Evolution of extreme wave statistics in surface elevation and velocity field over a non-uniform depth

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It was shown experimentally in Trulsen *et al.* (2012) that irregular water waves propagating over a slope may have a local maximum of kurtosis and skewness in surface elevation near the shallower side of the slope. Later on, Raustøl (2014) did laboratory experiments for irregular water waves propagating over a shoal and found the surface elevation could have a local maximum of kurtosis and skewness on top of the shoal, and a local minimum of skewness after the shoal for sufficiently shallow water. Numerical results by Sergeeva *et al.* (2011), Zeng & Trulsen (2012), Gramstad *et al.* (2013) and Viotti & Dias (2014) support the experimental results mentioned above. Just recently, Jorde (2018) did new experiment with the same shoal as in Raustøl (2014) but with additional measurement of the interior horizontal velocity. The experimental results from Raustøl (2014) and Jorde (2018) were reported in Trulsen *et al.* (2020) and it was found the evolution of skewness for surface elevation and horizontal velocity have the same behaviour but the kurtosis of horizontal velocity has local maximum in downslope area which is different with the kurtosis of surface elevation.

In present work, we utilize numerical simulation to study the effects of incoming significant wave height, peak wave frequency on evolution of wave statistics for both surface elevation and velocity field with more general bathymetry. Numerical simulations are based on High Order Spectral Method (HOSM) for variable depth Gouin *et al.* (2016) for wave evolution and Variational Boussinesq model (VBM) Lawrence *et al.* (2018) for velocity field calculation.

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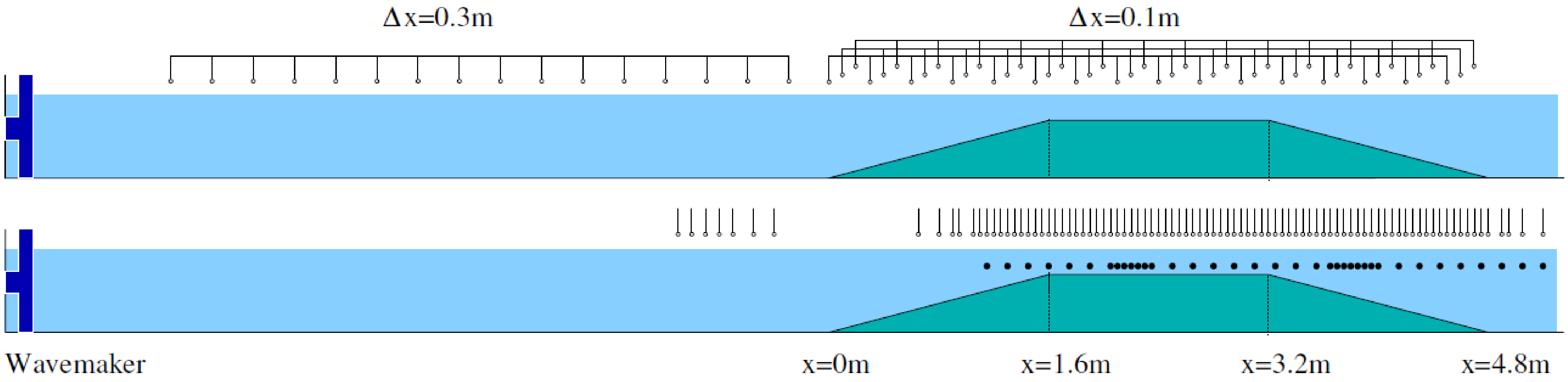
EGU General Assembly 2020

Lab experiments at UiO

Anne Raustøl
(2014)

Stian Jorde
(2018)



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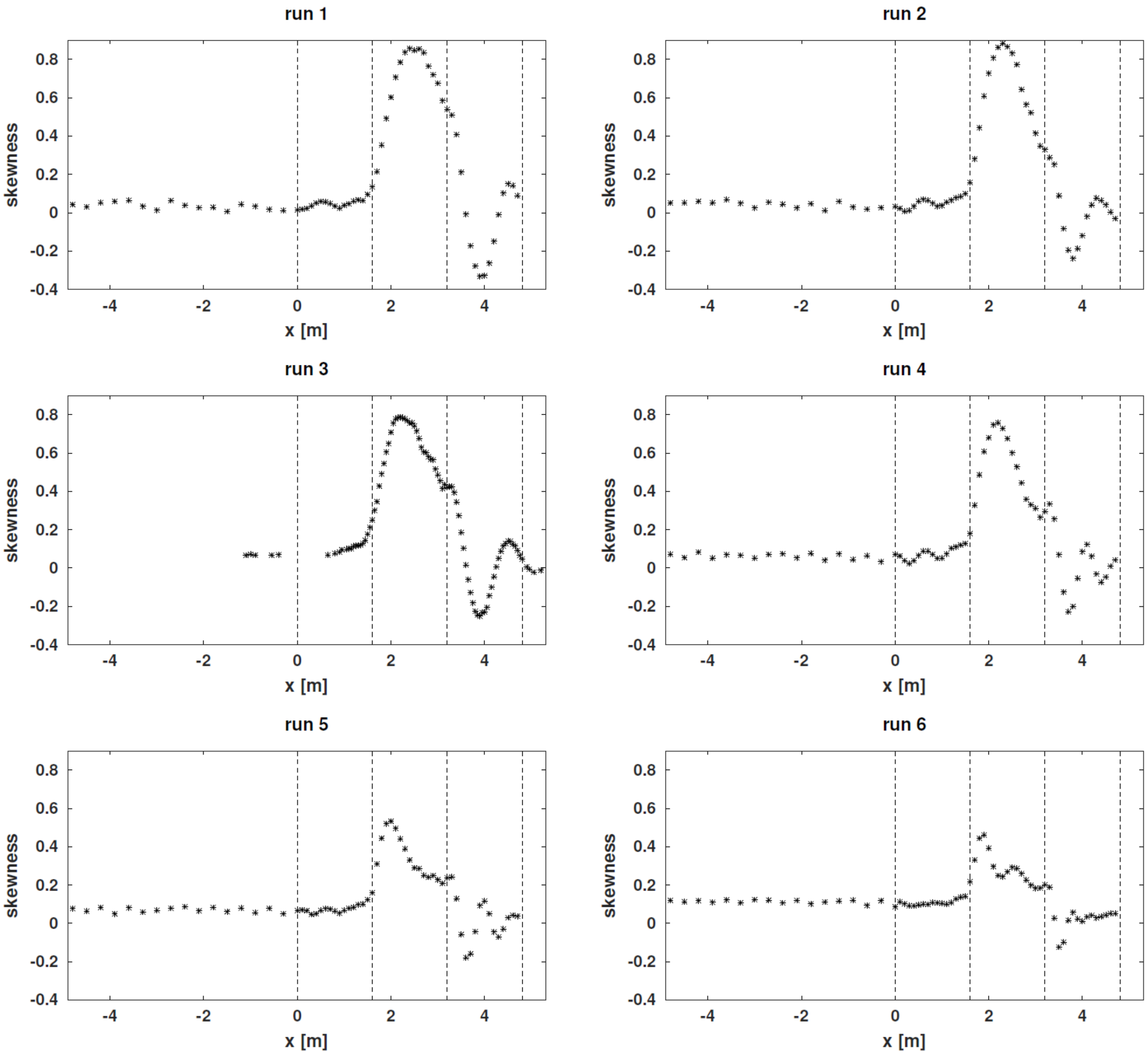
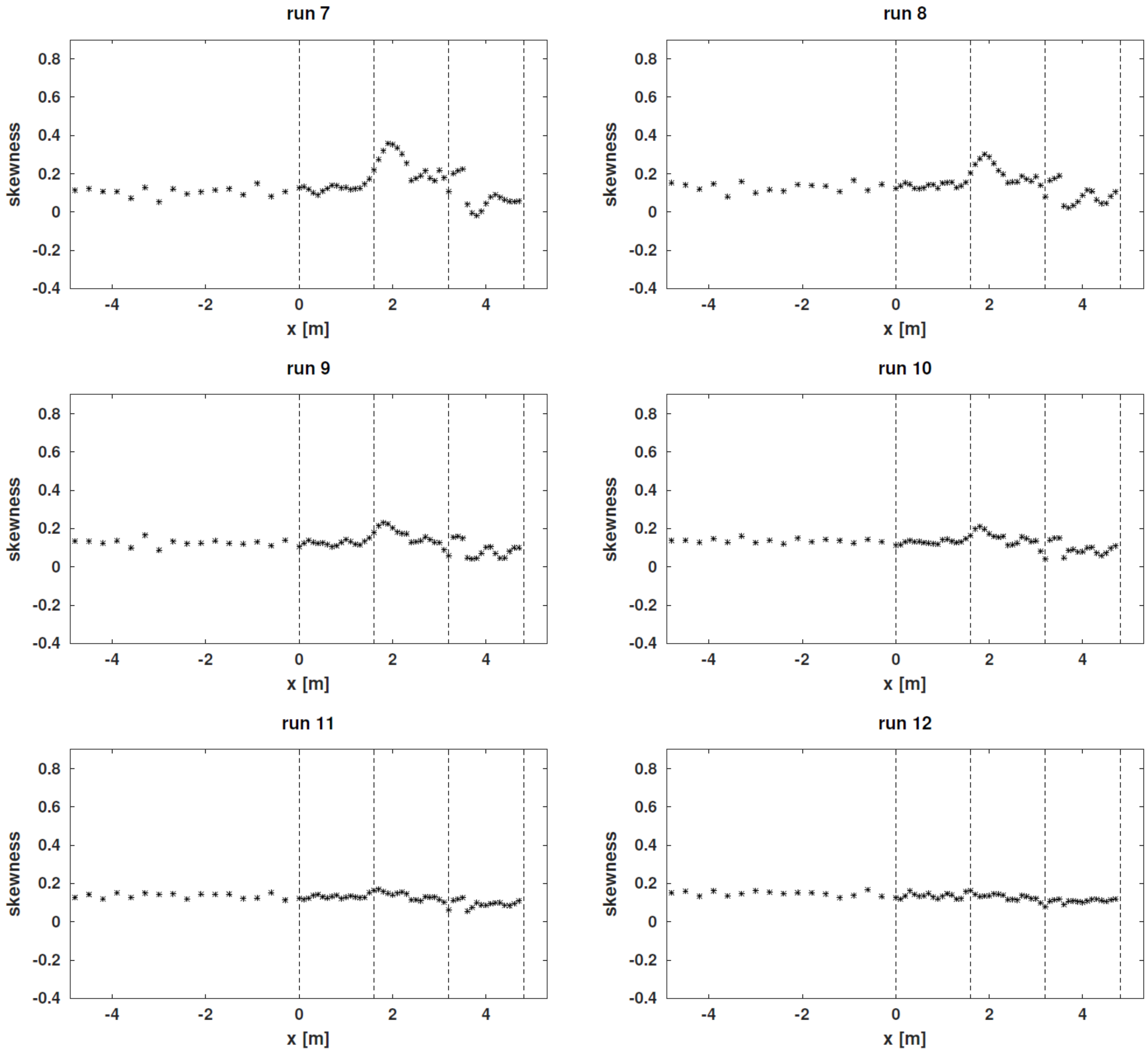
		deeper side					shallower side				
Run	T_p [s]	h_1 [cm]	$k_p h$	H_s [cm]	ϵ	Ur	h_2 [cm]	$k_p h$	H_s [cm]	ϵ	Ur
1	1.1	50	1.8	1.4	0.018	0.0033	8	0.54	1.5	0.036	0.22
2	1.0	50	2.1	1.6	0.024	0.0027	8	0.60	1.6	0.043	0.20
3	1.1	53	2.5	2.5	0.030	0.0020	11	0.65	2.5	0.052	0.19
4	0.9	50	2.5	1.7	0.031	0.0019	8	0.68	1.6	0.048	0.16
5	0.8	50	3.2	1.7	0.038	0.0012	8	0.78	1.5	0.052	0.11
6	0.7	50	4.1	1.7	0.049	0.00071	8	0.91	1.4	0.058	0.078
7	1.0	60	2.5	3.4	0.049	0.0033	18	0.97	3.1	0.060	0.064
8	0.9	60	3.0	3.2	0.056	0.0021	18	1.1	2.9	0.063	0.046
9	0.85	60	3.4	2.7	0.054	0.0014	18	1.2	2.5	0.058	0.033
10	0.8	60	3.8	2.6	0.058	0.0011	18	1.3	2.3	0.059	0.027
11	0.75	60	4.3	2.4	0.060	0.00076	18	1.4	2.1	0.059	0.020
12	0.7	60	4.9	2.1	0.062	0.00052	18	1.6	1.8	0.058	0.014

