On Determination of the Intensity and Size Frequency Distribution of Convective Vortices: Applications to Martian Dust Devils

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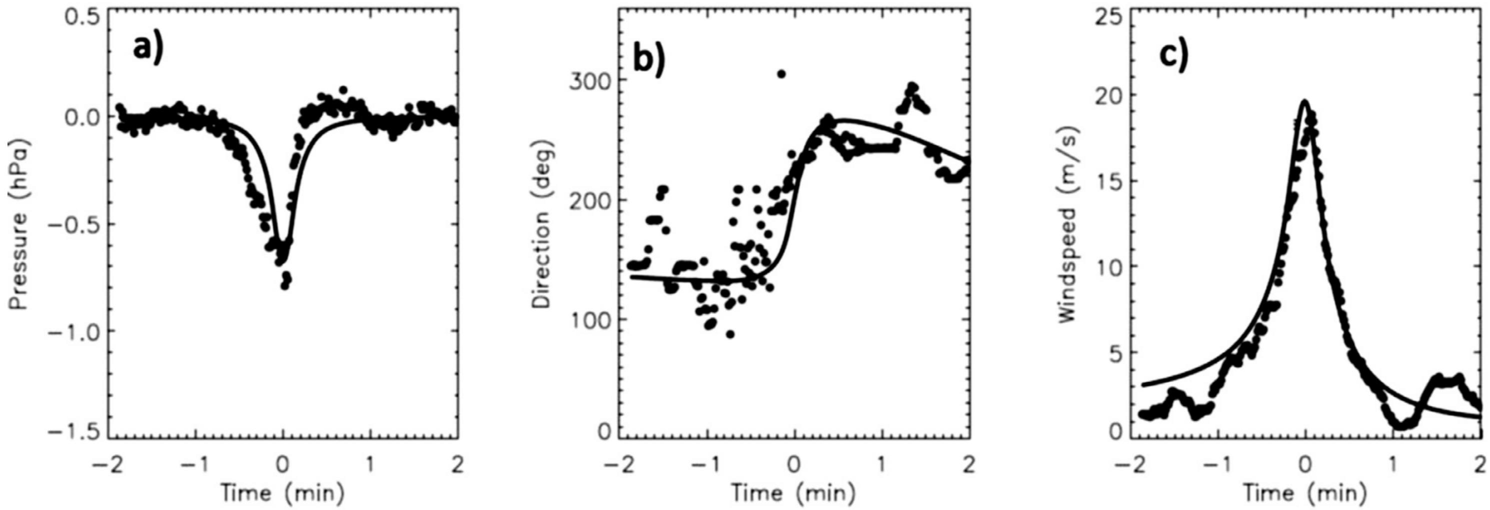
Introduction

- On the basis of the Abel transform application, a two-step methodology is proposed to infer both the intensity-frequency distribution and the size-frequency distribution of convective vortices, including dust devils, in pressure time-surveys on Mars. This methodology is applied with success to Mars Science Laboratory (MSL) convective vortices.



- An example of pressure (a), wind direction (b) and speed (c) measurements for a dust devil encounter in New Mexico during a month-long single-station survey in 2014. Data points are acquired at 1 s intervals, and the curves are a simultaneous fit to the three parameters by an analytical vortex model (from Lorenz, 2016).

Theory: intensity-frequency distribution (I)

- The radial distribution of the pressure drop in convective vortices is described by the Lorentzian profile

$$\Delta P(r) = \Delta P_0 \left(1 + 4r^2/W^2\right)^{-1}$$

- The statistical distribution (pdf) w of measured pressure drops ΔP and the statistical distribution (pdf) of core pressure drops ΔP_0 in detected vortices are related by the Abel integral equation

$$w(\Delta P) = \frac{\sqrt{\Delta P_{\min}}}{2(\Delta P)^{3/2}} \int_{\Delta P}^{\infty} \frac{\Delta P_0}{\sqrt{\Delta P_0 - \Delta P_{\min}}} \frac{\rho(\Delta P_0)}{\sqrt{\Delta P_0 - \Delta P}} \, \mathrm{d}(\Delta P_0)$$

- Here, $\Delta P_{\min}$ is the minimum detection level of the vortex signature against the background pressure noise

