

Shape asymmetry of rogue waves

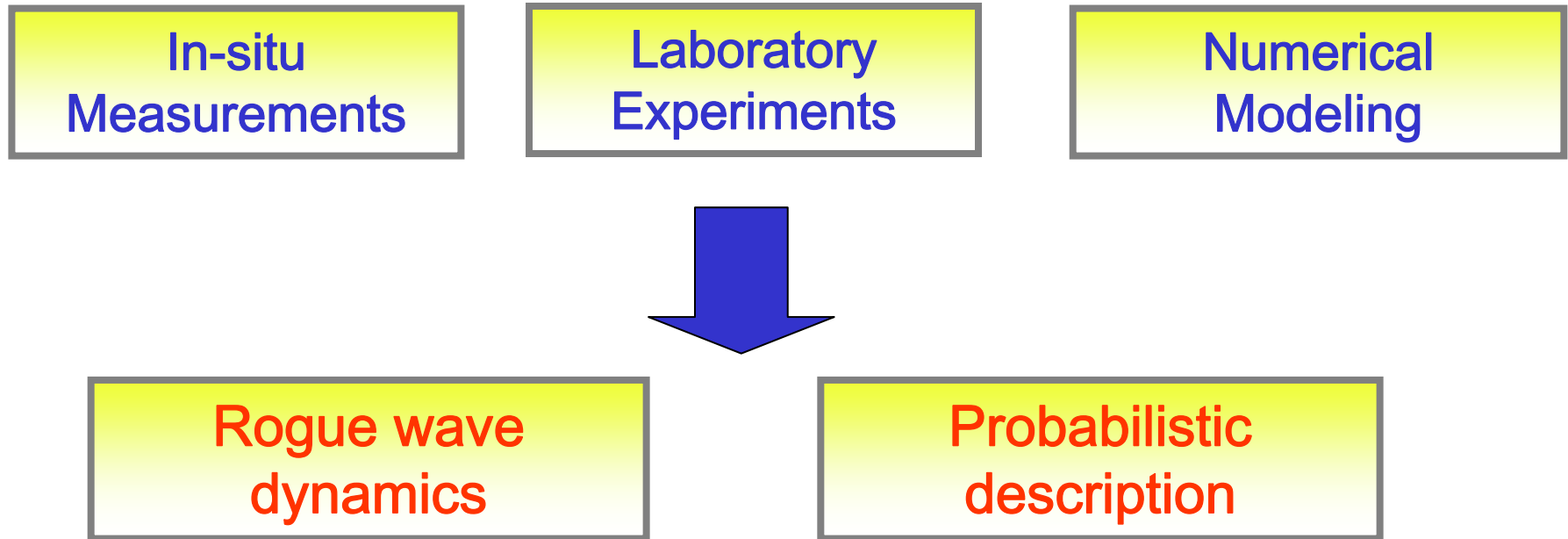
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Rogue Waves: Main approaches



Study: mechanisms of freak wave generation, conditions, characteristics of individual waves

Application: evaluation of wave impact on ships and marine stationary structures, safety methods,...

Study: probabilistic models, relations between spectral and statistical parameters

Application: forecast of hazardous weather and geophysical conditions, risk assessment of extreme waves, safety requirements of ship design and marine structures

Rogue Waves: Main approaches

Problems:

1. In-situ measurements :

- statistical inhomogeneity
- data unreliability
- one-point measurements

2. Approximate probabilistic models, kinetic equations:

- the use of approximations and assumptions questionable in the case of extreme waves

3. Laboratory experiments : expensive, time consuming, size limitation

4. Stochastic modelling

Direct numerical simulations of ensemble of quasi-random wave realizations:

- + Fully controllable conditions
- + Full information on waves
- + Accurate
- + Fast
- + Cheap

Direct numerical simulations

▪ Simulated by the **HOSM** (potential Euler eqs, West et al, 1987) with $M = 3$, **RK4** for the steps in time. The domain is double-periodic in space, 50×50 dominant wave periods.

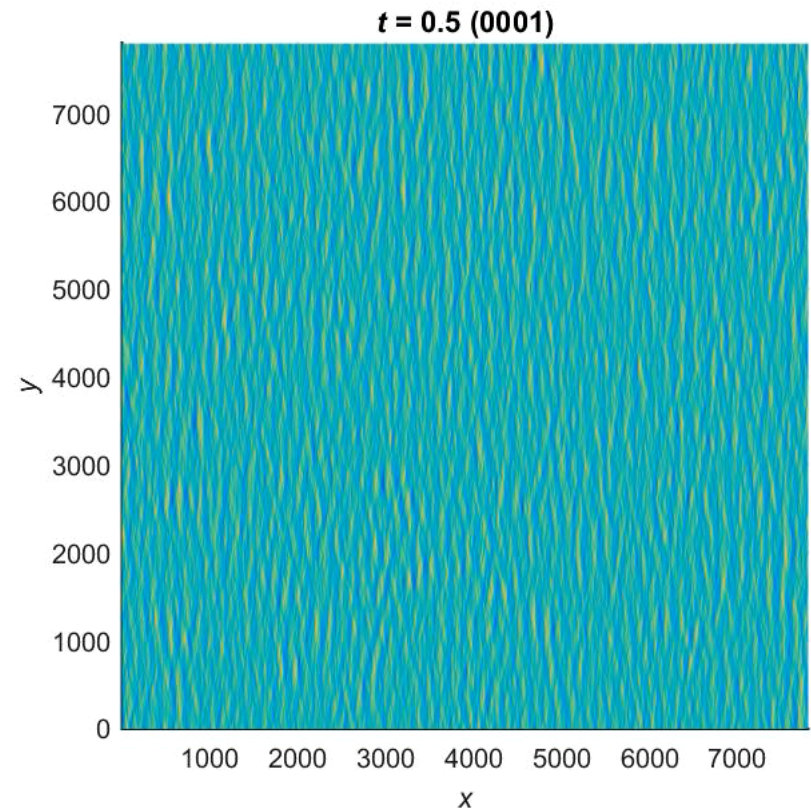
$t = 0$: Irregular waves with a given averaged **JONSWAP** spectrum and random phases.