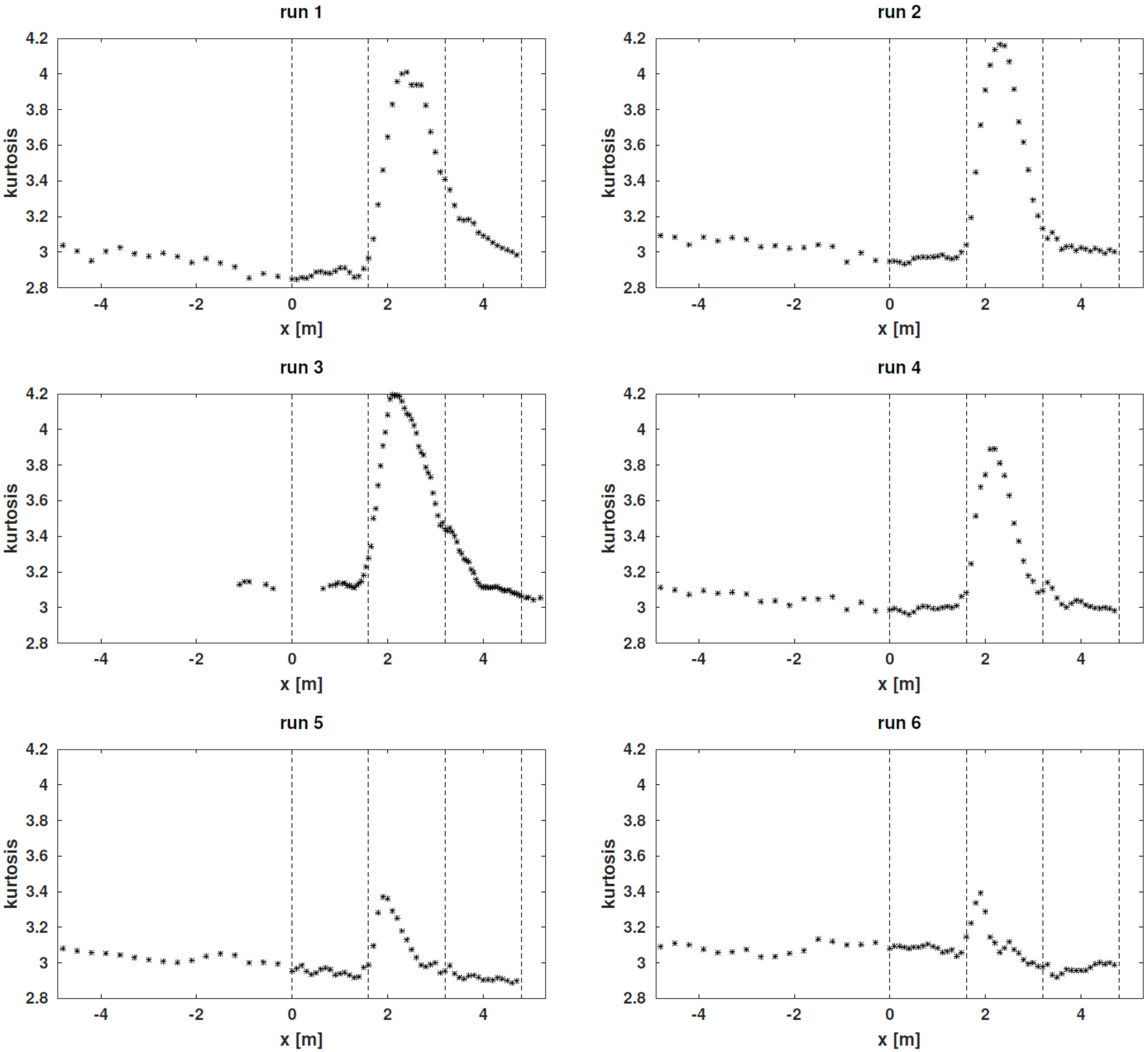
TABLE 1. Key parameters for all runs. The runs are numbered according to increasing dimensionless depth over the shoal. Run 3 belongs to the second campaign (Jorde 2018), the rest are from the first campaign (Raustøl 2014). The values of significant wave height H_s, steepness ϵ and Ursell number Ur are averages over all probes in front of or above the shoal, not including the probes at the edges of the sloping bottom.

