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On the Rapid Spectral Evolution of Steep Wave Groups

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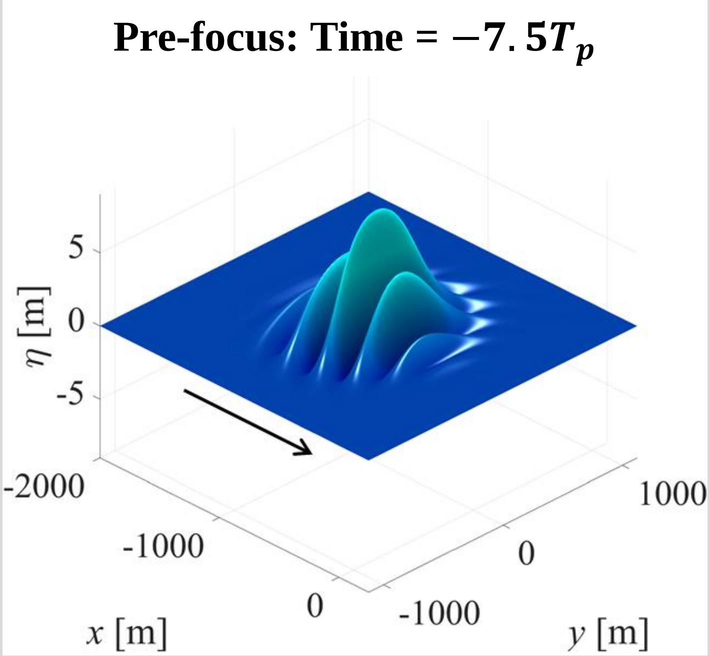
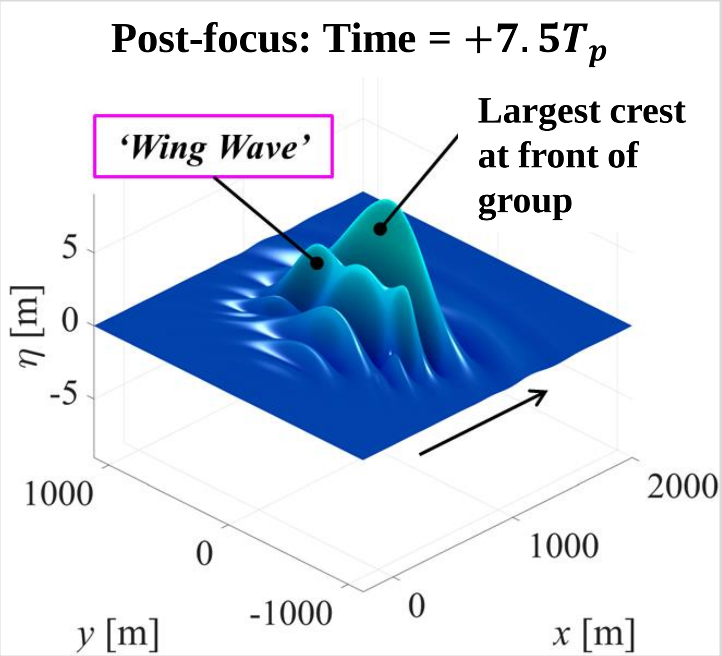
Potential Flow Simulations of Steep Wave Groups Reveal Directional Energy Transfers to Oblique Components

- Directional energy transfers associated with augmented kinematics, prolonged lifespan and 'wing wave' formation—localized oblique protrusions at edge of wave group





Gaussian wavenumber spectrum with Gaussian spreading function

Ak_p	k_p	k_w	σ
0.3	0.02796m^{-1}	0.004606m^{-1}	15°

Gibbs & Taylor (2005) Appl. Ocean Res., **27**.

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Numerical Simulations Performed with OceanWave3D Potential Flow Solver

- OceanWave3D solves fully nonlinear potential flow equations for water waves with finite difference spatial discretization & fourth-order Runge-Kutta (RK4) time-stepping
- Initial conditions based on Quasi-Determinism (QD) theory calculated 15 wave periods before focus according to linear theory and evolved for 30 wave periods with OceanWave3D

$$\eta(x,y,t) = A_L \frac{\sum_{i,j} F(k_i,\theta_j)\cos\left(k_i\cos\theta_j\, x + k_i\sin\theta_j\, y - \omega_i t + \varphi_0\right)}{\sum_{i,j} F(k_i,\theta_j)}$$