$H_s \approx 3.5 \div 7$ m, $T_p = 10$ s

$t = 0 \dots 200$ s: Slow nonlinearity adjustment according to Dommermuth (2000)

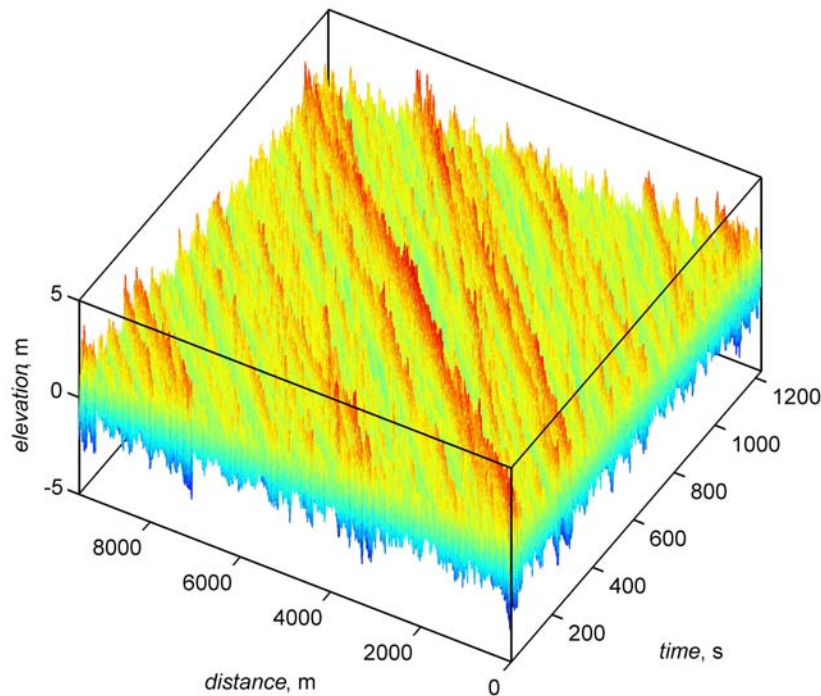
$t = 200 \dots 1400$ s: The simulation.
The data is recorded each 0.5 s of the simulated dynamics.

A. Slunyaev, A. Kokorina, Account of occasional wave breaking in numerical simulations of irregular water waves in the focus of the rogue wave problem. Water Waves. pp.1-20, (2019)



I. Unidirectional Waves

Initial conditions



The spectrum corresponds to the **North Sea** conditions (**JONSWAP**)

The output: sheets of wave fields in the domain **10 km x 20 min** with high resolution both in time and space

1 realization: 60 waves during 120 wave periods.

1 sea state: from 100 to 1000 realizations

A. Slunyaev, A. Sergeeva, I. Didenkulova, Rogue events in spatiotemporal numerical simulations of unidirectional waves in basins of different depth. Natural Hazards 84(2), 549-565 (2016).

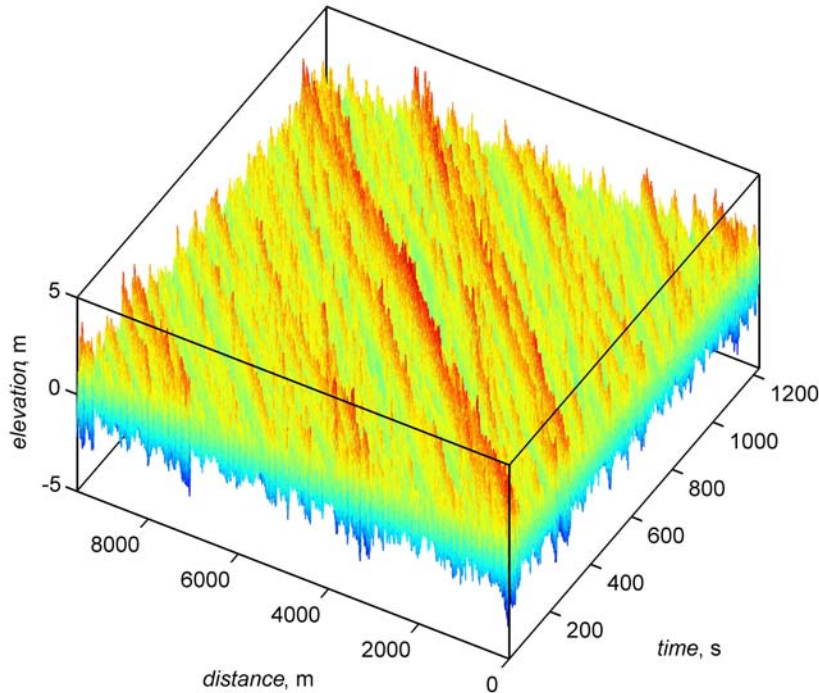
DOI: 10.1007/s11069-016-2430-x

A. Sergeeva, A. Slunyaev, Rogue waves, rogue events and extreme wave kinematics in spatio-temporal fields of simulated sea states. Nat. Hazards Earth Syst. Sci. Discuss. Vol. 1, 39-72 (2013).

doi:10.5194/nhessd-1-39-2013

I. Unidirectional Waves

Initial conditions



Objectives:

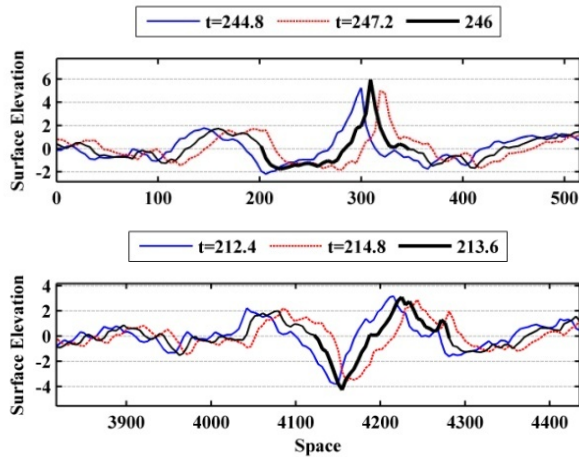
- Detection of rogue waves ($H > H_s$).
Probability functions
- Variety of rogue wave shapes
- Assembling rogue waves into 'rogue events'
- Joint statistics of surface elevation and velocity fields

A. Slunyaev, A. Sergeeva, I. Didenkulova, Rogue events in spatiotemporal numerical simulations of unidirectional waves in basins of different depth. Natural Hazards 84(2), 549-565 (2016). DOI: 10.1007/s11069-016-2430-x

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I. Unidirectional Waves

Portrait of rogue wave: Zero-crossing analysis for time series and space series



In-situ measurements at Brazilian coast

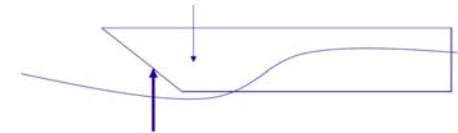
U.Ferreira, P.C. Liu and C. E. Parente GEOFIZIKA, V.21, 2004

in total 305 rogue waves

Rear slope waves :

197 waves (~65%)

