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The dependence of global super-rotation on planetary rotation rate

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with thanks to the Isca development team!

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Definition:

Super-rotation strength can be measured via super-rotation indices:

Global super-rotation index: $$S = \frac{\iiint \rho m \, \mathrm{d}V}{\iiint \rho \Omega a^2 \cos^2 \vartheta \, \mathrm{d}V} - 1$$

Local super-rotation index: $$s = \frac{m}{\Omega a^2} - 1$$ where $m = a \cos \vartheta (\Omega a \cos \vartheta + u)$.

S and s are defined so that $S = 0$ and $s \leq 0$ for an atmosphere in solid-body co-rotation with the underlying planet ($u = 0$ everywhere).

(Read 1986b, Read and Lebonnois 2018)

S and *s* are distinct! *S* is not mass-weighted integral of *s* over atmosphere:

- s compares m at a given location with the value it would have if transported there from the equator (initially at rest) whilst materially conserving m.
- S measures global integral of m relative to a state of solid-body co-rotation.

Aim:

We will investigate how global super-rotation depends on planetary rotation rate.

Hide's Theorem:

Consider the zonally-averaged zonal angular momentum budget:

$$\underbrace{\frac{\mathrm{D}\overline{m}}{\mathrm{D}t}}_{\substack{\text{Material}\\ \text{change in } m}} = \underbrace{-\frac{1}{\cos\vartheta}\frac{\partial}{\partial\vartheta}\left(\overline{m'v'}\cos\vartheta\right)}_{\substack{\text{Horizontal eddy}\\ \text{acceleration}}} \underbrace{-\frac{\partial\overline{m'\omega'}}{\partial p}}_{\substack{\text{Vertical eddy}\\ \text{acceleration}}} - F_\lambda$$

Friction

In an axisymmetric, inviscid atmosphere:

$$\frac{\mathrm{D}\overline{m}}{\mathrm{D}t} = -F_\lambda$$

Surface friction can inject m up to a maximum value $m = \Omega a^2$ (the value of m at the equatorial surface). This corresponds to $s = 0$.

Hide's theorem: ***s*** **cannot exceed zero at any location in an axisymmetric inviscid fluid unless it is initialised as such.**

(Hide 1969)

Implication for *S*:

The focus of this work is global super-rotation, what does Hide's theorem say about *S*?

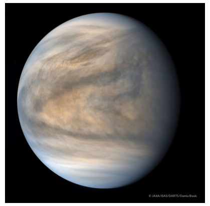
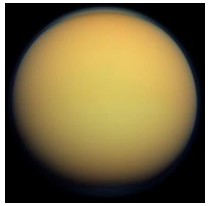
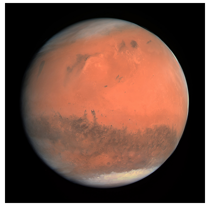
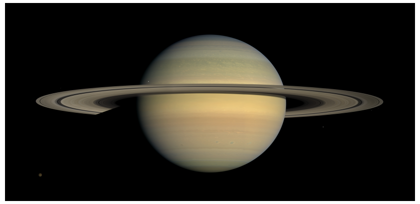
Hide's theorem says that $s \leq 0$ everywhere in an axisymmetric fluid. The limiting case of this is $s = 0$ everywhere, corresponding to $m = \Omega a^2$ everywhere. Substituting this into the definition of S yields $S_{\max} = 1/2$.

While an axisymmetric atmosphere may not permit local super-rotation, it can globally super-rotate to some degree, at least in principle.

Examples

S in the Solar System

Estimates from Read and Lebonnois (2018).