- Exact second-order correction of initial conditions based on Dalzell (1999)
- Linearisation of surface elevation to remove bound harmonics using phase-separation

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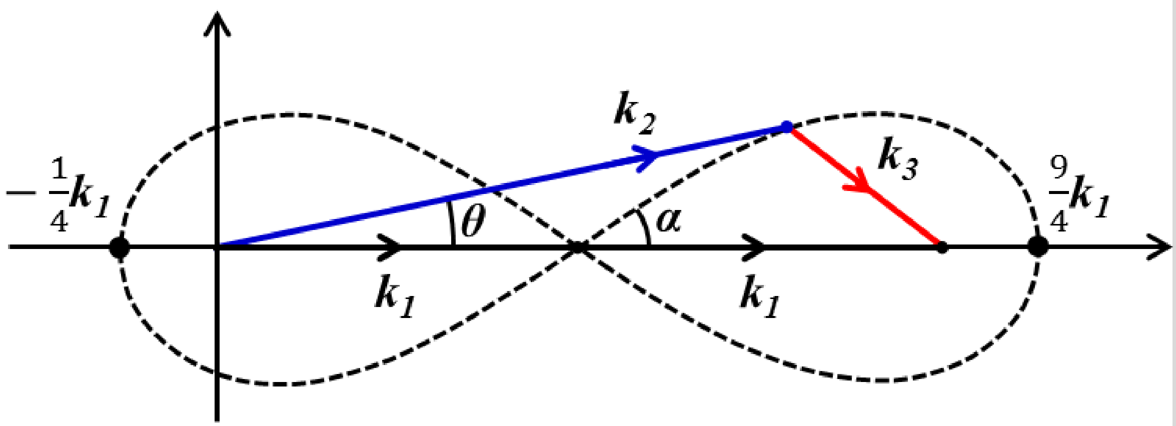
Phillips ‘Figure-of-Eight’ Resonance Loop Describes Energy Transfers of Degenerate Quartet

- Third-order resonance predicts energy transfer between wave components for deep-water:

$$\boldsymbol{k_1} - \boldsymbol{k_2} - \boldsymbol{k_3} + \boldsymbol{k_4} = 0 \quad \text{and} \quad \omega_1 - \omega_2 - \omega_3 + \omega_4 = 0$$

- Narrow-banded spectrum concentrates energy around spectral peak suggesting interactions resembling ‘*degenerate quartet*’ ($\boldsymbol{k_1} = \boldsymbol{k_4}$) of Phillips (1960)
- *Degenerate quartet* predicts energy transfer to oblique wave components at angle ±35.26°



$$n \equiv \frac{\omega_2 - \omega_1}{\omega_1},$$

approaches $\alpha = \pm 35.26°$ as n approaches zero

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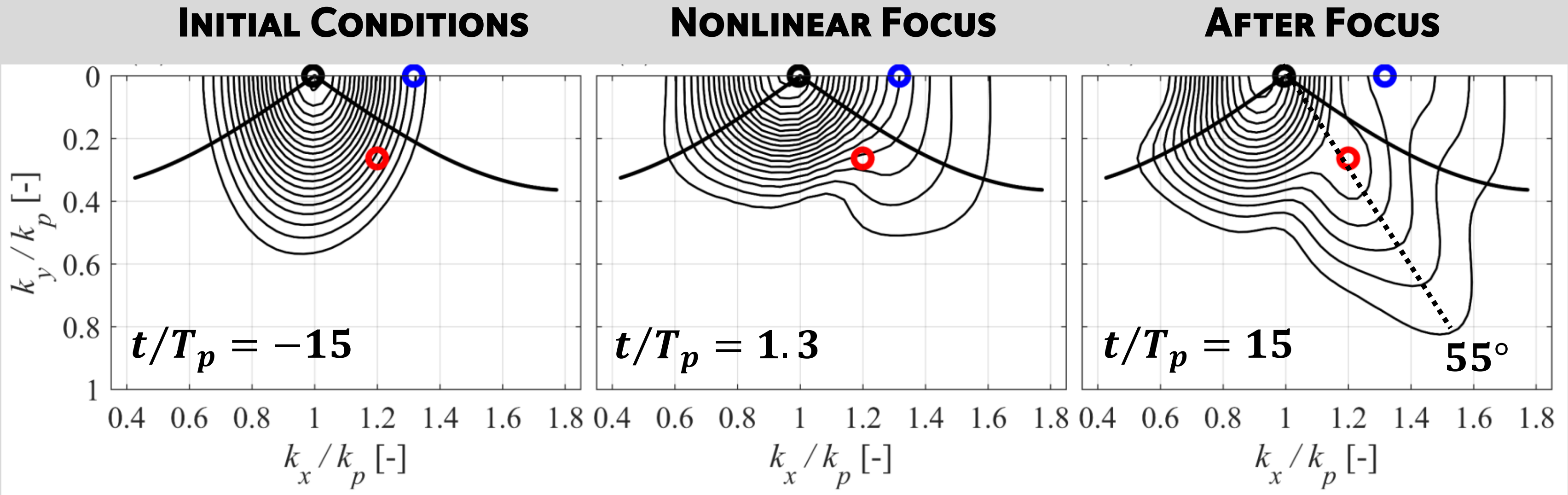
Directional Energy Transfers of Narrow-Banded Wave Group Qualitatively Resemble Phillips Resonance Loop

