Run-up of narrow and wide-banded irregular waves on a beach

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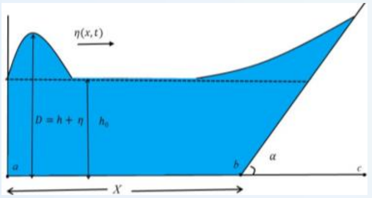
Problem Description

- The runup of initial Gaussian narrow-banded and wide-banded wave fields and its statistical characteristics are investigated using direct numerical simulations, based on the nonlinear shallow water equations, written in mass and momentum conservation:

$$\frac{\partial D}{\partial t}+\frac{\partial (Du)}{\partial x}=0, \qquad \frac{\partial}{\partial t}\left(Du\right)+\frac{\partial}{\partial x}\left(Du+\frac{g}{2}D^2\right)=gD\frac{dh}{dx}$$



Bathymetry sketch of numerical experiment

- Second order accurate finite volume scheme with the UNO2 spatial reconstruction
- shock-wave formation and propagation with the speed given by Rankine-Hugoniot jump conditions
- On the left extremity $x = a$ of the computational domain we impose the Dirichlet boundary condition on the total water depth, which turns out to be enough to obtain a well-posed initial boundary-value problem provided that the flow is subcritical at this point, which is always the case for Riemann waves [1, 2, 3]

- The considered boundary condition (wave field offshore) is distributed by the Gaussian distribution. To ensure this, all individual time-series have been verified by the Kolmogorov–Smirnov test.
- We use the narrow-band and wider band spectra (called 'wide-band' below) with bandwidths $\Delta f/f_0 = 0.1$ and $\Delta f/f_0 = 0.4$, respectively, where $f_0 = 1/10 = 0.1$ *Hz*.
- To address different levels of nonlinearity, the time series with five different significant wave heights are considered. Parameter of the nonlinearity for generated waves is estimated as H_s/h_0 and is changing from 0.03 (non-breaking waves) to 0.14 (nonlinear waves, strongly affected by wave breaking during their runup). The selected wave regimes allow to see the effects of wave breaking on wave statistics.
- Parallel computations have facilitated the calculation of the statistics of wave runup characteristics for 5000 hours, for each bandwidth, and 10000 hours in total. The numerical computations have been carried out in MATLAB and run on a cluster containing 28 workers.
- The characteristic parameter $kh_0 = 0.38$ is at the border of validity of the shallow water theory. Since we are focusing on wave runup, the choice of the theory is justifiable.
- The number of spatial grid points along the distance between [a] and [c] is fixed and equal to 1000 for all experiments.
- The time step is chosen to satisfy the CFL (stability) condition for all considered significant wave heights.
- Each of these time-series has a duration of 1000 hours (360 000 wave periods).

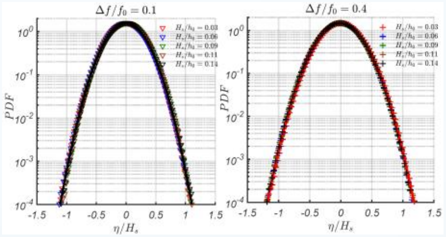
Conditional Weibull distribution

The large (extreme) wave runup heights, *Rextrm* = *R*/*Rs* ≥ *s*, where *s* is some threshold value, somehow behave similar to a conditional Weibull law whose density is given by :

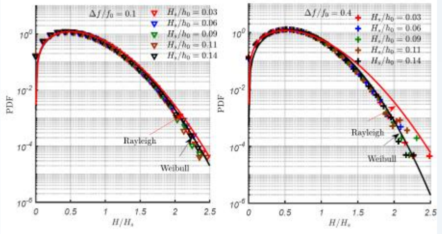
$$f\left(R_{extrm}\right)=\begin{cases}\dfrac{k}{\lambda}\left(\dfrac{R_{extrm}}{\lambda}\right)^{k-1}e^{-\left(R_{extrm}/\lambda\right)^k+\left(s/\lambda\right)^k}, & R_{extrm}\geq s\\ 0, & R_{extrm}<s\end{cases} \qquad (*)$$

A conditional Weibull law is characterized by three parameters: the shape k, the scale λ and the threshold s. Given the data ($R_{i\ extrm}$) = 1…*n*, *s* is fixed and k and λ are computed by maximum likelihood estimator.