Venus	Titan	Mars	Earth	Jupiter	Saturn
					

- $S_{\mathrm{V}} \sim 7.7, \quad S_{\mathrm{T}} \sim 2$
- $s_{\mathrm{V}} \sim 60, \quad s_{\mathrm{T}} \sim 12$

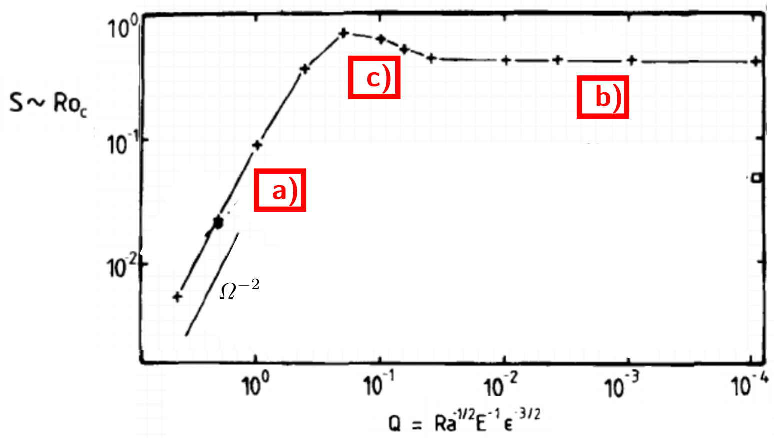
- $S_{\mathrm{M}} \sim 0.04, \quad S_{\mathrm{E}} \sim 0.0135$
- $s_{\mathrm{M}} \sim s_{\mathrm{E}} \sim 0.02$ (transient.)

Zonal velocity too sparsely observed in vertical to estimate S.

- $s_{\mathrm{J}} \sim 0.0075, \quad s_{\mathrm{S}} \sim 0.04$

S in 2D (quasi-axisymmetric) models

Read (1986a) model of cylindrical rotating annulus:



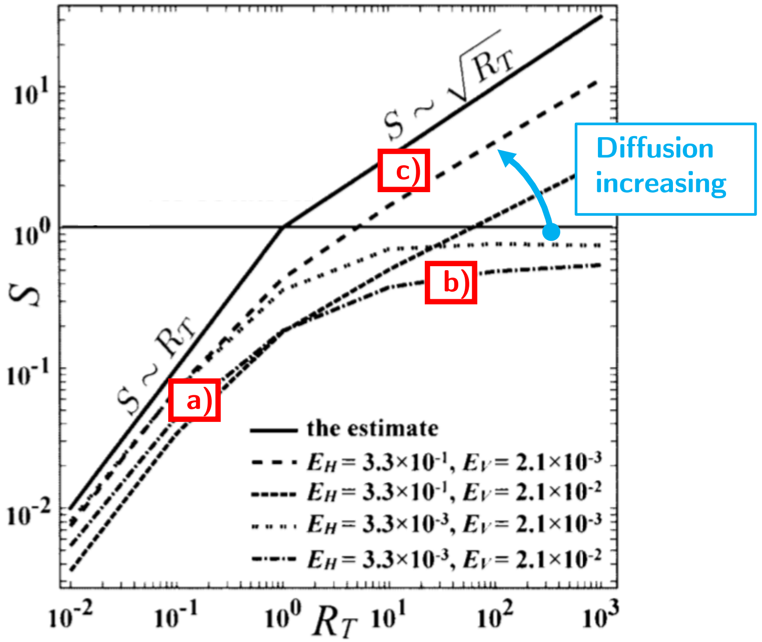
Rotation rate increasing

Yamamoto et al. (2009, 2013) Boussinesq model on sphere:

a) 'geostrophic regime' (Earth and Mars in this regime)

b) 'angular momentum conserving regime'

c) 'cylostrophic regime' (Venus and Titan in this regime)



Rotation rate increasing

'Quasi-axisymmetric' refers to an axisymmetric (2D) model with diffusion parametrisation that permits up-gradient transport of m (i.e., performing the role of eddies in a 3D circulation).

What about 3D models?

Numerous modelling studies using 3D General Circulation Models (GCMs) have shown that local super-rotation emerges ($s > 0$) in an Earth-like atmosphere when

$$\mathcal{R} = \frac{R_{\mathrm{d}} \Delta T_{\mathrm{eq}}}{(\Omega a)^2}$$

is made large [e.g. Mitchell and Vallis (2010), Kaspi and Showman (2015), Wang et al. (2018)].

None, however, have calculated the global super-rotation index S.

This work

What is the dependence of global super-rotation on rotation rate in a 3D (Earth-like) GCM?