- Quasi-degenerate interactions transfer energy to high-wavenumber components at angle of ±35° to spectral peak until nonlinear focus
- Broadening of spectrum by quasi-degenerate interactions facilitates non-degenerate interactions after focus, forming a high-wavenumber ridge at ±55° to spectral peak



WAVENUMBER–AMPLITUDE SPECTRA BASED ON LINEARISED SURFACE ELEVATION

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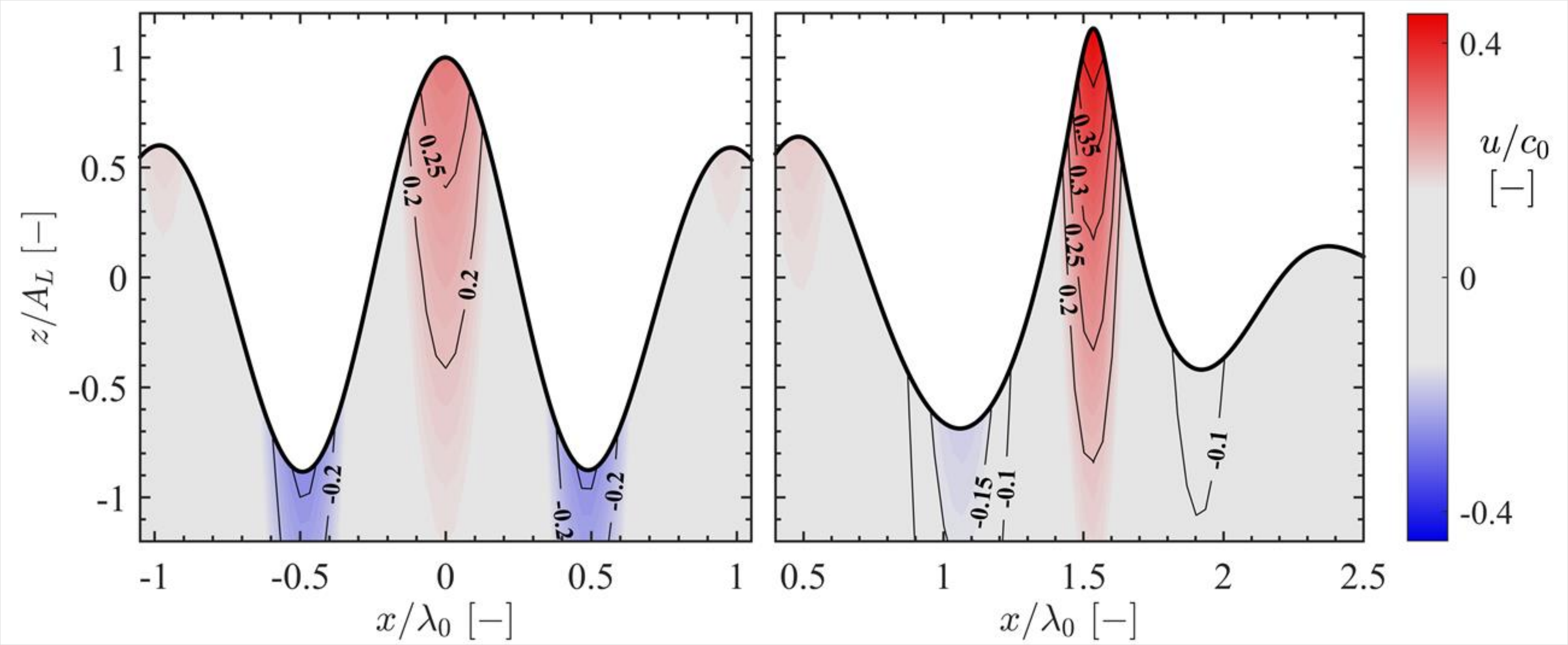
Energy Transfer to High-Wavenumber Components Augments Kinematics and Loads

