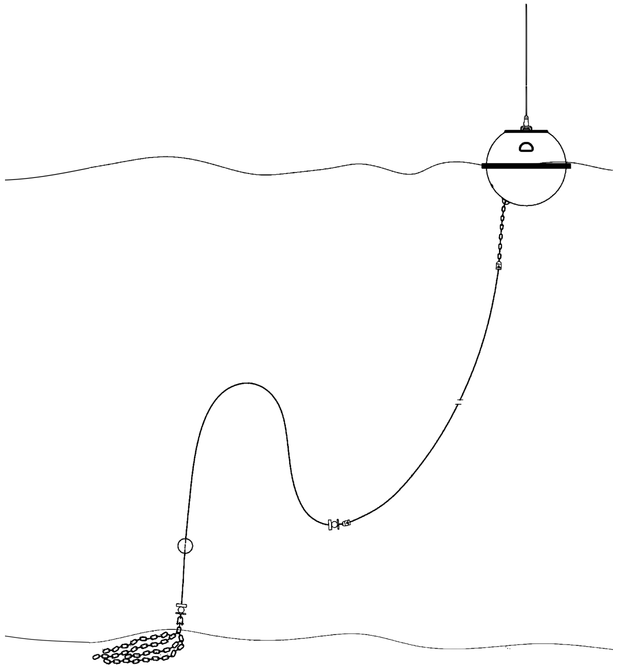
An Experimental Study of the Systematic Underestimation of Wave Crests Measured by Lagrangian Buoys, and a Retrospective Correction Method

Mark L. McAllister and Ton S. van den Bremer

University of Oxford

EGU 4th May 2020



Lagrangian Measurement of Waves by Buoys

- Motivation
- Second-Order Motion of a Wave-Following Measurement Buoy
- Statistical Properties of Directionally Spread Ocean Waves Measured by Buoys
- Approximate retrospective correction method for crest heights

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EPSRC
Engineering and Physical Sciences Research Council

wave energy SCOTLAND

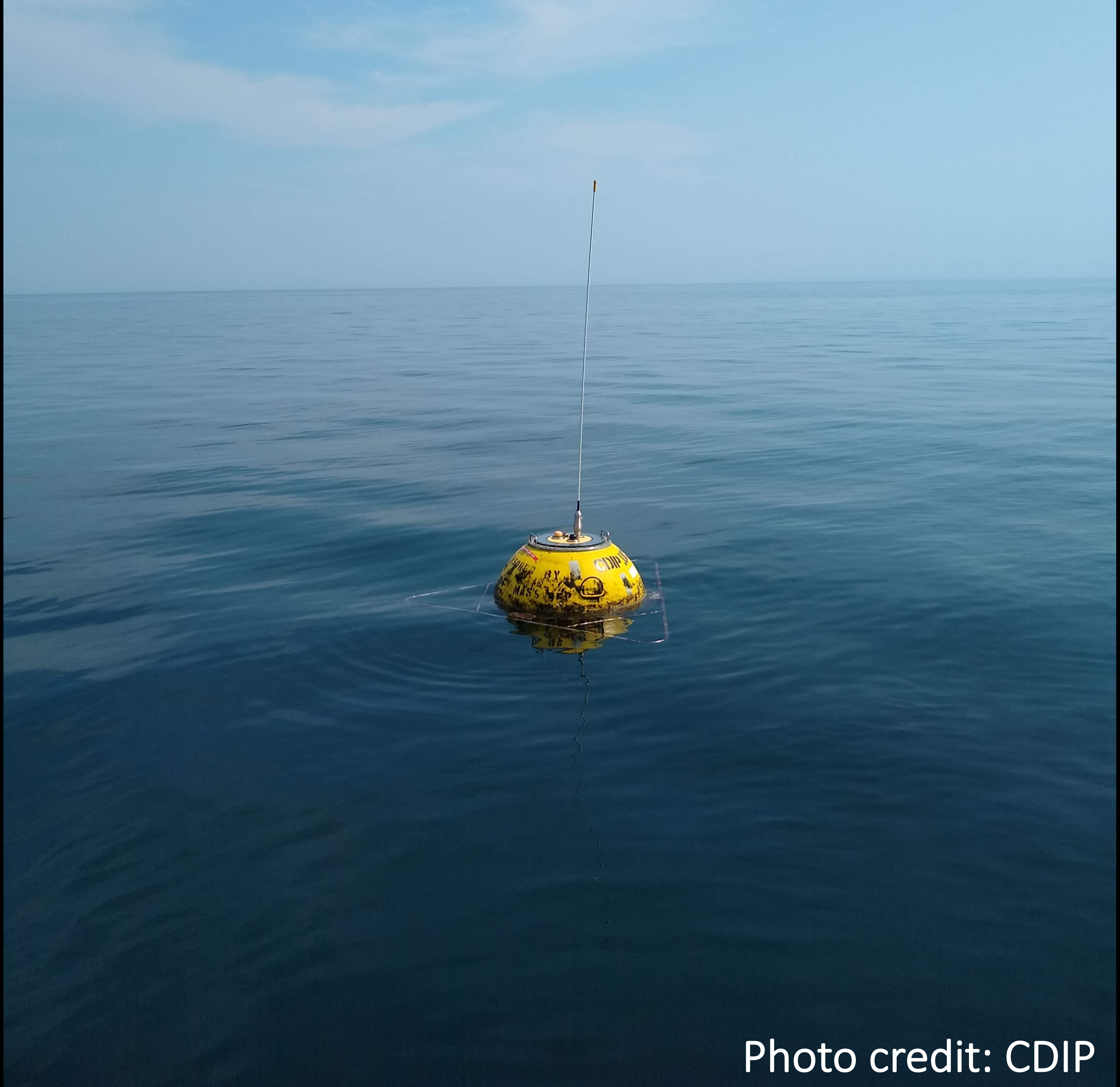
Photo credit: CDIP

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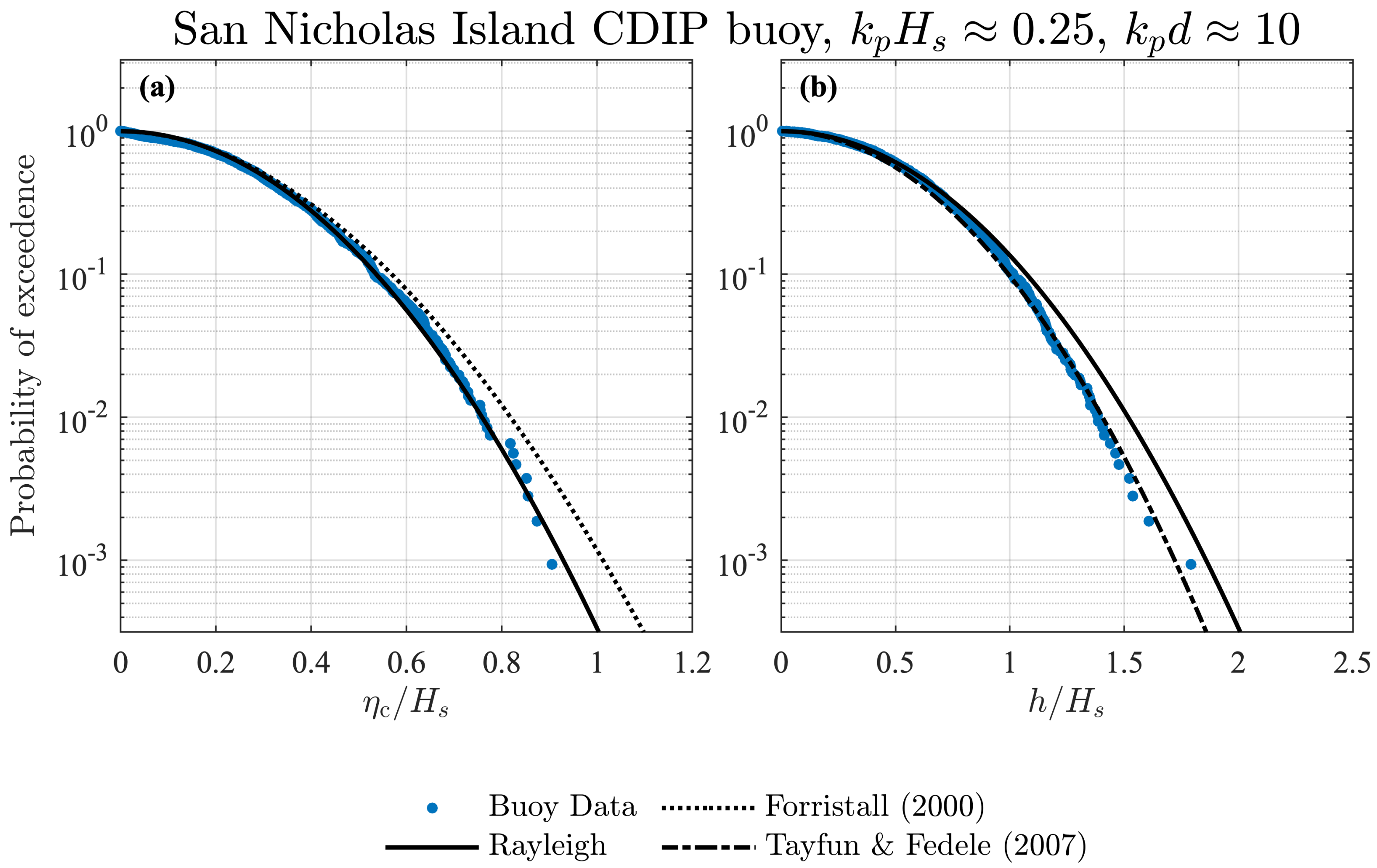
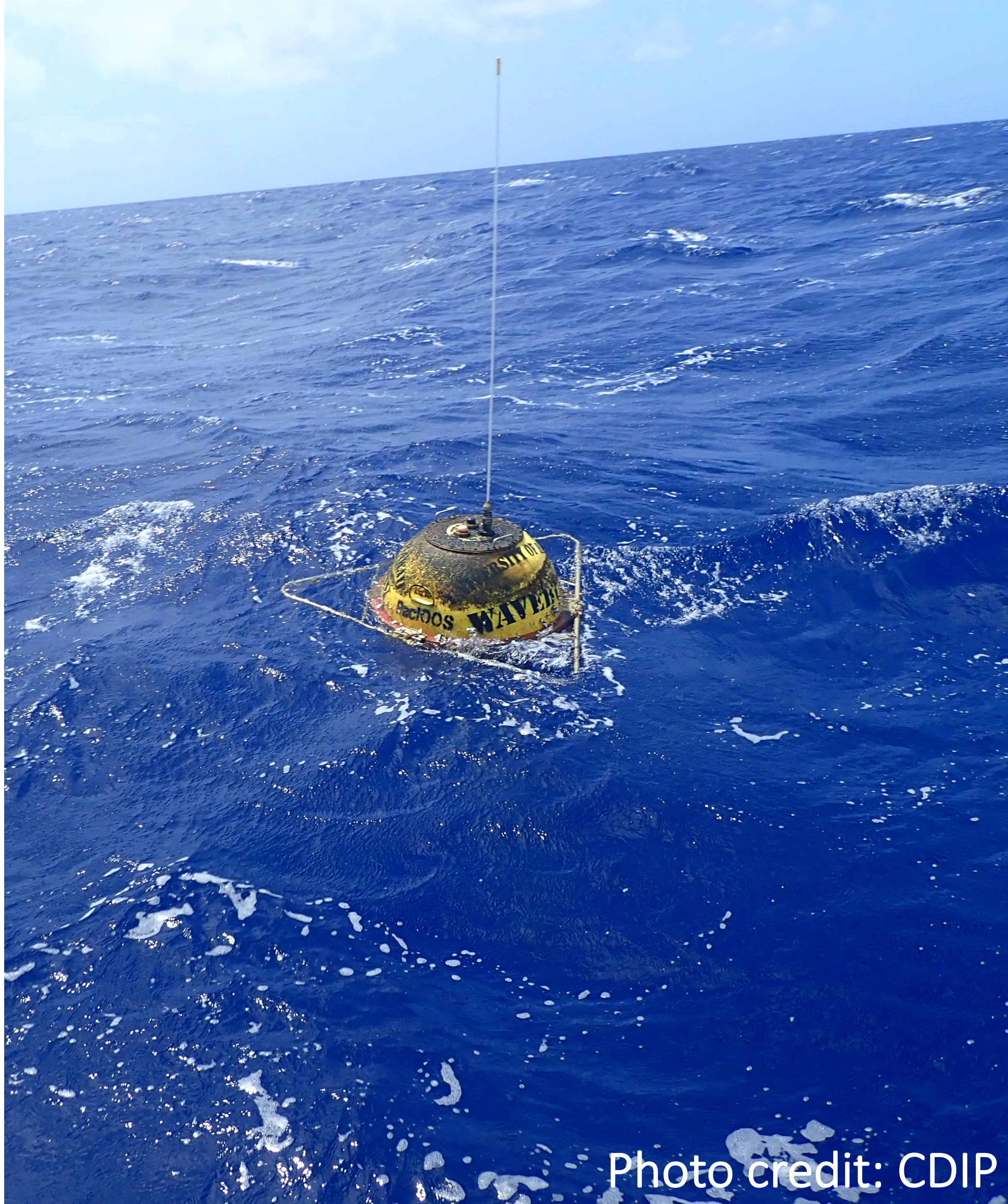
Motivation