Theory: intensity-frequency distribution (II)

- In the case of no correlation between ΔP_0 and the vortex core width W the probability density functions for recorded vortices and for the whole population of vortices are related via

$$\rho\left(\Delta P_0\right)=\frac{\sqrt{\Delta P_0-\Delta P_{\min}}}{\overline{\sqrt{\Delta P_0-\Delta P_{\min}}}}\,\rho^*\left(\Delta P_0\right)$$

- Here, an overbar denotes averaging for the whole population of vortices and the Abel integral equation reads

$$w\left(\Delta P\right)=\frac{1}{2\left(\Delta P\right)^{3/2}\,\overline{\sqrt{\Delta P_0/\Delta P_{\min}-1}}}\int\limits_{\Delta P}^{\infty}\frac{\rho^*\left(\Delta P_0\right)}{\sqrt{\Delta P_0-\Delta P}}\,\Delta P_0\,\mathrm{d}\left(\Delta P_0\right)$$

Results: intensity-frequency distribution (I)

- For power distributions and in case of no correlation between ΔP_0 and W

$$\rho^*(\Delta P_0) = A(\Delta P_{\min}/\Delta P_0)^m \qquad w(\Delta P) = A(\Delta P_{\min}/\Delta P)^m$$

i.e. the measurements provide an unbiased estimate.

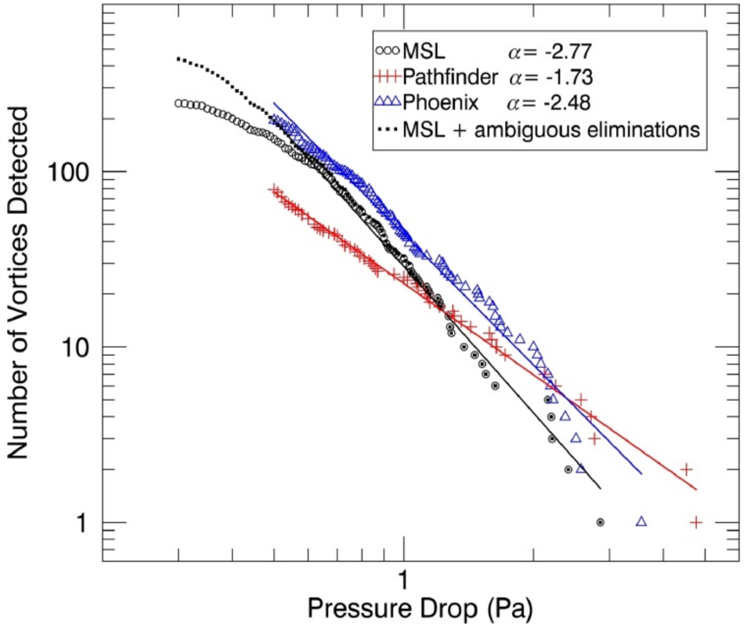
- In general cases and in the limit $\Delta P_0 >> \Delta P_{\min}$

$$\rho(\Delta P_0) \propto \left(\frac{\Delta P_{\min}}{\Delta P_0}\right)^{m-1/2}$$

- The distribution of central pressure drops in the population of detected vortices has thus a less steep slope than the distribution of measured pressure drops.

Results: intensity-frequency distribution (II)

- Cumulative distribution of pressure drop magnitudes. Three distributions of pressure drop magnitudes from three separate Mars missions are shown as the total number of vortices detected above a given pressure drop magnitude in pascals. MSL vortices are open circles, Pathfinder vortices are plus signs, and Phoenix vortices are triangles. The small filled squares show the MSL detected vortices plus the candidates which were eliminated because they were classified as too ambiguous to be confident detections. Power law fits were applied to the cumulative distributions of each mission (solid lines) and the magnitude of each power law slope, α, is shown in the legend (MSL $\alpha=-2.77$, Pathfinder $\alpha=-1.73$, and Phoenix $\alpha=-2.48$ (from Steakley and Murphy, 2016; their Fig. 9)).



Results: intensity-frequency distribution (III)

- **Table 1**
- Inferred value of exponents in the power-law complementary cumulative distribution of pressure drops in Martian convective vortices, including dust devils. In the last column, the first values are derived under an assumption of no correlation between the vortex width *W* and the central pressure drop ΔP_0; the values in parentheses correspond to assumed proportionality between the vortex width squared and the central pressure drop.

