

Optimizing Image Analysis Processing in Thin Transparent Aquifers

Application to Pixel Wise Regression of Salt-Water Intrusion

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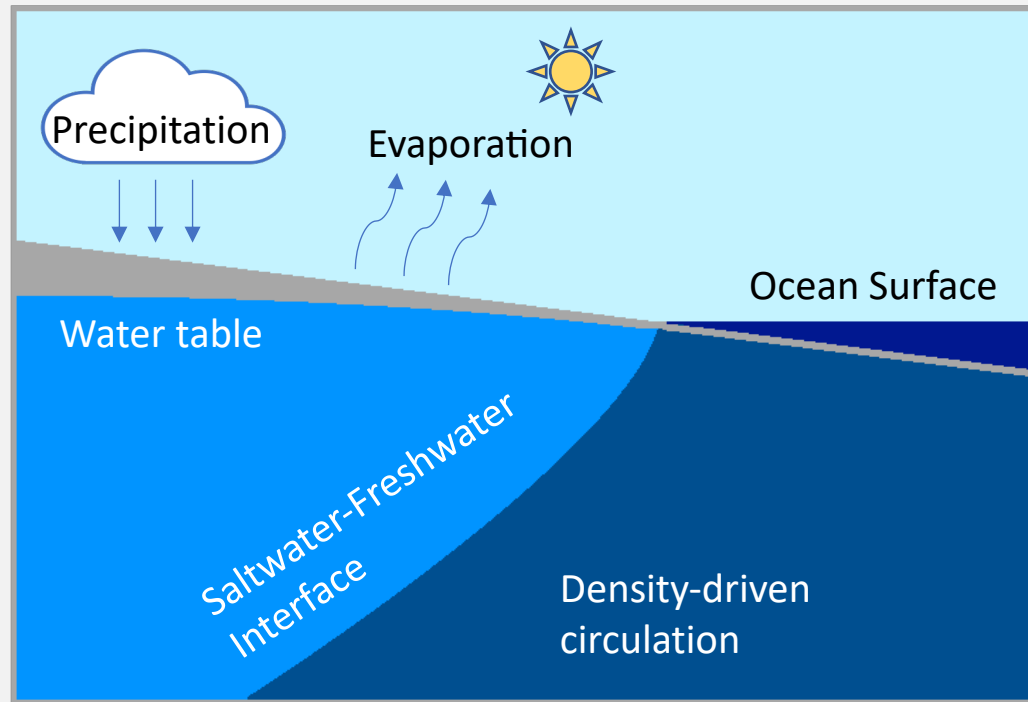
Outline

- Experiment & Motivations
- Pixel-wise regression procedure:
 - Power-law fitting (grayscale, standard)
 - Objectives
- Novel alternatives:
 - Laurent series fitting (grayscale)
 - Bead-correcting Beer-Lambert fitting (monochromatic)
 - Reduction-deviation metric fitting (color image)
- Comparison & Summary



Lab device & camera setup

Background: Salt Water Intrusion (SWI)

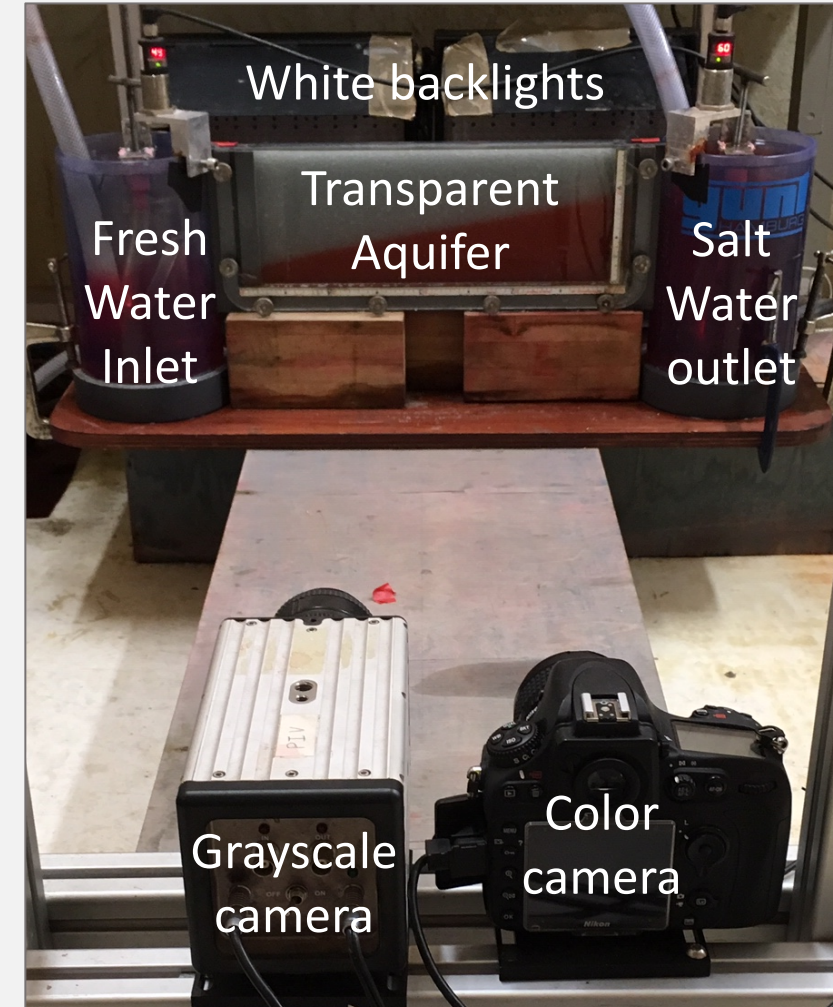


Salt water intrusion in a coastal aquifer

- SWI increases via anthropogenic environmental changes:
 - Pumping
 - Evapotranspiration (e.g. higher temperatures and crops)
 - Urban runoff
 - Sea level rise
- Physical complexity of systems
 - Pressure-density-diffusion transport in porous media
 - Ordered heterogeneity: Stratification and fractures
 - Random heterogeneity
 - Tidal/seasonal variability
 - Pumping
- *Important and complex system that needs rigorous experimental study*

