Transformation of envelope solitons propagating over a bottom step

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Abstract

The problem of the **weakly nonlinear wave transformation on a bottom step** is studied **analytically** and **numerically** by means of the direct simulation of the Euler equation. It is assumed that the quasi-linear wave packets can be described by the nonlinear Schrödinger equation for surface waves in finite-depth water. The process of wave transformation in the vicinity of the bottom step can be described within the framework of the linear theory and the transformation coefficients (the transmission and reflection coefficients) can be determined by the approximate formula suggested in [1]. The fate of transmitted and reflected wave trains emerging from the incident envelope soliton can be determined with the help of the Inverse Scattering Technique [2, 3]. The parameters of secondary envelope solitons (their number, amplitudes, and speeds) asymptotically forming in the far-field zone are obtained analytically and compared against the numerically calculated ones, as the functions of the depth drop h_2/h_1, where h_1 and h_2 are the undisturbed water depths in front of and behind the bottom step, respectively. It is shown that the **wave amplitudes can notably increase** when the envelope soliton travels **from the relatively shallow to much deeper water**. The amplitudes of secondary solitons **can exceed more than twice** the amplitude of the incident wave.

The direct numerical simulation of envelope soliton transformation was undertaken by means of the High Order Spectral Method [4, 5]. The comparison of approximate analytical solutions with the results of numerical simulations reveals the domains of **very good agreement** between the data where the approximate theory is applicable. In the meantime, the **noticeable disagreement** between the approximate nonlinear theory and the direct simulations is found when the theory is inapplicable.

References

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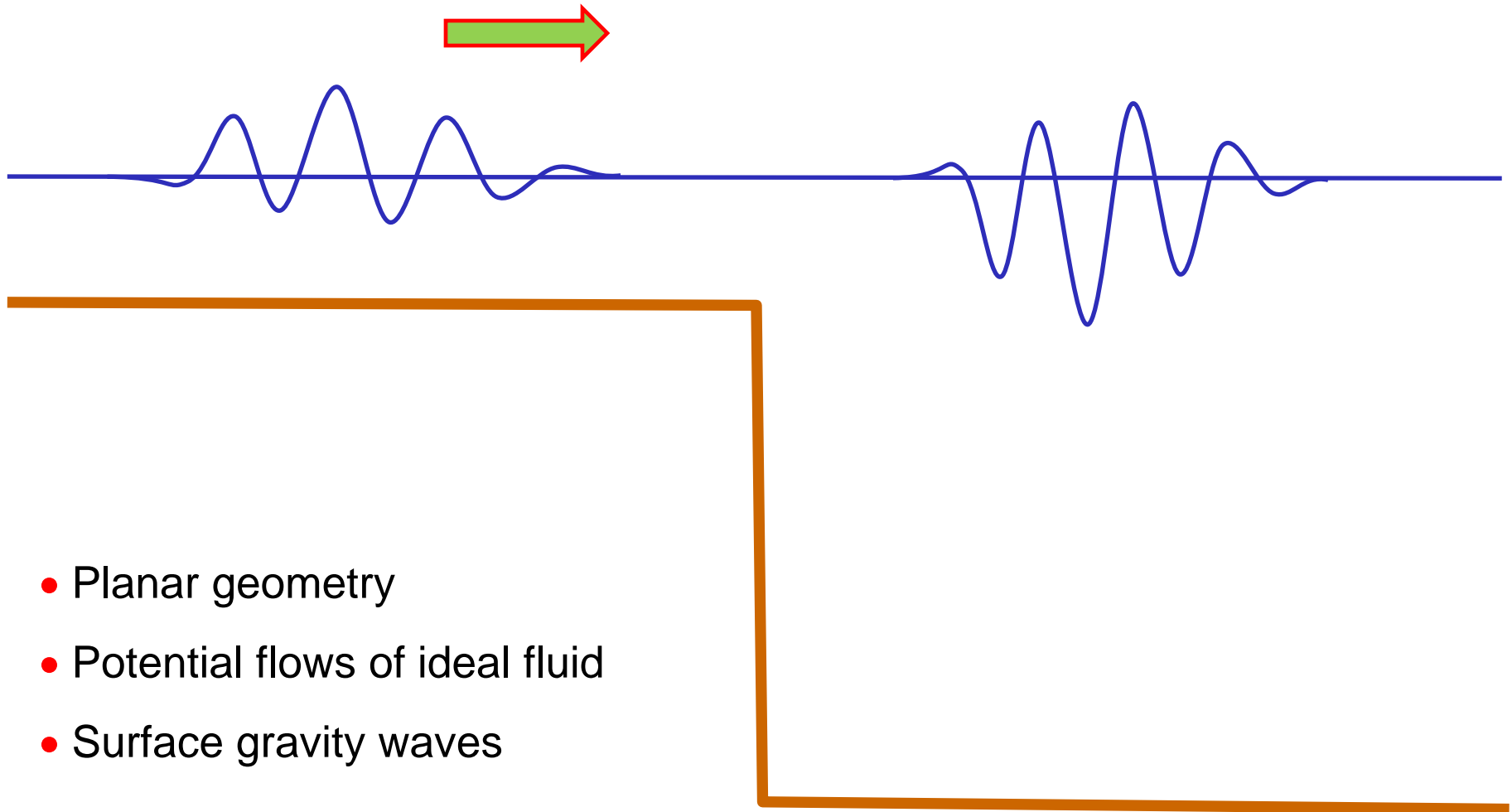
- Planar geometry