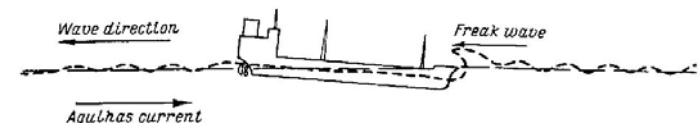
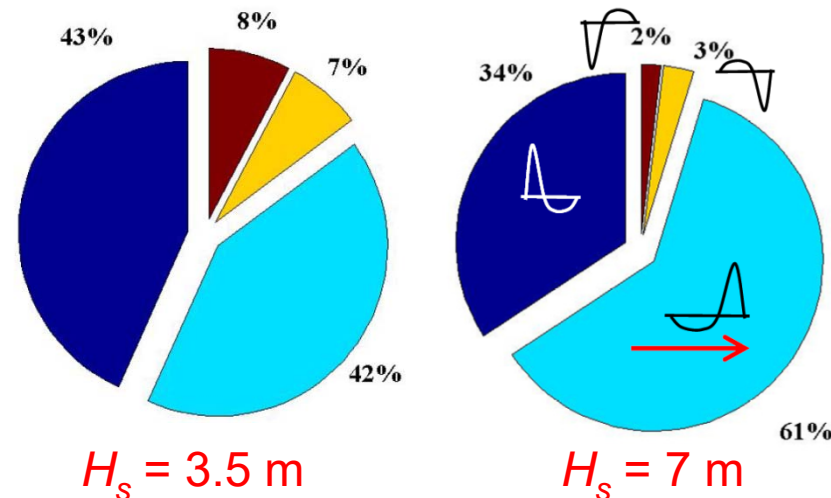
in numerical simulations ~ 63%



Front slope waves:

108 waves (~35%)

in numerical simulations ~37%



[Sergeeva & Slunyaev, 2013;
Slunyaev et al, 2016]

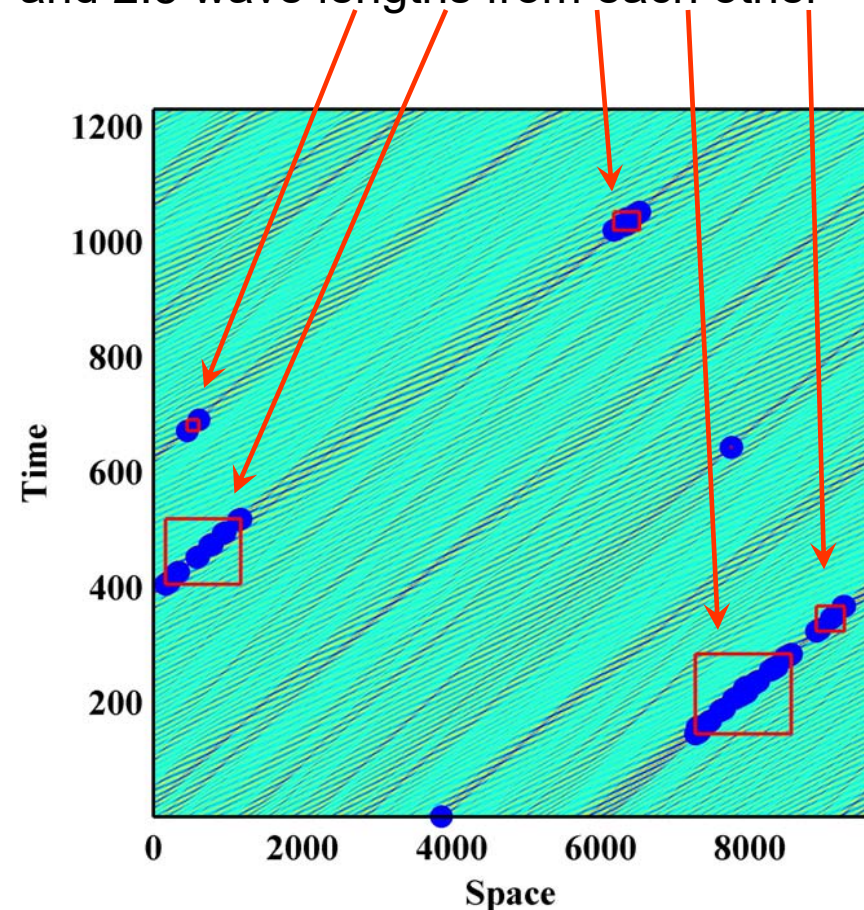
I. Unidirectional Waves

Rogue wave evolution: Rogue events

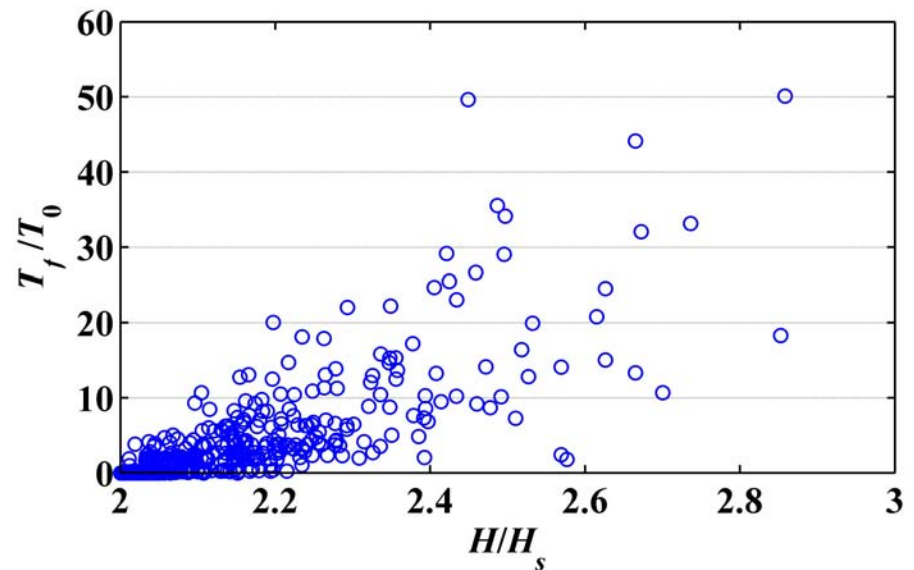
Step 1: detection of rogue waves ($H > 2H_s$)

Step 2: gathering rogue waves in events

Individual rogue waves are within 2.5 wave periods and 2.5 wave lengths from each other



$T_0 \sim 10$ s



Duration of a rogue event is up to 10 min!

More intense waves live for longer time

II. Directional Waves

Examples of 3D rogue wave events

Snapshots of the surface in the co-moving frame and the corresponding longitudinal wave sections during a long-living rogue wave event.

The sections are taken along the points where rogue waves are detected, shown on the surfaces by circles with strokes.

The circles show locations of the rogue wave crests; the strokes show directions to the deep troughs.