Experimental System

- Transparent aquifer (1 cm thick) is filled with glass beads of approximately 1 mm diameter
- Head-driven fresh water flows from left
- Salt and red dye (Allura Red) pre-mixed with water in separate tank
- Salt water recirculates by density-driven flow from right reservoir
- White LED backlights illuminate the transparent aquifer
- Cameras (grayscale and color) record the state of the aquifer
- *Transparent aquifer setup gives precise information on length of SW intrusion and mixing zone of interface*



Smaller SWI sandbox apparatus with original (left) and new (right) cameras

Pixel-wise Image Post-Processing

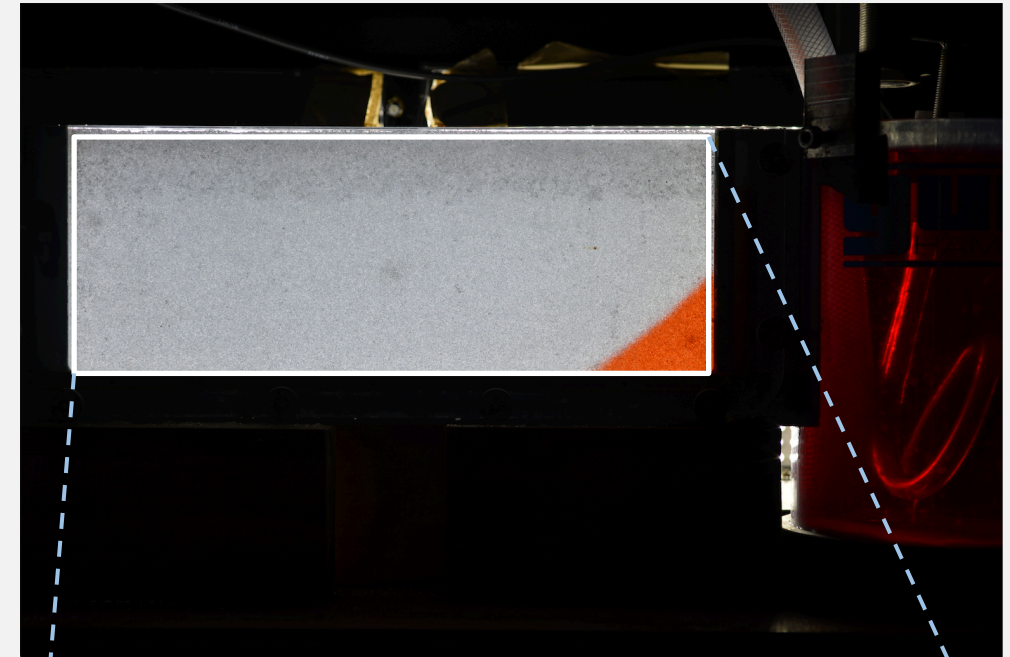
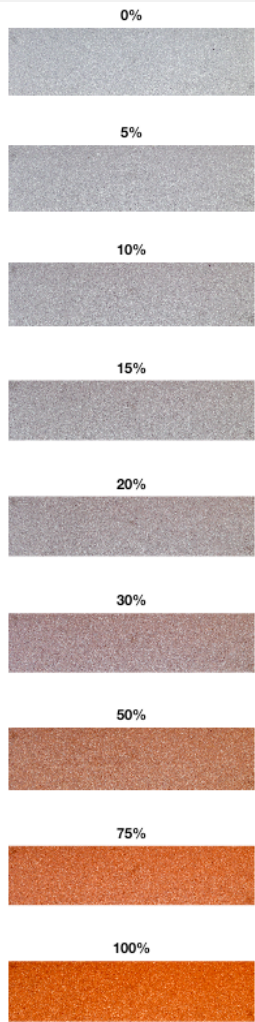
Experimental Procedure

- Prior to intrusion test: 8 calibration images taken of aquifer filled with fractional amounts of saltwater
- Aquifer flushed to pure water, then SW fills reservoir to intrude into aquifer
 - Images taken every 5 minutes to steady state interface
- Experiments prior to this work were carried out with grayscale camera only.
- To take calibrations images, 1 intrusion test requires an extended workday.

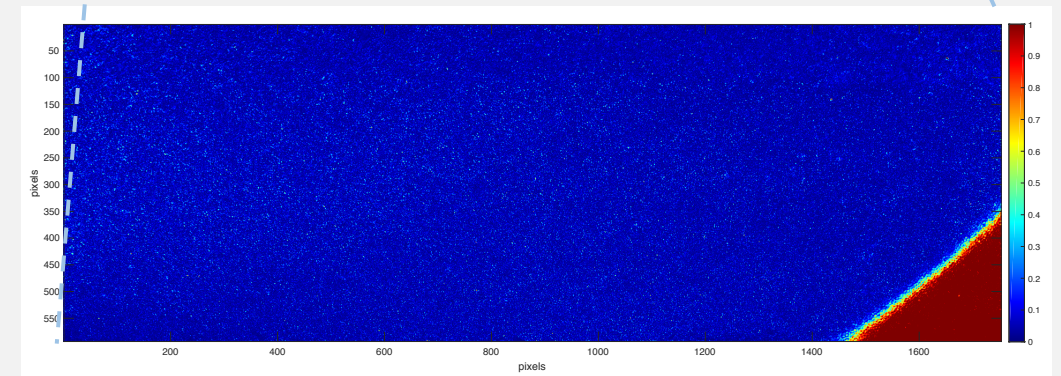
Post-processing Procedure

- Crop image set automatically
- At each pixel, calibrate the fitting function via the dilution images.
- Apply fitting sets to intrusion images

RGB calibration images



Experiment Image (RGB camera)



