Strongly coherent dynamics of stochastic waves causes abnormal sea states

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Abstract

The **dynamic kurtosis** (i.e., produced by the free wave component) is shown to contribute essentially to the abnormally large values of the full kurtosis of the surface displacement, according to the direct numerical simulations of realistic directional sea waves within the HOSM framework. In this situation the **free wave stochastic dynamics is strongly non-Gaussian**, and the kinetic approach is inapplicable. Traces of **coherent wave patterns** are found in the Fourier transform of the directional irregular sea waves. They strongly **violate the classic dispersion relation** and hence lead to a greater spread of the actual wave frequencies for given wavenumbers.

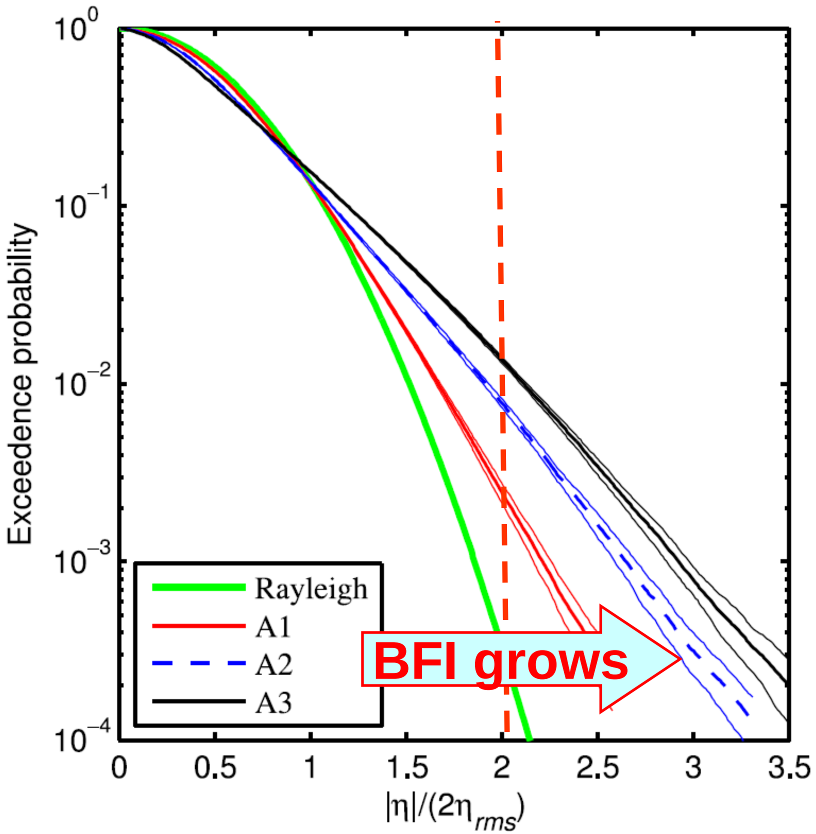
References

- **A. Slunyaev, A. Kokorina,** The method of spectral decomposition into free and bound wave components. Numerical simulations of the 3D sea wave states. *Geophys. Research Abstracts*, V. 21, EGU2019-546 (2019).
- **A.V. Slunyaev, A.V. Kokorina,** Spectral decomposition of simulated sea waves into free and bound wave components. *Proc. VII Int. Conf. "Frontiers of Nonlinear Physics"*, 189-190 (2019).
- **A. Slunyaev, A. Kokorina, I. Didenkulova,** Statistics of free and bound components of deep-water waves. Proc. 14th Int. *MEDCOAST Congress on Coastal and Marine Sciences, Engineering, Management and Conservation* (Ed. E. Ozhan), Vol. 2, 775-786 (2019).
- **A. Slunyaev,** Strongly coherent dynamics of stochastic waves causes abnormal sea states. *arXiv:* 1911.11532 (2019).

Rogue wave problem

Wave height probability and the Benjamin – Feir instability

Routine forecast of the BFI maps



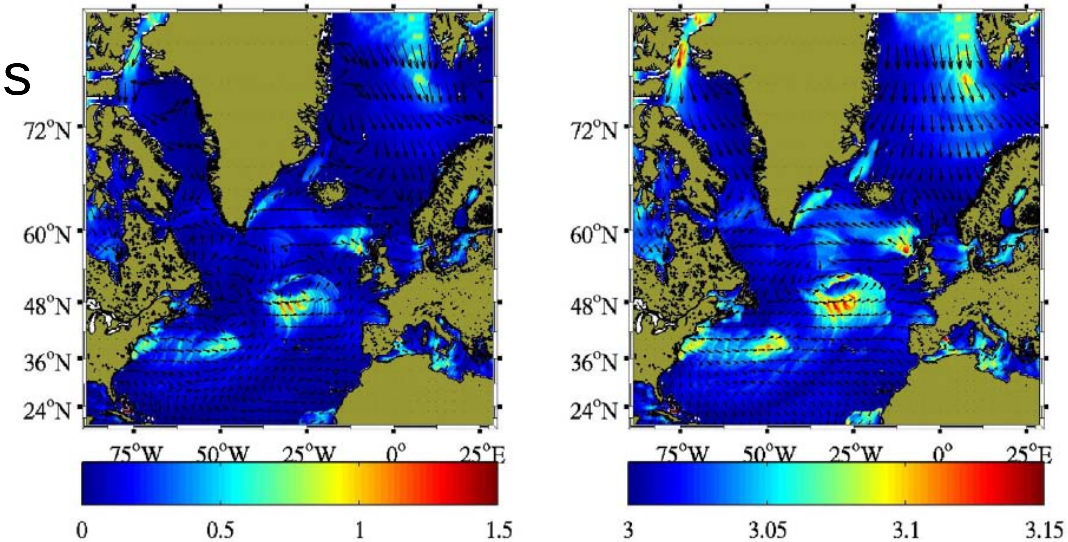
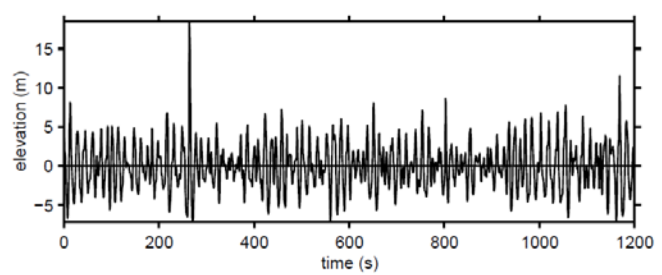
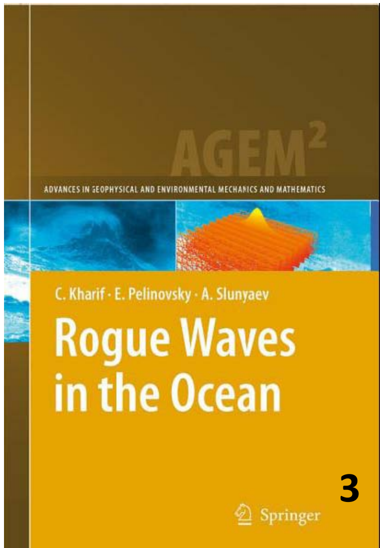


Fig. 6. BFI (left) and the kurtosis (right) spatial distribution. Date: 9th February 2007 at 12 UTC.