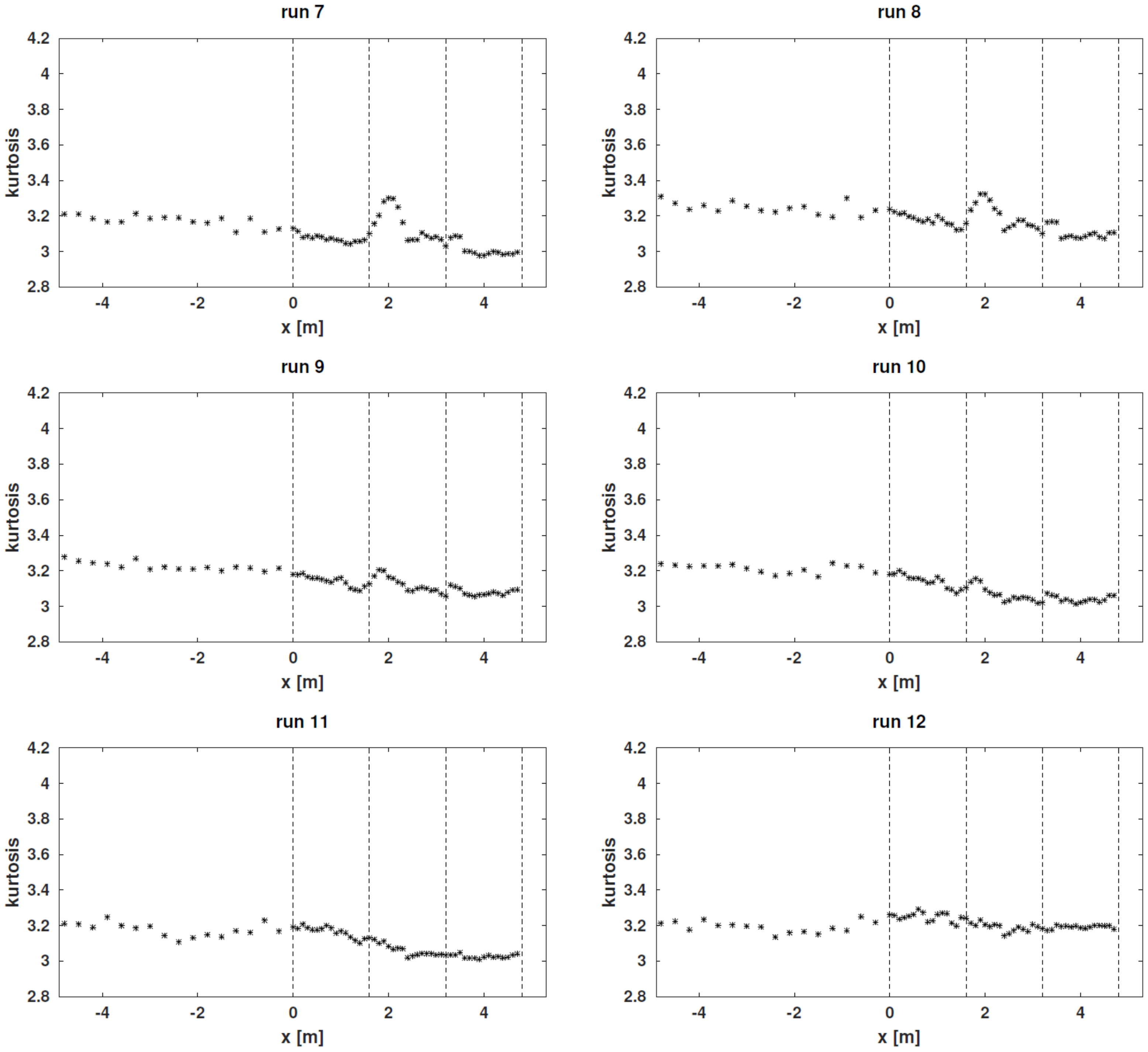
K.Trulsen, A., Raustøl, S. Jorde, and L. B. Rye. Extreme wave statistics of longcrested irregular waves over a shoal. J. Fluid Mech. 882:R2,2020.


run 1
run 2
run 3
run 4
run 5
run 6
skewness
x [m]
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-4
-2
0
2
4



run 7
run 8
run 9
run 10
run 11
run 12
skewness
x [m]
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-4
-2
0
2
4


run 1
run 2
run 3
run 4
run 5
run 6
kurtosis
x [m]
4.2
4
3.8
3.6
3.4
3.2
3
2.8
-4
-2
0
2
4



run 7
run 8
run 9
run 10
run 11
run 12
kurtosis
x [m]
4.2
4
3.8
3.6
3.4
3.2
3
2.8
-4
-2
0
2
4

Numerical model

- High Order Spectral Method for wave evolution over variable depth

M. Gouin, G. Ducrozet, and P. Ferrant. Development and validation of a non-linear spectral model for water waves over variable depth. European Journal of Mechanics-B/Fluids, 57:115-128, 2016.

M. Gouin G. Ducrozet,and P. Ferrant. Propagation of 3D nonlinear waves over an elliptical mound with a high-order spectral method. European Journal of Mechanics-B/Fluids,63:9-24, 2017.

- Variational Boussinesq model for kinematics calculation

G. Klopman, E. van Groesen and M. W. Dingemans. A variational approach to boussinesq modeling of fully non-linear water waves. J. Fluid Mech, 657:36-63, 2010.

C. Lawrence, D. Adytia and E. van Groesen. Variational Boussinesq model for strongly nonlinear dispersive waves. Wave Motion, 76:78-102, 2018.

High Order Spectral Method

Dynamic equations

$$\frac{\partial \eta}{\partial t} = W\left(1 + \left|\frac{\partial \eta}{\partial x}\right|^2\right) - \frac{\partial \phi}{\partial x}\frac{\partial \eta}{\partial x}$$

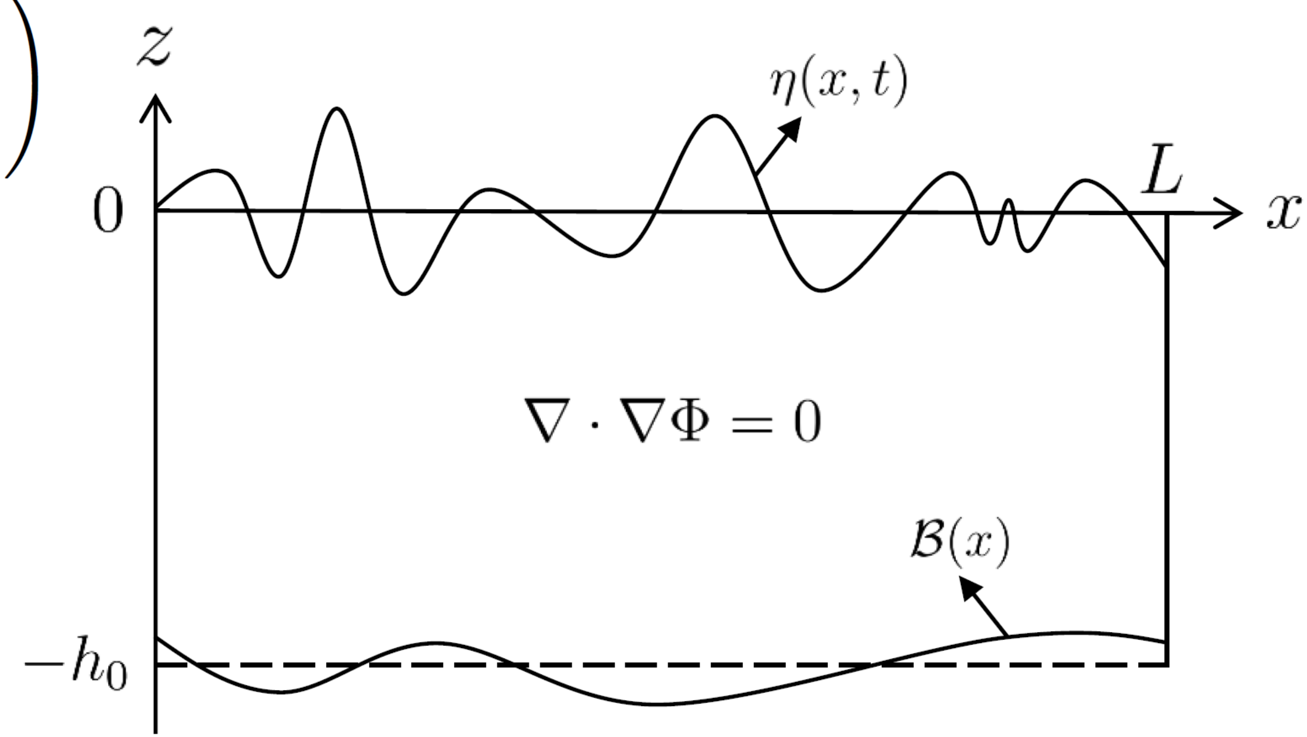
$$\frac{\partial \phi}{\partial t} = -g\eta - \frac{1}{2}\left|\frac{\partial \phi}{\partial x}\right|^2 + \frac{1}{2}W^2\left(1 + \left|\frac{\partial \eta}{\partial x}\right|^2\right)$$