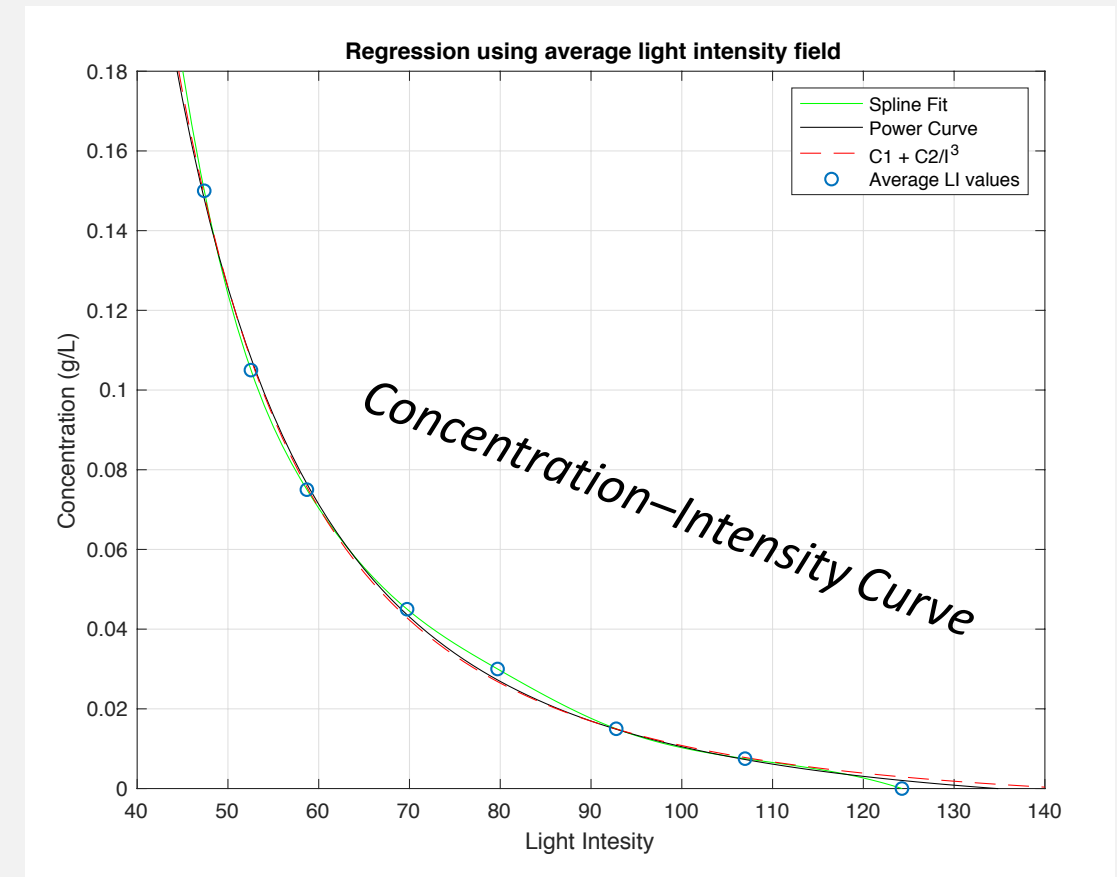
Power Law Fitting

- Power Law Fitting (PLF) on grayscale images:

$$C = x_1 I^{x_2} - x_3$$

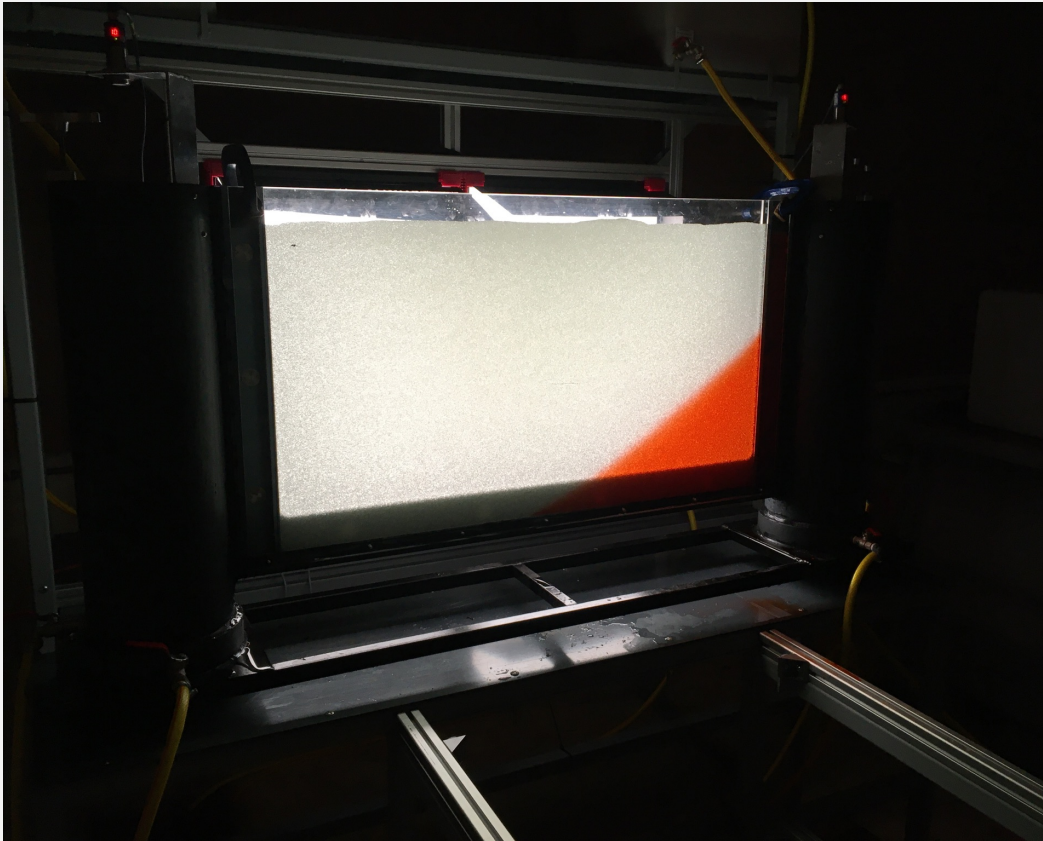
- Applied at each pixel to determine coefficient values, x_i , for light curve
- Takes > **1 day** to calibrate full RGB image!
- This specific power-law equation cannot be linearized
 - Non-linear optimization is **computationally expensive** relative to linear optimization
- For average data: $x_2 = -2.86$
- Less accurate simplification of fitting:

$$C = x_1 + x_2 I^{-3}$$



Power Laws fitting example calibration data
GR power law (black) & simplified law (dashed)