[De Ponce & Guedes Soares, 2014]





Large BFI corresponds to high probability of large waves

[Kharif, Pelinovsky, Slunyaev, 2009]

Rogue wave problem

Wave height probability and the Benjamin – Feir instability

PHYSICS OF FLUIDS 17, 078101 (2005)

Modulational instability and non-Gaussian statistics in experimental random water-wave trains

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Real world ocean rogue waves explained without the modulational instability

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Francesco Fedele[1,2], Joseph Brennan[3], Sonia Ponce de León[3], John Dudley[4] & Frédéric Dias[3]

Since the 1990s, the modulational instability has commonly been used to explain the occurrence of rogue waves that appear from nowhere in the open ocean. However, the importance of this instability in the context of ocean waves is not well established. This mechanism has been successfully studied in
…athematical studies, but there is no consensus on what actually takes
…e question the oceanic relevance of this paradigm. In particular, we
…n various European locations with various tools, and find that the main
…waves is the constructive interference of elementary waves enhanced
…ities and not the modulational instability. This implies that rogue
…nces of weakly nonlinear random seas.

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European Journal of Mechanics B/Fluids •••

Extreme waves, modulational instability
wave flume experiments on ir

M. Onorato [a,*], A.R. Osborne [a], M. Serio [a], L. Cavaleri
[a] *Dipartimento di Fisica Generale, Università di Torino, Via P.*

…ments of Rogue Water Waves

…ARIOS CHRISTOU*
…*ns International B.V., Rijswijk, Netherlands*
KEVIN EWANS
…*ll Berhad, Kuala Lumpur, Malaysia*

…September 2013, in final form 17 April 2014)

ABSTRACT

…ty control, and analysis of single-point field measurements from
…ms. In total, the quality-controlled database contains 122 million
…aves. Geographically, the majority of the field measurements were
…tary data from the Gulf of Mexico, the South China Sea, and the
…nt wave height ranged from 0.12 to 15.4 m, the peak period ranged
… was 18.5 m, and the maximum recorded wave height was 25.5 m.
This paper will describe the offshore installations, instrumentation, and the strict quality control procedure employed to ensure a reliable dataset. An examination of sea state parameters, environmental conditions, and local characteristics is performed to gain an insight into the behavior of rogue waves. Evidence is provided to demonstrate that rogue waves are not governed by sea state parameters. Rather, the results are consistent with rogue waves being merely extraordinary and rare events of the normal population caused by dispersive focusing.

PRL 102, 114502 (2009) PHYSICAL REVIEW LET…

Statistical Properties of Directional Ocean Waves: The Ro… in the Formation of Extreme E…

M. Onorato,[1] T. Waseda,[2] A. Toffoli,[3] L. Cavaleri,[4] O. Gramstad,[5] P. A. E…
N. Mori,[9] A. R. Osborne,[1] M. Serio,[1] C. T. Stansberg,[10] H…
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Numerical simulations of irregular sea waves

Nonlinear simple deep-water wave

Stokes wave

free wave

bound / phase-locked / slave waves

1 1+1–1 1+1 1+1+1

A AAA^* AA AAA

$$k_0\eta(x,t)=\varepsilon\left(1+\frac{1}{8}\varepsilon^2+\dots\right)\cos\theta+\frac{1}{2}\varepsilon^2\cos 2\theta+\frac{3}{8}\varepsilon^3\cos 3\theta+O\left(\varepsilon^4\right)$$

Surface displacement

first harmonic / fundamental / primary component

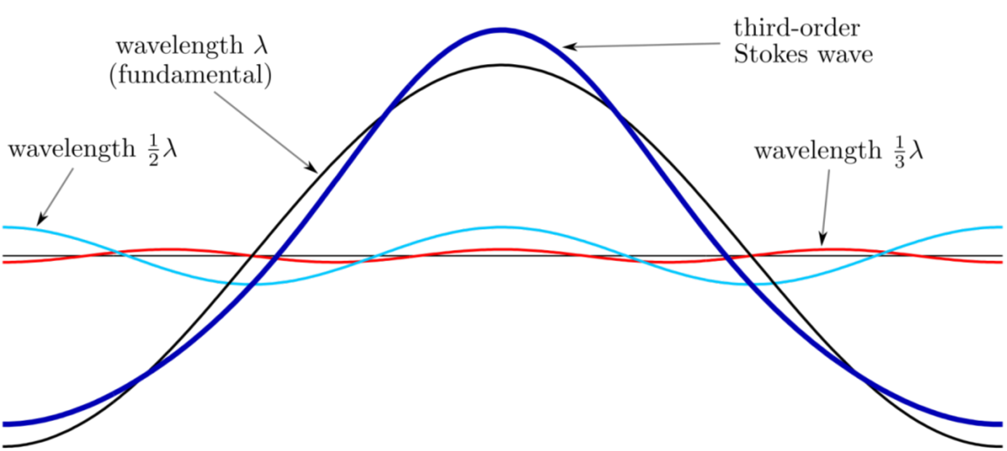
2nd harmonic

3rd harmonic

…

$\varepsilon = k_0 a$ steepness

$\theta = k_0 x - \omega t$ phase



Nonlinear Stokes waves are not sinusoidal due to the multiple harmonics

Narrow-banded weakly nonlinear waves

Statistical moments for the surface displacement

Fourth statistical moment, the kurtosis

$$\lambda_4 = \frac{\langle \eta^4 \rangle}{\langle \eta^2 \rangle^2}$$

$$\lambda_4 \approx 3 + 24\varepsilon^2 + \underbrace{\frac{\pi}{\sqrt{3}} BFI^2}_{\text{May be } O(1)}$$

[Mori & Janssen, JPO2006]

The Gaussian statistics

Bound wave contribution (wave unsinusoidality)

Dynamic part: Benjamin-Feir instability (quasi-resonant interactions) [Onorato et al, 2001]

Exceedance probability for wave heights *H*

$$P(H) \approx \exp\left(-\frac{H^2}{8\sigma^2}\right)\left[1 + (\lambda_4 - 3)B\left(\frac{H}{\sigma}\right)\right]$$

$$B(\xi) = \frac{1}{384}\xi^2\left(\xi^2 - 16\right)$$

The probability of large waves increases when the kurtosis surpasses the value of three

Large kurtosis is a signature of dangerous wave conditions

[Mori & Janssen, JPO2016]

Dynamical spectral theory

Hamiltonian weakly nonlinear Zakharov's equation

Non-resonant terms are eliminated with the help of the canonical transformation of variables ($(\eta, \Phi) \to a(\mathbf{k}, t) \to b(\mathbf{k}, t)$).
Higher than 4-wave resonances are not resolved (wave slopes should be mild).

$$i\frac{\partial b_0}{\partial t} = (\omega_0 + i\gamma_0)b_0 + \iiint T(\vec{k}_0, \vec{k}_1, \vec{k}_2, \vec{k}_3) b_1^* b_2 b_3 \delta(\vec{k}_0 + \vec{k}_1 - \vec{k}_2 - \vec{k}_3) d\vec{k}_1 d\vec{k}_2 d\vec{k}_3$$

$$b(\vec{k}, t) = \frac{1}{\sqrt{2}}\left(\sqrt{\frac{\omega(\vec{k})}{|\vec{k}|}}\eta(\vec{k}, t) + i\sqrt{\frac{|\vec{k}|}{\omega(\vec{k})}}\Phi(\vec{k}, t)\right) + HOT \qquad \omega(\vec{k}) = \sqrt{g|\vec{k}|}$$

Dynamic kurtosis (resonant and near-resonant interactions)

$$\lambda_4^{dyn} = \frac{\frac{3}{4}\iiiint \sqrt{\omega_0\omega_1\omega_2\omega_3}\langle b_0^* b_1^* b_2 b_3\rangle d\vec{k}_0 d\vec{k}_1 d\vec{k}_2 d\vec{k}_3 + c.c.}{\sigma^4} \qquad \sigma^2 = \int \omega_0 N_0 d\vec{k}_0$$