$$\phi = \Phi(x, z = \eta, t)$$

$$W = \left.\frac{\partial \Phi}{\partial z}\right|_{z=\eta}$$

Bottom boundary condition

$$\frac{\partial \Phi}{\partial x}\frac{\partial \mathcal{B}}{\partial x} - \frac{\partial \Phi}{\partial z} = 0 \quad \text{at} \quad z = -h_0 + \mathcal{B}(x)$$



High Order Spectral Method

The velocity potential is truncated as power series

$$\Phi(x,z,t) = \sum_{m=1}^{M} \Phi^{(m)}(x,z,t)$$

$$\Phi^{(m)} = \Phi_0^{(m)} + \Phi_B^{(m)}$$

$$\Phi_0^{(m)} = \sum_j A_j \frac{\cosh\left(k_j(z+h_0)\right)}{\cosh\left(k_j h_0\right)} e^{ik_j x}$$

$$\Phi_B^{(m)} = \sum_j B_j \frac{\sinh\left(k_j z\right)}{\cosh\left(k_j h_0\right)} e^{ik_j x}$$

Variational Boussinesq model (VBM)

$$\Phi(x,z) \approx \phi(x) + \sum_{m=1}^{N} \left(\frac{\cosh \kappa_m (z+h)}{\cosh \kappa_m (\eta + h)} - 1 \right) \psi_m(x)$$

Solve the Laplace via Dirichlet principle (minimize the kinetic energy)

$$K(\phi, \eta) = Min(D(\Phi) | \, \Phi = \phi \; at \; z = \eta)$$

$$D(\Phi) = \int \int \frac{1}{2} |\nabla \Phi|^2 \, dz dx$$

Statistics of irregular waves propagating over a shoal

Simulation setup

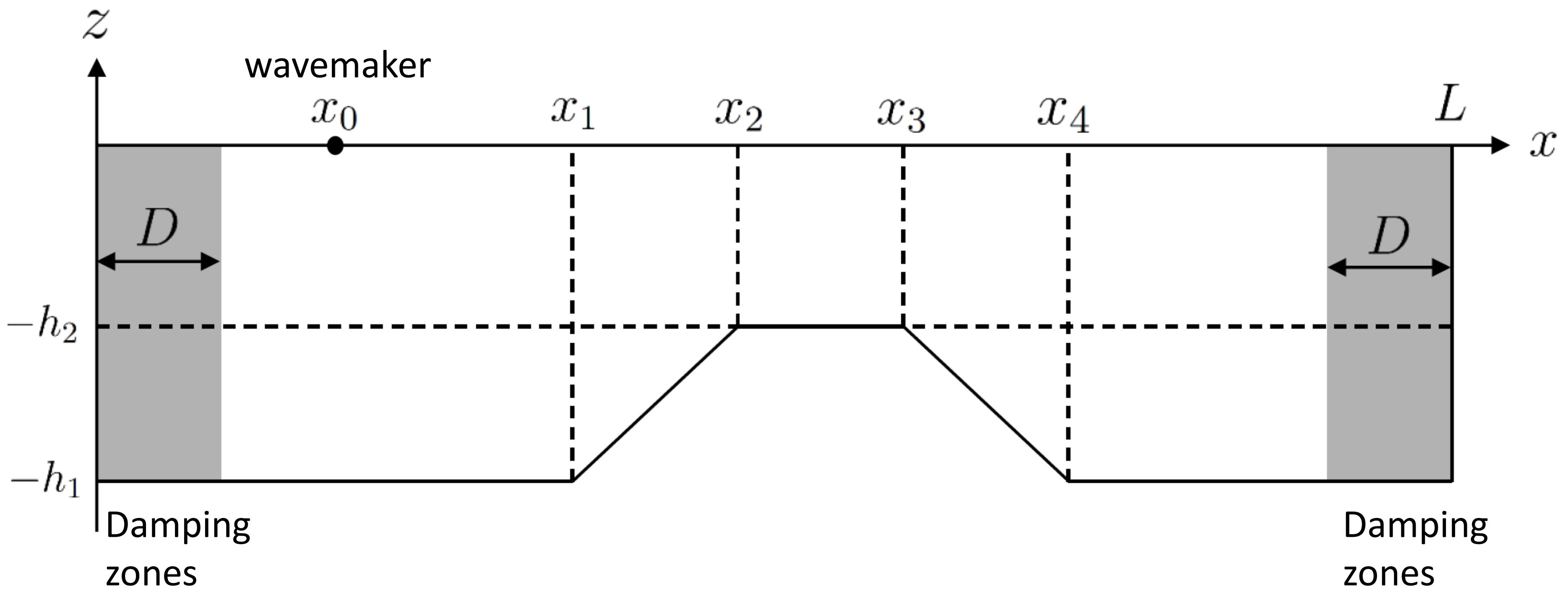
Incoming waves with JONSWAP spectrum: Hs=2.5cm, Tp=1.1s, γ =3.3

100 different realizations with time series of 200Tp are used to calculated statistical quantities (kurtosis and skewness)

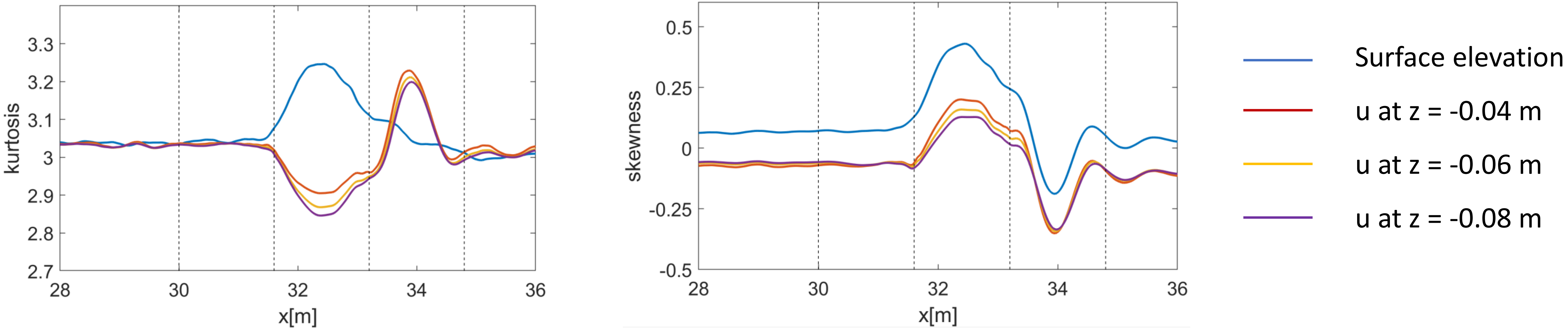
The bathymetry is the same with laboratory experiments.

