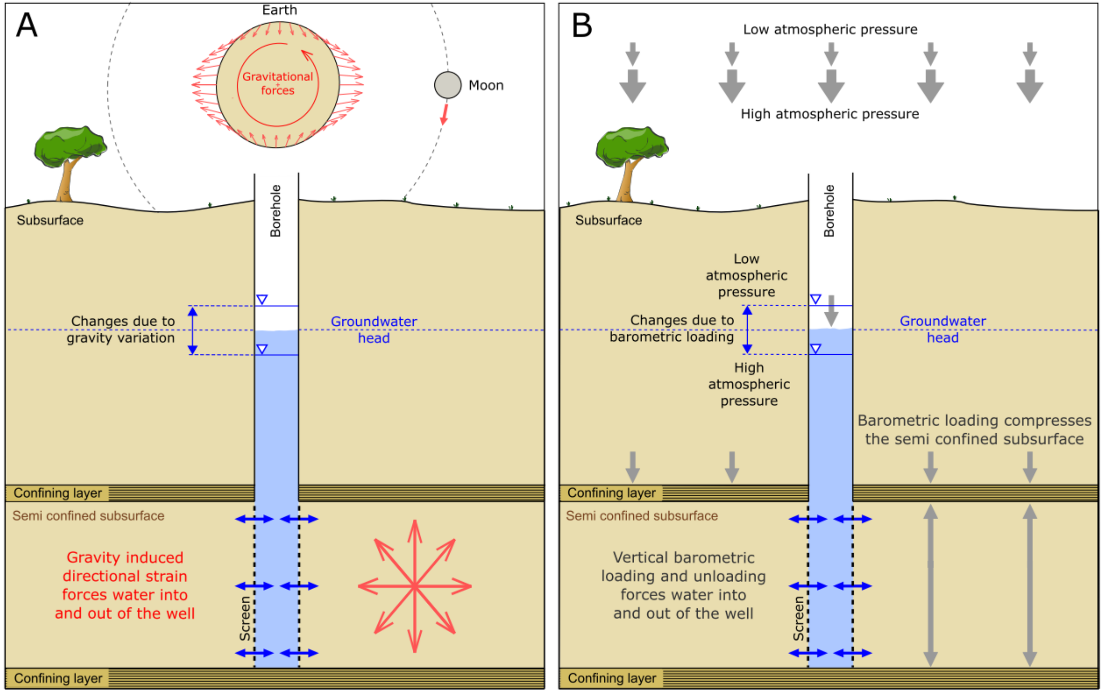


What is Tidal Subsurface Analysis (TSA)?

Karlsruher Institut für Technologie

- Earth and atmospheric tides cause subsurface compression and expansion at well-known cycles (i.e. tides)
- By knowing these drivers (tides), the groundwater response can be inverted to quantify **in-situ subsurface hydro-geomechanical properties**
  - Hydraulic conductivity
  - Specific storage
  - Porosity
  - Bulk modulus

Earth tides | Atmospheric tides



McMillan et al. (2019) *Reviews of Geophysics*

Role of Tidal Constituents

Karlsruher Institut für Technologie

- Tidal constituents occur at known frequencies grouped around **1** or **2** cylces per day (cpd)
  - Most impactful: **S2** and **M2**
- To quantify subsurface hydro-geomechanical properties, the **amplitude and phase** of the constituents need to be estimated from noisy measurements:

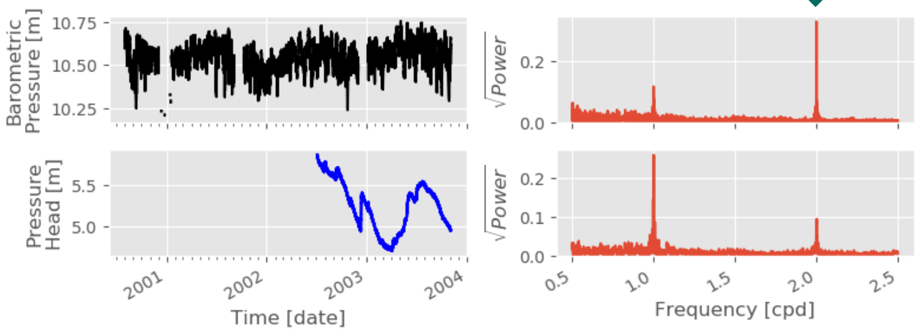
Table 1
Table of Major Tidal Components Ordered According to Frequency in Cycles per Day (cpd)