	Observed distributions of ΔP (Steakley and Murphy, 2016)	Inferred distributions of ΔP_0 for encountered vortices	Inferred distributions of ΔP_0 for the whole population of vortices
Pathfinder vortices	–1.73	–1.23	–1.73 (–2.23)
Phoenix vortices	–2.48	–1.98	–2.48 (–2.98)
MSL vortices	–2.77	–2.27	–2.77 (–3.27)

Theory: size-frequency distribution(I)

- The statistical distribution (pdf) *w* of measured pressure profile "full width at half maximum" Γ and the statistical distribution (pdf) of the width *W* of detected vortices are related by the Abel integral equation

$$w(\Gamma)=\Gamma\int_0^{\Gamma}\int_{2\Delta P_{\min}\Gamma^2/W^2}^{\infty}\frac{\rho(\Delta P_0)}{\sqrt{\dfrac{\Delta P_0}{2\Delta P_{\min}}-1}}\cdot\frac{\rho(W)}{W\sqrt{\Gamma^2-W^2}}\,\mathrm{d}\Delta P_0\,\mathrm{d}W$$

- In case of no correlation between ΔP_0 and *W* this equation acquires a more compact form; $\rho^*(\Delta P_0)$ and $\rho^*(W)$ refer to the whole population of vortices:

Theory: size-frequency distribution(II)

$$w(\Gamma) = \frac{\Gamma}{\overline{D}} \int_0^{\Gamma} \int_{2\Delta P_{\min}\Gamma^2 / W^2}^{\infty} \rho^*(\Delta P_0) \frac{\rho^*(W)}{\sqrt{\Gamma^2 - W^2}} \mathrm{d}\Delta P_0 \, \mathrm{d}W$$

- For by a power law with the exponent

$$\rho^*(\Delta P_0) = C\left(2\Delta P_{\min} / \Delta P_0\right)^k$$

- the emerging Abel equation

$$w(\Gamma) = \frac{\Gamma}{\overline{D}} \int_0^{\Gamma} \frac{W^{2k-2}}{\Gamma^{2k-2}} \cdot \frac{\rho^*(W)}{\sqrt{\Gamma^2 - W^2}} \, \mathrm{d}W$$

- can be inversed

$$\frac{\rho^*(W) W^{2k-2}}{\overline{D}} = \frac{2}{\pi} \frac{\mathrm{d}}{\mathrm{d}W} \int_0^{W} w(\Gamma) \Gamma^{2k-2} \frac{\mathrm{d}\Gamma}{\sqrt{W^2 - \Gamma^2}}$$

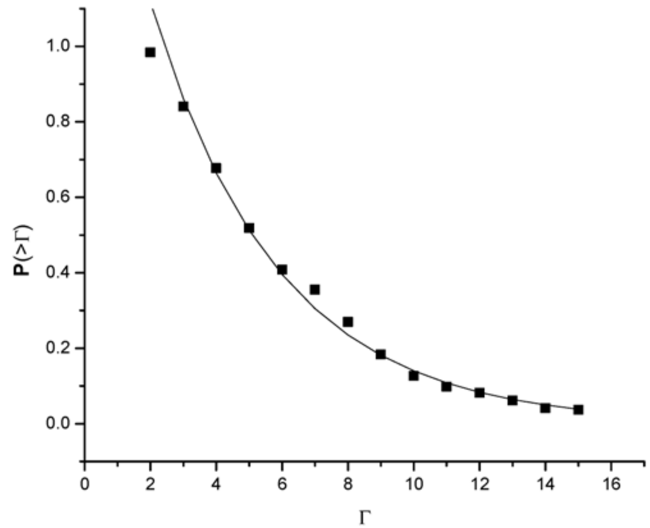
Results: size-frequency distribution; applications to MSL vortices (I)

- We follow an approximate matrix method (see, a classical paper by Wicksell (1925) and further references in, e.g., Pretzler et al. (1992) and De Micheli (2017)), which is used when $w(\Gamma)$-values are given as a table of *N* numbers, and apply the matrix method to MSL vortices (Steakley and Murphy, 2016) possessing a power law intensity-frequency distribution with the exponent k=3.77.
- **Table 2**
- The number of vortices N that have a pressure "profile full width at half maximum" exceeding the fixed Γ-value (in seconds); calculated from the supplementary material to (Steakley and Murphy, 2016)

Γ(sec)	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
N (>Γ)	245	243	241	206	166	127	100	87	66	45	31	24	20	15	10	9

Results: size-frequency distribution; applications to MSL vortices (II)

- The complementary cumulative function **P**(>Γ) inferred from data in **Table 2** (small black squares) and the exponential fit to it (solid line). The values of Γ are in seconds.