$x_1 = 30$ m
$x_2 = 31.6$ m
$x_3 = 33.2$ m
$x_4 = 34.8$ m

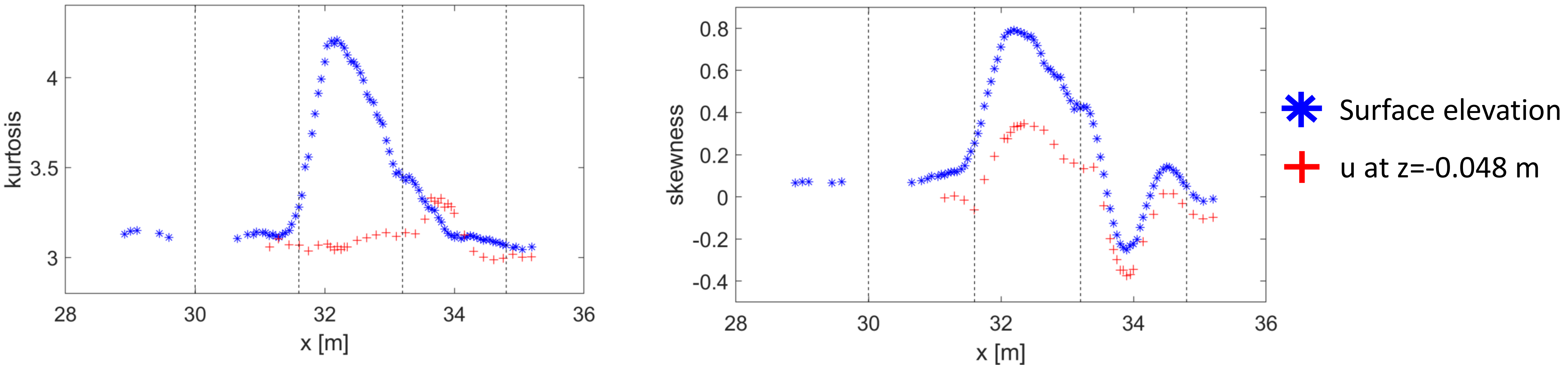
$h_1 = 0.53$ m
$h_2 = 0.11$ m



Simulation



Experiment

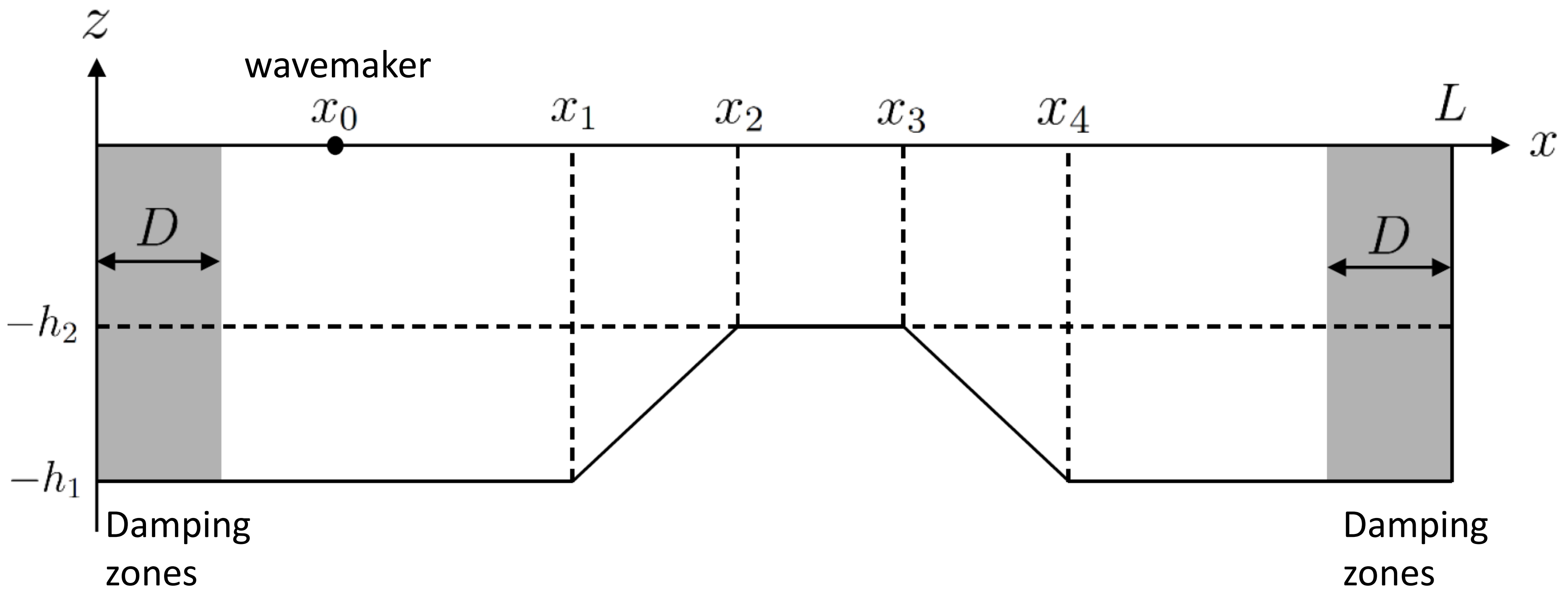


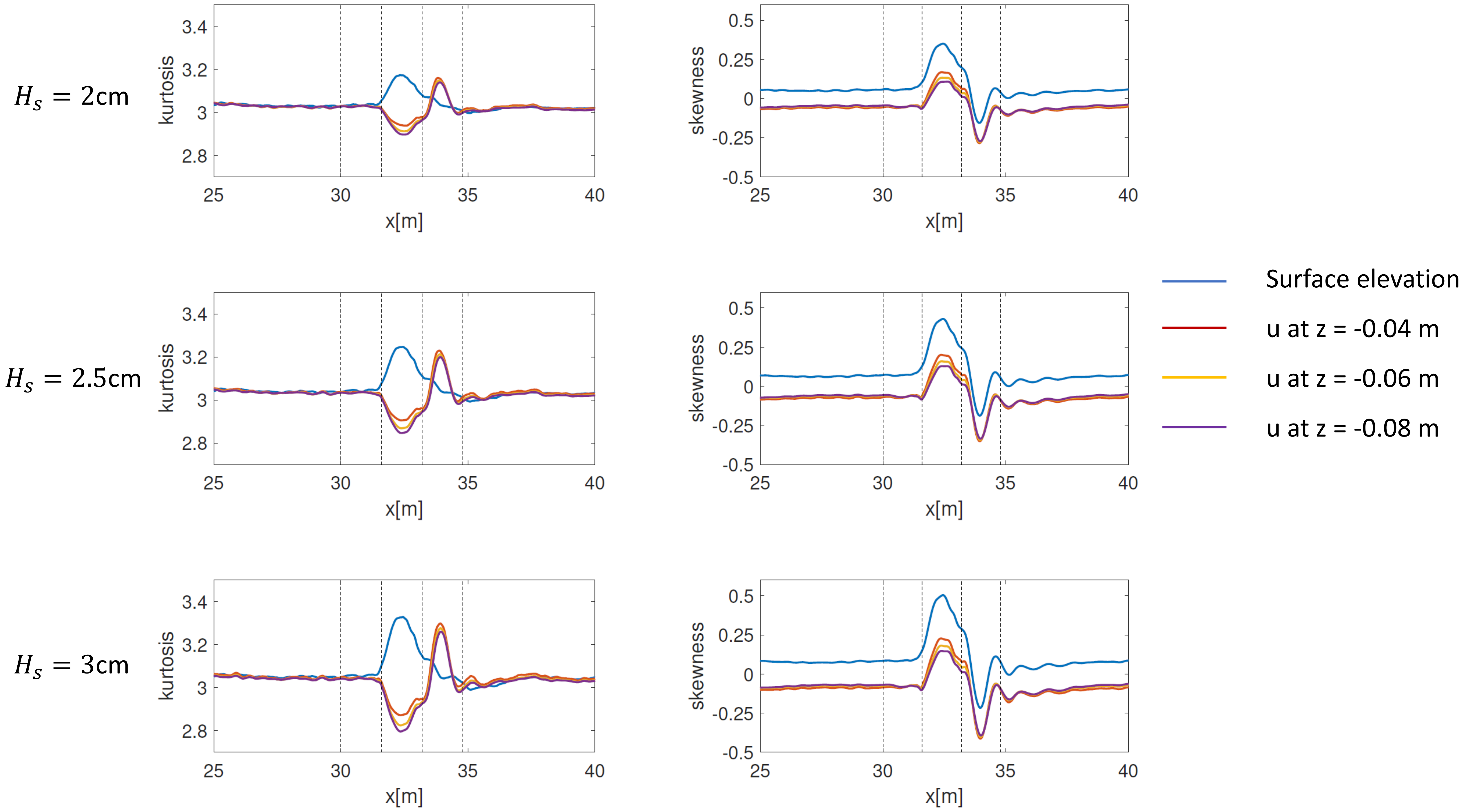
Effect of H_s

Incoming waves with JONSWAP spectrum: Hs=2, 2.5, 3.0 cm, Tp=1.1s, γ =3.3

$x_1 = 30$ m
$x_2 = 31.6$ m
$x_3 = 33.2$ m
$x_4 = 34.8$ m

$h_1 = 0.53$ m
$h_2 = 0.11$ m



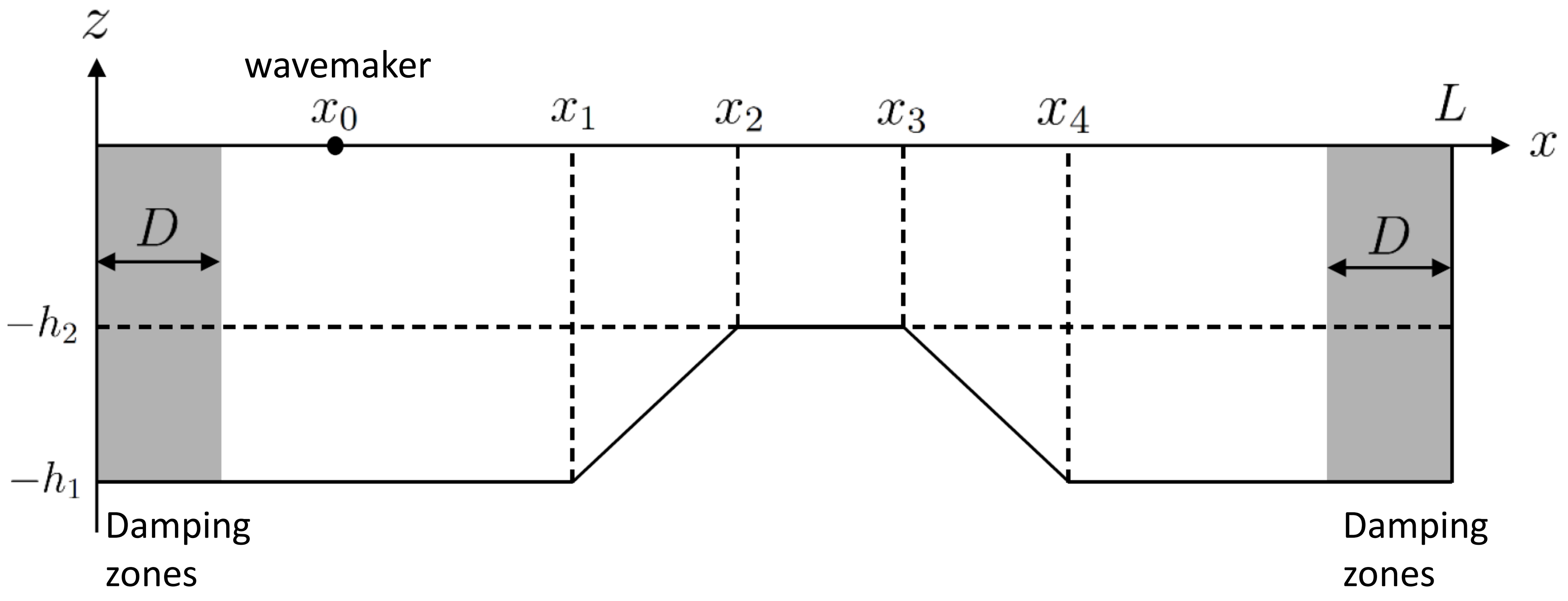

$H_s = 2$cm