Bound wave kurtosis (non-resonant interactions)

$$N_0 = \langle b_0^* b_0\rangle$$

$$\lambda_4^{bound} = \frac{12\int F(\vec{k}_0, \vec{k}_1, \vec{k}_2)\omega_0\omega_1\omega_2 N_0 N_1 N_2 d\vec{k}_0 d\vec{k}_1 d\vec{k}_2}{\sigma^4}$$

[Zakharov, 1968; Krasitskii, 1994; Janssen, 2008]

Kinetic spectral theory

Phase-averaged equations

$$\underbrace{\frac{\partial N_{\vec{k}}}{\partial t} + \underbrace{\nabla_{\vec{k}}\omega_{\vec{k}}}_{\text{group velocity}}\nabla_{\vec{r}} N_{\vec{k}}}_{\text{advection}} = \underbrace{S_{nl}\left(N_{\vec{k}}\right)}_{\text{nonlinear effects}} + \underbrace{S_{in}\left(N_{\vec{k}}\right)}_{\text{forcing}} + \underbrace{S_{diss}\left(N_{\vec{k}}\right)}_{\text{dissipative terms}}$$

wave action $\quad N_{\vec{k}} = \frac{E_{\vec{k}}}{\omega\left(\vec{k}\right)}$

dispersion relation $\quad \omega\left(\vec{k}\right)$

Cannot describe effects of coherent wave dynamics

$$S_{nl}\left[N_{\mathbf{k}}\right] = \int_{\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3} |T_{\mathbf{k},\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3}|^2 \{N_2N_3(N+N_1) - NN_1(N_2+N_3)\} \times\delta(\mathbf{k}+\mathbf{k}_1-\mathbf{k}_2-\mathbf{k}_3)\delta(\omega+\omega_1-\omega_2-\omega_3)d\mathbf{k}_1 d\mathbf{k}_2 d\mathbf{k}_3$$

~ Boltzmann collision integral

$$N_{\vec{k}} = \left\langle b^*_{\vec{k}} b_{\vec{k}} \right\rangle$$

phase averaging
assuming random incoherent phases

[Hasselmann, JFM1962]

Kinetic spectral theory

Phase-averaged equations

$$\frac{\partial N_{\vec{k}}}{\partial t} + \nabla_{\vec{k}} \omega_{\vec{k}} \nabla_{\vec{r}} N_{\vec{k}} = S_{nl}\left(N_{\vec{k}}\right) + \cancel{S_{in}\left(N_{\vec{k}}\right)} + \cancel{S_{diss}\left(N_{\vec{k}}\right)}$$

Conservative equations in what follows (no wind, no dissipation)

Wave evolution within different frameworks

Evolution of the total kurtosis

J. Fluid Mech. (2018), vol. 844, pp. 766–795. © Cambridge University Press 2018 766
doi:10.1017/jfm.2018.185

Spectral evolution of weakly nonlinear random waves: kinetic description versus direct numerical simulations

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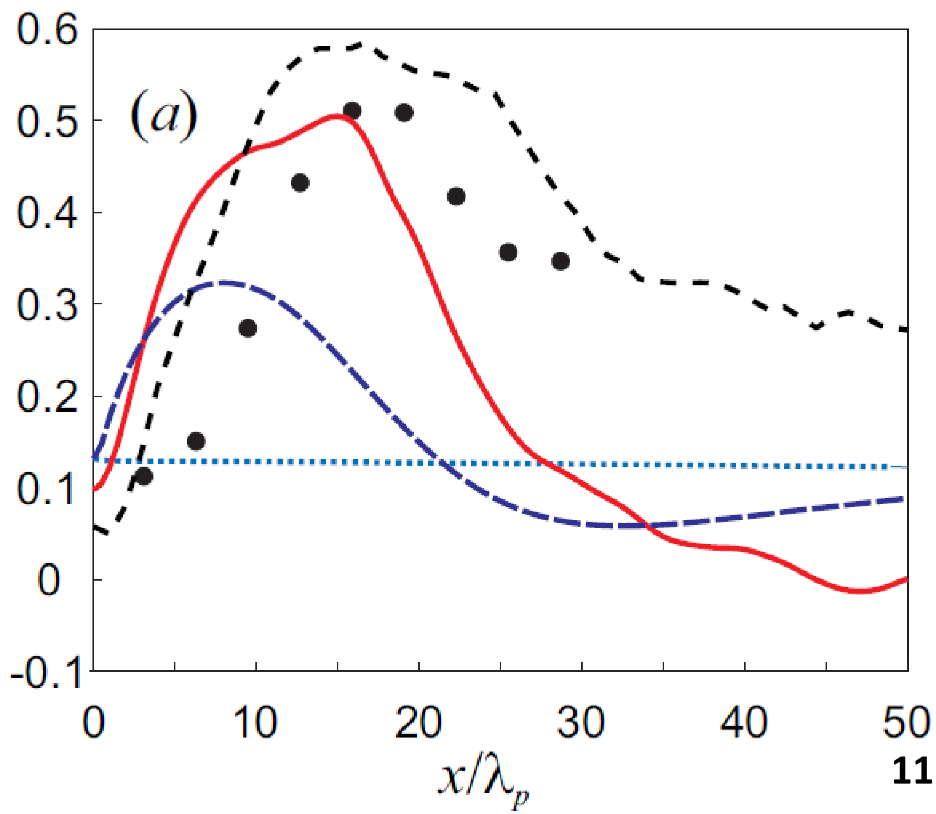
$$\lambda_4 = \frac{\langle \eta^4 \rangle}{\langle \eta^2 \rangle^2} - 3$$



The curves differ noticeably. The reason is unclear

- - - HOS (Euler eqs.) —— DNS-ZE — — — gKE

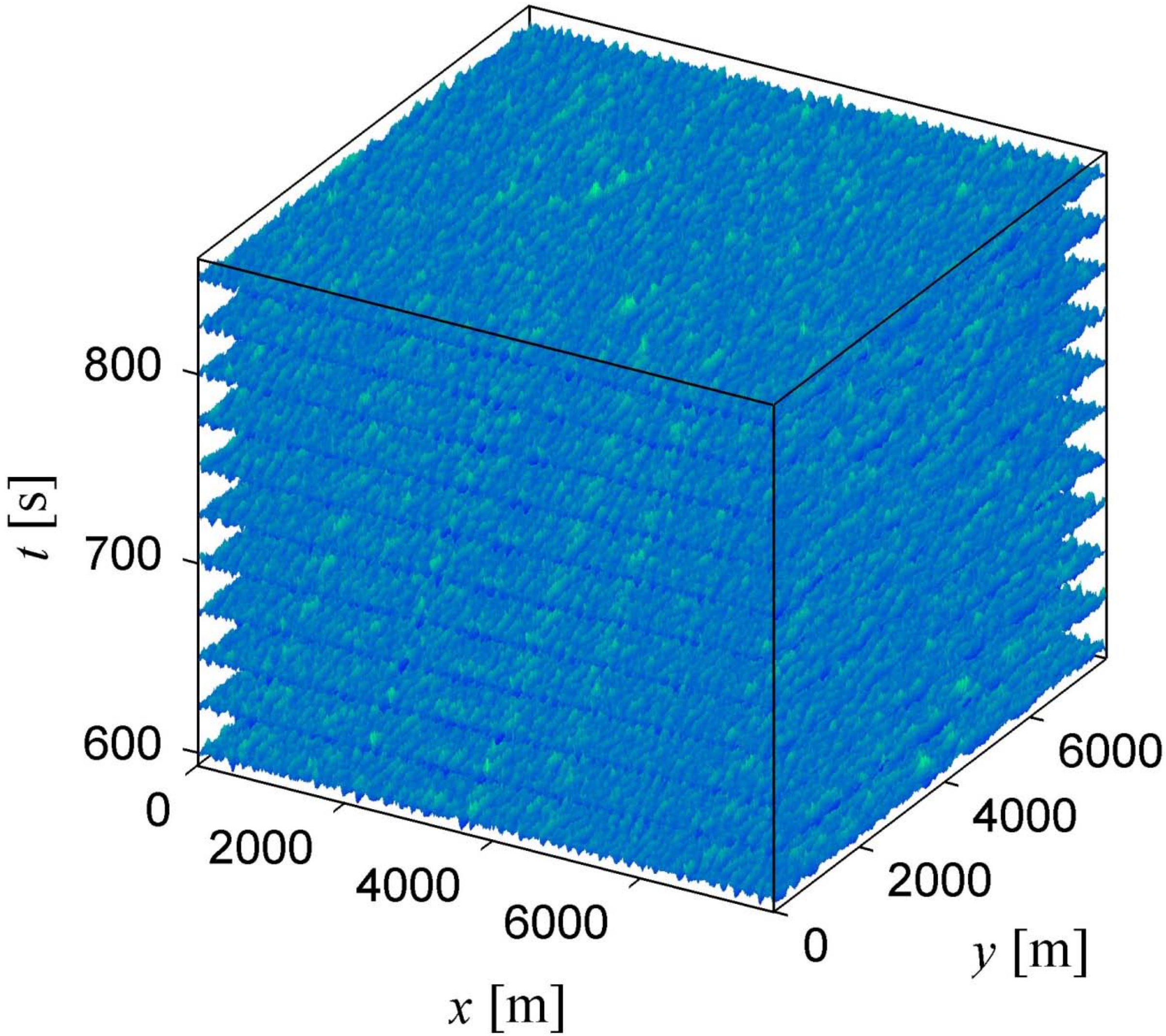
The method of wave decomposition into free wave and bound wave constituents