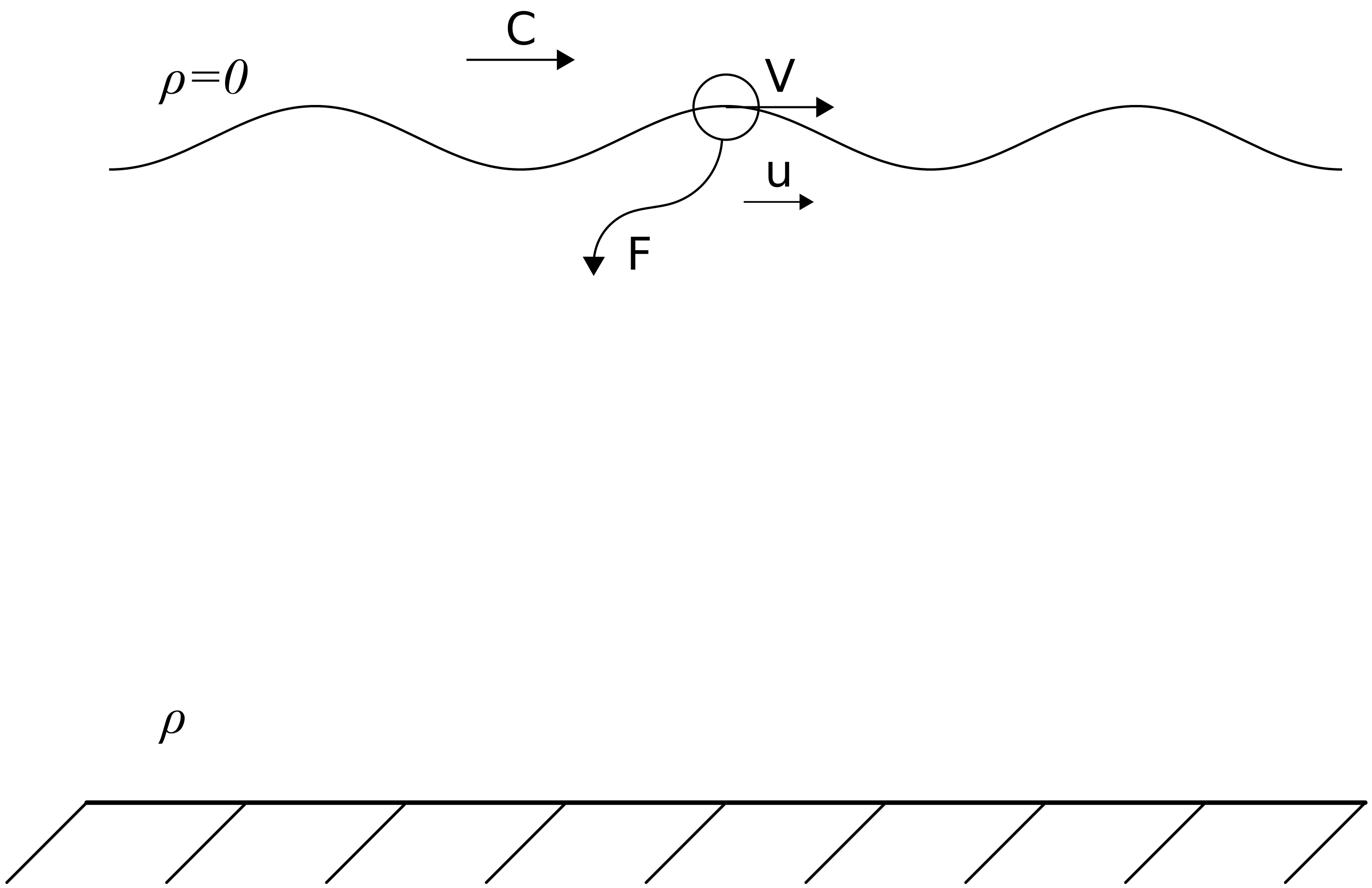
San Nicholas Island CDIP buoy, $k_p H_s \approx 0.25$, $k_p d \approx 10$
(a)
(b)
Probability of exceedence
10^{0}
10^{-1}
10^{-2}
10^{-3}
0
0.2
0.4
0.6
0.8
1
1.2
0.5
1.5
2
2.5
η_c/H_s
h/H_s
Buoy Data
Forristall (2000)
Rayleigh
Tayfun & Fedele (2007)

Motivation

- Buoys avoid large crest
- Lack or mooring compliance drags buoys under crests
- Low sampling rate misses crests
- Lagrangian motion “linearises” crests
- Instrumentation and signal processing



Second-Order Motion of a Wave-Following Measurement Buoy



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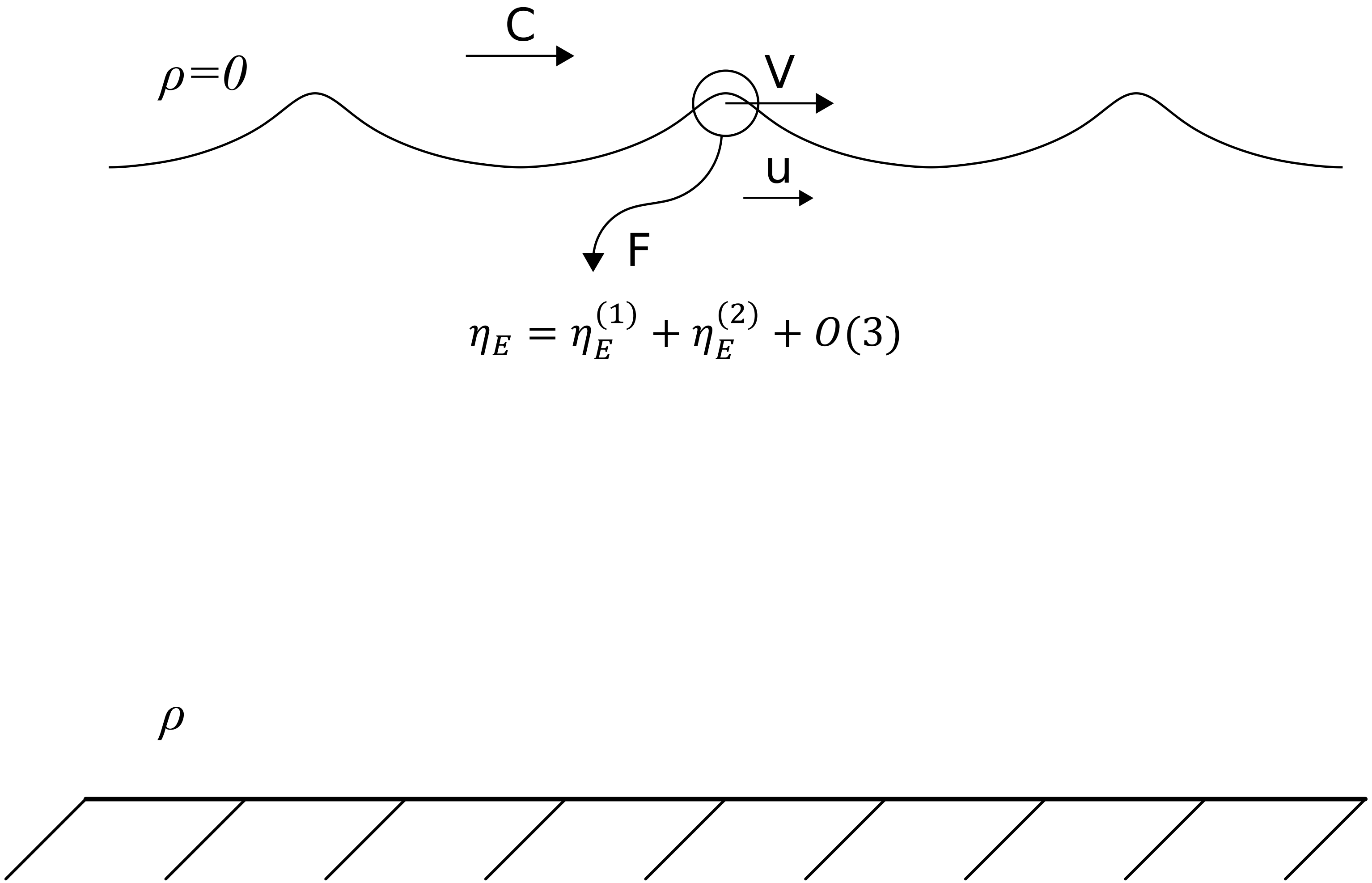
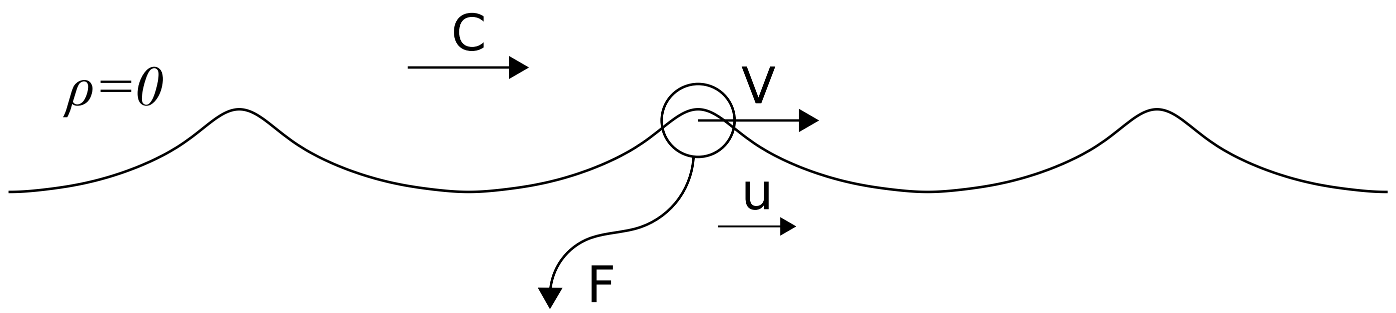