II. Directional Waves

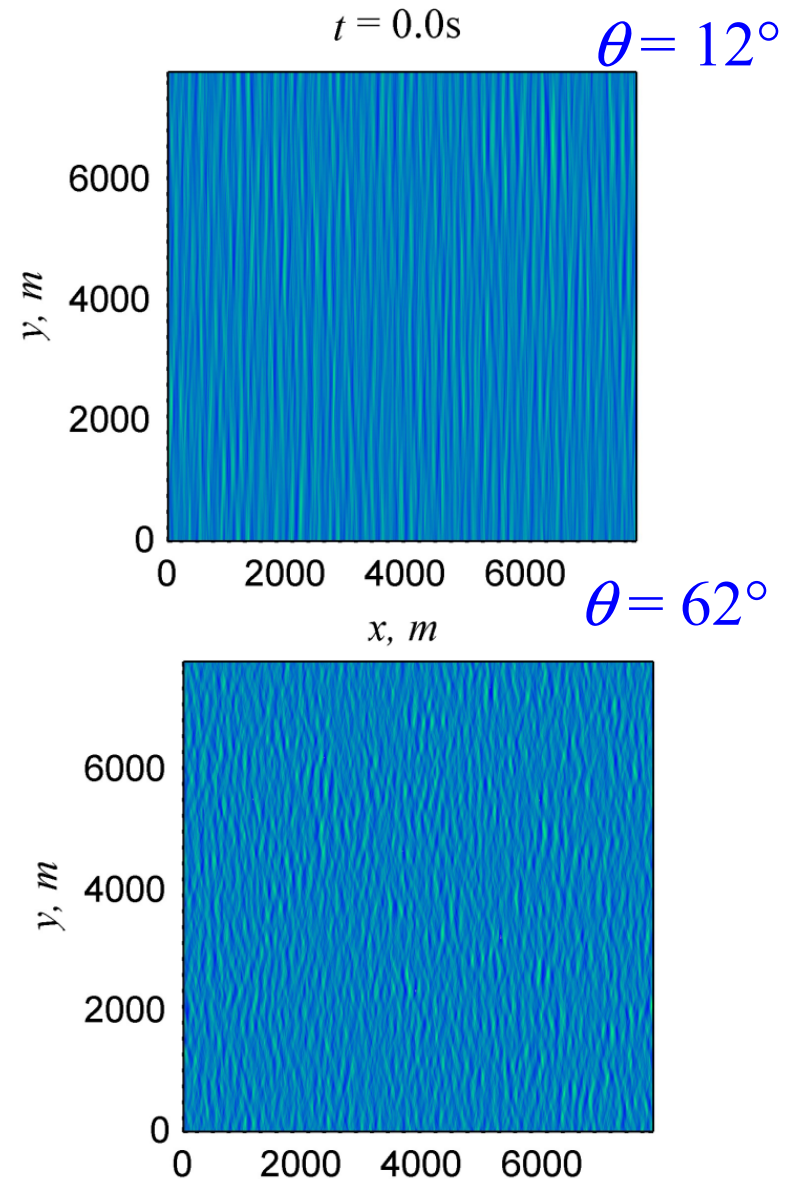
Initial conditions

Irregular waves with a given averaged JONSWAP spectrum ($T_p = 10\text{s}$, $\gamma = 3, 6$), random Fourier amplitudes and random phases $H_s \approx 3.5 - 7\text{ m}$.

The directional spreading is according to the \cos^2 distribution with $\theta = 5^\circ, 12^\circ, 62^\circ$ (following Xiao et al, 2013)

$$D(\chi) = \begin{cases} \frac{2}{\theta} \cos^2\left(\frac{\pi\chi}{\theta}\right), & |\chi| \leq \frac{\theta}{2} \\ 0, & |\chi| > \frac{\theta}{2} \end{cases}$$

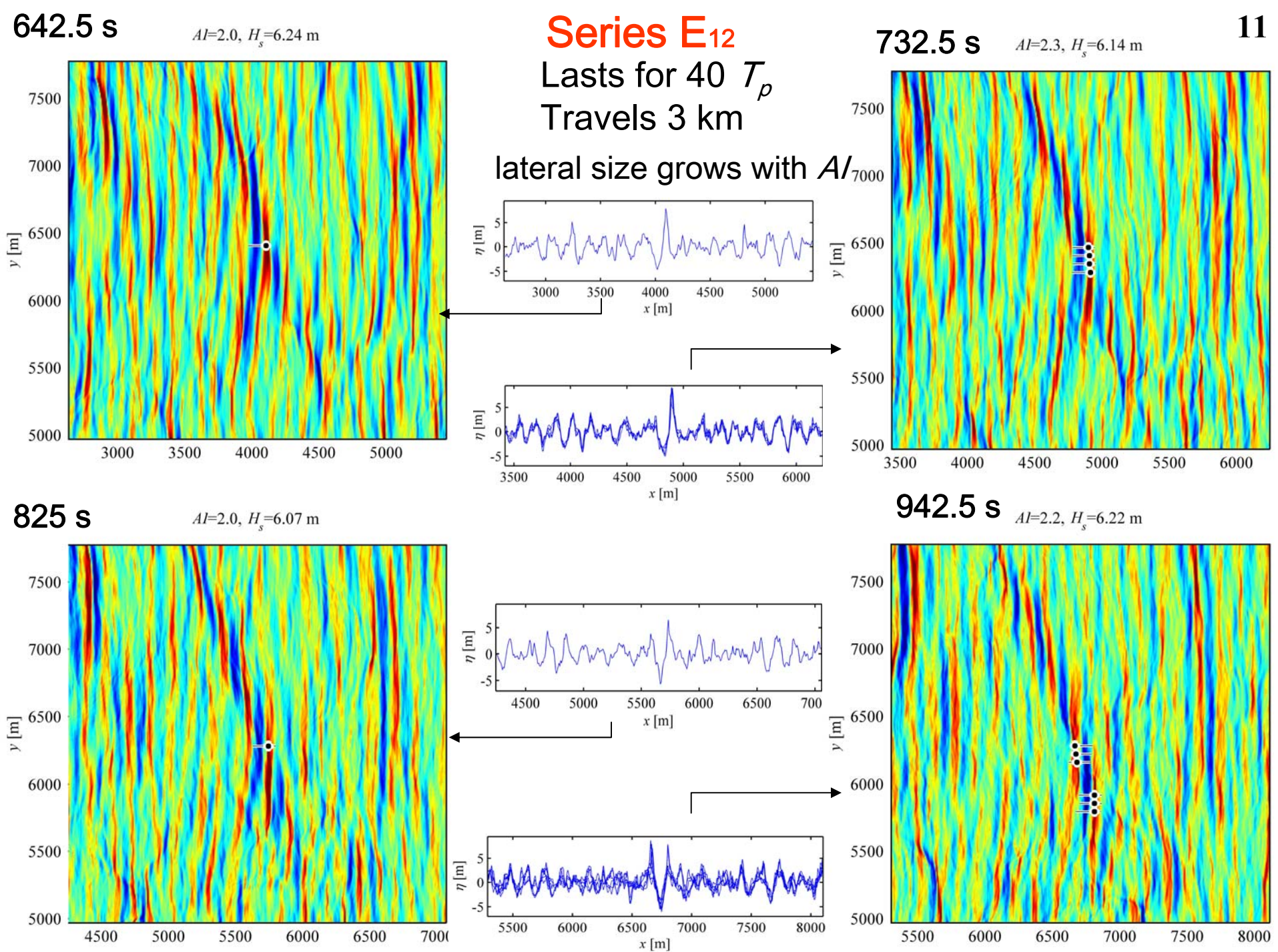
Simulated by the HOSM (West et al, 1987) with $M = 1, 3, 4$. The domain is double-periodic in space, it is of the size 50×50 dominant wave periods.