- Velocities and accelerations scale with powers of component angular frequency.
- Thus, increase in drag and inertial loads according to the Morison equation:

$$F = \int_{-d}^{\eta} \frac{1}{2} C_D \rho A \, u(z)|u(z)| \, dz + \int_{-d}^{\eta} C_M \rho V \frac{Du(z)}{Dt} \, dz$$

- Drag loads 24% higher for nonlinear focused event, compare with linear focused event.

LINEAR FOCUS **NONLINEAR FOCUS**



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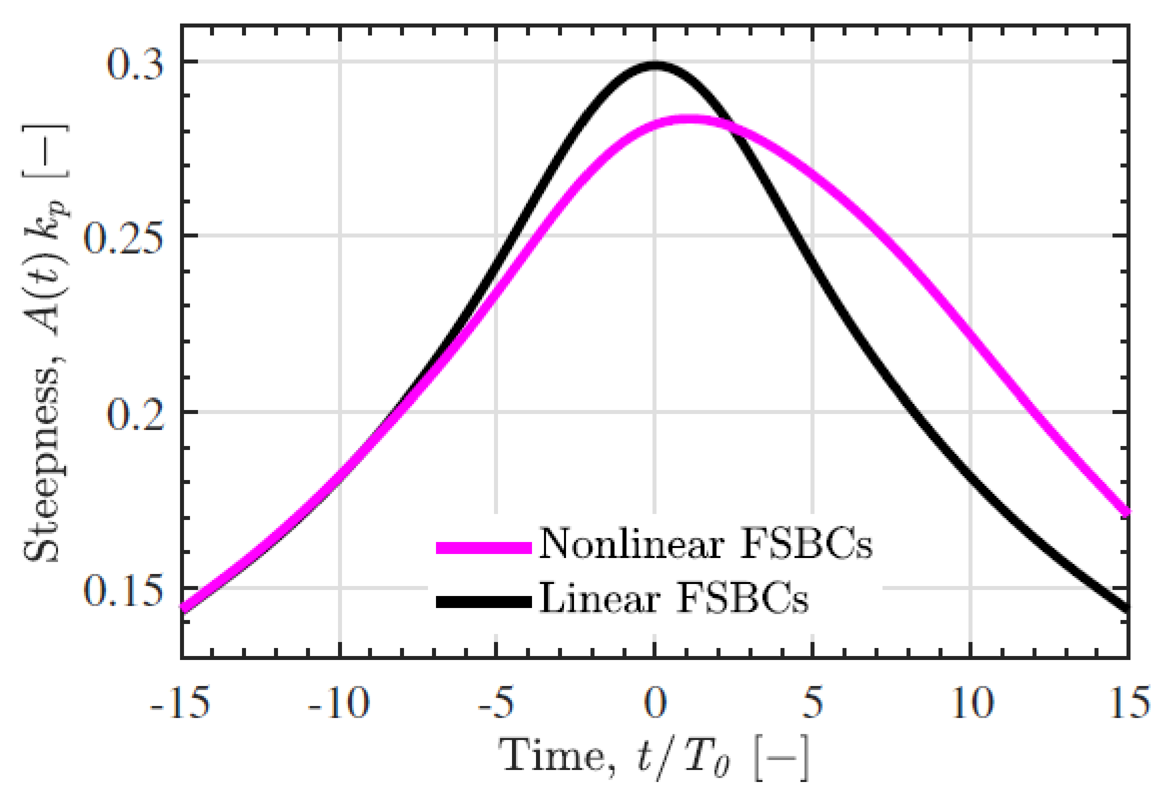
Spectral Evolution Extends Lifespan of Focused Wave Event

- Nonlinear interactions counteract dispersion due to modified phase velocity of components
- Oblique energy transfers at ±35.26° associated with suppressed linear dispersion as shown by Steer et. al. (2019)