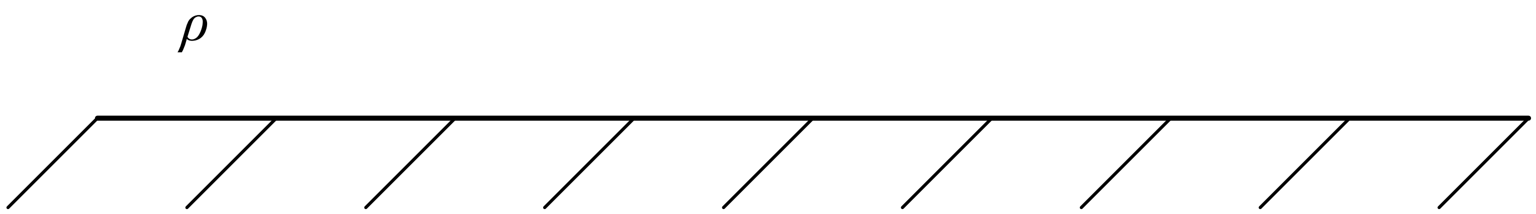
$$\eta_E = \eta_E^{(1)} + \eta_E^{(2)} + O(3)$$

Second-Order Motion of a Wave-Following Measurement Buoy



$$\eta_E = \eta_E^{(1)} + \eta_E^{(2)} + O(3)$$

$$\eta_L^{(2)} = \eta_E^{(2)} + \Delta\eta_L^{(2)}$$



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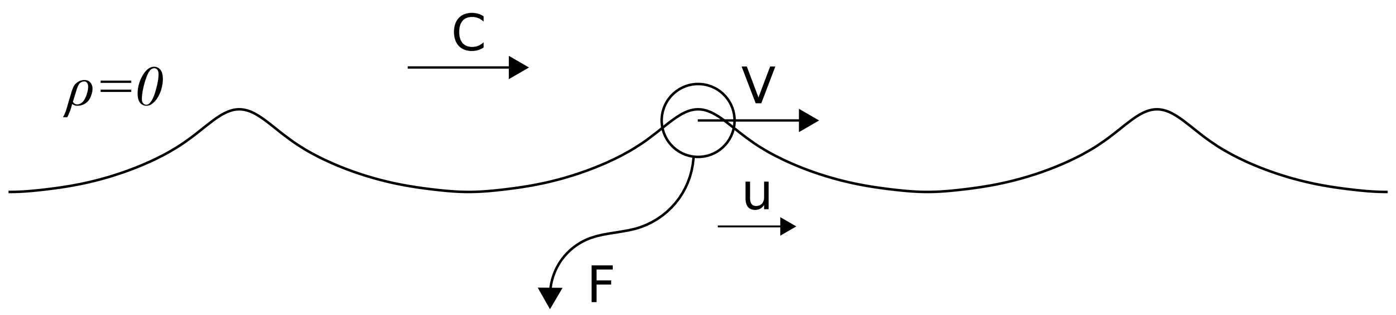

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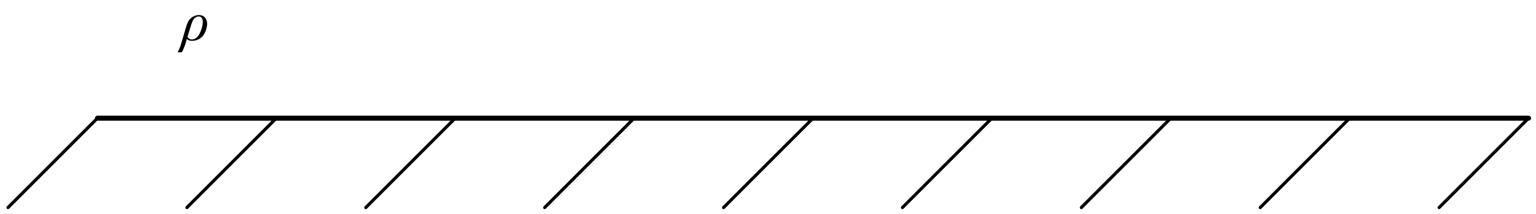
$$\Delta\eta_L^{(1)} = \Delta x_H^{(1)} \cdot \nabla_H \eta^{(1)}$$



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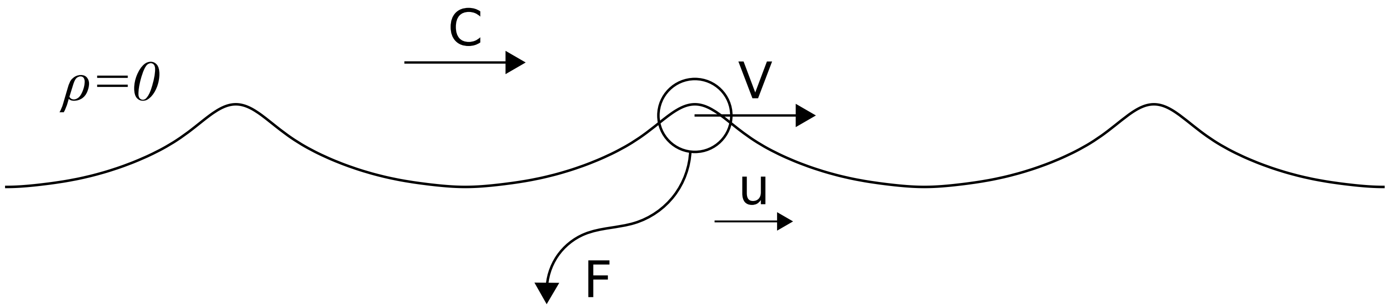

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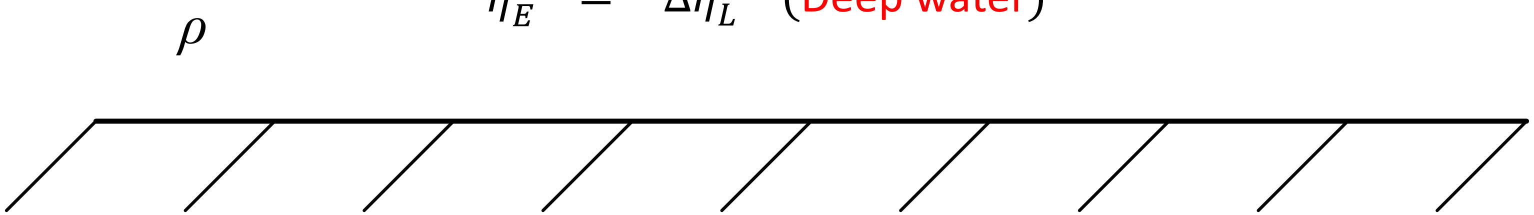
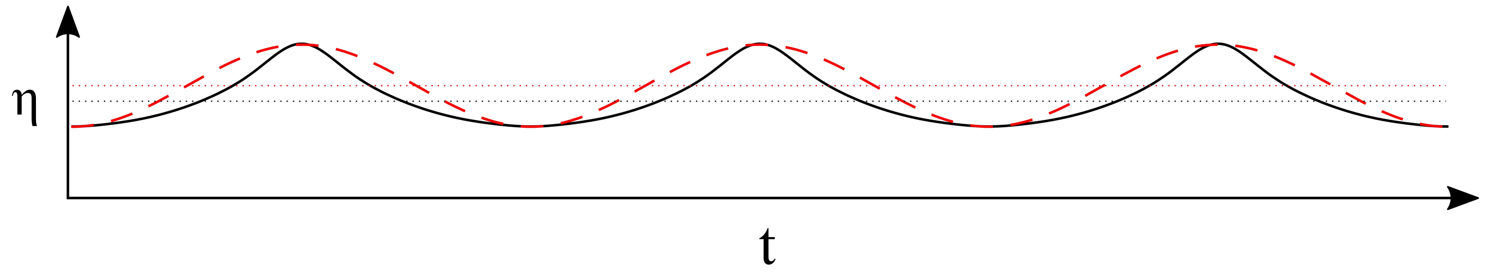
$\eta_E^{(2)} = -\Delta\eta_L^{(2)}$ (Deep water)