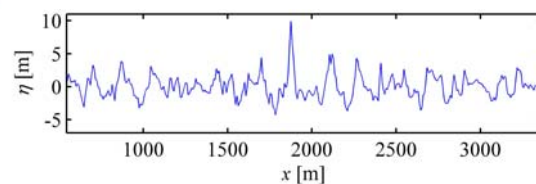
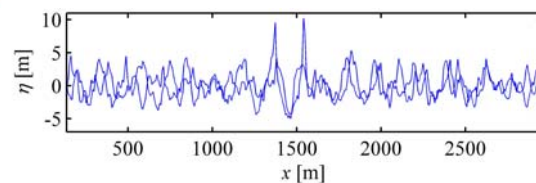
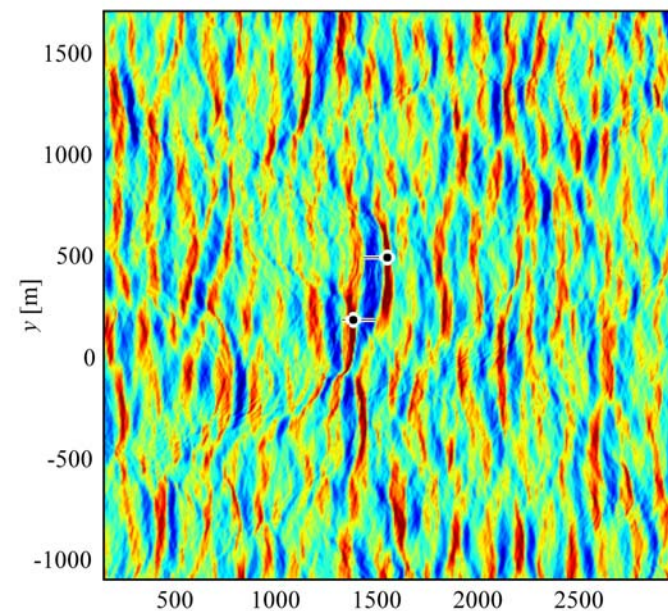
Series E₁₂

Lasts for 40 T_p
Travels 3 km

lateral size grows with $A/$



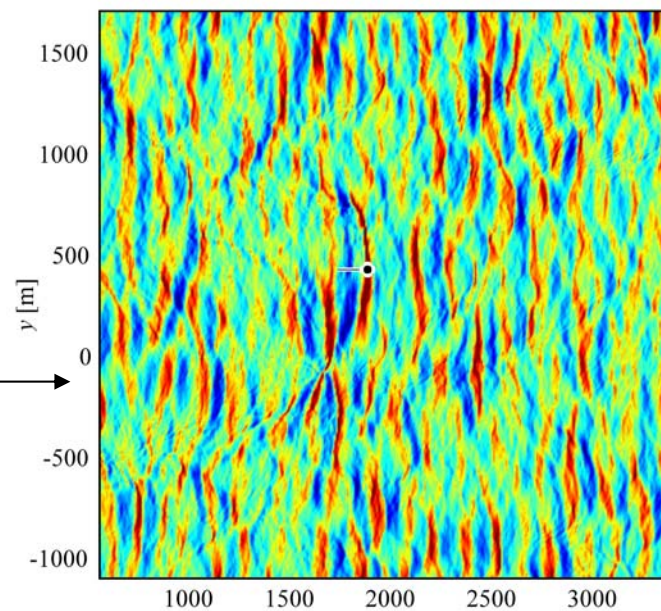
339 s

 $Al=2.1$, $H_s=7.05$ mSeries E³₆₂Lasts for $24\tau_\rho$ 

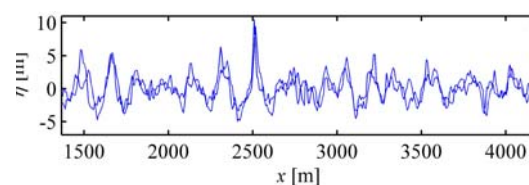
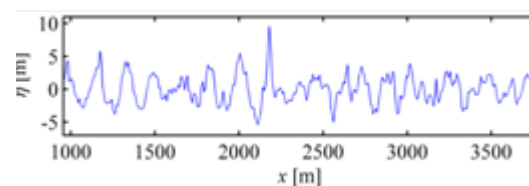
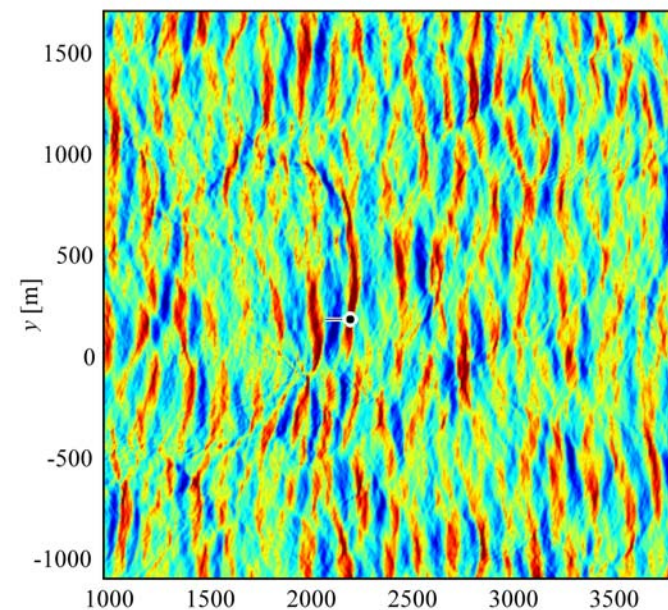
379 s