Direct numerical sims + Triple Fourier transform

Deep-water gravity waves obeying the JONSWAP spectrum are simulated by the High Order Spectral Method (HOSM, the potential Euler equations with truncated order of nonlinearity).

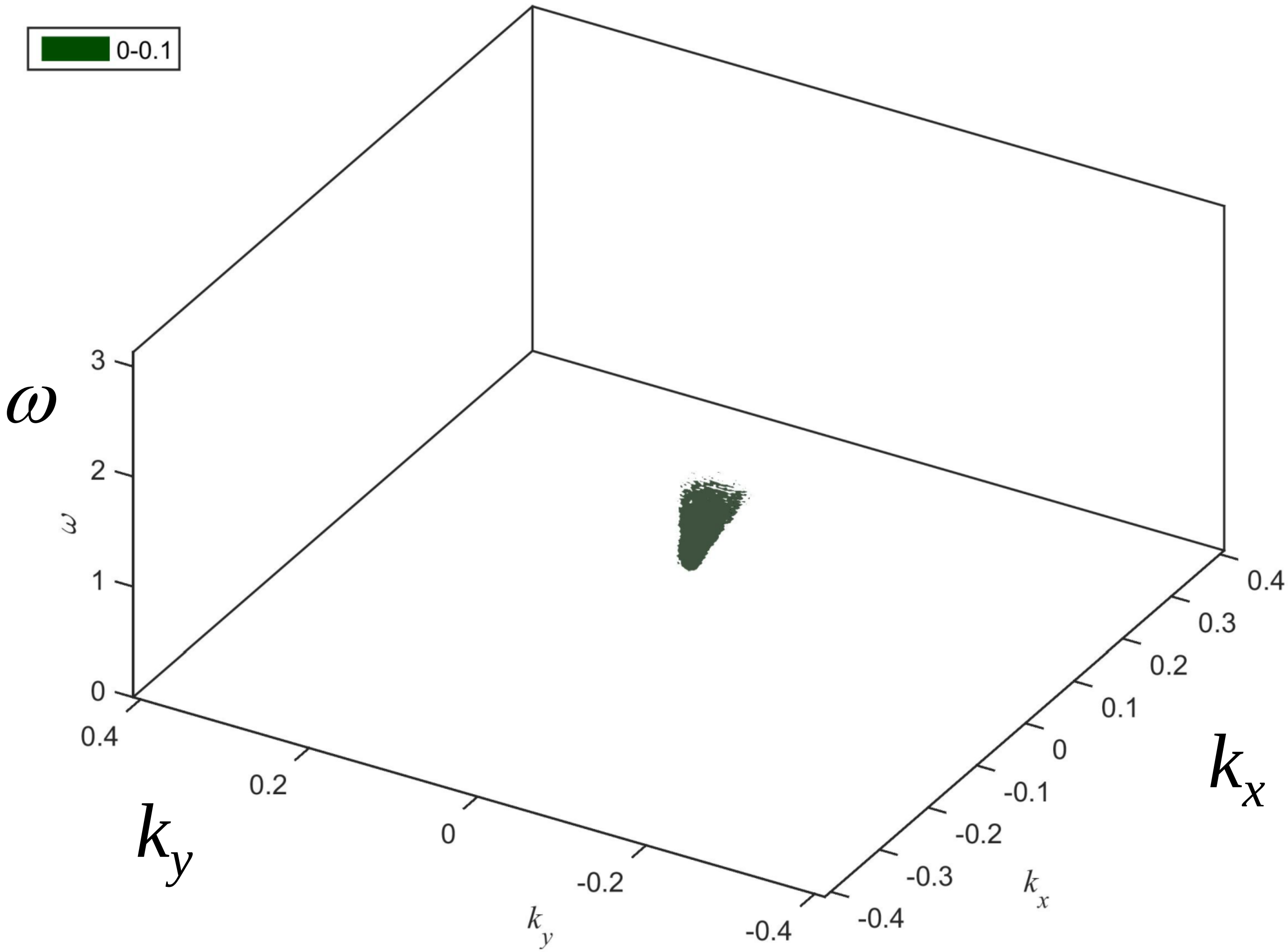
No winds.
Almost no dissipation.

A sequence of snapshots of the water surface represents a real-valued field in a space-time domain, periodic along the two horizontal axes.
50 × 50 dominant wave lengths × 25 periods.