Second-Order Motion of a Wave-Following Measurement Buoy



Eulerian η =

Lagrangian η =

$$\eta_L^{(2)} = \eta_E^{(2)} + \Delta\eta_L^{(2)}$$

$$\Delta\eta_L^{(1)} = \Delta x_H^{(1)} \cdot \nabla_H \eta^{(1)}$$

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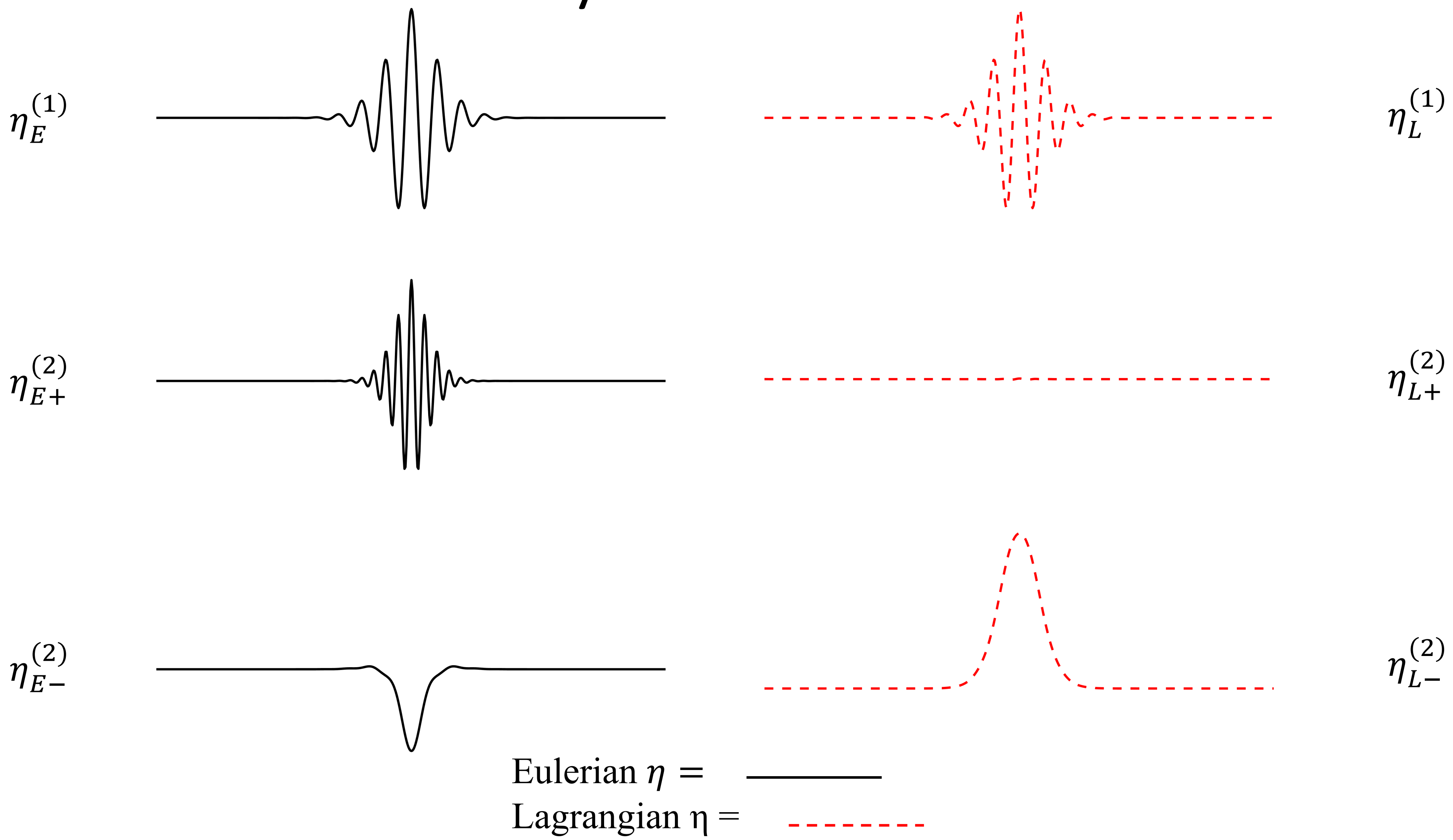
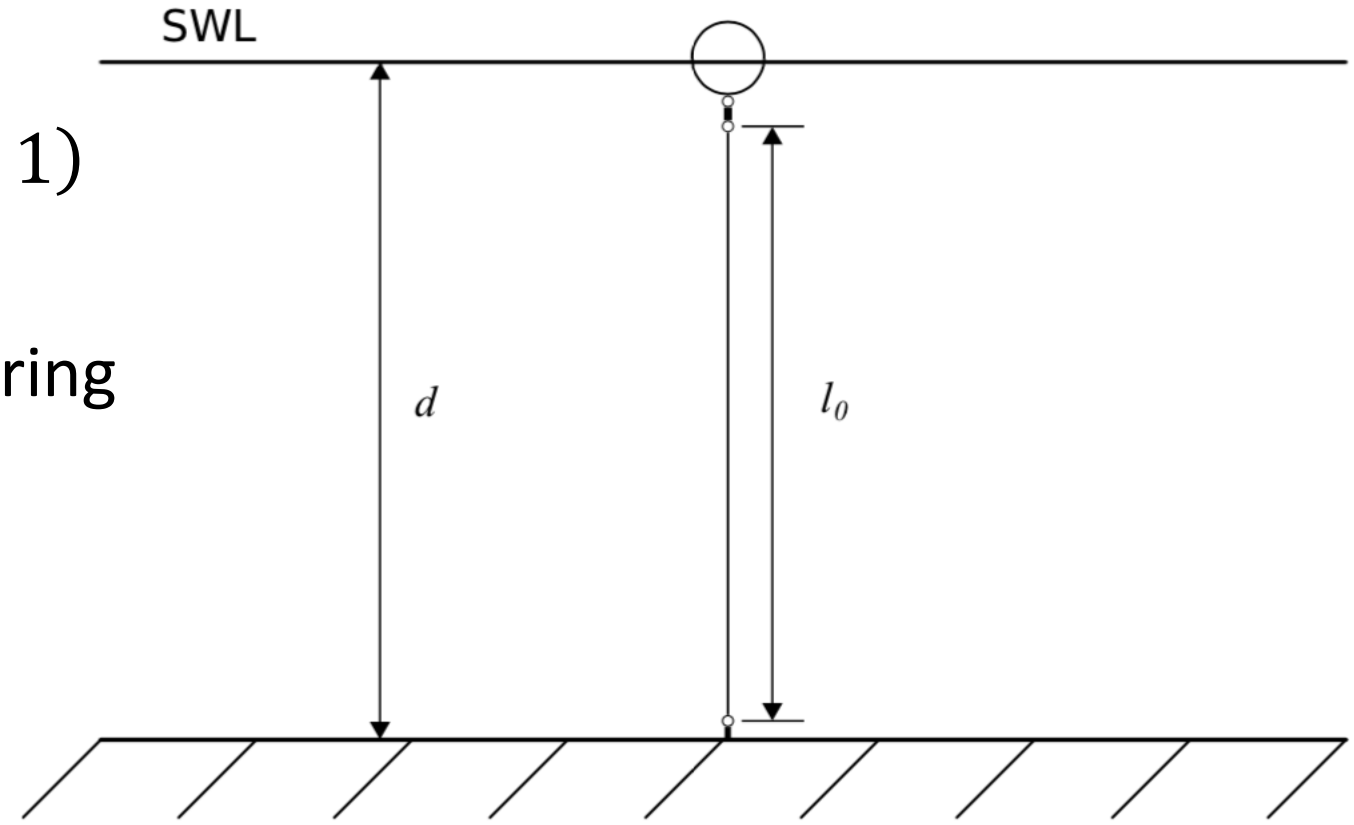
Second-Order Motion of a Wave-Following Measurement Buoy



Second-Order Motion of a Wave-Following Measurement Buoy

- Diameter 0.07m $(D/\lambda_0 \ll 1)$
- Density $\approx 0.5\rho_w$
- Highly-flexible taught mooring
- Depth 2 m $(k_0 d = 3 - 4)$
- Wave amplitude ≈ 0.2 m $(k_0 a_0 / k_p H_s \approx 0.3)$



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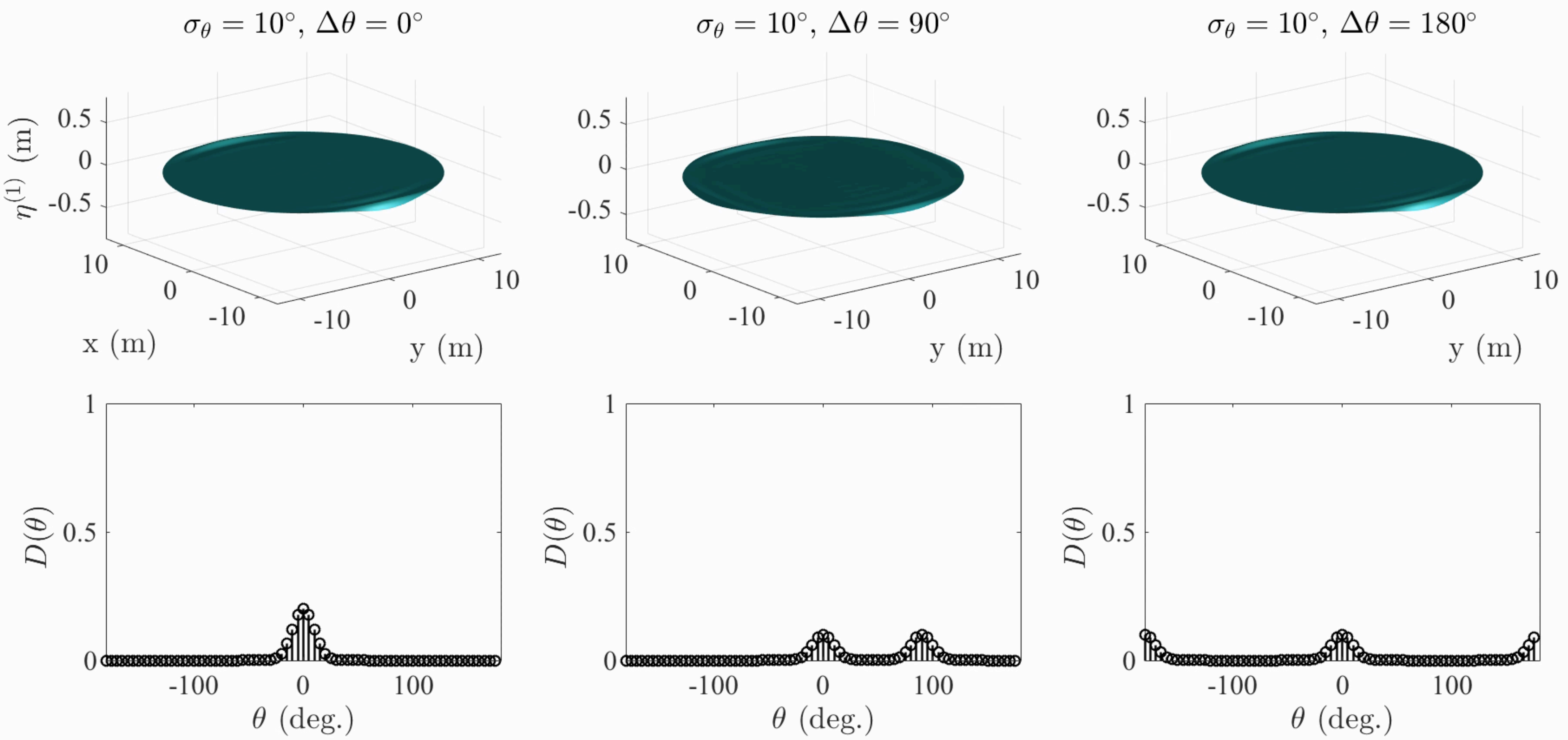

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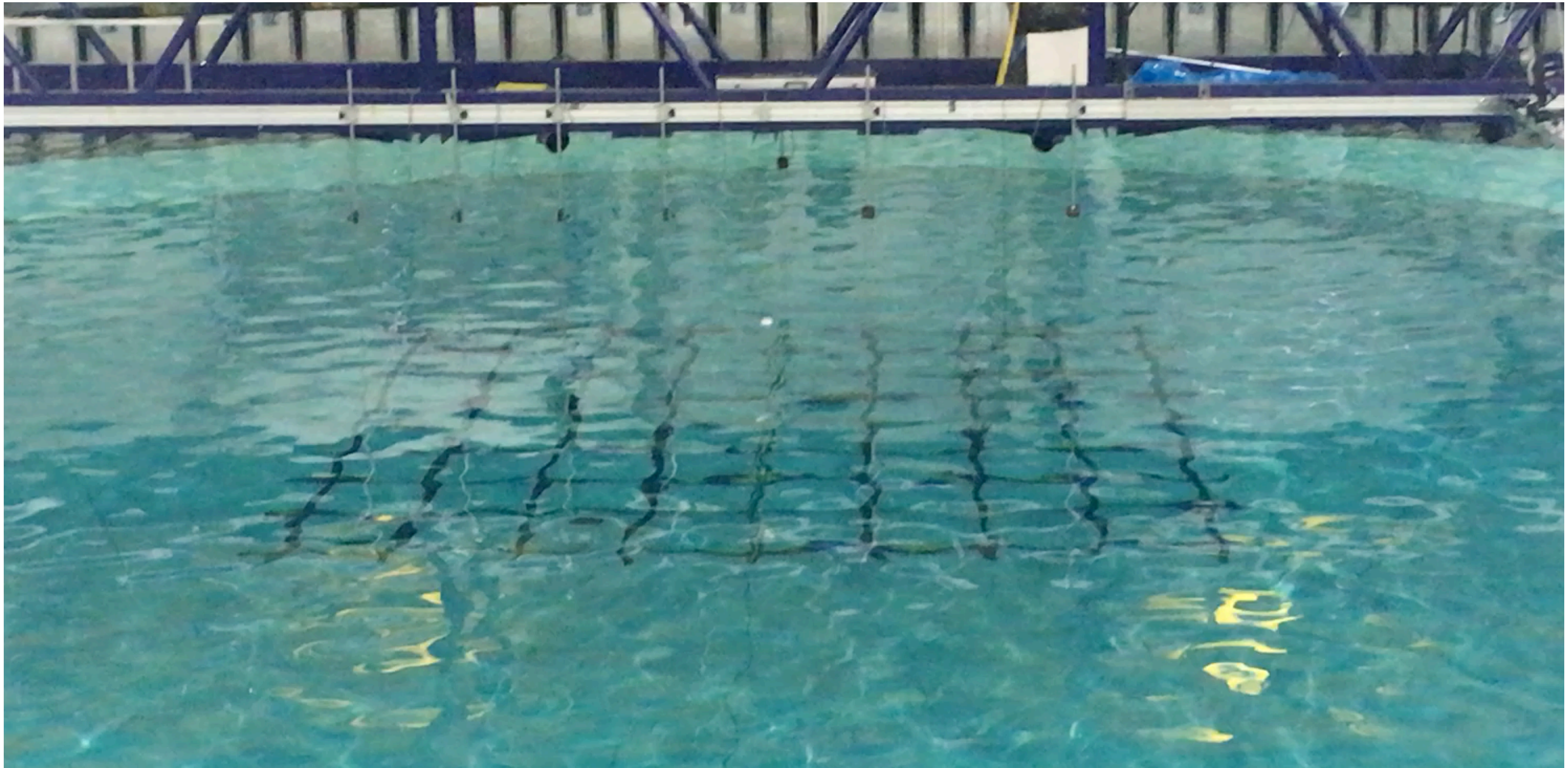