Darwinian name	Frequency (cpd)	Tidal potential (m^2/s^2)	Tidal gravity variation (m/s^2)	Tidal dilation (–)	Description	Attribution
Diurnal						
O_1	0.929536	5.363385	8.26E-06	3.347E-08	Principal lunar diurnal	Earth
M_1	0.966446	10.286769	1.58E-05	6.419E-08	Lunar diurnal	Earth
P_1	0.997262	7.407625	1.14E-05	4.622E-08	Diurnal lunar perigee	Earth
S_1	1.000000				Principal solar atmospheric pressure (thermal)	Atmosphere
K_1	1.002738	22.924982	3.53E-05	1.431E-07	Lunar solar diurnal	Earth
Semidiurnal						
N_2	1.895982	12.963403	1.996E-05	8.089E-08	Lunar elliptic semidiurnal (variation in Moon distance)	Earth
M_2	1.932274	42.060943	6.477E-05	2.625E-07	Principal lunar semidiurnal	Earth
S_2	2.000000	19.309855	2.973E-05	1.205E-07	Principal solar semidiurnal	Atmosphere/Earth
K_2	2.005476	11.791770	1.816E-05	7.358E-08	Lunar Solar Semidiurnal	Earth

BUT: How accurate and reliable are estimates, and what signal analysis methods work best?

Atmospheric pressure record

Groundwater responds



Signal analysis methods

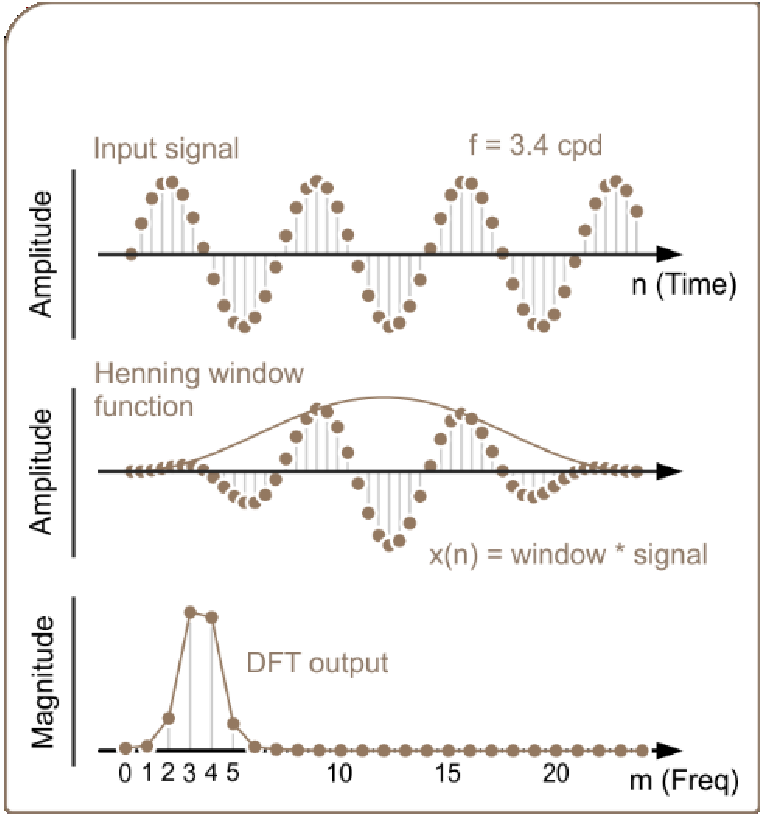
Karlsruher Institut für Technologie

Discrete Fourier Transform (DFT)

- converts a finite sequence of uniformly-spaced samples in the time domain (e.g. groundwater head record) into a same-length sequence of uniformly-spaced samples in the frequency domain.

BUT:

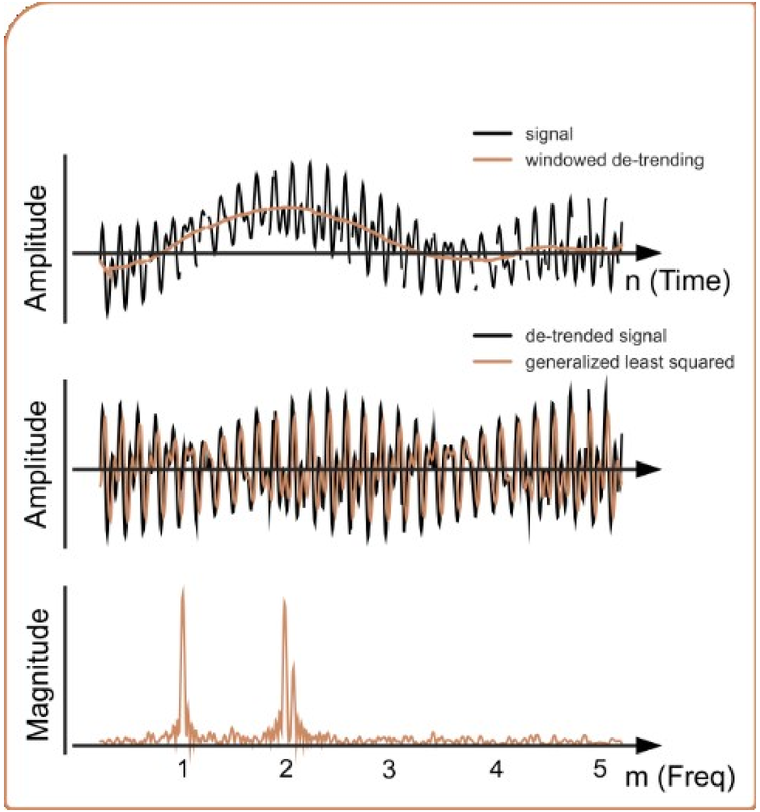
- Frequency resolution depends on record length
  - M2 and S2 at nearby frequencies - not reliably separated
- Records often contain gaps and irregularly spaced sampling
  - Data treatment required (interpolation or resampling)



Signal analysis methods

Generalized least squares amplitude and phase estimation (APES)

- Uses non-linear least squares to fit a harmonic function to the discrete time series measurement



Advantages:

- Handles missing values and data gaps (no interpolation), non-uniform sampling (no resampling)

1. How well does APES perform in estimating amplitude and phase when compared to DFT?

2. What are the practical data requirement for APES (sampling frequency, record duration, signal-to-noise ratio, signal quantisation and data gaps) for which an accurate extraction of harmonic constituents is guaranteed?

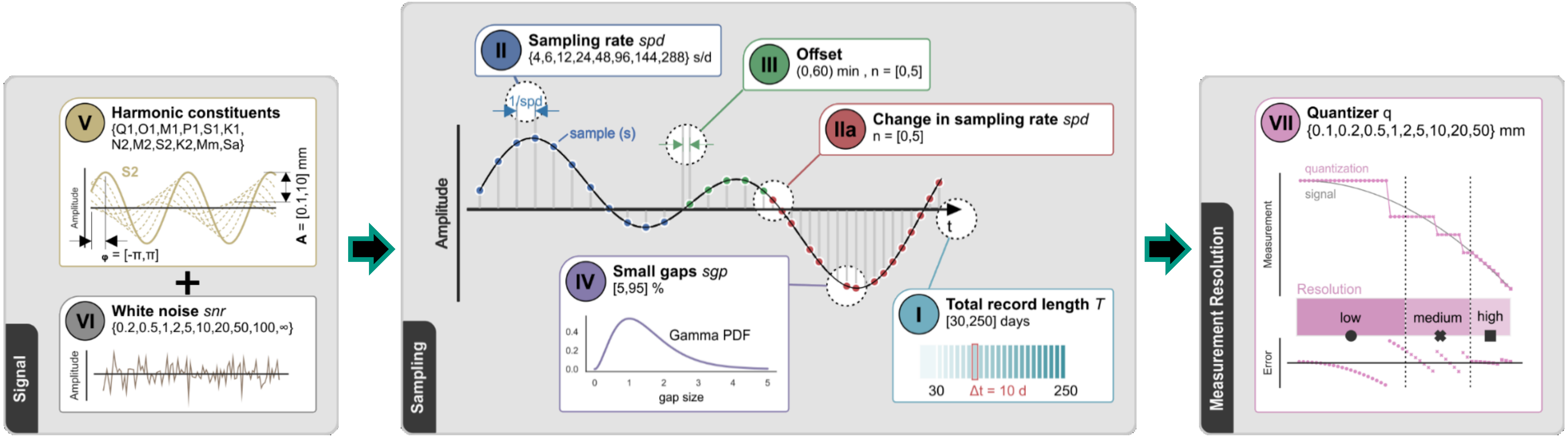
QUESTIONS

Workflow: generate synthetic data sets

Two general types of data set configurations:

- uniformly sampled data with no missing values
- non-uniformly sampled data with missing values, varying sampling rates and sampling time offsets.