- Potential flows of ideal fluid
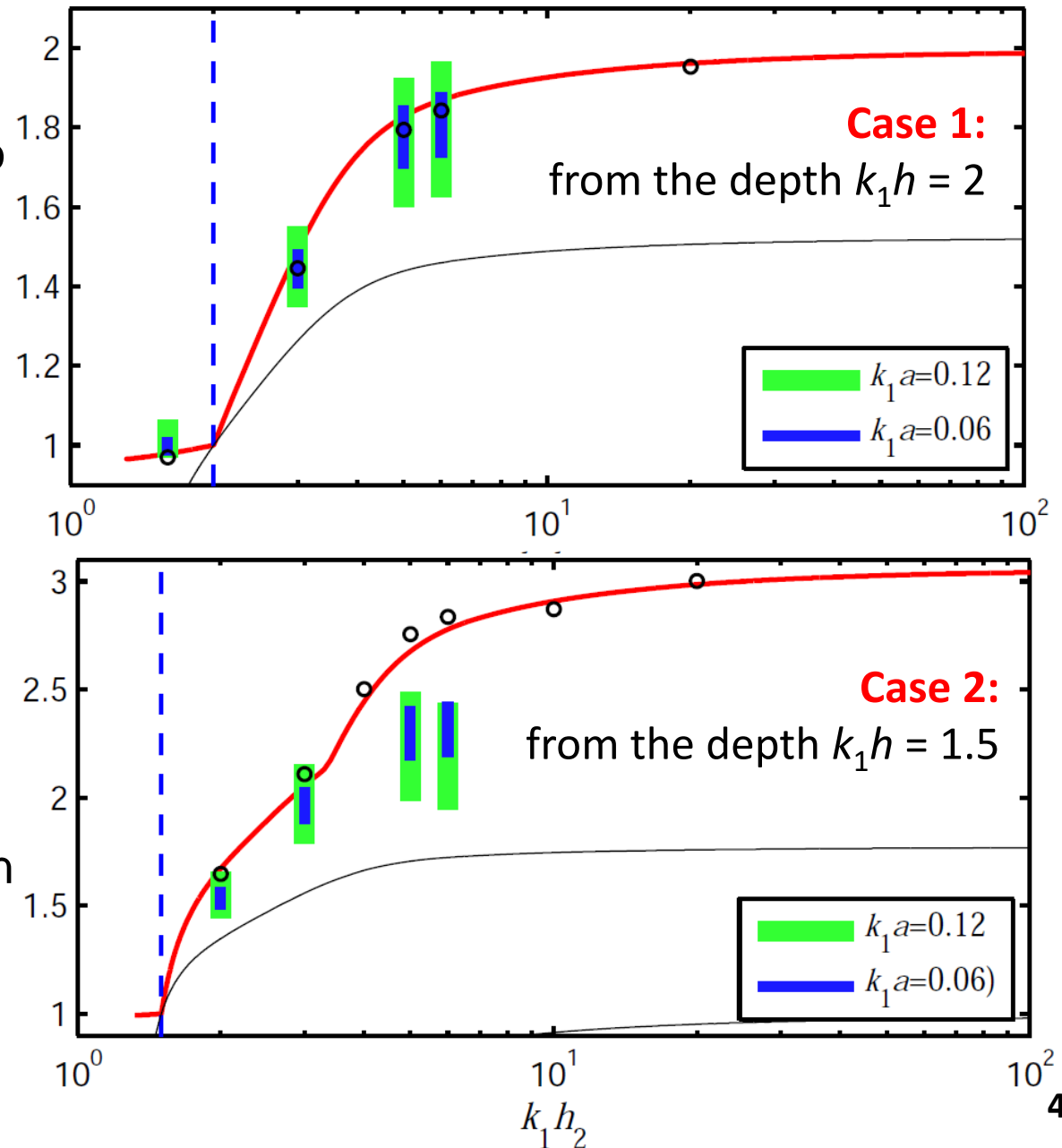
- Surface gravity waves

Wave amplification tests

The two cases correspond to the wave deepening from two dimensionless depths $k_1 h$ in the left-hand-side domain:

Wave groups with two initial stepnesses $k_1 a$ were tested

Case 1:
from the depth $k_1 h = 2$

(green bar)	$k_1 a = 0.12$
(blue bar)	$k_1 a = 0.06$

Red line – analytic solution

Black circles – direct numerical simulation of the NLS equation

Bars – HOSM simulations

Case 2:
from the depth $k_1 h = 1.5$

(green bar)	$k_1 a = 0.12$
(blue bar)	$k_1 a = 0.06$)

$k_1 h_2$

The paper is in progress…