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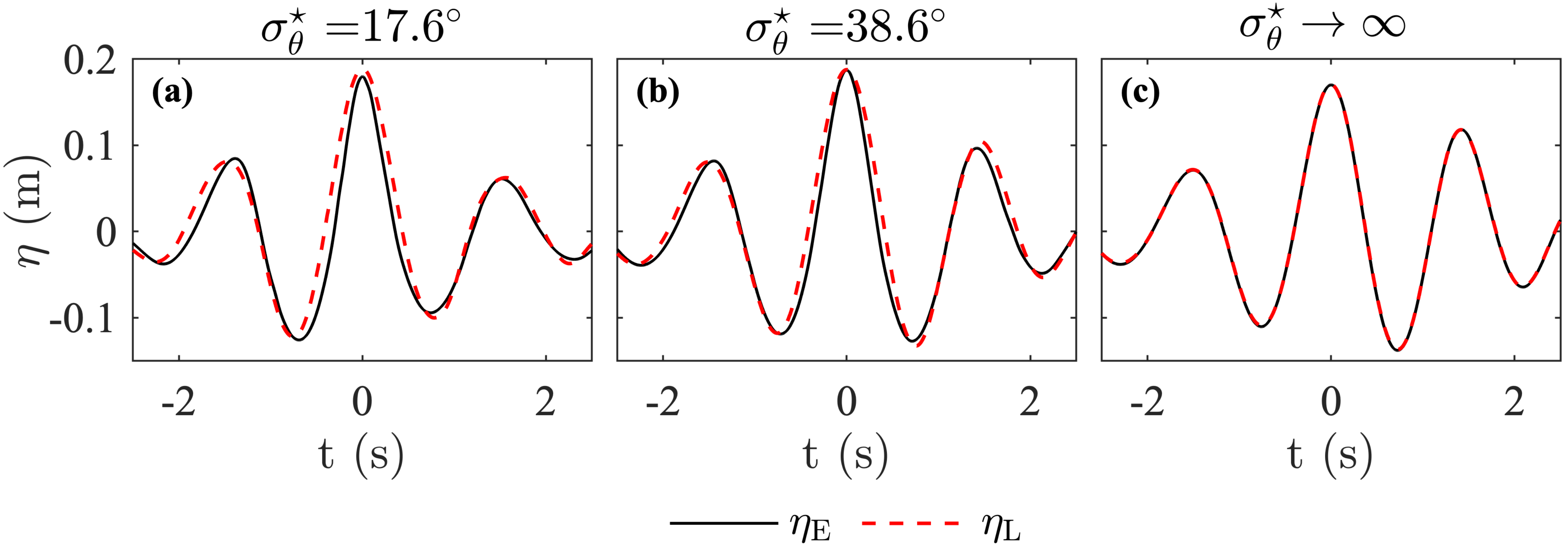

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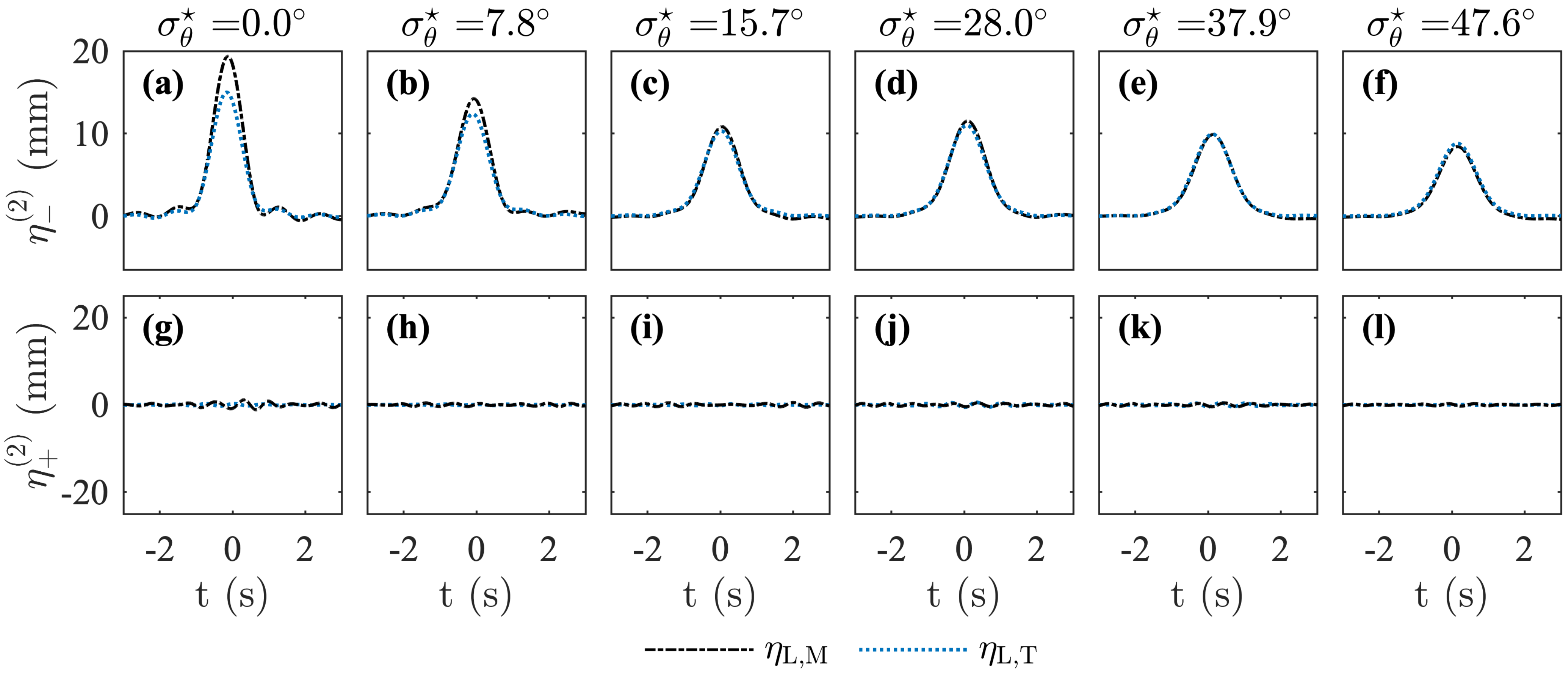

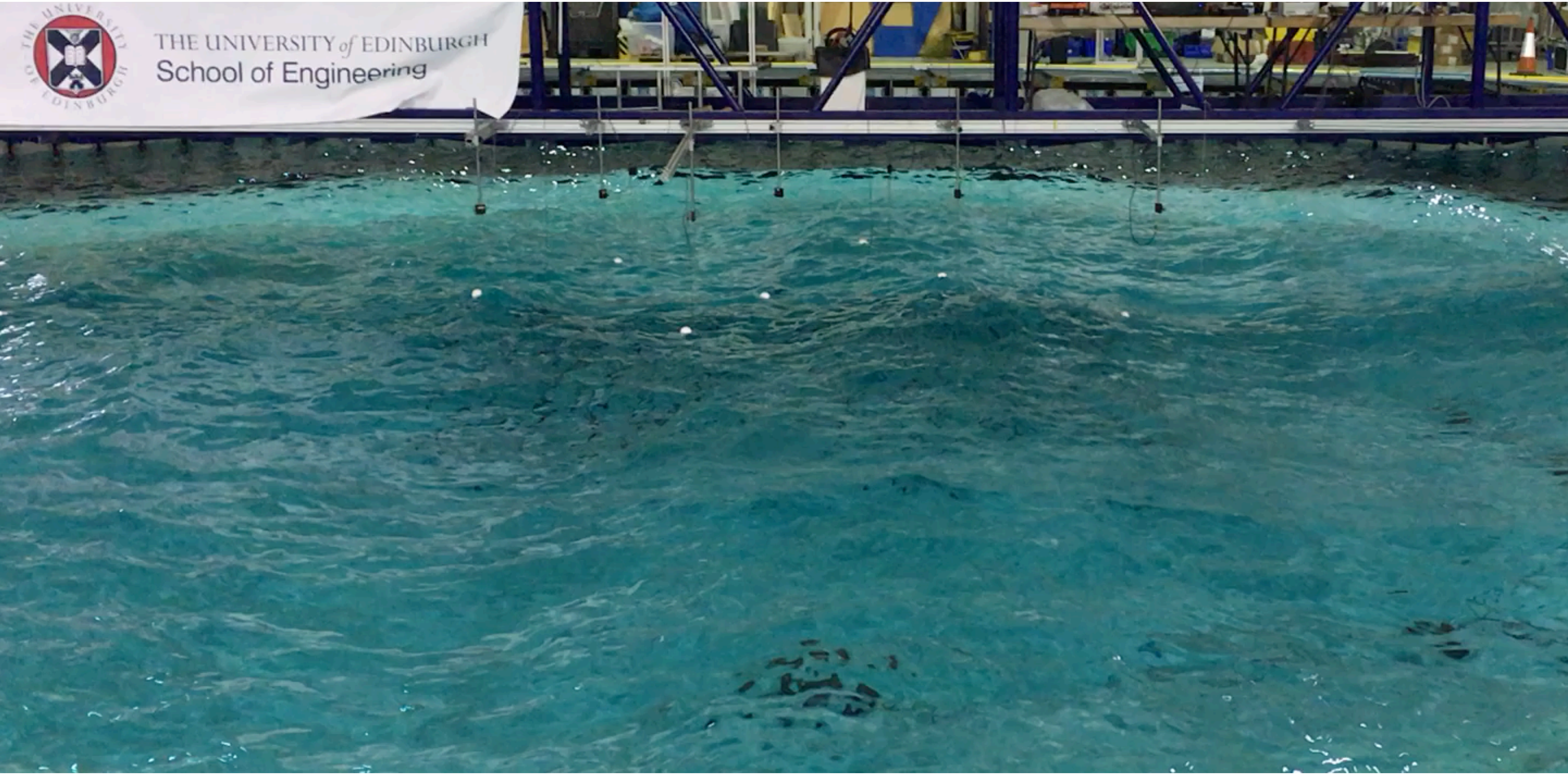
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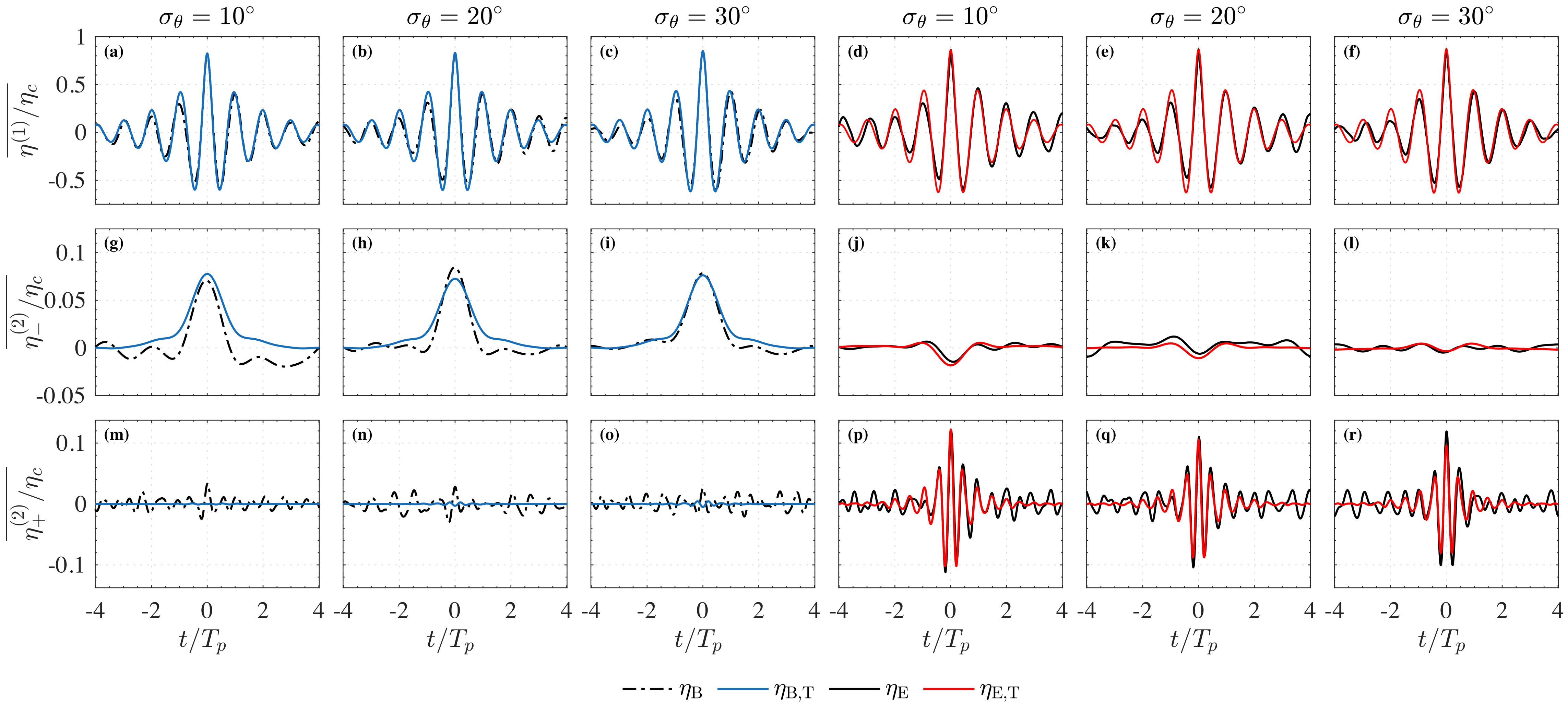
School of Engineering