Objectives



**New larger-scale experimental apparatus
(10x larger aquifer volume)**

Goals for Optimization

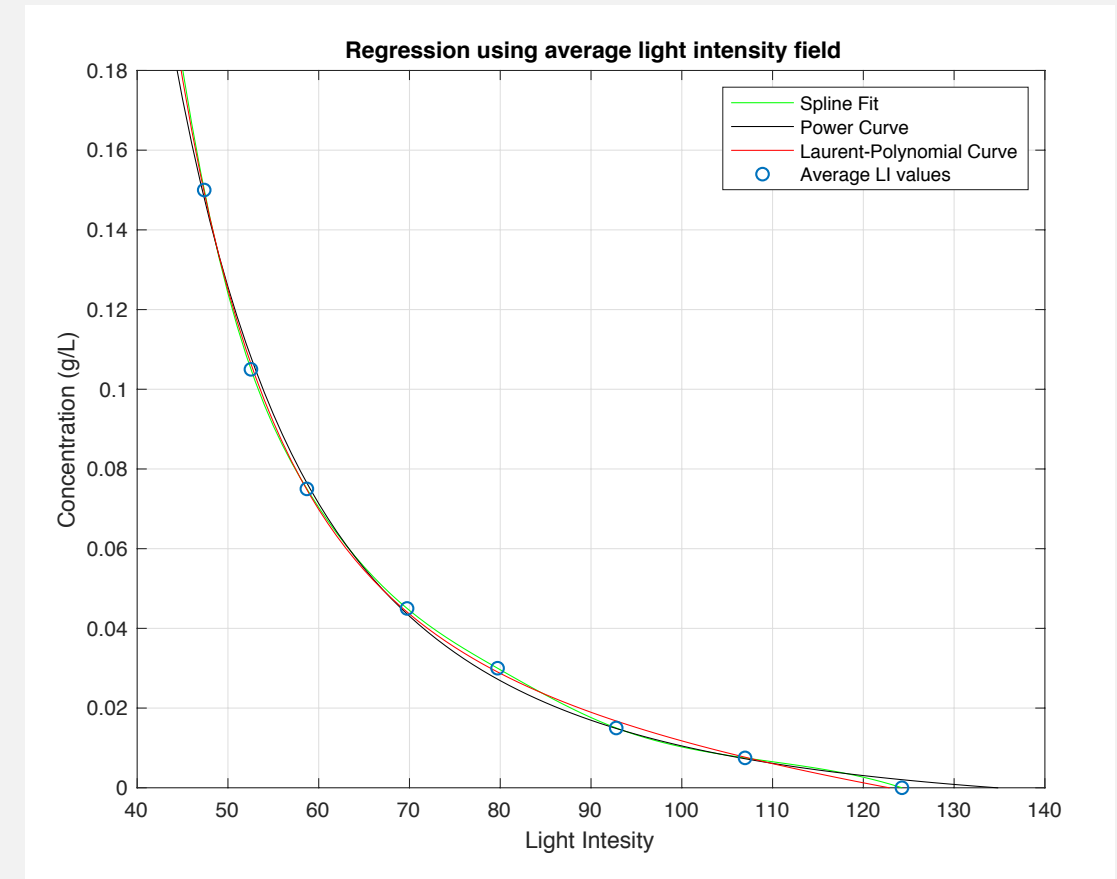
- *Speed up, simplify, and improve accuracy of experimental process at all stages: Setup & calibration, intrusion runs, and data processing*
- **Simplicity:** Focus on *pixel-wise* methods using simple, fast fitting functions
- **Speed up:** Improve post-processing times by optimizing numerical methods
- **Efficiency:** Reduce number of calibrations to speed up time of experiment, and decrease water and salt resource use

Laurent Series Fitting

- Expanding linear regression equation into Laurent Series (LSF)

$$C = x_1 + x_2 I^{-1} + x_3 I^{-2} + x_4 I^{-3}$$

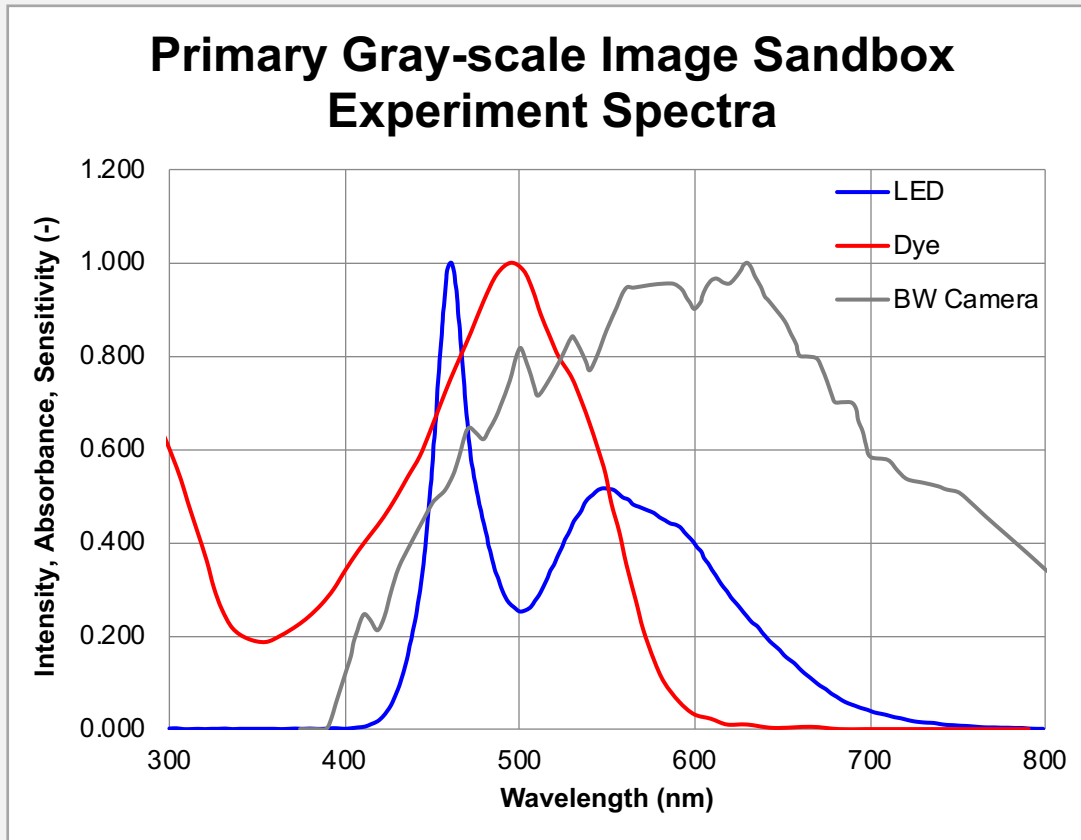
- Significant benefits to LSF
 - Simpler computational method via **linear regression** solution
 - Radical speedup: **3 minutes** ($>10^3$ x **faster** than power law!)
 - Improved accuracy near extremes
- Some concerns with LSF method
 - Not a physically motivated method
 - Subject to errors near low-concentration limit (non-monotonic)
 - Does not simplify experimental procedure (similar # of calibrations)