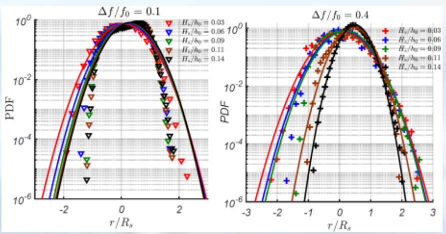
Data analysis and the main results



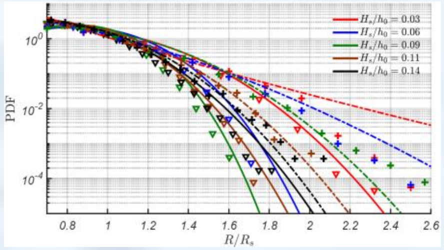
Probability distribution functions of narrow-band (left) and wide-band (right) wave fields offshore for different nonlinearities, *Hs*/*h0*. Solid lines correspond to Gaussian distributions fitted to the corresponding datasets.



Probability distribution functions of trough-to-crest wave heights of the initial narrow-band and wide-band wave fields offshore, *Hs*/*h0*. Red solid line corresponds to the Rayleigh distribution. Black solid line corresponds to the Weibull distribution.



Probability distribution functions of runup oscillations. Solid lines correspond to Gaussian distributions, fitted to the corresponding datasets.



Probability distribution functions of large runup heights ($R \geq 0.7Rs$) for narrow-banded (triangles) and wide-banded (pluses) waves. Lines correspond to a conditional Weibull distributions [Eq. (*)] fitted to narrow-banded (solid lines) and wide-banded (dashed lines) datasets.

Freak events

e.g freak runup of initial narrow-band wave field ($\Delta f/f_0$ = 0.1) for weak nonlinearity ($H_s/h_0 = 0.1$)

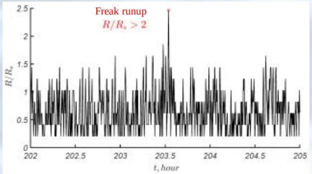
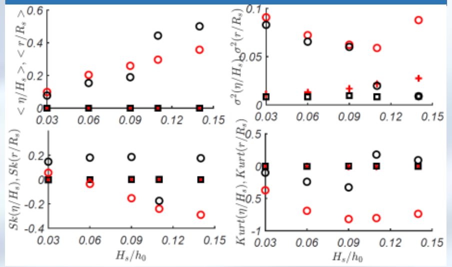


Table: The number of freak events in the sea coastal zone and on a beach for different wave regimes.

	$\Delta f/f_0 = 0.1$			$\Delta f/f_0 = 0.4$		
H_s/h_0	Number of waves	Freak waves offshore	Freak runups	Number of waves	Freak waves offshore	Freak runups
0.03	362255	125	61	389232	51	118
0.06	362380	117	0	389385	45	76
0.09	362096	89	0	389444	49	62
0.11	362319	88	0	389263	53	2
0.14	362302	102	0	389728	34	1

Statistical moments



Statistical moments of runup oscillations (*r*/*Rs*) of narrow-banded (red circles) and wide-banded (black circles) waves on a beach, *r*, versus nonlinearity, *Hs*/*h0*. Red crosses show statistical moments of narrow-band and wide-band wave fields offshore and black squares respectively.

Conclusion

- For both narrow-band and wide-band cases we observe the effect of wave set-up (increase in the mean value of runup oscillations), which increases with an increase in wave nonlinearity. However, for wide-banded waves this increase is significantly stronger than for narrow-banded ones.
- What regards extreme, so-called "freak events", their statistics in the initial narrow-banded wave signal offshore is more representative, than on the beach ("freak runups") even for non-breaking waves. Therefore, for narrow-banded waves, gentle beaches reduce the number of freak events as compared to the sea coastal zone, and work as a 'low-pass filter' for extreme wave heights.
- A conditional Weibull distribution has been suggested to describe statistics of extreme wave runup heights on a gentle beach. It gives reasonable results and may be used for assessment of extreme inundations on a beach (freak runups).

References

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