$H_s = 2.5$cm
$H_s = 3$cm
kurtosis
skewness
x[m]
Surface elevation
u at z = -0.04 m
u at z = -0.06 m
u at z = -0.08 m

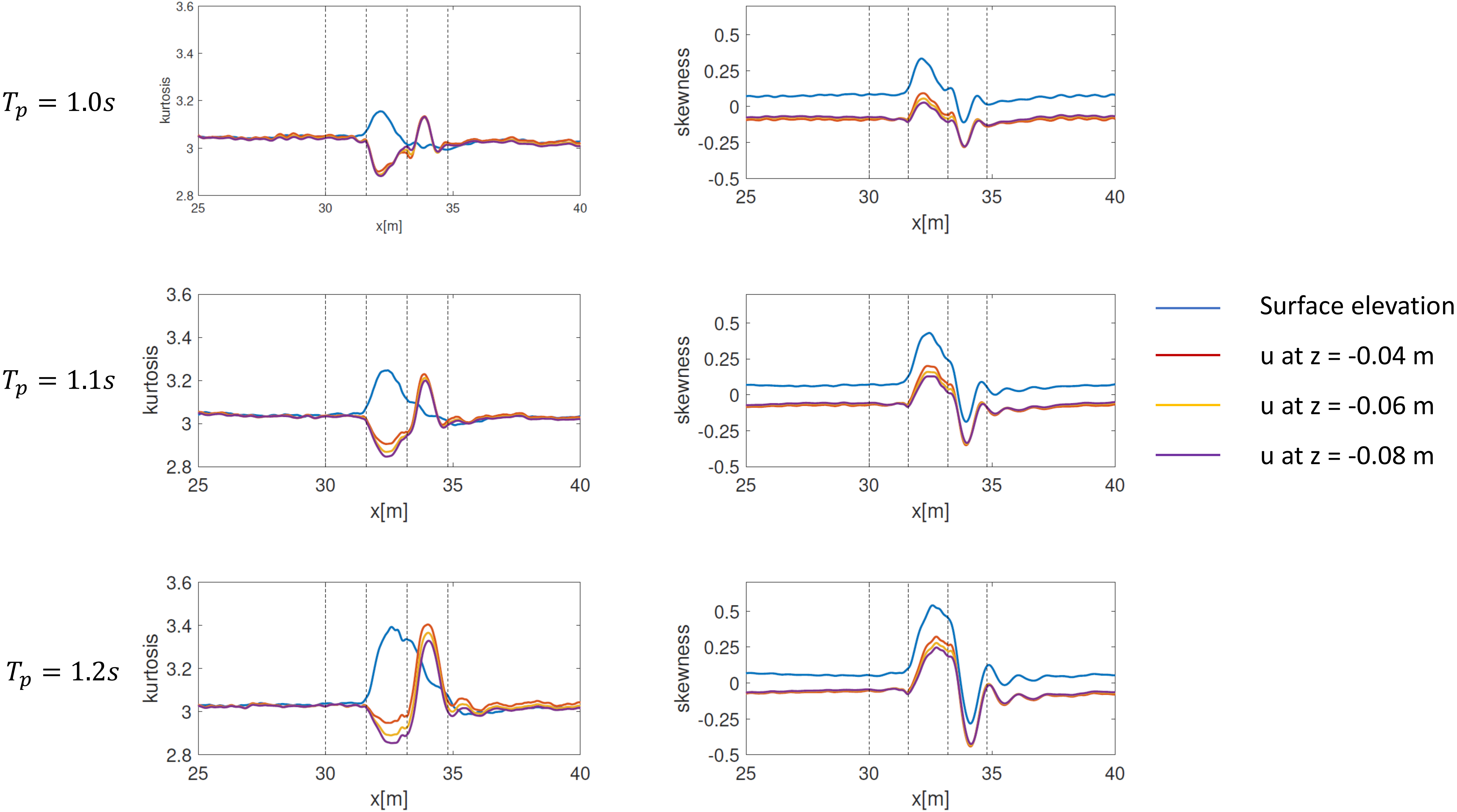
Effect of T_p

Incoming waves with JONSWAP spectrum: Hs=2.5 cm, Tp=1,1.1,1.2 s, γ =3.3

$x_1 = 30$ m
$x_2 = 31.6$ m
$x_3 = 33.2$ m
$x_4 = 34.8$ m

$h_1 = 0.53$ m
$h_2 = 0.11$ m





$T_p = 1.0s$
$T_p = 1.1s$
$T_p = 1.2s$
kurtosis
skewness
x[m]
Surface elevation
u at z = -0.04 m
u at z = -0.06 m
u at z = -0.08 m