Fourier domain

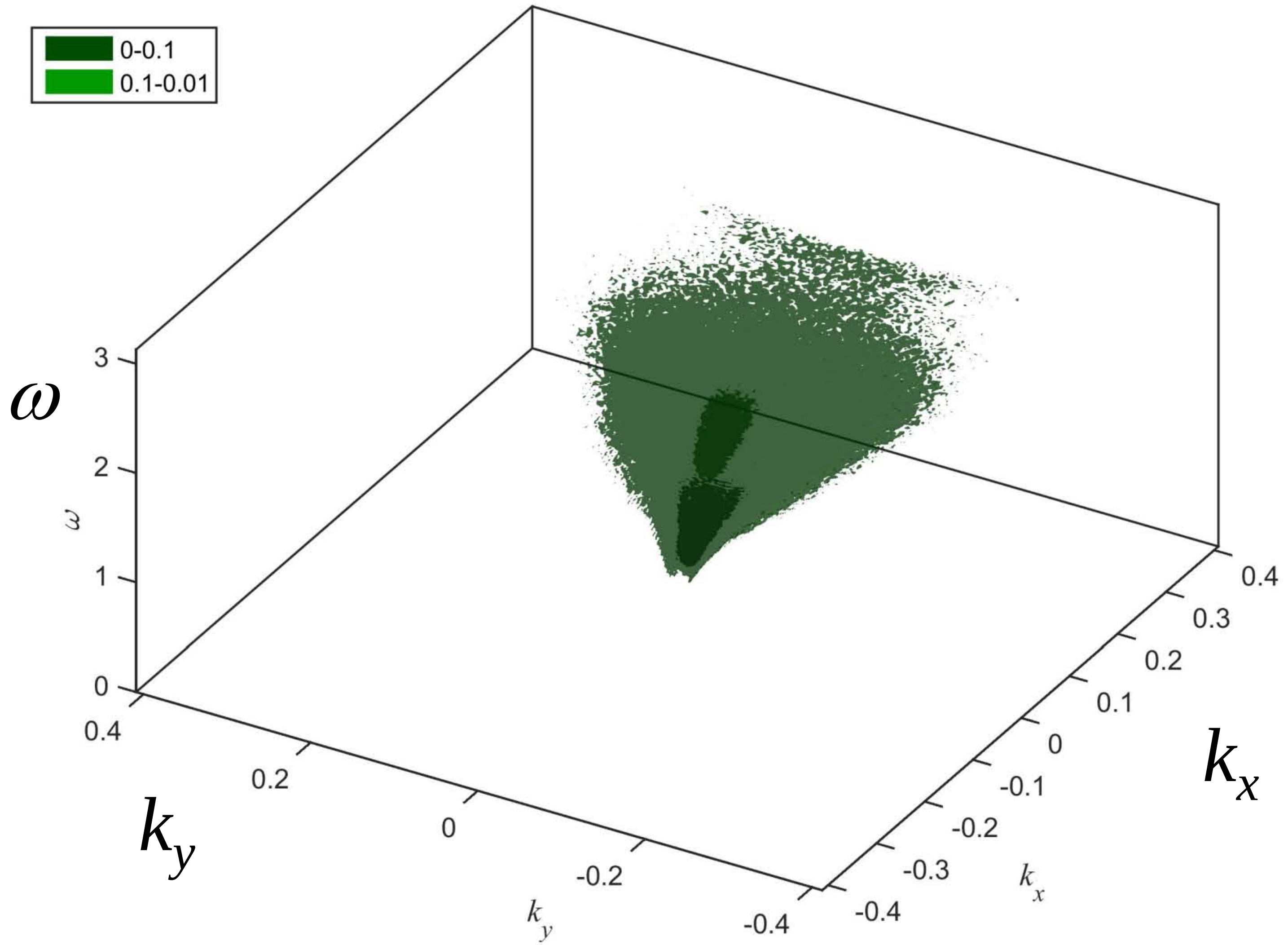
We plot contours for the normalized Fourier amplitudes by different colors (0 … –10 Db)



JONSWAP, H_s = 7 m, γ = 3, Θ = 62°

Fourier domain

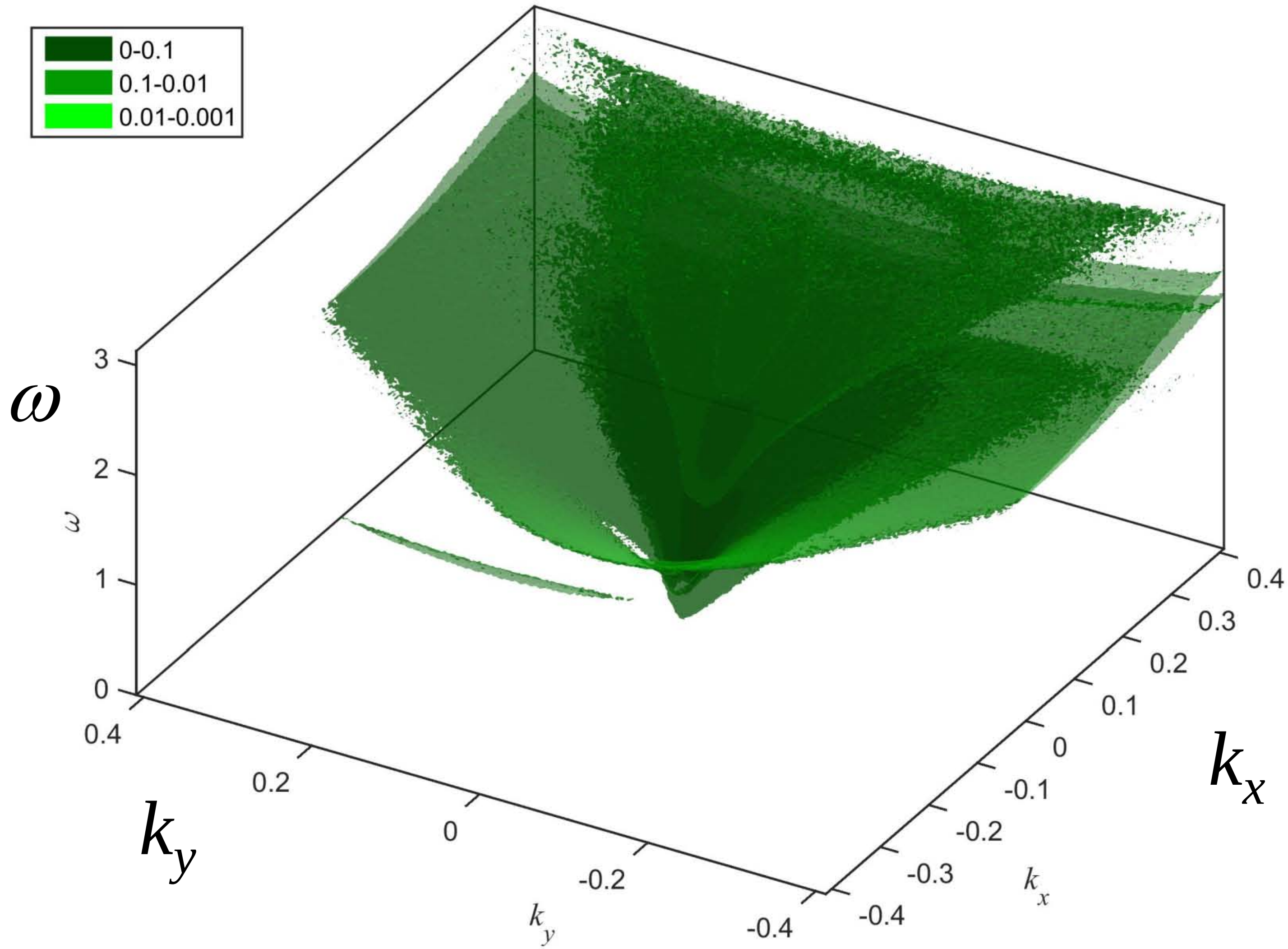
We plot contours for the normalized Fourier amplitudes by different colors (0 … –20 Db)