Statistical Properties of Directionally Spread Ocean Waves Measured by Buoys



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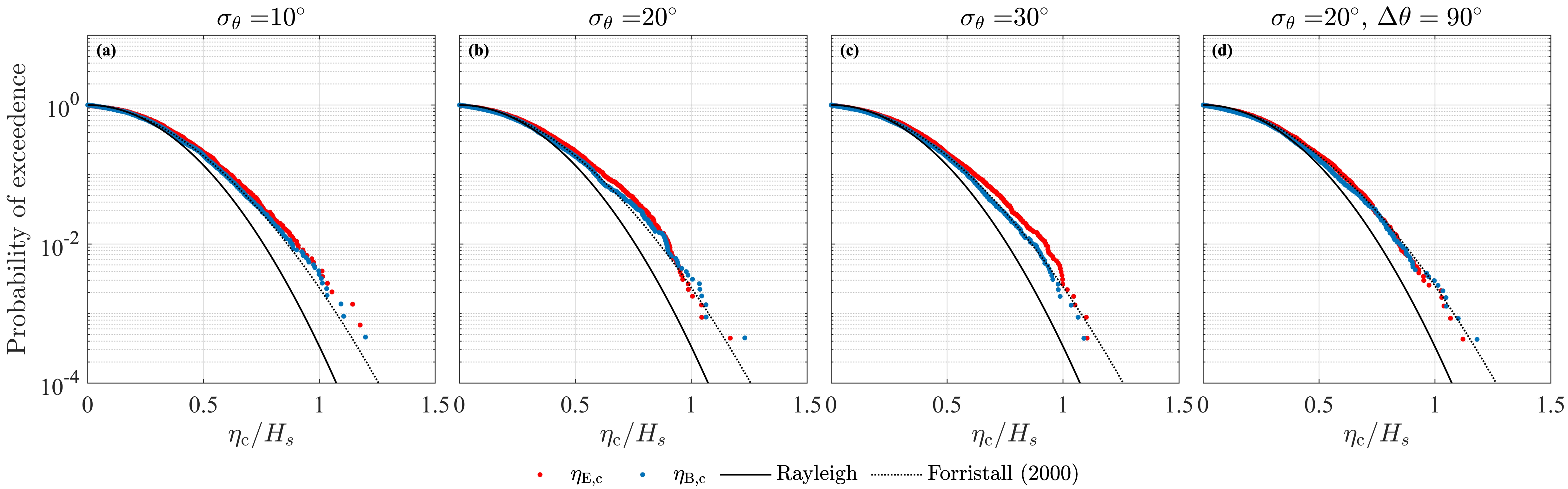

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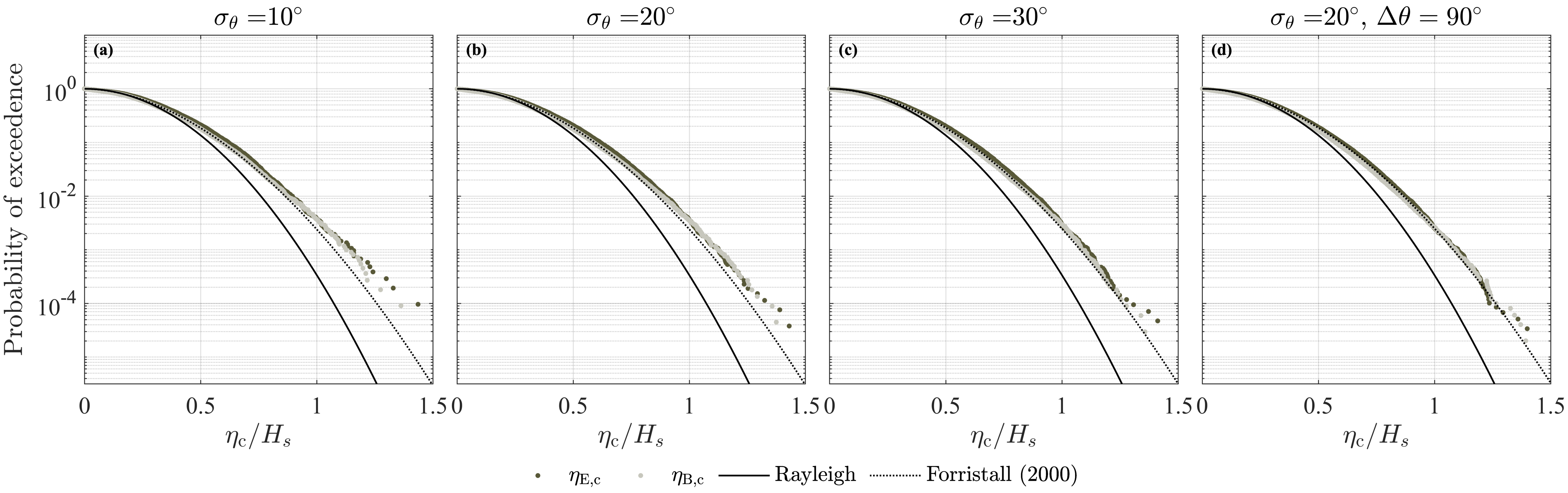
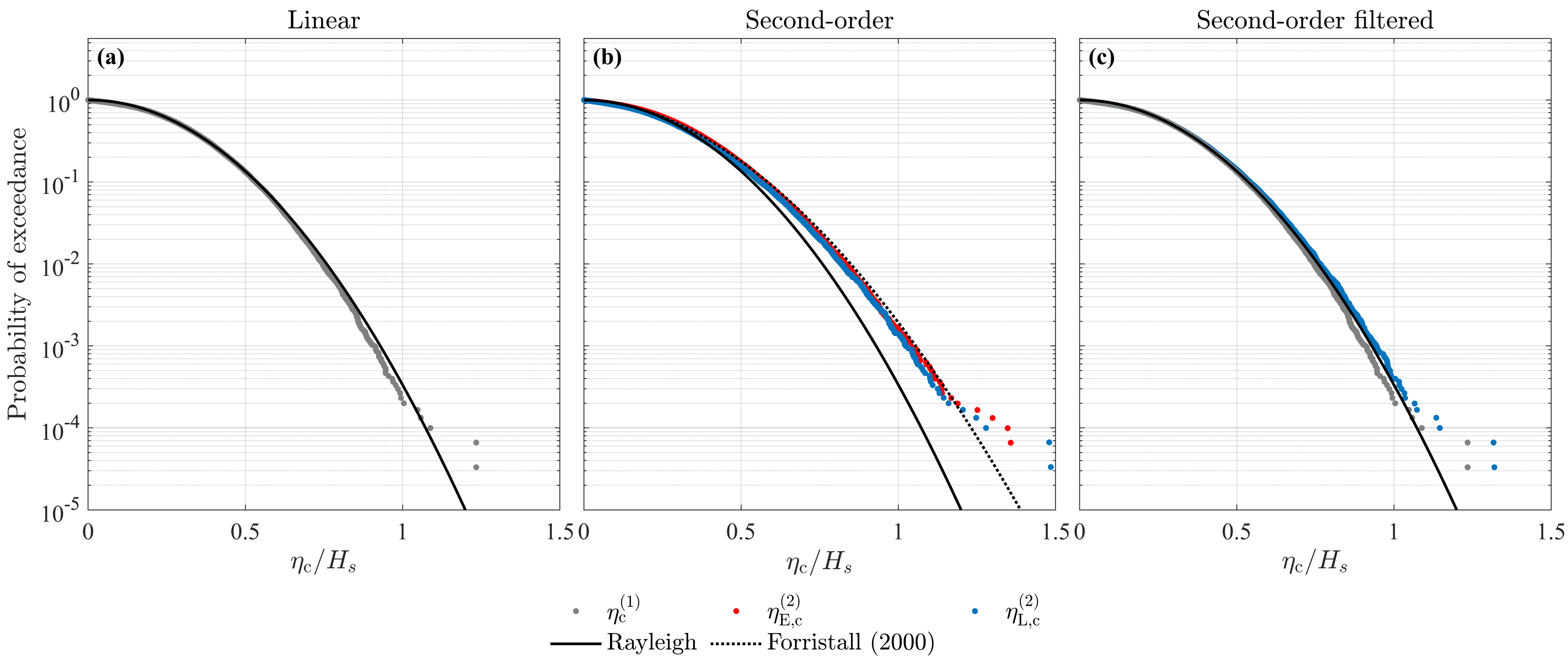