Total of ~ 300,000 datasets with varying signal and sampling parameter combinations

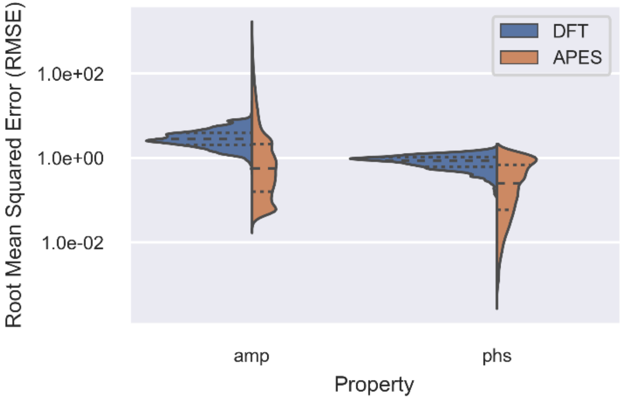


 06.05.2020 Institut für Angewandte Geowissenschaften

APES vs DFT: for uniformly sampled data

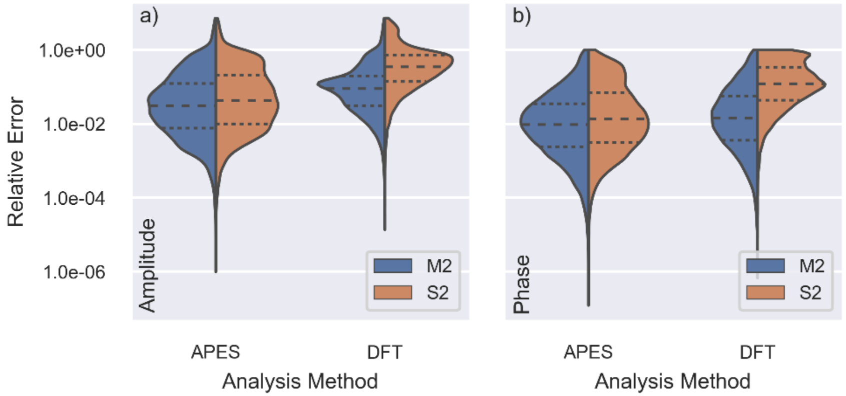
Overall

- APES generally performs better in estimating both Phase and Amplitude, but has larger spread



M2 and S2

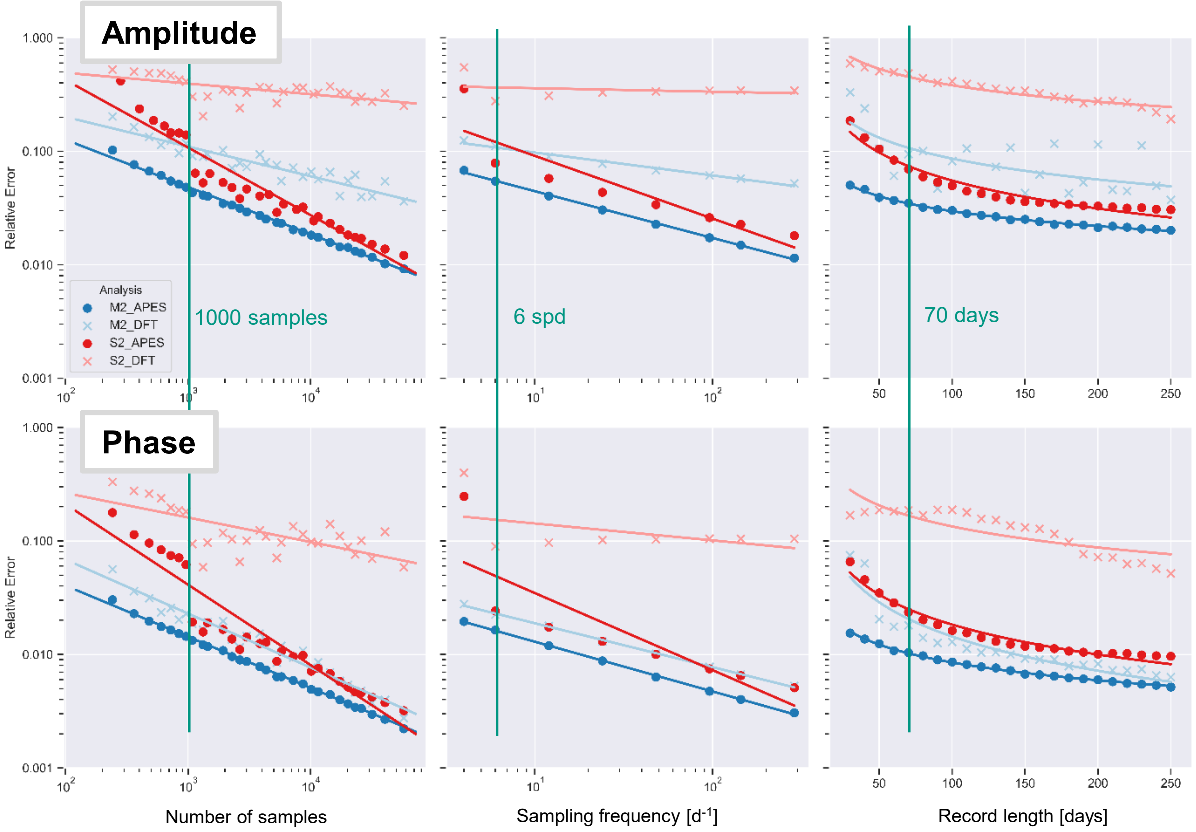
- APES superior in distinguishing amplitude and phase of close by frequencies
  -> better S2 estimate