JONSWAP, $H_s = 7$ m, $\gamma = 3$, $\Theta = 62°$

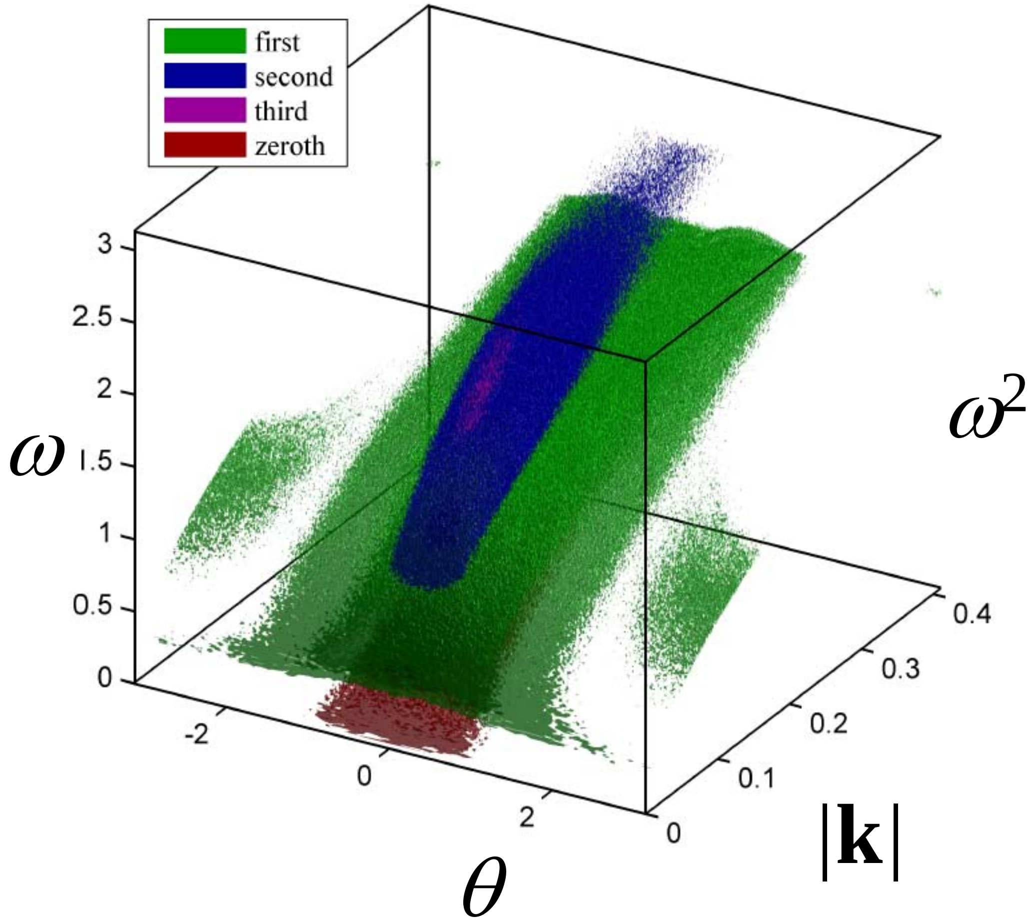
Fourier domain

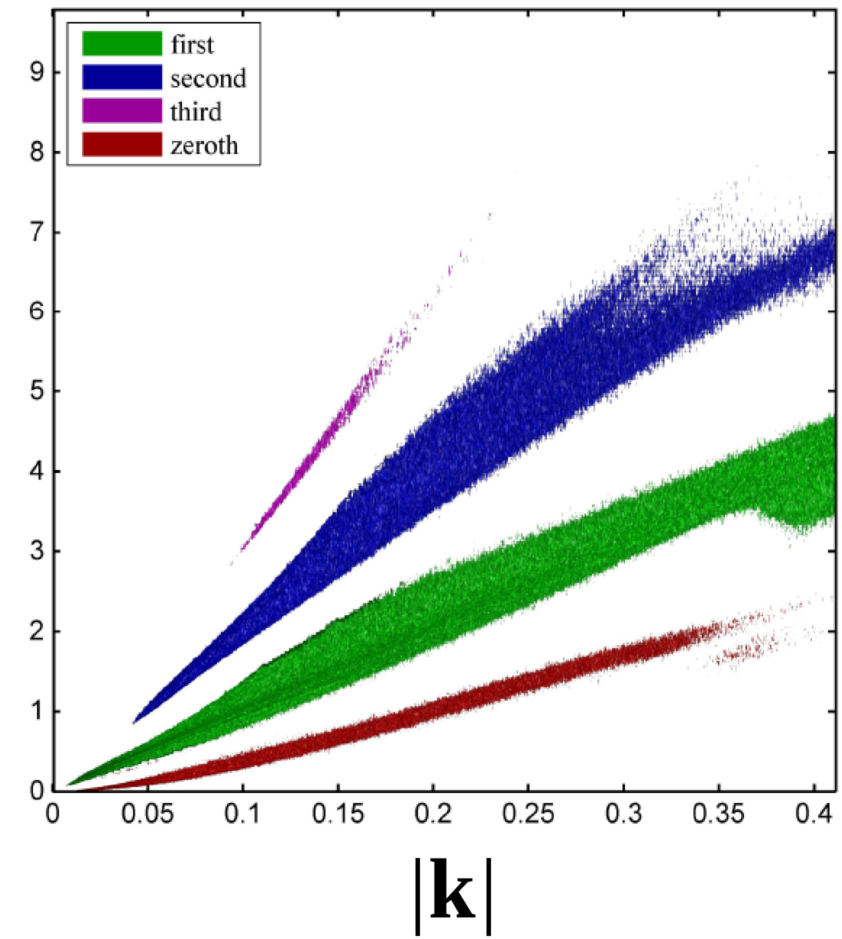
We plot contours for the normalized Fourier amplitudes by different colors (0 … –30 Db)



JONSWAP, $H_s = 7$ m, $\gamma = 3$, $\Theta = 62°$

Fourier domain





JONSWAP, H_s = 7 m, γ = 3, Θ = 62°

Fourier domain

Weakly nonlinear narrow-banded theory

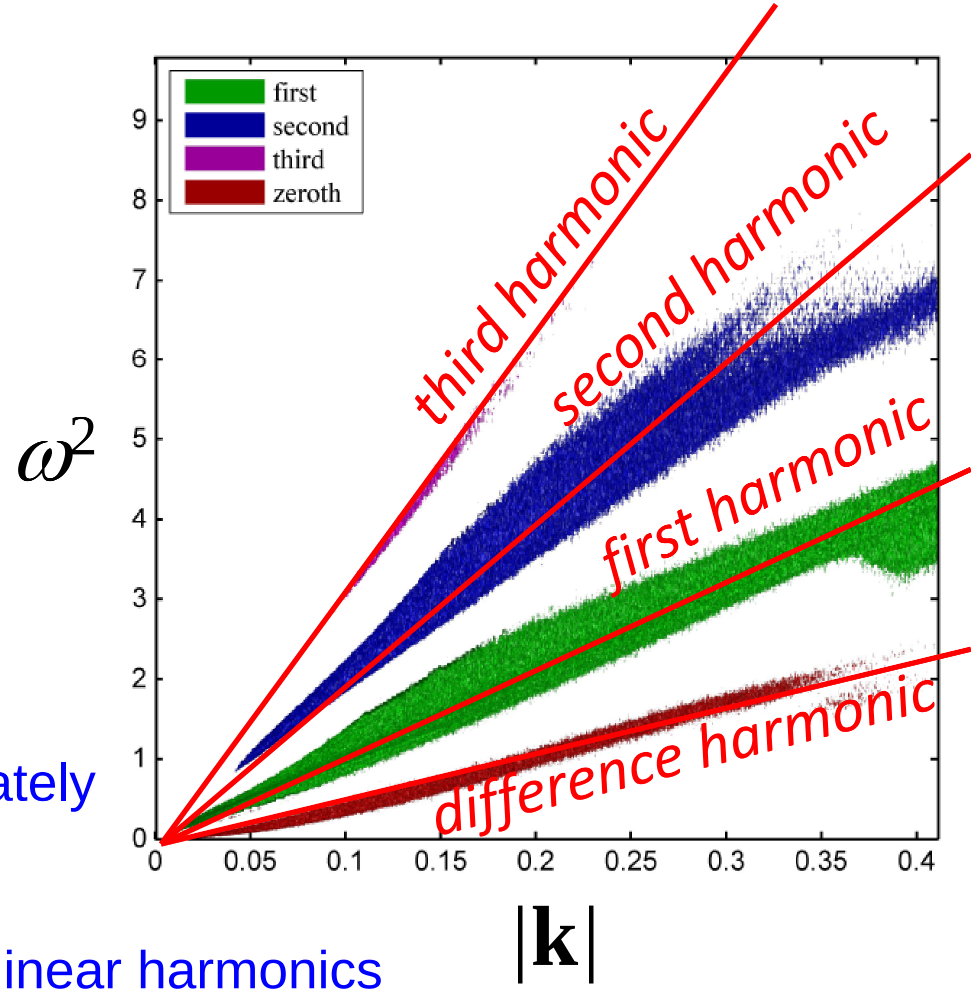
Nonlinear harmonics:

$$\omega_n = n\Omega(nk)$$

$$\Omega(k) \equiv \sqrt{gk} \qquad k = \sqrt{k_x^2 + k_y^2}$$

n = 2, 3, … – order of nonlinearity
n = 1/2 – the difference harmonic

$$\boxed{\omega_n^2 = n^2 gk}$$



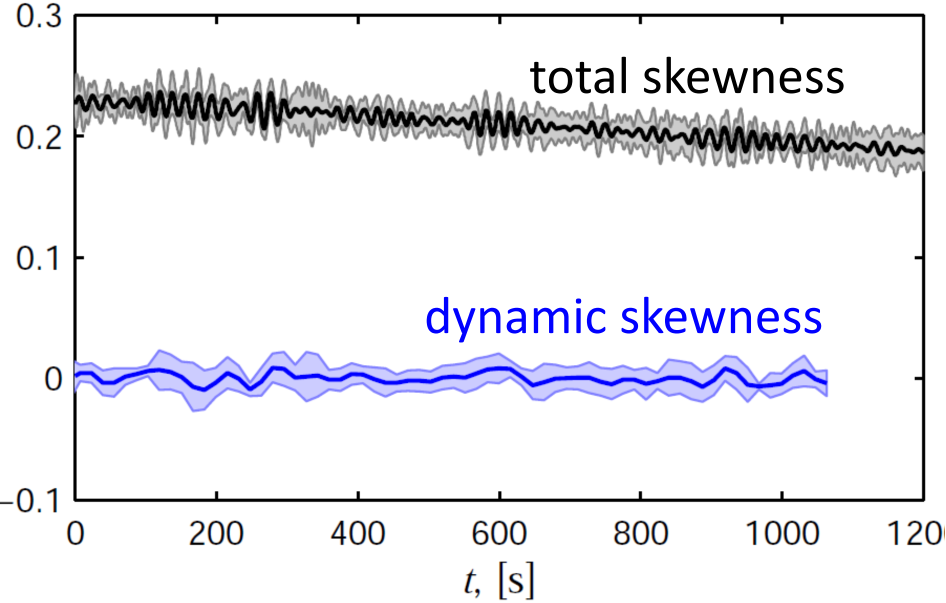
- Actual nonlinear harmonics approximately follow the narrow-band theory

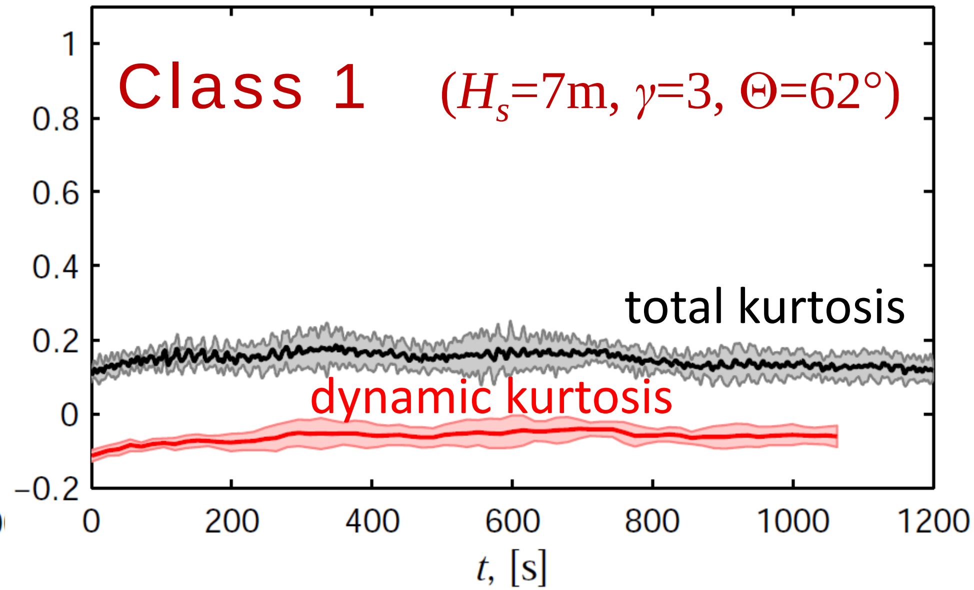
- Spectral filters can select wanted nonlinear harmonics

- The free wave component is reconstructed via inverse triple Fourier transform

Total and ‘dynamic’ statistical moments

Two classes of sea states





$$\lambda_3^{tot} = \frac{\langle \eta^3 \rangle}{\langle \eta^2 \rangle^{3/2}}$$

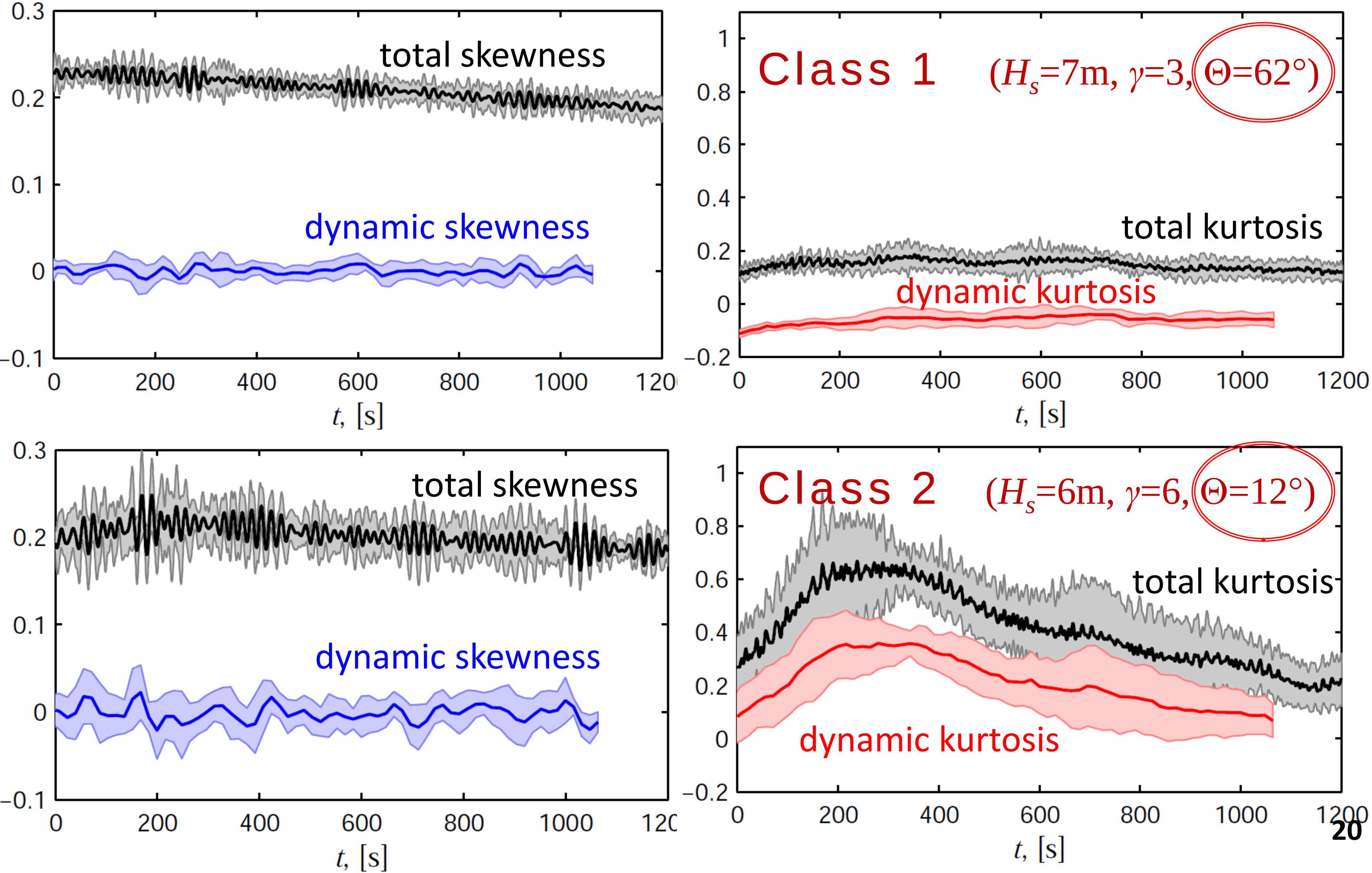
$$\lambda_4^{tot} = \frac{\langle \eta^4 \rangle}{\langle \eta^2 \rangle^{2}} - 3$$