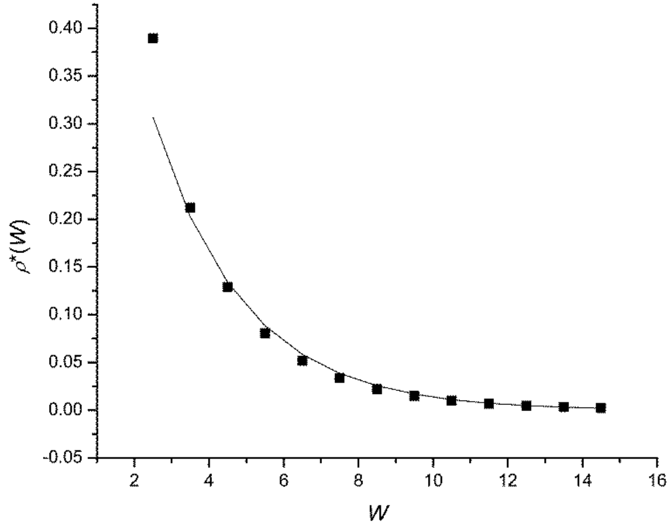


- The differential distribution computed using an exponential fit

$$w(\Gamma) = -\mathrm{d}\mathbf{P}(>\Gamma)/\,\mathrm{d}\Gamma = 0.48671\exp(-\Gamma/3.85362\,)$$

Results: size-frequency distribution; applications to MSL vortices (III)

- The normalized to unity differential distribution function $\rho^*(W)$ inferred by the matrix method (small black squares) and the exponential fit to it (solid line). The values of W are in seconds.



- Exponential fit: $$\rho^*(W) = 0.8655\exp(-W/2.40805)$$
- As one would expect a priori, the distribution $\rho^*(W)$ is steeper than $w(\Gamma)$.

Discussion and summary

- Using the Abel transform, a two-step method has been developed to determine the marginal statistical distributions of convective vortices, including dust devils, on their intensity (pressure drop in the vortex center) and size (diameter), based on statistics of transient pressure drops recorded when the vortices pass near a pressure sensor placed on the planet's surface. This two-step technique has been applied with success to Mars Science Laboratory (MSL) convective vortices.
- A separate difficult problem is the recalculation of Γ -values, and consequently of *W*-values, from time intervals to spatial dimensions. In principle, it is necessary knowing the translational velocity *U* of the vortex relative to the pressure sensor for each detectable vortex. Unfortunately, such data are not available for all MSL vortices and we use for the recalculation that U=7.6 m/s which is an approximate average value of the median wind speed (Kahanpää et al., 2016).
- We note how the Abel inverse transform is mathematically classified as a "mildly" ill-posed problem (cf. De Micheli, 2017), but a stable solution can be obtained after regularization of the problem, particularly for monotonic differential distributions, as shown in this contribution.
- The proposed method can also be used for post-processing the data obtained in pressure time-series surveys for dust devils in arid and semi-arid locations on Earth (Lorenz and Lanagan, 2014; Jackson and Lorenz, 2015) and, more generally, for inferring statistical properties of populations of atmospheric vortices of convective origin based on meteorological in situ measurements.

Acknowledgments & References

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- More details on the applied methodology and the references to the used literature sources can be found in:

Kurgansky, M.V., 2019. On the statistical distribution of pressure drops in convective vortices: Applications to Martian dust devils. Icarus 317, 209–214.

Kurgansky, M.V., 2020. On determination of the size-frequency distribution of convective vortices in pressure time-series surveys on Mars. Icarus. Volume 335, 2020, 113389.