Laurent series fitting example calibration data

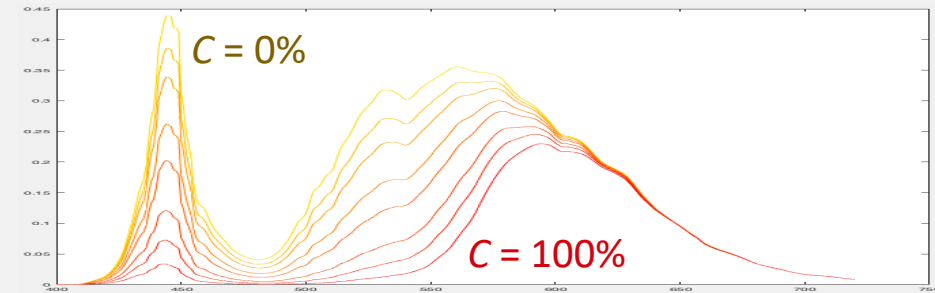
Laurent series (red) visually gives best fit among methods

Color Absorption Behavior Prediction with Beer–Lambert Law

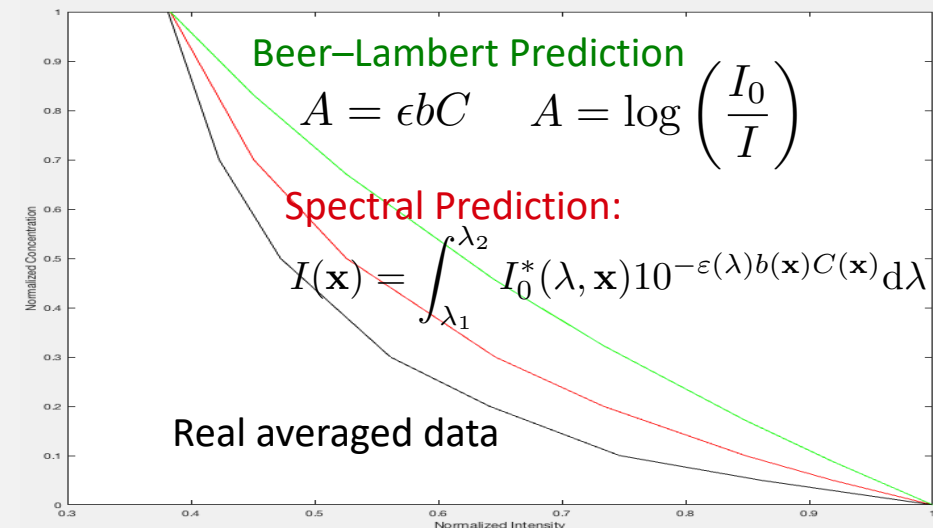


Backlights, Red dye, and BW-camera spectra

Sensitivity of camera is poorly correlated with dye absorption



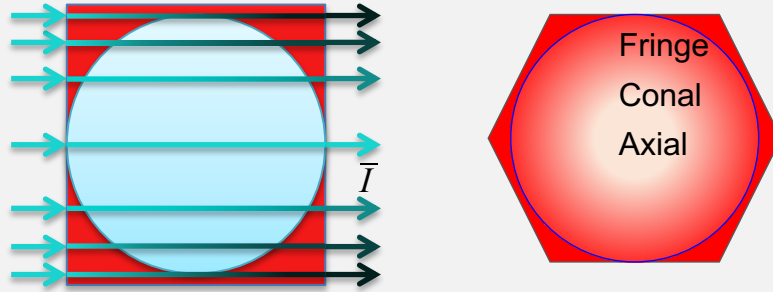
Spectral prediction for BW-camera after white LED light passes through red dye