$$\lambda_3^{dyn} = \frac{\langle \eta_{free}^3 \rangle}{\langle \eta_{free}^2 \rangle^{3/2}}$$

$$\lambda_4^{dyn} = \frac{\langle \eta_{free}^4 \rangle}{\langle \eta_{free}^2 \rangle^{2}} - 3$$

Total and ‘dynamic’ statistical moments

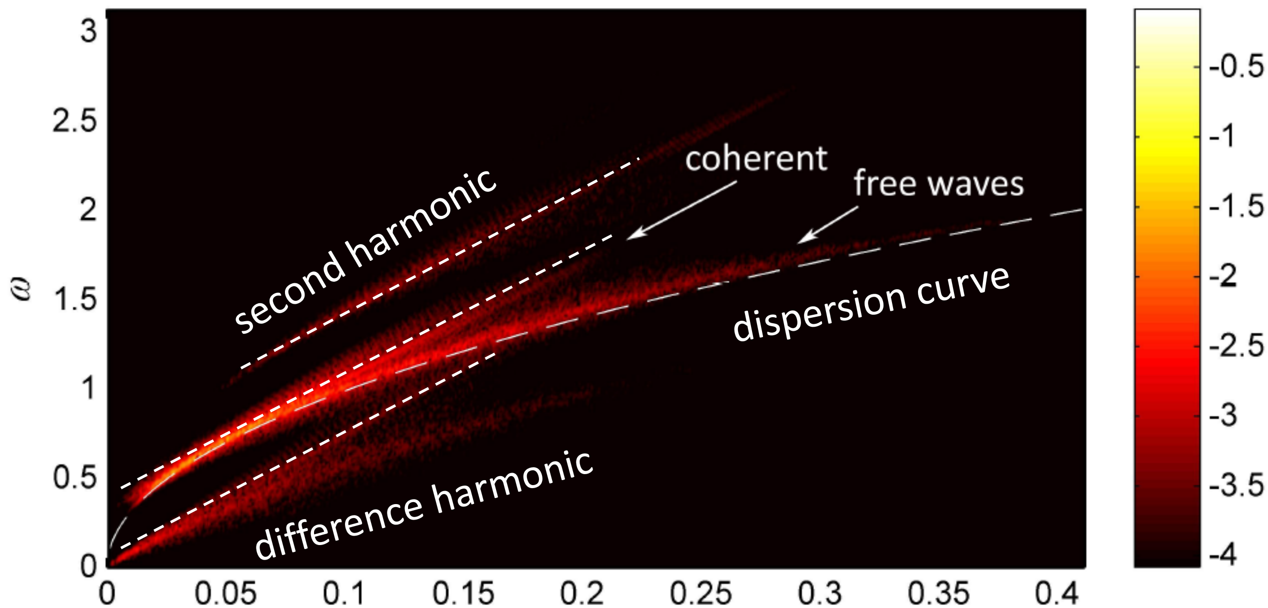
Two classes of sea states



Evidence of coherent wave patterns

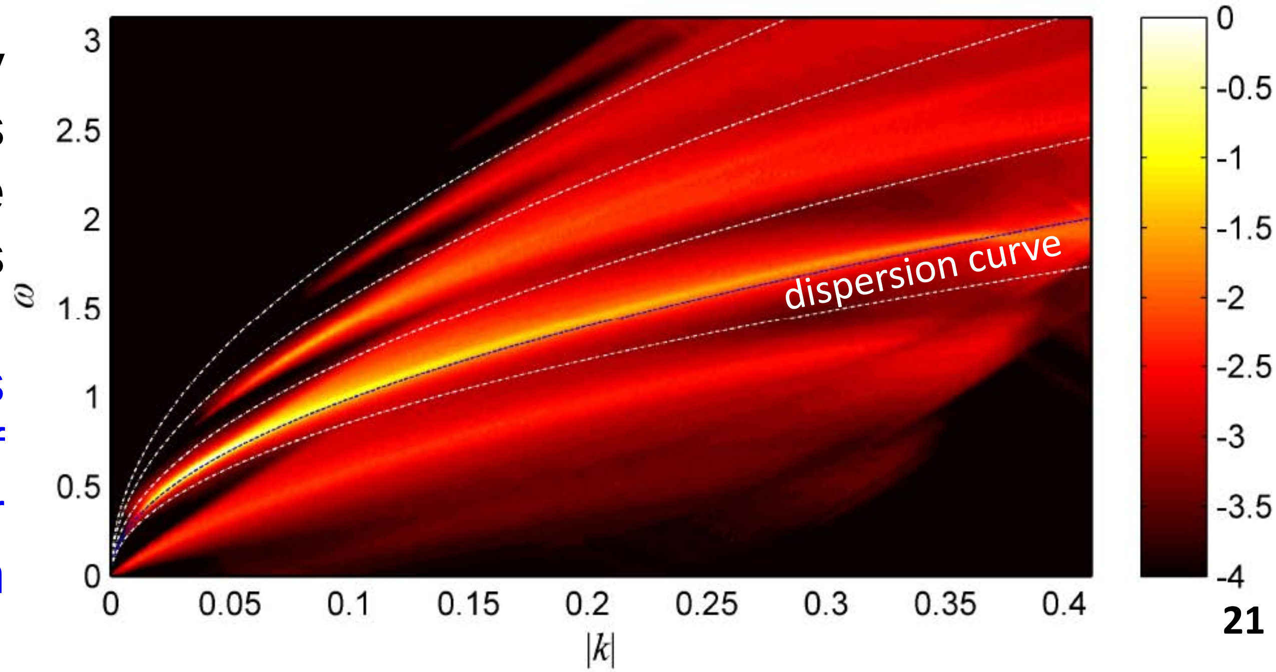
Wavenumber-frequency Fourier amplitudes along the wave direction $\theta \approx 14°$

Manifestation of coherent wave patterns, which violate the dispersion law



Wavenumber-frequency Fourier amplitudes integrated along all wave directions

The coherent patterns lead to the spread of energy in the Fourier domain



Conclusions

The method to calculate the free wave component from the wave data is suggested

The strongly non-Gaussian dynamics of the free wave component is shown to occur under realistic sea conditions

It occurs under the conditions favorable for the Benjamin – Feir instability (intense waves with narrow spectrum)

It cannot be simulated by phase-averaging models

The evidence of generation of nonlinear coherent patterns in directional irregular sea surface waves is presented

The new effect leading to the spread of the irregular wave dispersion is shown