Specifically, at low rotation rate, does the 3D GCM enter the angular momentum conserving regime (weak super-rotation), or the cyclostrophic regime (strong super-rotation)?

[In the quasi-axisymmetric (2D) models, if up-gradient viscous transport of m is weak, the circulation enters a regime determined by conservation of angular momentum. If viscous transport is strong, it enters a regime determined by cyclostrophic balance (see previous slide)].

Experiment design

We use Isca, a framework for the idealised modelling of planetary atmospheres.

Built on-top of the GFDL spectral dynamical core.

Vallis et al. (2018)

For this work, we use a configuration similar to the Held–Suarez benchmark.

Dry dynamical core forced by diabatic heating parametrised via Newtonian cooling, and subject to linear friction at the surface.

Planetary parameters (e.g. radius, gravity, surface pressure) and Newtonian cooling reference profile set to those of the Earth (this is what ‘Earth-like’ means).

We run GCM experiments over a wide range of rotation rates: $8\Omega_E$ – $\Omega_E/512$.

For each rotation rate, we run an axisymmetric and a 3D experiment.

Results

S vs. R

S increases proportionally with R at high rotation rate, and saturates at low rotation rate.

These scalings correspond to the 'geostrophic' and 'angular momentum conserving' regimes identified in Read (1986a)

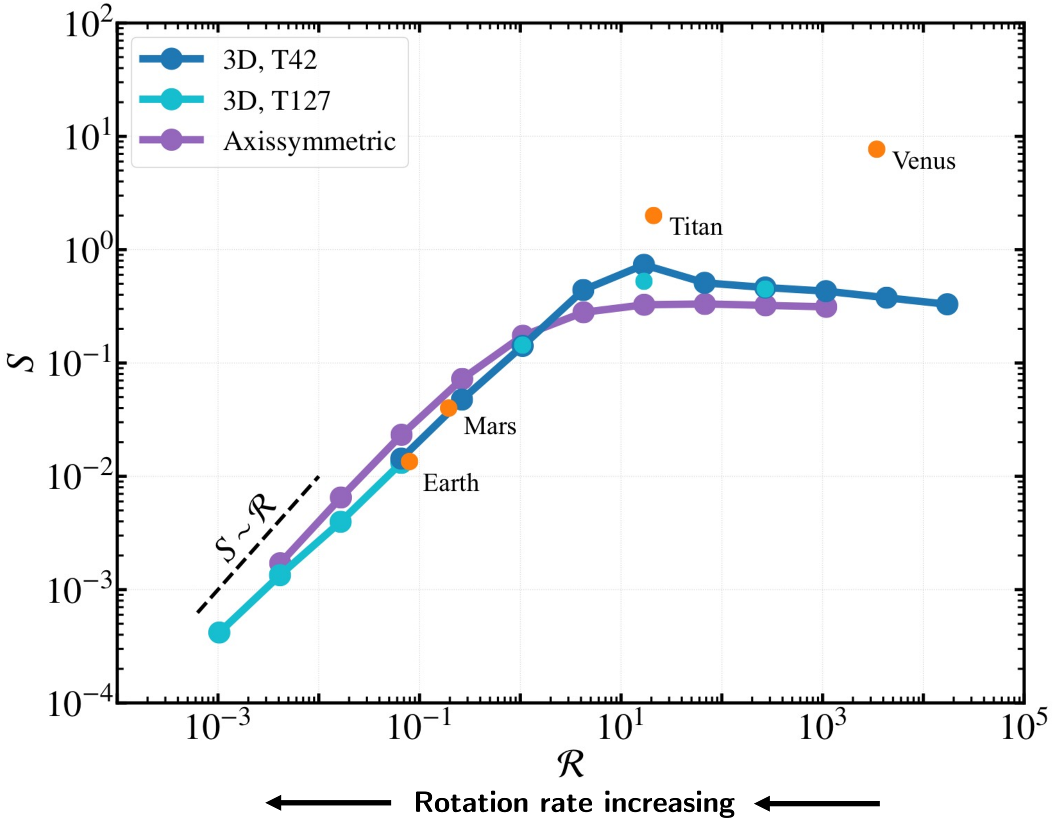
$S_{3D} < S_{ax}$ at high rotation rate, and $S_{3D} > S_{ax}$ at low rotation rate.