APES vs DFT: for uniformly sampled data

Sampling parameters:

- APES more robust than DFT across full range of sampling parameters
- S2 generally more difficult to estimate! -> interferance with other constituents?
- Minimum general criteria:
  - Sharp decrease in S2 relative error at around **1000 sampling points** and **>= 6 samples per day**
  - Similar tendency for a record duration of **~ 70 days**.



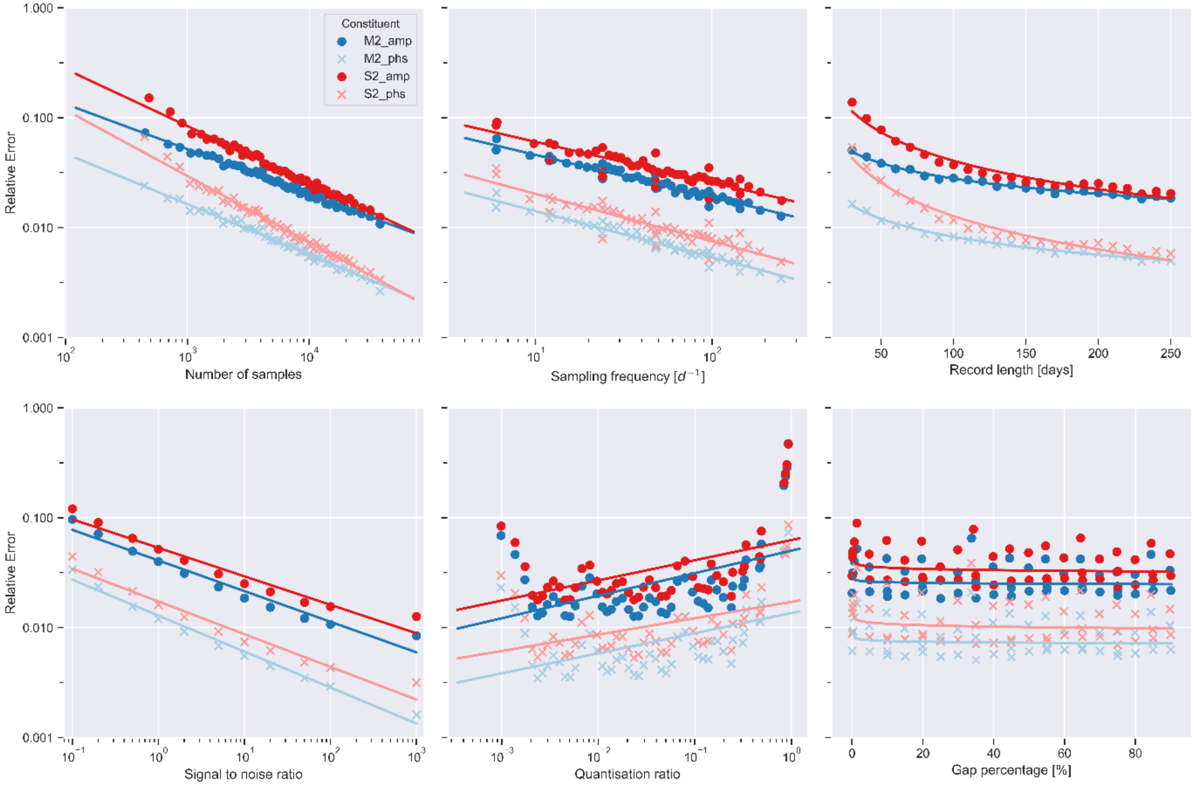
APES: for non-uniformly sampled data

Non-uniformities:

- Percentage of gaps has no overall effect on performance of the APES

General trends:

- An increase in number of samples has a strong overall effect on the error.
- Quantisation becomes important where its value is similar to the power of the signal.



Tanks for your attention!!!