C-I Curve for spectral prediction

Mismatch indicates additional effects in spectrum

Bead-Correcting Beer–Lambert Fitting

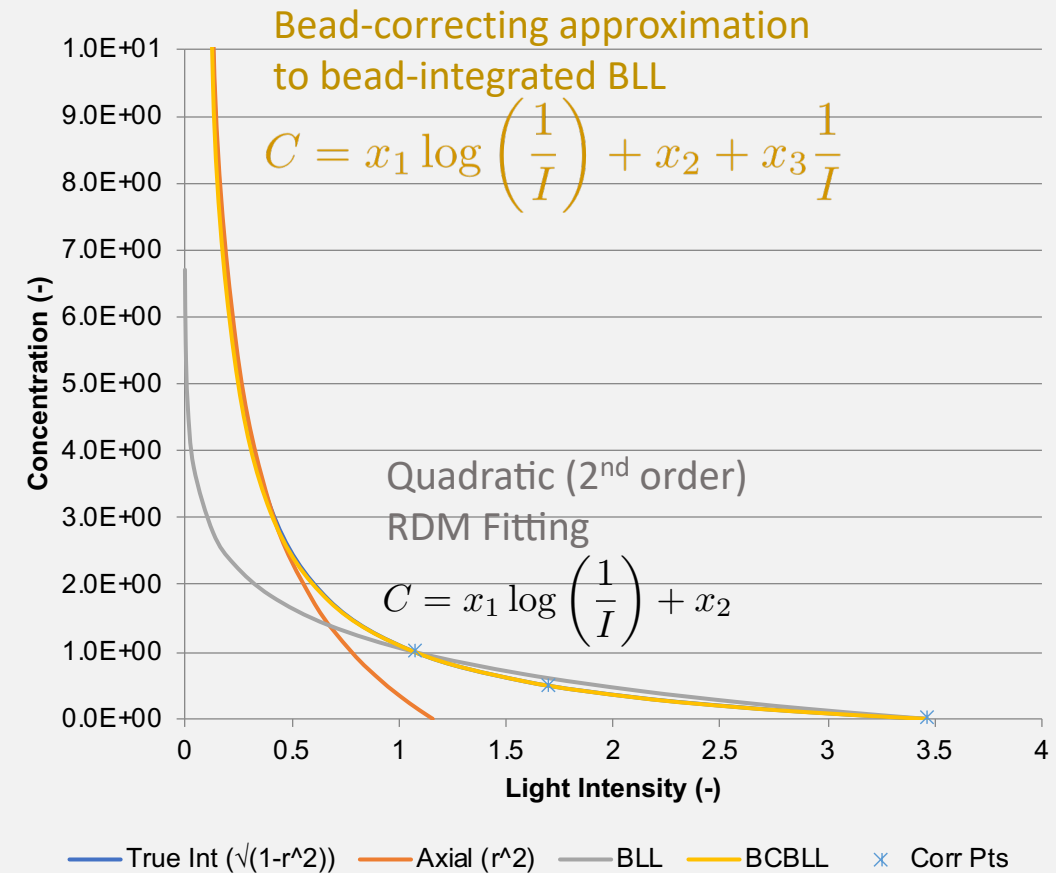


Light transmission through cross-section

- Lowest-order light decay is $1/C$ going through axis, other terms are higher order with e^{-C} like in the Beer–Lambert Law
- Solid-solid contact points hypothesized to dominate light transmission in high-concentrations
- Suggests adding inverse intensity to BLL

$$C = x_1 \log \left(\frac{1}{I} \right) + x_2 + x_3 \frac{1}{I}$$

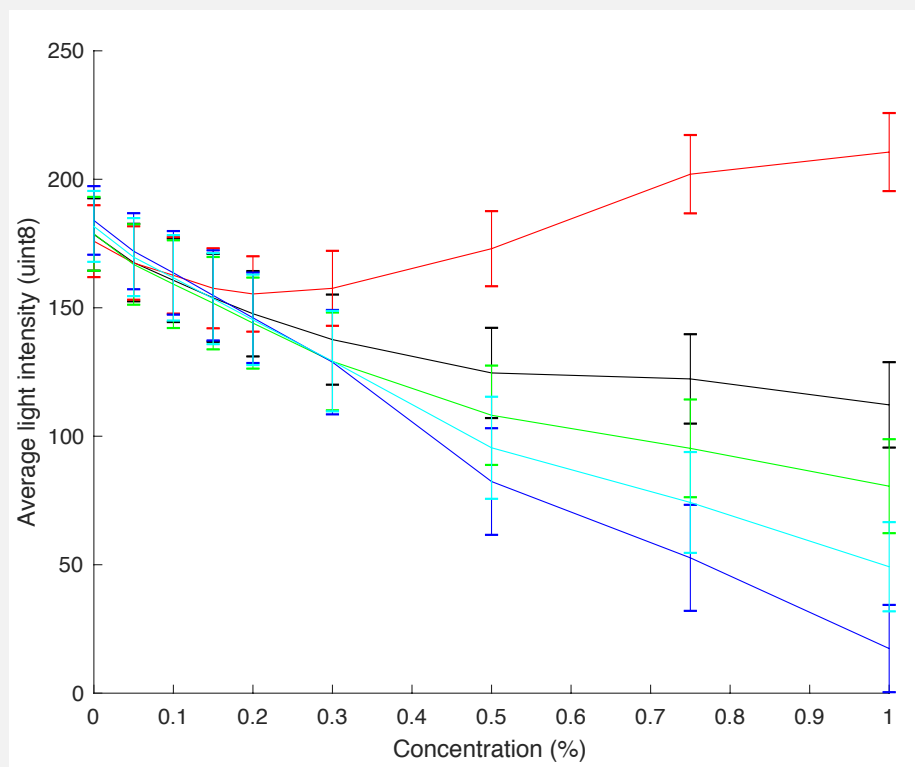
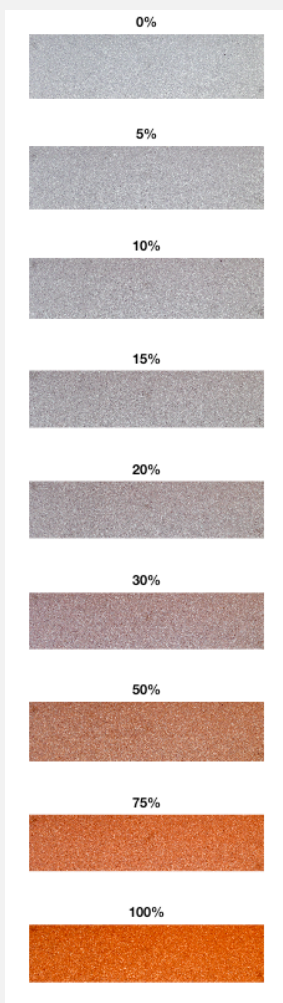
- Asymptotic expansion gives a Laurent series.
- Ideally applied to filtered monochrome pictures



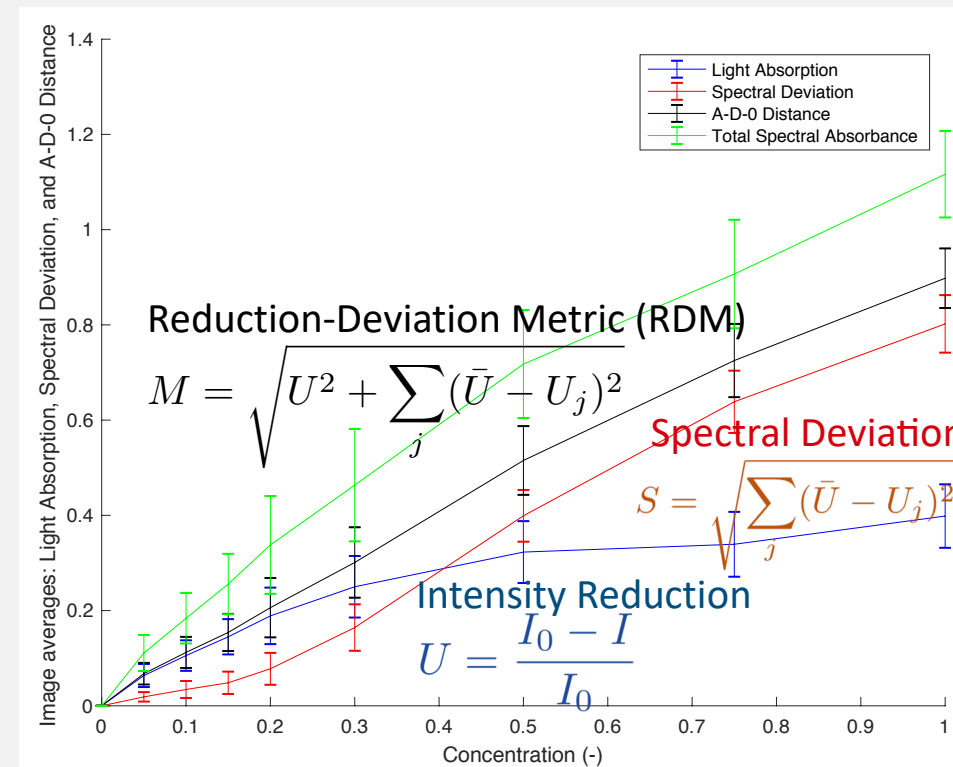
C-I Curve: Comparison of numerical integrals of theoretical monochromatic data with BLL & BCBLL