Eddies can alter the amount of angular momentum 'lifted' into the atmosphere by the Hadley circulation (during spin-up). For a slowly rotating planet, the eddies cause more angular momentum to be lifted up relative to the axisymmetric case (see pre-print for more details).

The 3D experiments do not reach a Venusian level of super-rotation at very low rotation rate.

This is because the equatorial wave associated with the acceleration of super-rotation at moderate rotation rate [$R = O(1)$] collapses at low/very-low rotation rate (see pre-print for discussion on the cause of this).

This suggests that more than just slow rotation is required to generate strong super-rotation similar to that in Venus' atmosphere.



S vs. R. Blue lines are for 3D experiments. Purple lines are for 2D experiments. Orange dots indicate S estimated for the Solar System planets (taken from Read and Lebonnois, 2018).

We plot S vs. $\mathcal{R} = R_{\mathrm{d}}\Delta T_{\mathrm{eq}}/(\Omega a)^2$ to facilitate comparison between the Isca experiments and real planets, which have different radii to the Earth (as well as different rotation rates).

Results

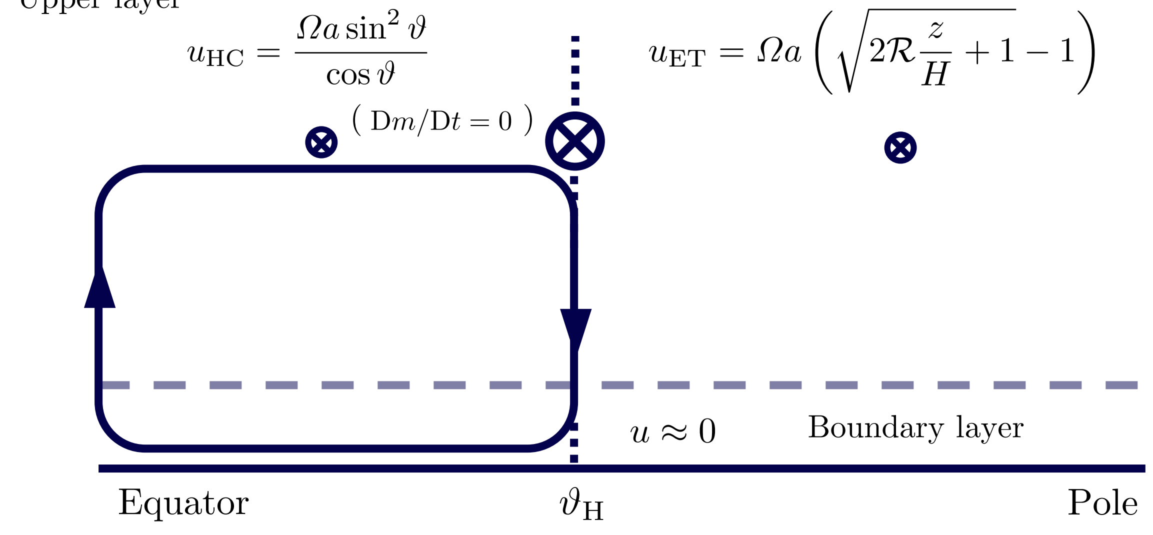
Scaling theory for S_{ax}

Scaling for axisymmetric case

S in our 3D experiments remains close to that in our axisymmetric experiments.

A good model for the zonally averaged circulation of an axisymmetric atmosphere is that of Held and Hou (1980; HH). We will use the HH model to develop a theory for S.

In the HH model, the boundary latitude ϑ_{H} is determined in terms of $\mathcal{R}$:



$$\mathcal{R} = \frac{3}{4}\left[\frac{1}{3} + \frac{1}{x_{\mathrm{H}}^2} + \frac{x_{\mathrm{H}}^2}{1-x_{\mathrm{H}}^2} - \frac{1}{2x_{\mathrm{H}}^3}\ln\left(\frac{1+x_{\mathrm{H}}}{1-x_{\mathrm{H}}}\right)\right]$$

$x_{\mathrm{H}} = \sin\vartheta_{\mathrm{H}}$, and $\mathcal{R} = \Delta_h g H/(\Omega a)^2$ in the Boussinesq framework.