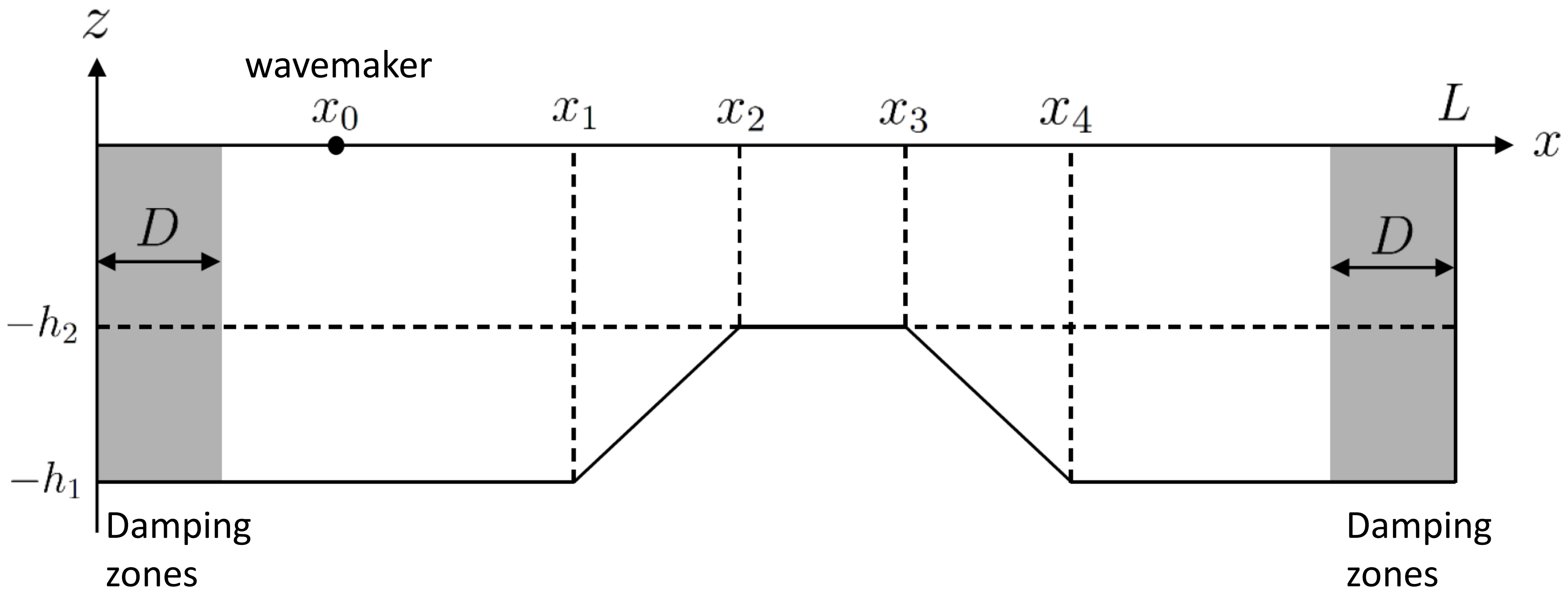
Effects of upslope

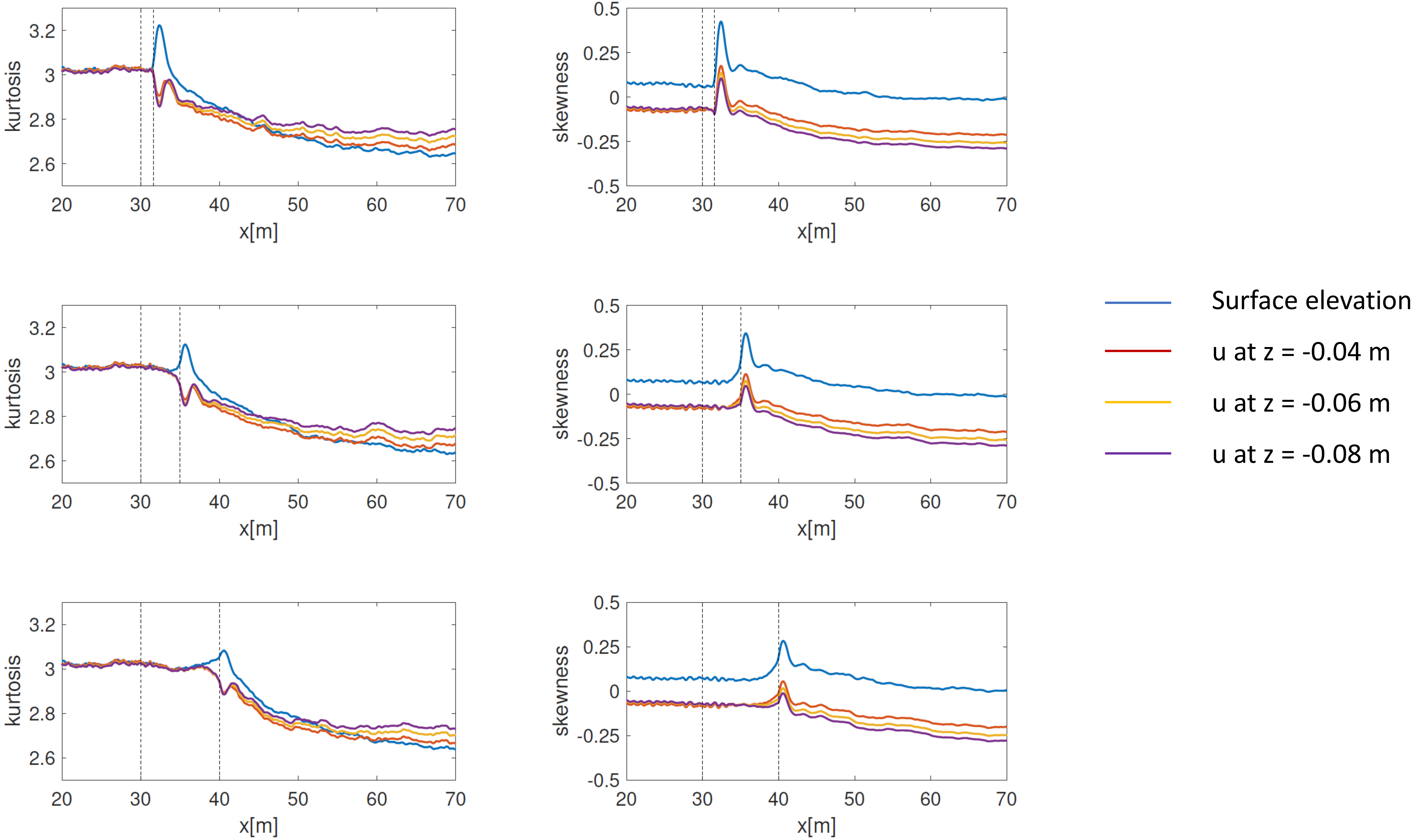
Incoming waves with JONSWAP spectrum: Hs=2.5cm, Tp=1.1s, γ =3.3

Three different upslope were investigated with numerical simulations

$x_1 = 30$ m
$x_2 = 31.6, 35, 40$ m
$x_3 = 100$ m
$x_4 = 110$ m

$h_1 = 0.53$ m
$h_2 = 0.11$ m





kurtosis
skewness
x[m]
Surface elevation
u at z = -0.04 m
u at z = -0.06 m
u at z = -0.08 m