Real Color Absorption-Emission Behavior

RGB calibration images

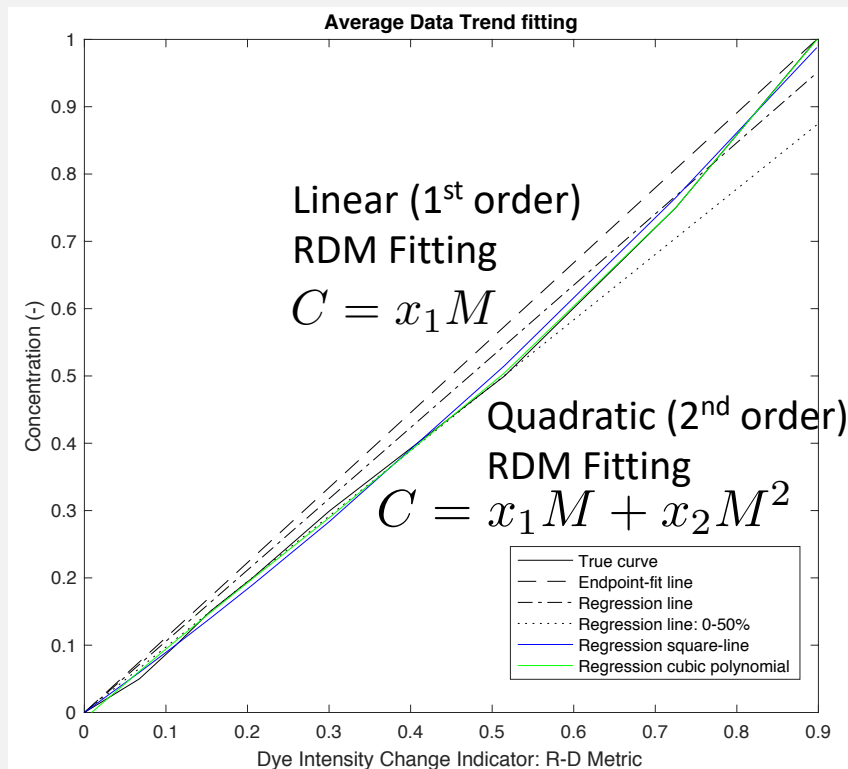


RGB Intensity vs. Concentration curve
 Increasing Red intensity
 Deviation between color

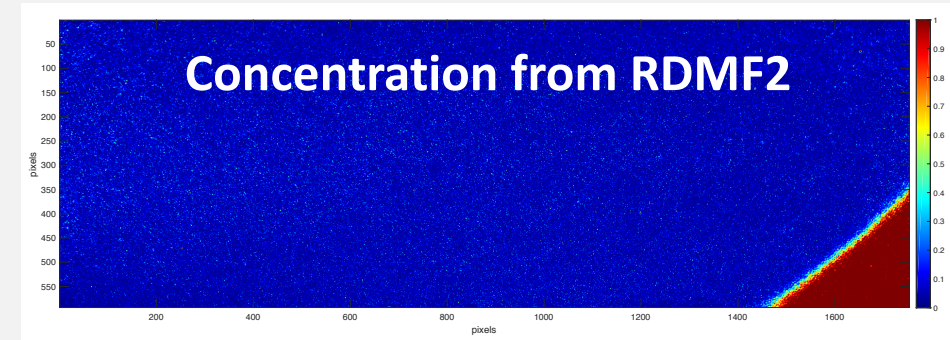


Dye effect quantities with salt concentration
 Combination of reduction and deviation
 (RDM) is nearly linear