If we substitute the HH expressions for u into the definition of S we obtain

$$S = S_{\mathrm{HC}} + S_{\mathrm{ET}} = \frac{2}{5}\sin^3\vartheta_{\mathrm{H}} + \frac{1}{12}\left[\frac{(2\mathcal{R}+1)^{\frac{3}{2}} - 1}{2\mathcal{R}} - \frac{3}{2}\right](8 - 9\sin\vartheta_{\mathrm{H}} - \sin 3\vartheta_{\mathrm{H}})$$

This expression determines S solely in terms of $\mathcal{R}$ as ϑ_{H} is determined by $\mathcal{R}$. The first term is the contribution to S from inside the Hadley cell, and the second term is the contribution from outside the Hadley cell (the extra-tropics).

(see pre-print for details of derivation.)

Results

Theoretical S vs. R

Theoretical prediction for S agrees well with 2D Isca experiments.

$R \ll 1$: Geostrophic regime.

When $R \ll 1$ then $\sin^3\vartheta_H \to 0$, and $\frac{(2\mathcal{R}+1)^{\frac{3}{2}}-1}{2\mathcal{R}} - \frac{3}{2} \approx \frac{3\mathcal{R}}{4}$ so that

$$S \approx \frac{1}{2}\mathcal{R}$$

In this limit, the contribution to S from S_{HC} disappears, and S is dominated by S_{ET}.

The R dependence in S_{ET} is from geostrophic thermal wind balance with the temperature field.

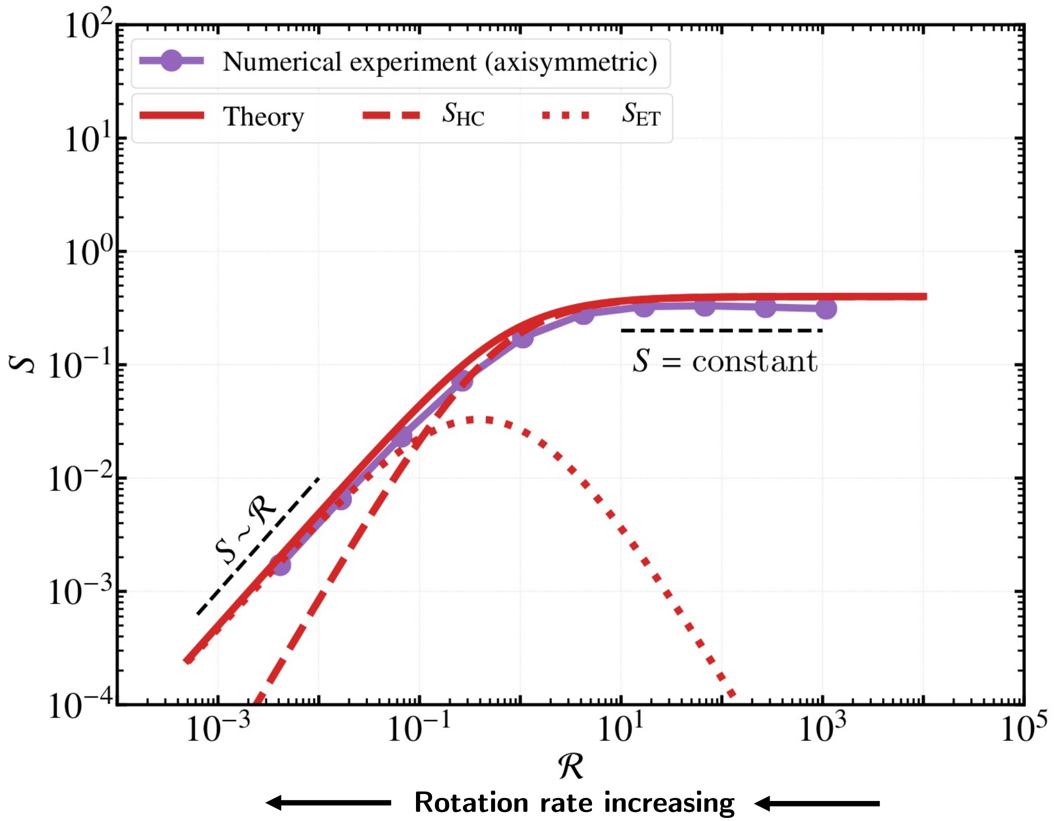
$R \gg 1$: Angular momentum conserving regime.