Approximate retrospective correction method for crest heights



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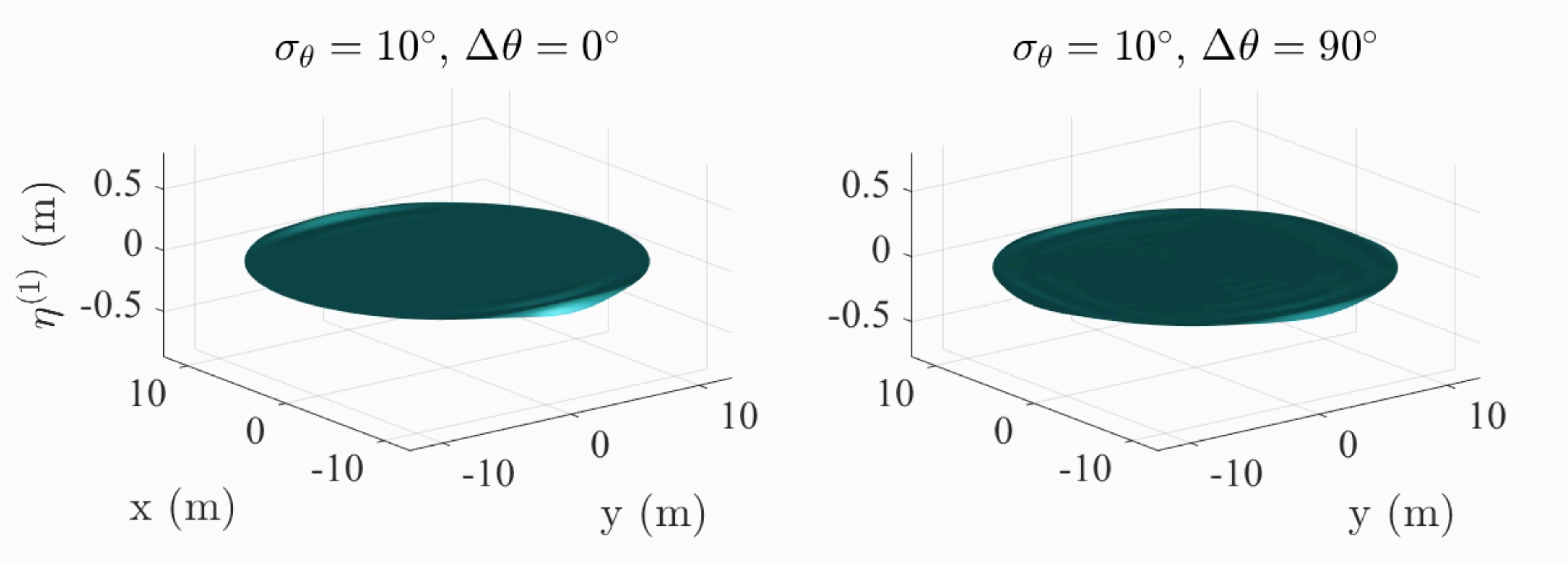


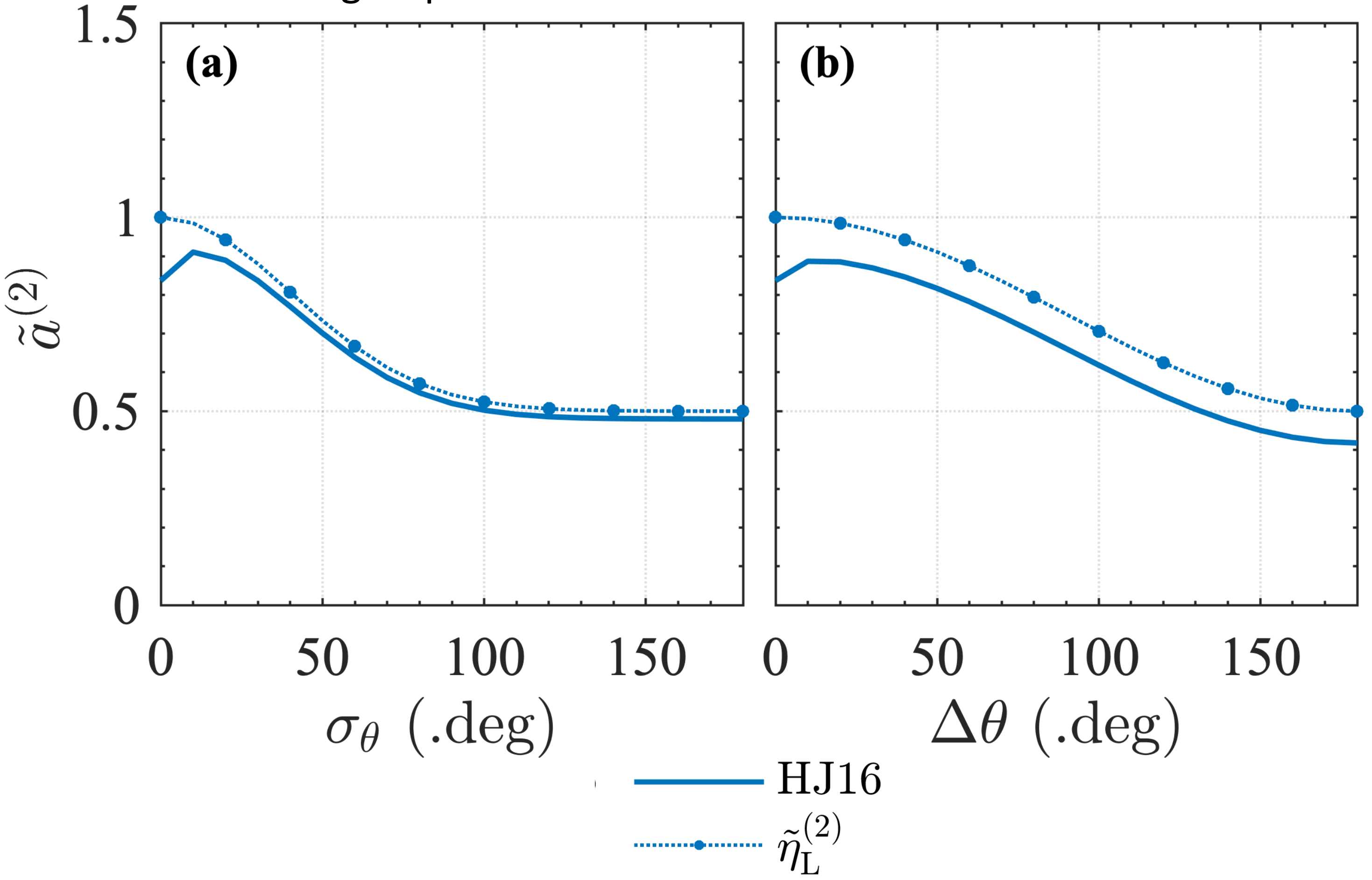
$$\tilde{\eta}^{(2)}(\sigma_\theta) = \frac{1}{2}\left(1 + e^{-\sigma_\theta^2}\right)$$

$$\tilde{\eta}^{(2)}(\Delta\theta) = \frac{1}{2} + \frac{1}{4}\left(1 + \cos(\Delta\theta)\right)$$

Approximate retrospective correction method for crest heights

A directionally spread wave group

Two crossing wave group



Approximate retrospective correction method for crest heights

$$\eta_c \approx \eta_c^{(1)}$$

$$\eta_{Corr,c} = \eta_c + \eta_{c-}^{(2)}$$

Approximate retrospective correction method for crest heights

$$\eta_c \approx \eta_c^{(1)}$$

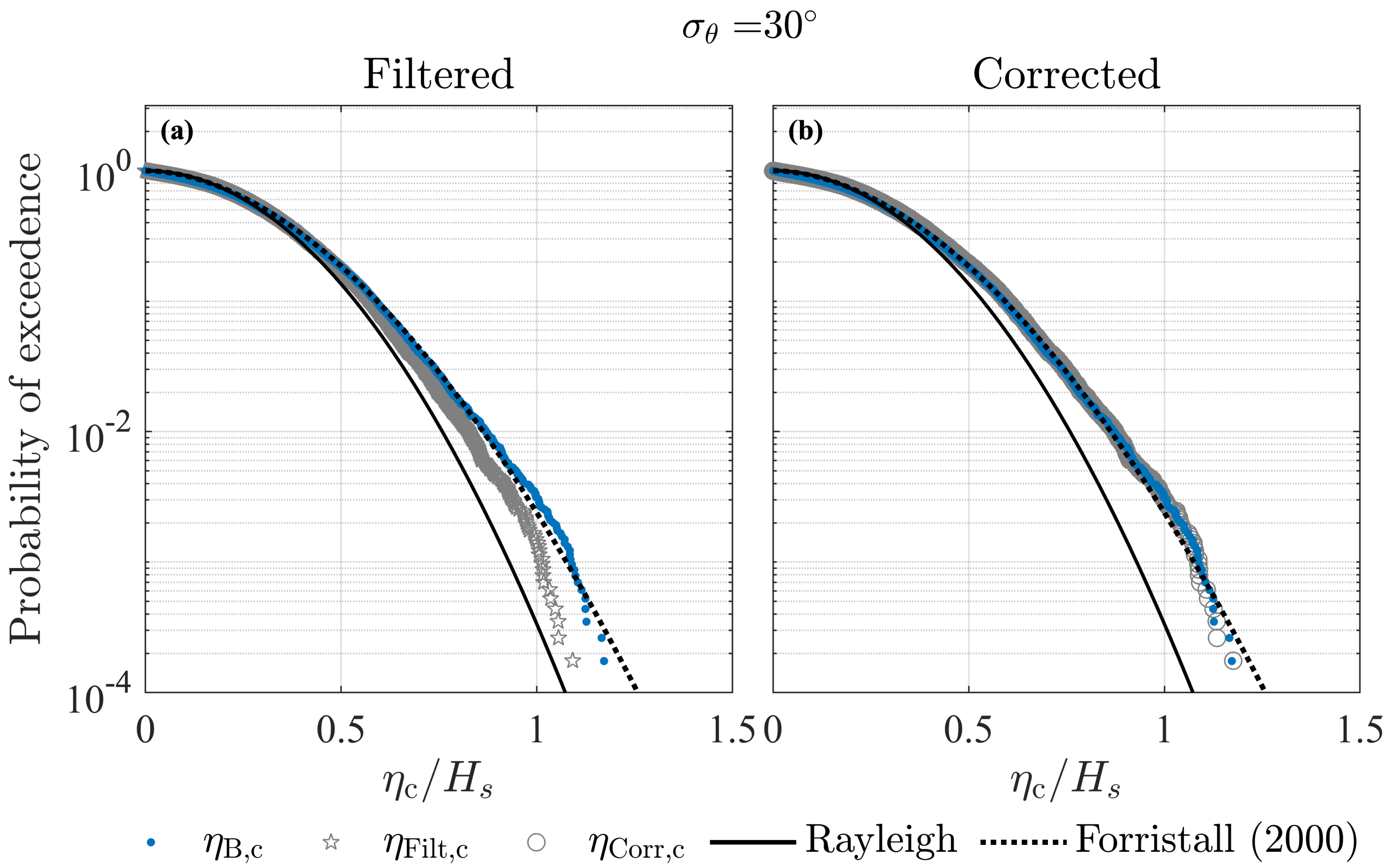
$$\eta_{Corr,c} = \eta_c + \eta_{c-}^{(2)}$$

$$\eta_{c-}^{(2)} = \frac{1}{2}(\eta_c)^2\, k_0 \tilde{\eta}^{(2)}(\sigma_\theta, \Delta\theta)\left(1 - \varepsilon_{hpf}\right)$$

$$\tilde{\eta}^{(2)}(\sigma_\theta) = \frac{1}{2}\left(1 + e^{-\sigma_\theta^2}\right)$$

$$\tilde{\eta}^{(2)}(\Delta\theta) = \frac{1}{2} + \frac{1}{4}(1 + \cos(\Delta\theta))$$

Approximate retrospective correction method for crest heights



Conclusions

- In deep water (ocean waves), second-order Lagrangain motion causes the cancelation of super-harmonic (sigma theta -> 0) and an increase in sub-harmonic contribution to crest height
- O(2) effects alone will not result in a change to crest height, however, this constitutes a shifting of bound energy from low to high
- For deterministic extreme (non breaking) wave groups buoy motion is essentially purely Lagrangian