Reduction–Deviation Metric Fitting

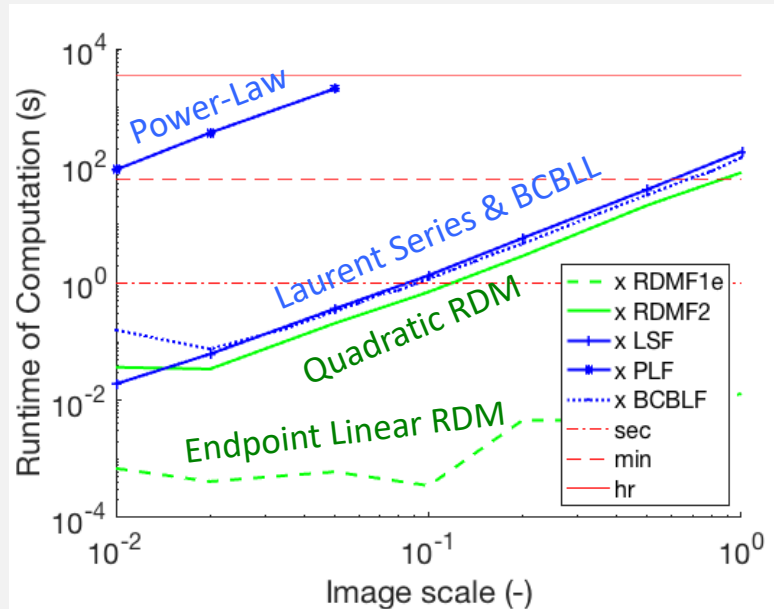


**Concentration regression on metric
(C-M curve)**



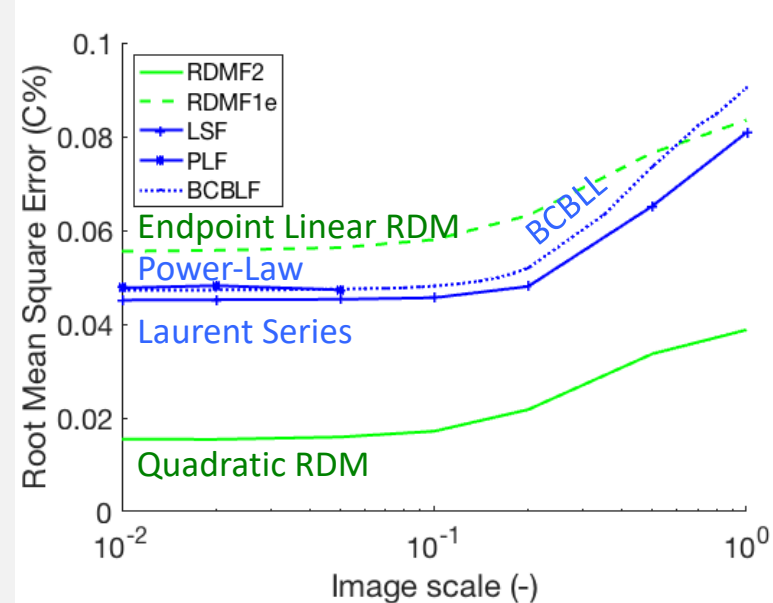
- Low-order functions of the Reduction-Deviation metric reproduce accurate concentrations
- Near-linear influence of dye on metric-based light curve; requires less calibration
- Calculation of RDM field over image and pixel-wise fitting coefficients are fast
- Fitting functions are stable and monotonic
- Requires RGB camera
- *Significant improvements over grayscale methods*

Performance of Methods



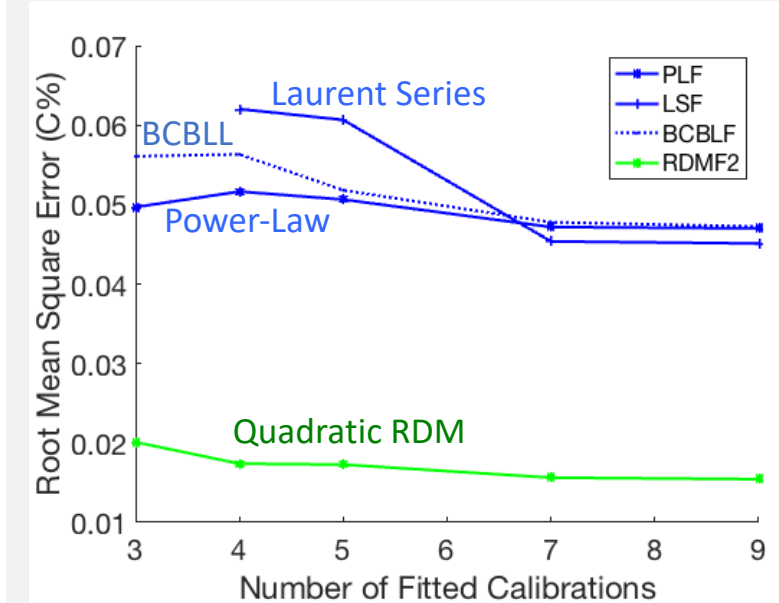
Fitting coefficient computation time vs. downscaling

- Power law fitting is three orders of magnitude slower than the other fitting methods
- Linear RDM is extremely fast



Calibration error vs. downscaling

- Grayscale methods have very similar error values
- The quadratic RDM is much more accurate than the other fittings
- Linear RDM is nearly as good as others



RMSE vs. number of calibration images (0.01 ds)

- For most, fewer fittings have only small impact on accuracy
- At least one more image than number of fitting function coefficients is best

Summary of Methods

Method	Post-processing time	Minimum Calibration Images	Accuracy	Fitting Function Stability	Sensor Requirements
Power Law (Standard) $C = x_1 I^{x_2} - x_3$	>1 day (nonlinear!)	5	Acceptable (5% error)	Stable, Monotonic	N/A (grayscale image)
Laurent Series $C = x_1 + x_2 I^{-1} + x_3 I^{-2} + x_4 I^{-3}$	3 minutes	6	Acceptable (5% error)	Non-monotonic	N/A (grayscale image)
Bead-Correcting BLL $C = x_1 \log\left(\frac{1}{I}\right) + x_2 + x_3 \frac{1}{I}$	2 minutes	5	Acceptable (5% error)	Non-monotonic	(Monochrome light filter?)
1 st order Reduction– Deviation Metric $C = x_1 M$	0.01 second	2	Acceptable (6% error)	Linear, monotonic	RGB camera
2 nd order Reduction– Deviation Metric $C = x_1 M + x_2 M^2$	1 minute	4	Best (1.5% error)	Monotonic	RGB camera

Conclusions

- Linear regression methods are much faster than nonlinear.
 - The Laurent Series Fitting has similar accuracy to the Power-Law Fitting, but is orders of magnitude more rapid to postprocess.
- Beads may significantly effect the experimental light curve.
 - The simpler, physics-based linear regression adapting the Beer–Lambert Law to this experiment performs as well as the other fitting functions.
- Allura Red dye causes spectral effects in RGB camera via spectral-variable absorbance and red colored fluorescence.
 - RGB specific methods perform better than grayscale intensity methods (e.g. PLF & LSF).
 - The second order Reduction-Deviation Metric method reduces the error by two-thirds.
- All novel methods improve post-processing times and can reduce the calibration cost of the experiment.
 - In particular, the first-order reduction-deviation metric fitting reduces the concentration estimation to two calibrations, while maintaining comparable accuracy.

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References

- Robinson, G., G. A. Hamill, and Ashraf A. Ahmed. “Automated image analysis for experimental investigations of salt water intrusion in coastal aquifers.” *Journal of Hydrology* 530 (2015): 350–360.
- Abdoulhalik, Antoifi, and Ashraf A. Ahmed. “Transience of seawater intrusion and retreat in response to incremental water-level variations.” *Hydrological processes* 32.17 (2018): 2721-2733.



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