As R increases, the contribution from S_{ET} drops off as the Hadley cell becomes larger. In the limit $R \gg 1$, $\sin\vartheta_H$ and $\sin^3\vartheta_H \to 1$, while $\sin 3\vartheta_H \to -1$. Then,

$$S \approx \frac{2}{5} = \text{constant}$$

In this limit, the contribution to S is dominated by S_{HC} .

The domain is filled by the Hadley circulation and $m = \Omega a^2$ everywhere, aside from the narrow boundary layer.



S vs. R. Red lines are for theory. Purple lines are for 2D Isca experiments.

The dotted red line is the contribution to S from the Hadley cell. The dashed red line is the contribution to S from the extra-tropical region.

This is the $S = 1/2$ limit discussed in the introduction, reduced by a factor $4/5$ by the quiescent boundary layer.

Results

Theoretical S vs. R

$R \gg 1$: Comment on cyclostrophic regime.

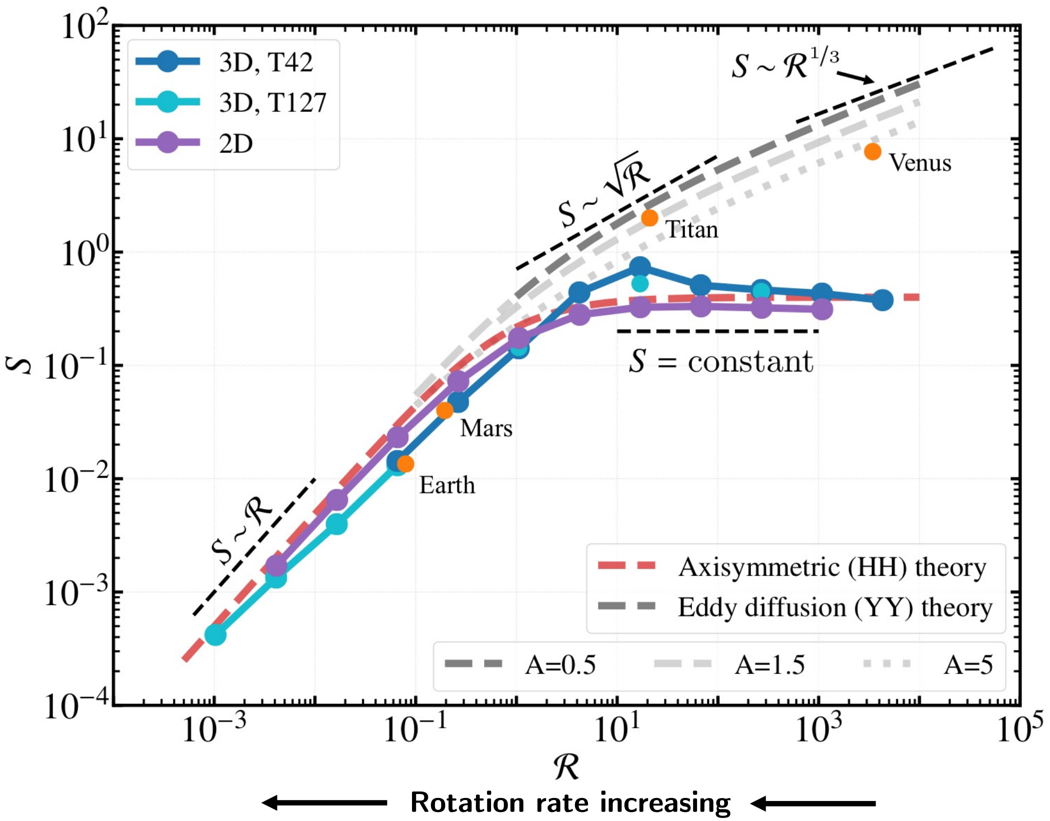
In the 3D Isca experiments, eddies do not induce strong enough up-gradient transport of m to enter the cyclostrophic regime identified by Yamamoto et al. (2009, 2013). This regime is clearly accessible in a 3D atmosphere, however, as evidenced by Venus and Titan.