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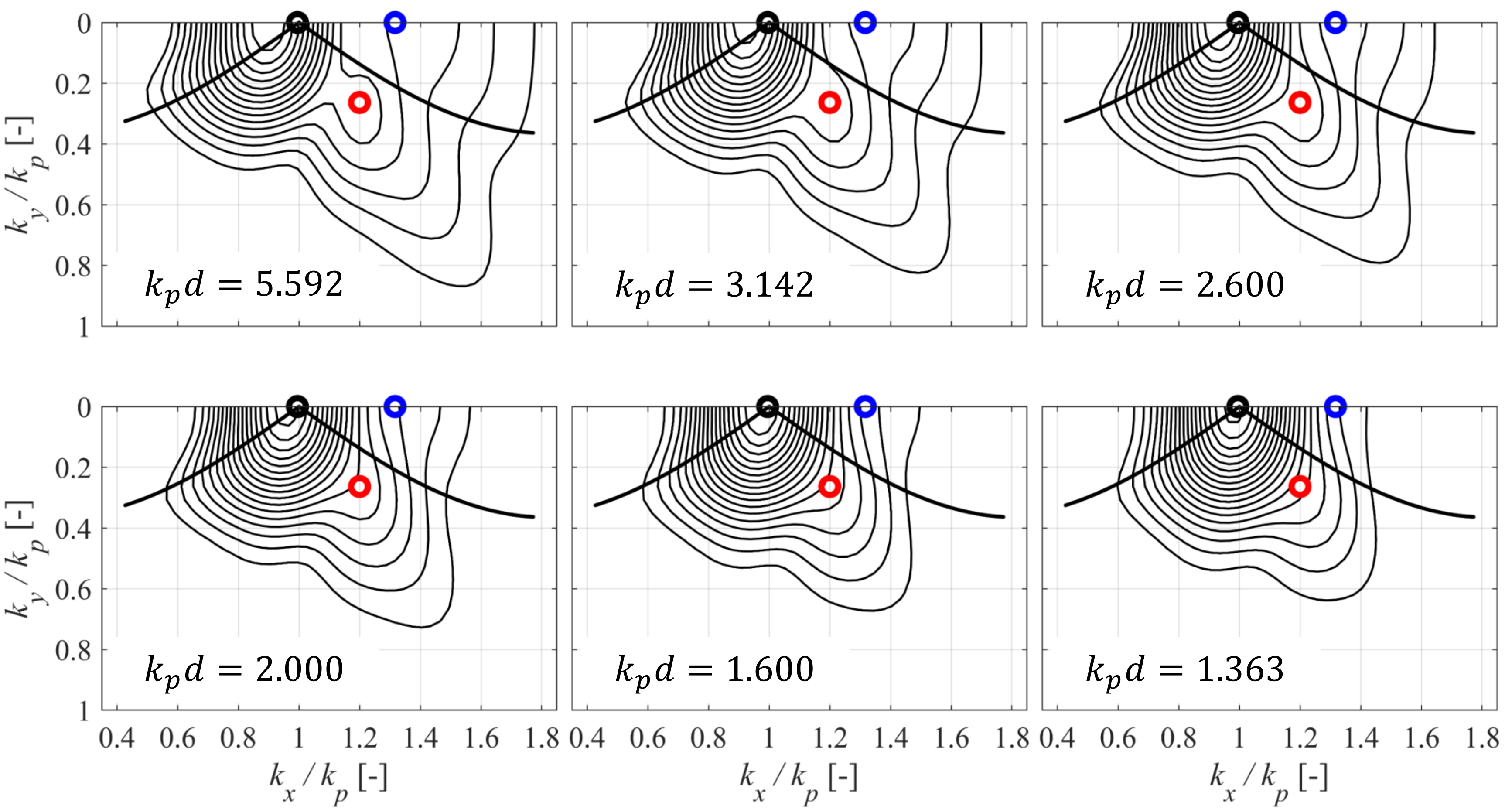
All Forms of Spectral Evolution Weakened by Finite Depth

- Directional energy transfers at ±35° almost disappear. However, energy transfers at ±55° less suppressed by depth and remain apparent at intermediate depths.
- Depth sensitivity observed within the range of ‘deep water’ ($5.592 > k_p d \geq 3.142$)



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Conclusions

- Spectral evolution augments kinematics and extends lifespan of simulated wave events.
- *Quasi-degenerate interactions* dominate spectral evolution until nonlinear focus.
- Spectral broadening facilitates *non-degenerate interactions*, which dominate after focus.
- All forms of spectral evolution weakened by depth. However, quasi-degenerate interactions exhibit greater depth sensitivity than non-degenerate interactions.