 $Al=2.0$, $H_s=6.95$ m

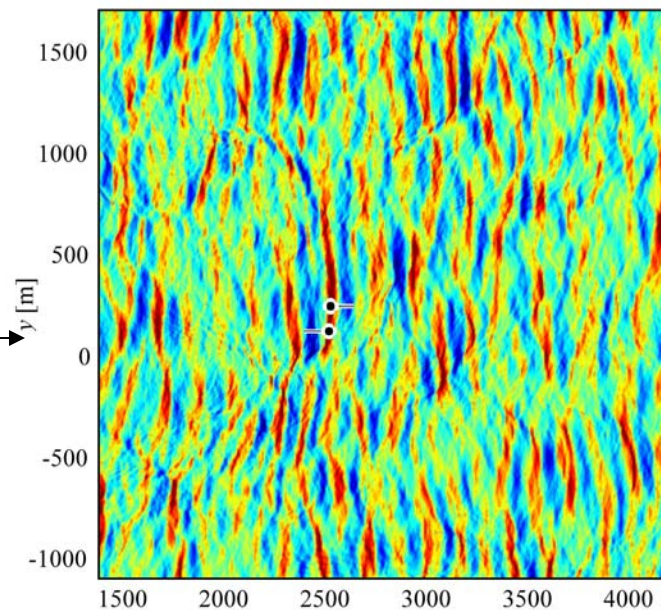
12



419 s

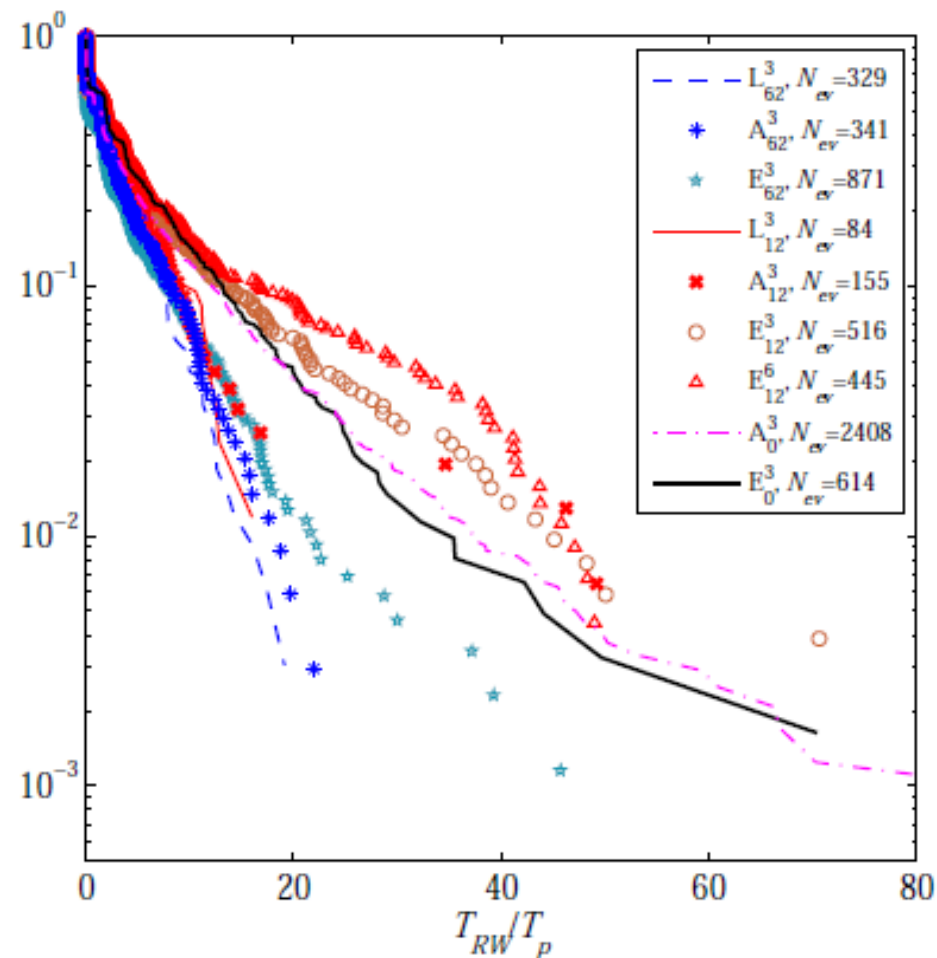
 $Al=2.1$, $H_s=7.02$ m

459 s

 $Al=2.2$, $H_s=7.11$ m

II. Directional Waves

Life-time distributions for 3D rogue events



Probability distribution of the duration of rogue-wave events T_{RW}

$$P(T_n) = \frac{n}{N_{ev} + 1}$$

$$n = 1, 2, \dots, N_{ev}$$

A_{θ}^{γ} - small steepness, $H_s = 3.5$ m

E_{θ}^{γ} - large steepness, $H_s = 7$ m

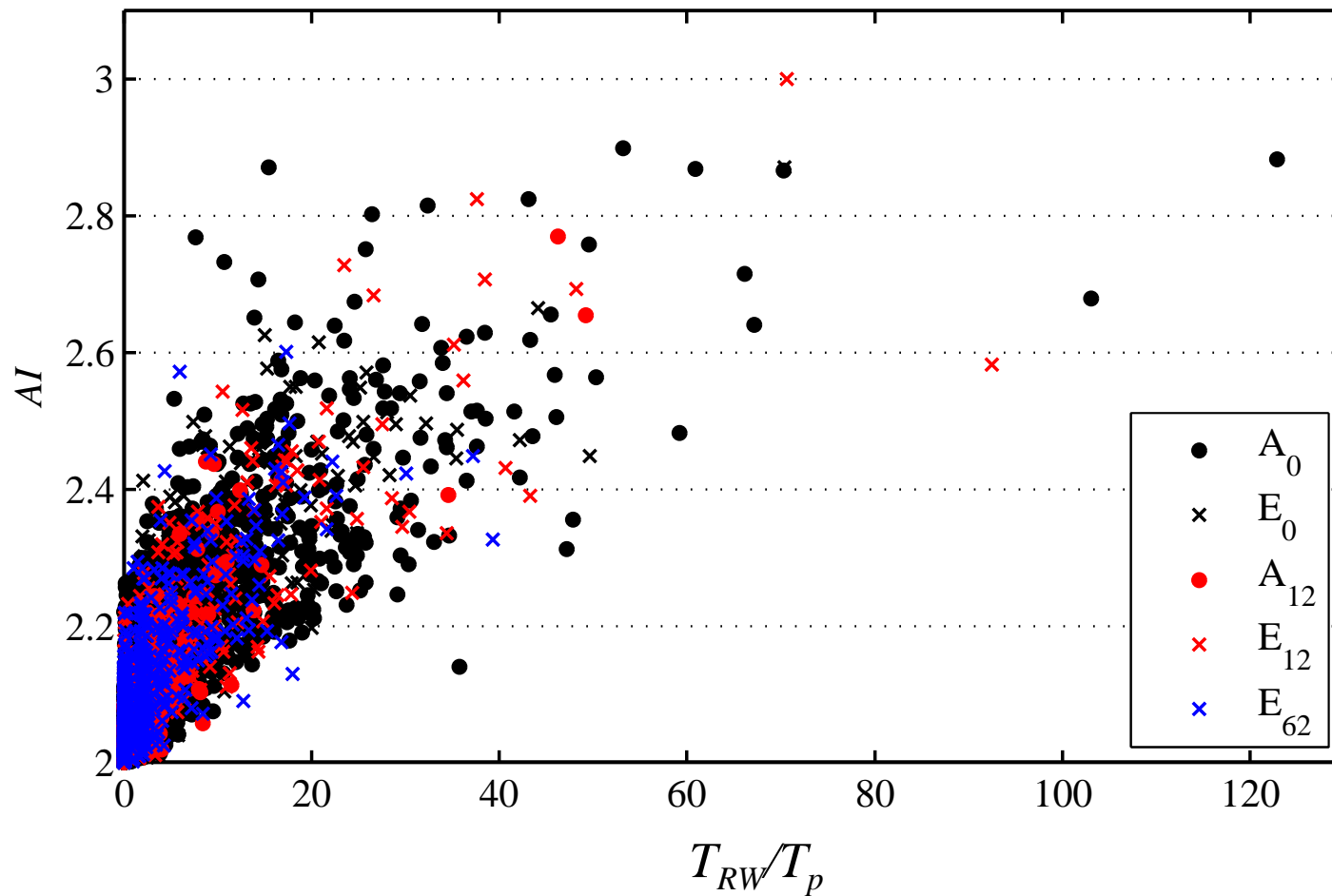
L_{θ}^{γ} - linear case

N_{ev} - number of rogue events

A.Kokorina, A.Slyunyaev, Lifetimes of Rogue Wave Events in Direct Numerical Simulations of Deep-Water Irregular Sea Waves. *Fluids*, 4(2), 70, (2019) <https://doi.org/10.3390/fluids4020070>