- Spectral parameters (Hs, Tp. Etc.) are not significantly different between buoys and gauge measurements
- Filtering slightly affects measured Hs, and significantly reduces measured skewness λ^3
- Wave and crest height measured by buoys and gauges follow the same distributions

- Simplified expressions for second-order contribution to crest height can be used to retrospectively correct measurements and remove the effects of filtering

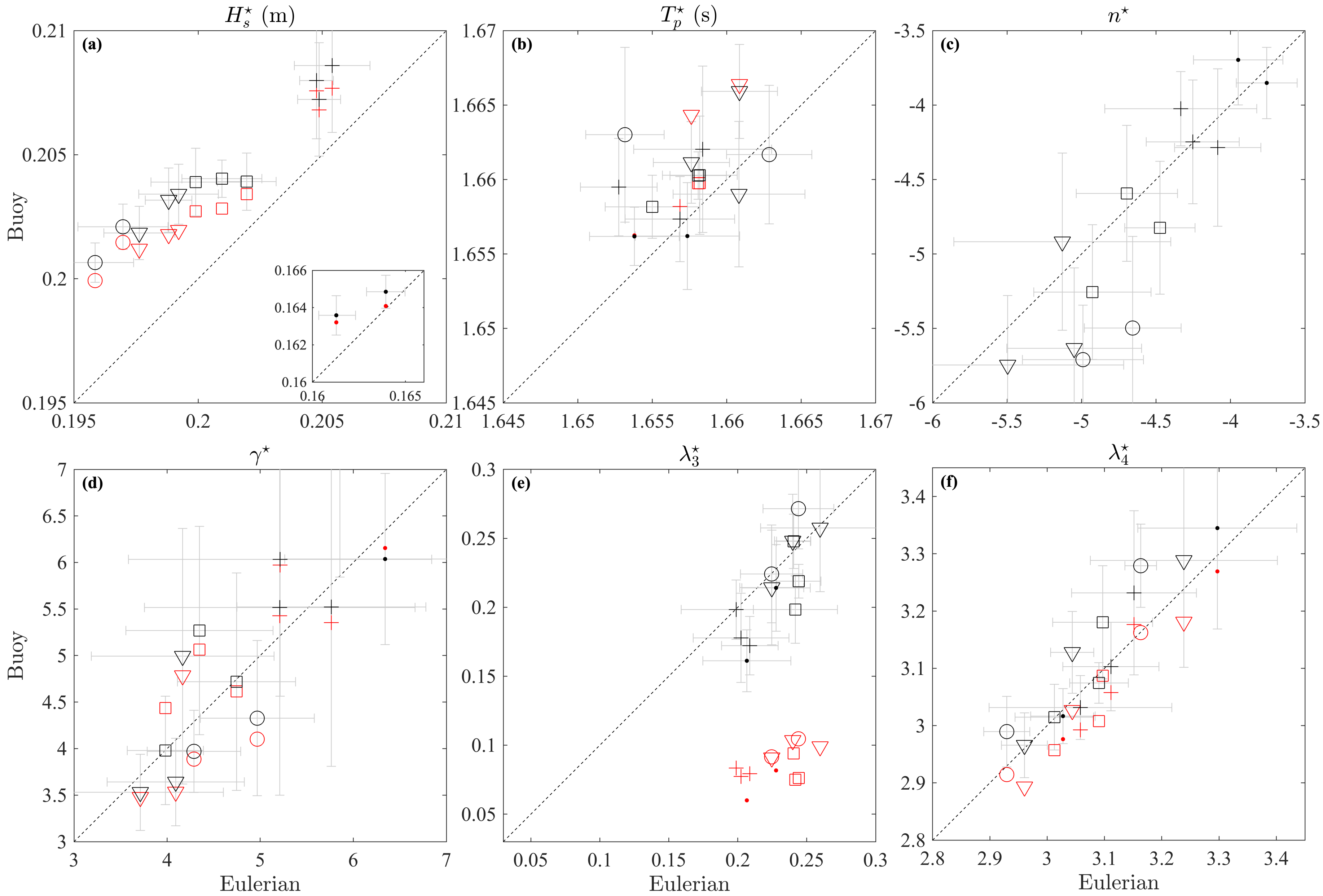
- These experiments do not consider a realistic mooring configuration, however, if a lack of mooring compliance was to cause an underestimation of crests we believe this would also affect measured wave heights

Thanks for your attention!

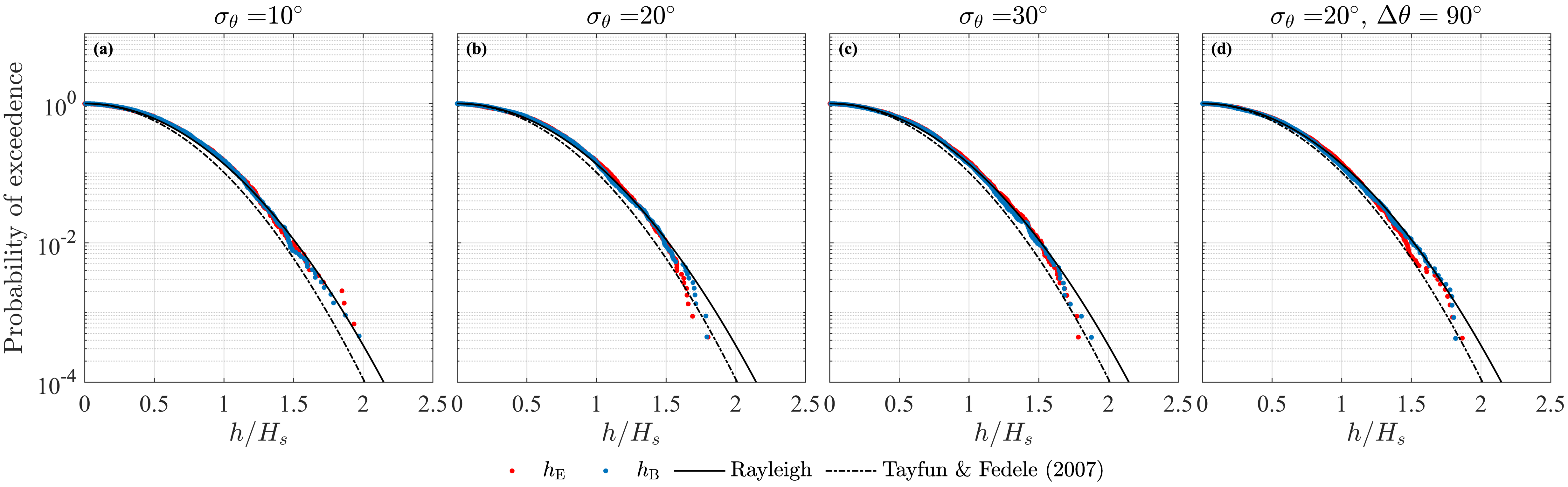
1. M. L. McAllister, and T. S. van den Bremer “Lagrangian Measurement of Steep Directionally Spread Ocean Waves: Second-Order Motion of a Wave-Following Measurement Buoy” **J. Phys. Oceanogr. (in press)**
2. M. L. McAllister, T. S. van den Bremer “Experimental Study of the Statistical Properties of Directionally Spread Ocean Waves Measured by Buoys” **J. Phys. Oceanogr.** (in press)

Photo credit: CDIP

Spectral Parameters

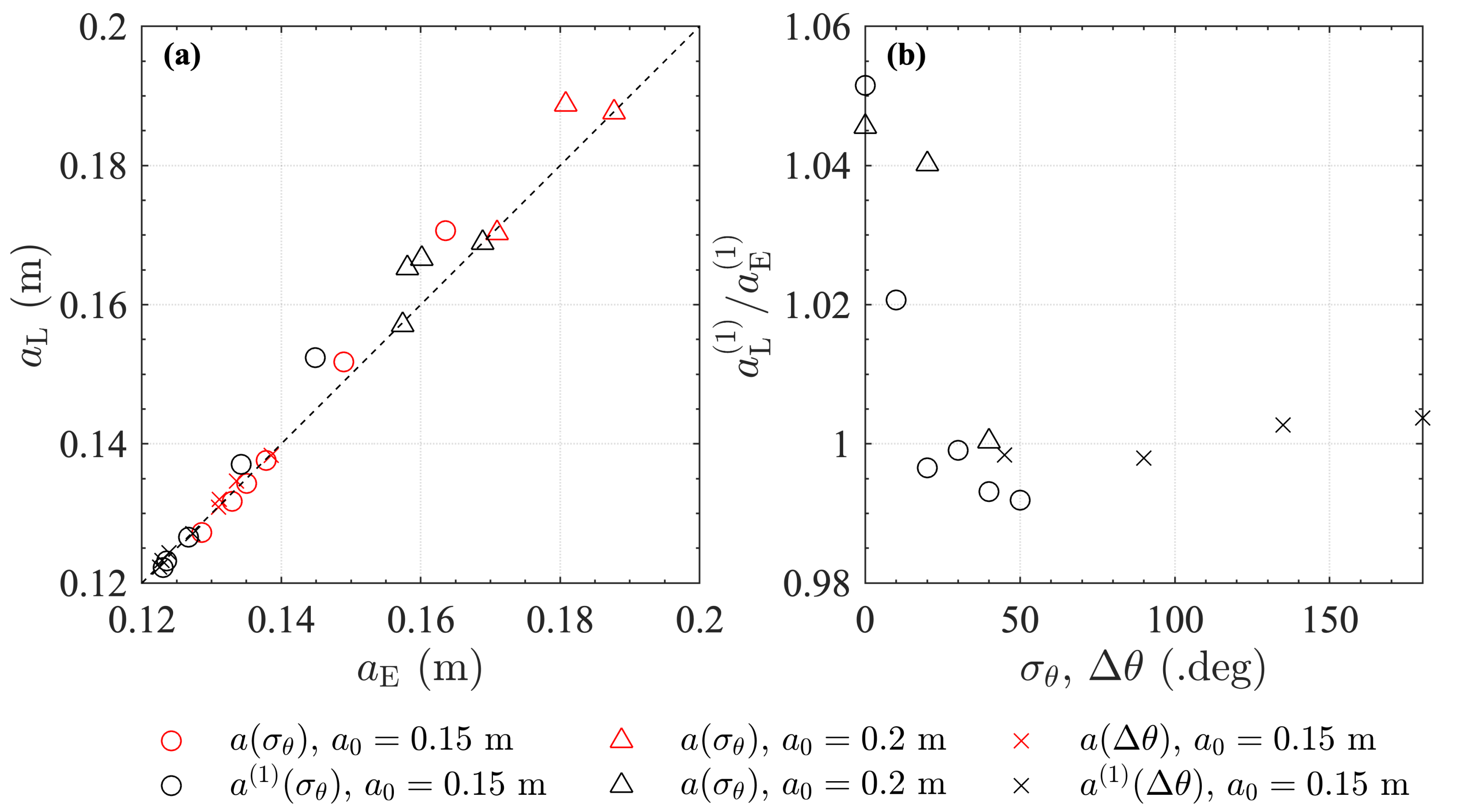


Wave height



$\sigma_\theta = 10^\circ$
$\sigma_\theta = 20^\circ$
$\sigma_\theta = 30^\circ$
$\sigma_\theta = 20^\circ,\ \Delta\theta = 90^\circ$
(a)
(b)
(c)
(d)
Probability of exceedence
10^0
10^{-2}
10^{-4}
0
0.5
1
1.5
2
2.5
h/H_s
h_{E}
h_{B}
Rayleigh
Tayfun & Fedele (2007)

Amplitude/Period



Frequency attenuation