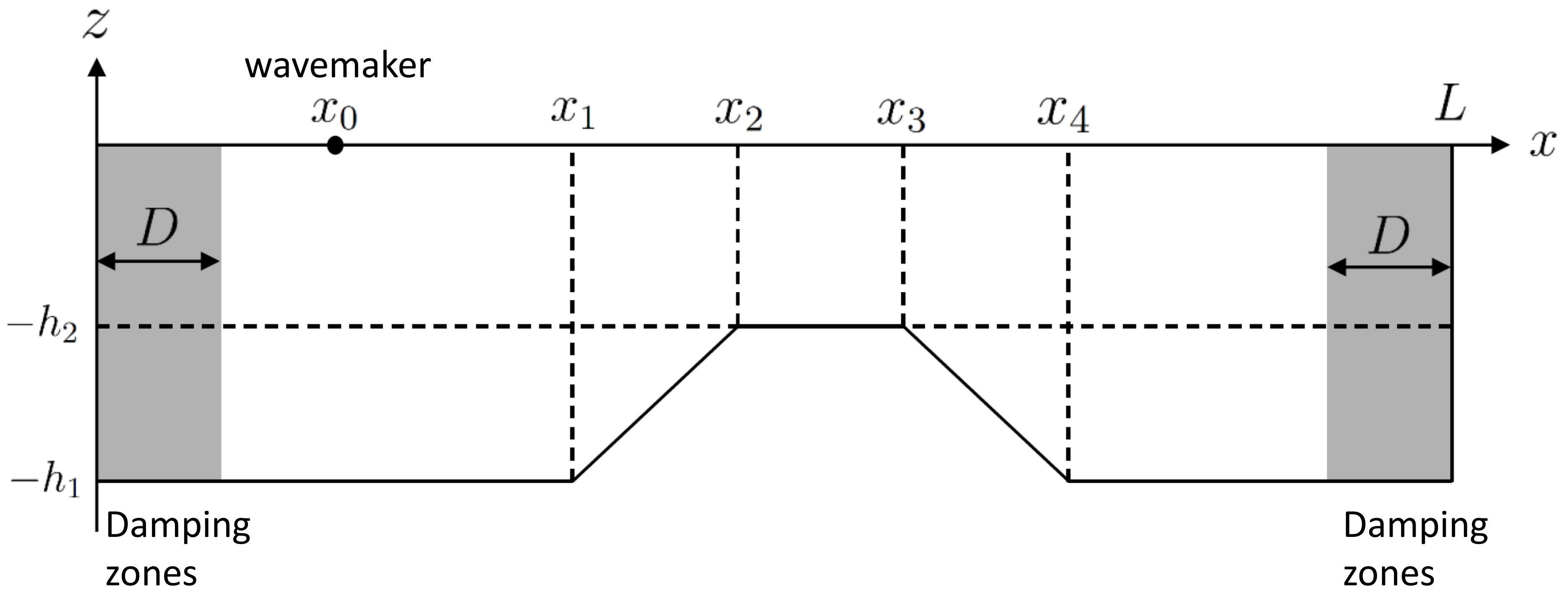
Effects of downslope

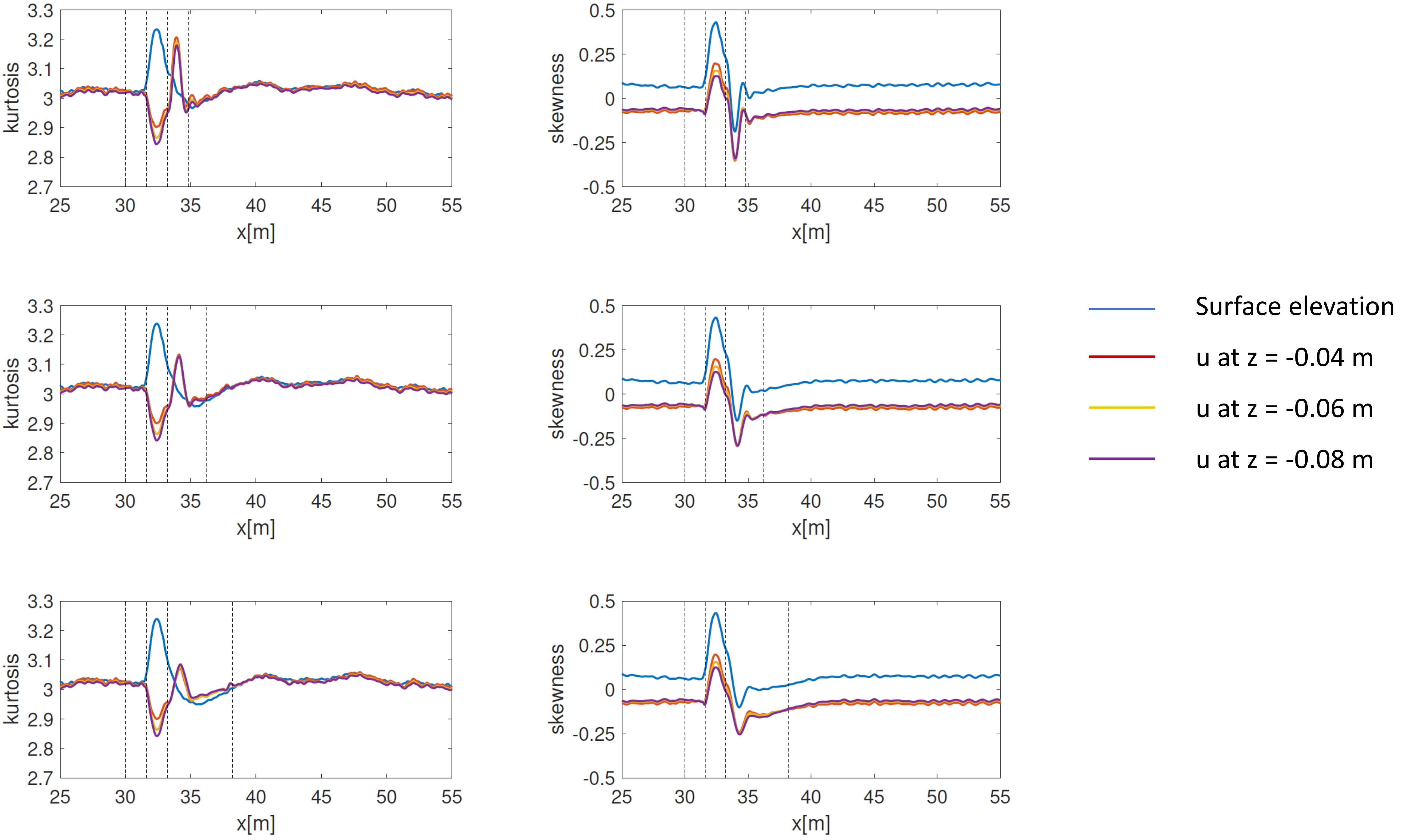
Incoming waves with JONSWAP spectrum: Hs=2.5cm, Tp=1.1s, γ =3.3

Three different downslope were investigated with numerical simulations

$x_1 = 30$ m
$x_2 = 31.6$ m
$x_3 = 33.2$ m
$x_4 = 34.8, 36.2, 38.2$ m

$h_1 = 0.53$ m
$h_2 = 0.11$ m





kurtosis
skewness
x[m]
Surface elevation
u at z = -0.04 m
u at z = -0.06 m
u at z = -0.08 m

Conclusion

- For the first time according to the authors knowledge, the numerical simulations are able to reproduce statistical properties of wave kinematics of irregular waves propagating over a shoal as in the laboratory experiments [1] .
- For irregular waves propagating over a shoal in sufficiently shallow water, the surface elevation has local maximum of skewness and kurtosis near the edge of the upslope on shallower side and a local minimum of skewness on the downslope of the shoal. Meanwhile, the horizontal velocity has local maximum on downslope of the shoal and local minimum at the same location with local maximum of kurtosis of surface elevation.
- The local effects on kurtosis and skewness may disappear if the length of slope is sufficiently long.

References

[1] K.Trulsen, A., Raustøl, S. Jorde, and L. B. Rye. Extreme wave statistics of longcrested irregular waves over a shoal. J. Fluid Mech. 882:R2,2020.

[2] M. Gouin, G. Ducrozet, and P. Ferrant. Development and validation of a non-linear spectral model for water waves over variable depth. European Journal of Mechanics-B/Fluids, 57:115-128, 2016.

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