A.Slyunyaev, A. Kokorina, Numerical Simulation of the Sea Surface Rogue Waves within the Framework of the Potential Euler Equations. *Izvestiya, Atmospheric and Oceanic Physics*, Vol. 56, No. 2, pp. 179–190, (2020).

Dependence between rogue event lifetimes T_{RW} and amplification factor AI

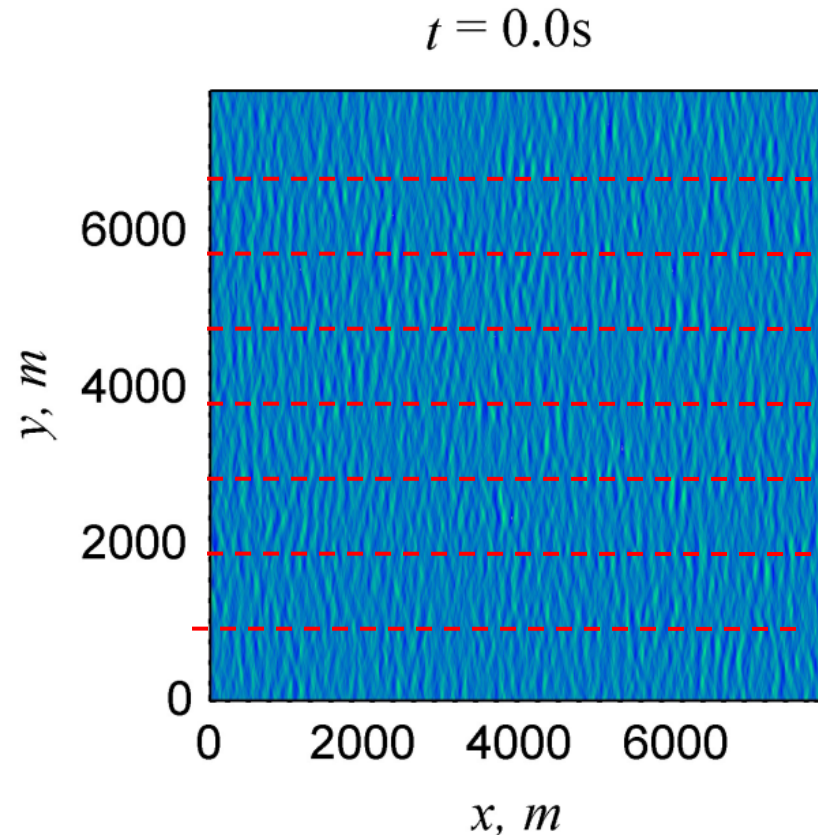
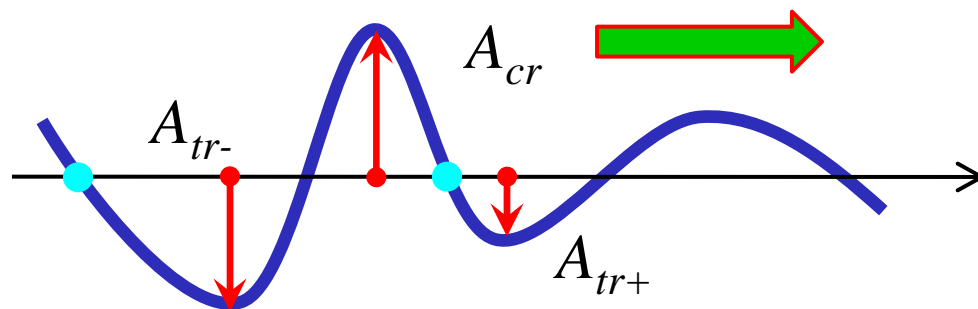


III. Typical shapes of rogue waves

The procedure:

- 1) cut along the direction of wave propagation
- 2) zero-crossing analysis of the space series → individual waves
- 3) selection of rogue waves according to the height criterion, $H > 2H_s$,

$$H = A_{cr} + A_{tr}, \quad A_{tr} = \max(|A_{tr-}|, |A_{tr+}|)$$



- 4) asymmetry properties:

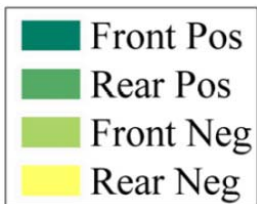
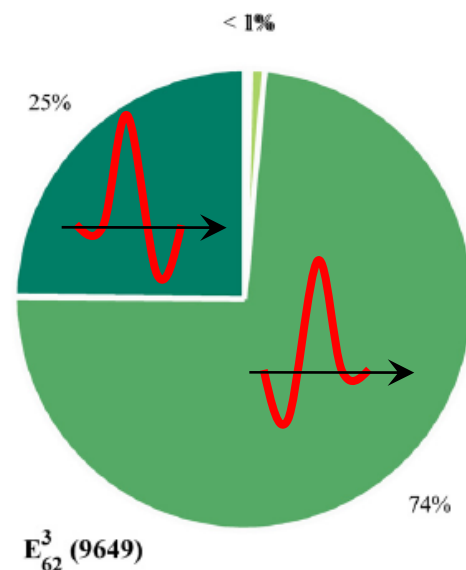
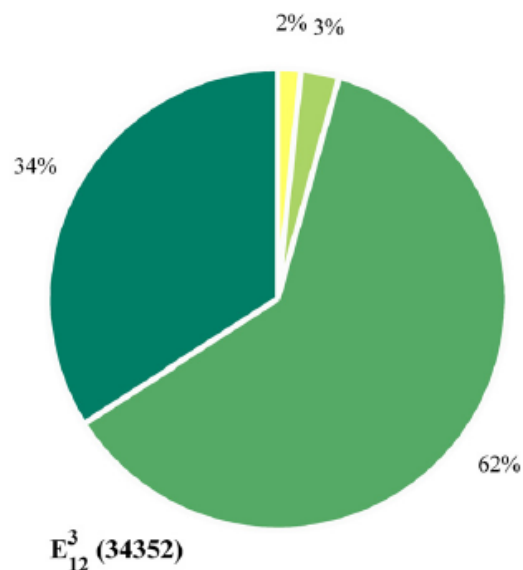
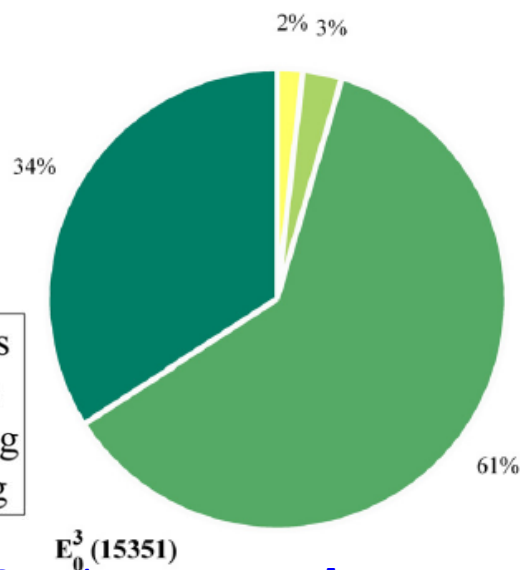
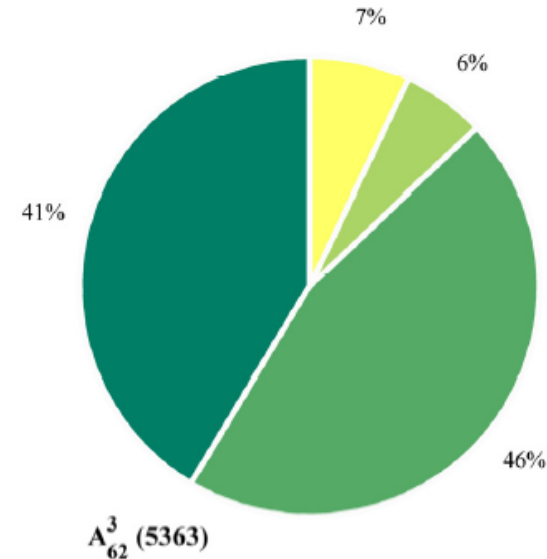
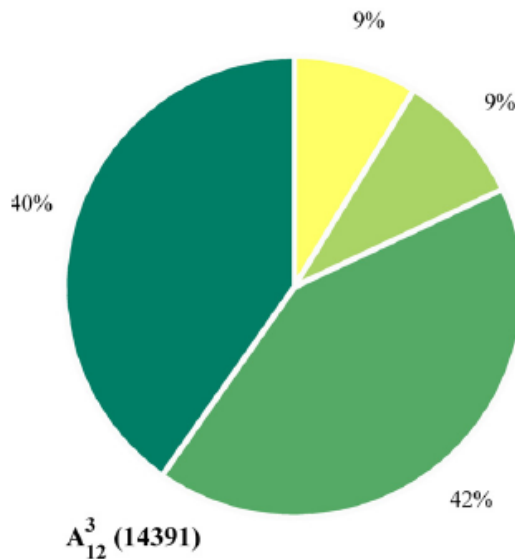
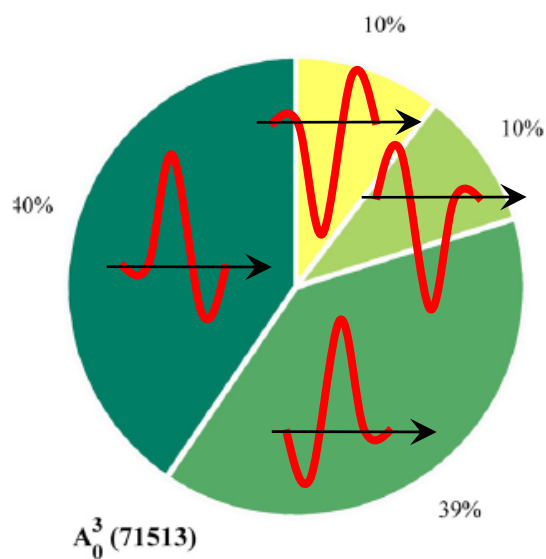
vertical asymmetry: $A_{cr} > A_{tr}$ or $A_{cr} < A_{tr}$

horizontal asymmetry: $A_{tr+} > A_{tr-}$ or $A_{tr+} < A_{tr-}$

III. Typical shapes of rogue waves

directional spread

sea state steepness



Conclusions

- Distribution of life-times of rogue waves is **exponential in the linear framework**.
- Life-time distribution of **short-crested** rogue waves approximately agree with the **linear** limit.
- Rogue waves under the conditions of **large wave steepness and narrow angle spectrum** live remarkably longer.
- Vertical and horizontal asymmetries **increase** when waves are **steeper or have broader angle spectrum**.
- For $H_s = 7$ m, $\theta = 62^\circ$, $\gamma = 3$ **three fourths** of rogue waves are represented by crests which are higher than troughs with deeper troughs behind.