To obtain a theory for S in the cyclostrophic regime, we would need to parametrise up-gradient transport of m in our axisymmetric model. Yamamoto and Yoden 2013 do this with an eddy viscosity.

They come up with a theory for S, from which predictions are plotted in the Figure to the right.

Scaling at really slow rotation rates is less than $\sqrt{\mathcal{R}}$. This is due to the weakening of eq-pole temperature contrast with respect to the radiative-equilibrium temperature contrast.

The YY theory shows that a slowly rotating Held-Hou like atmosphere can super-rotate strongly like Venus if eddy transport made sufficiently large. Our 3D experiments show that changing rotation rate alone doesn't give you the required eddy transport, however.



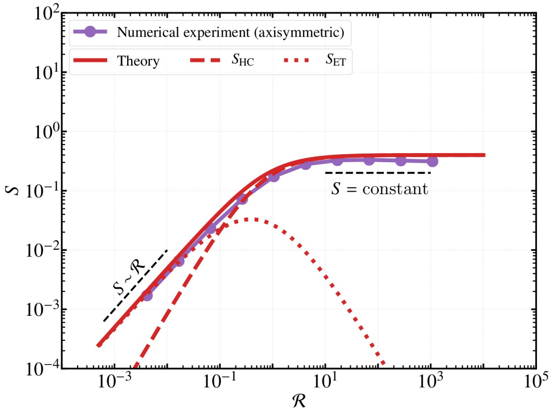
Summary of S vs. R curves. Blue lines are 3D Isca experiments. Purple line is 2D Isca experiments. Orange dots are estimates for real planets.

The red dashed line is the prediction for our axisymmetric theory. The grey dashed lines are predictions from Yamamoto and Yoden (2013)'s theory assuming strong up-gradient transport of m.

Summary

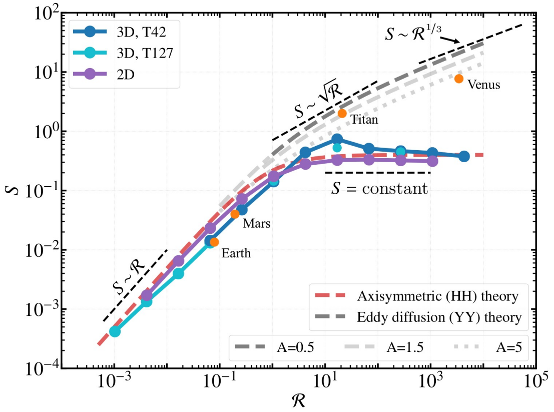
We have studied the dependence of S on planetary rotation rate using an idealised Earth-like GCM.

The 2D experiments occupy a geostrophic regime at high rotation rate, and an angular momentum conserving regime at low rotation rate. We have developed an analytic theory that captures this.



S in the 3D experiments remains close to the 2D experiments for all rotation rates.

At low rotation rate, eddies exist in the 3D experiments that can transport m up-gradient. This transport is not strong enough for the 3D experiments to enter the cyclostrophic regime.



So what?

It is often suggested that slow planetary rotation is synonymous with strong super-rotation. We argue that other processes in addition to slow rotation are *required* for strong super-rotation. Slow planetary rotation in isolation should be associated with weak global super-rotation *determined* by conservation of angular momentum.

Pre-print:

Lewis, N. T., G. J. Colyer, and P. L. Read, 2020: The dependence of global super-rotation on planetary rotation rate. Submitted to *J. Atmos. Sci.,* arXiv e-print: https://arxiv.org/abs/2004.08414.

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