Detecting subsurface interfaces with a physics-based level-set segmentation and additional geological constraints

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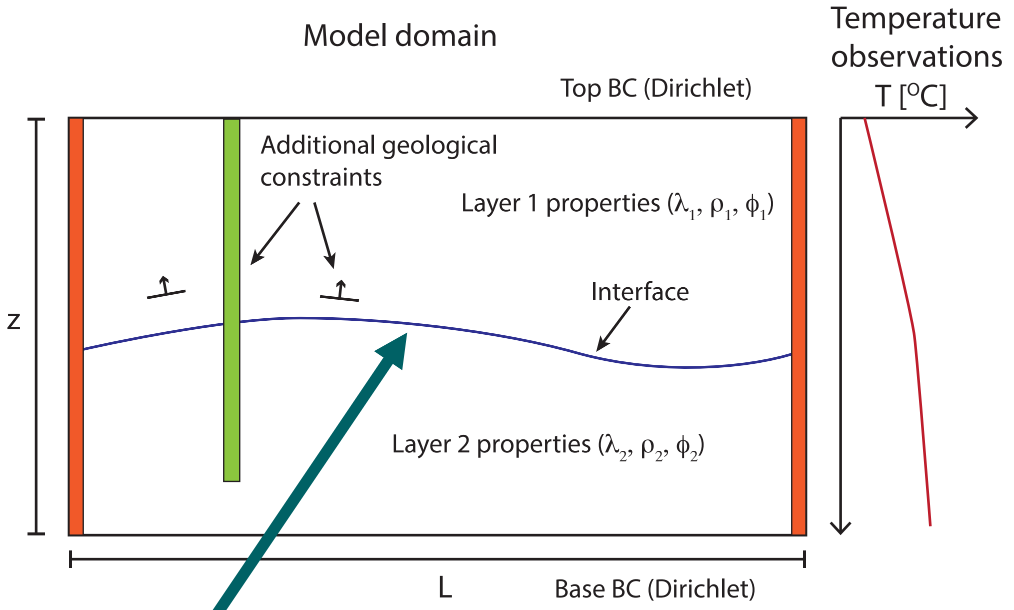
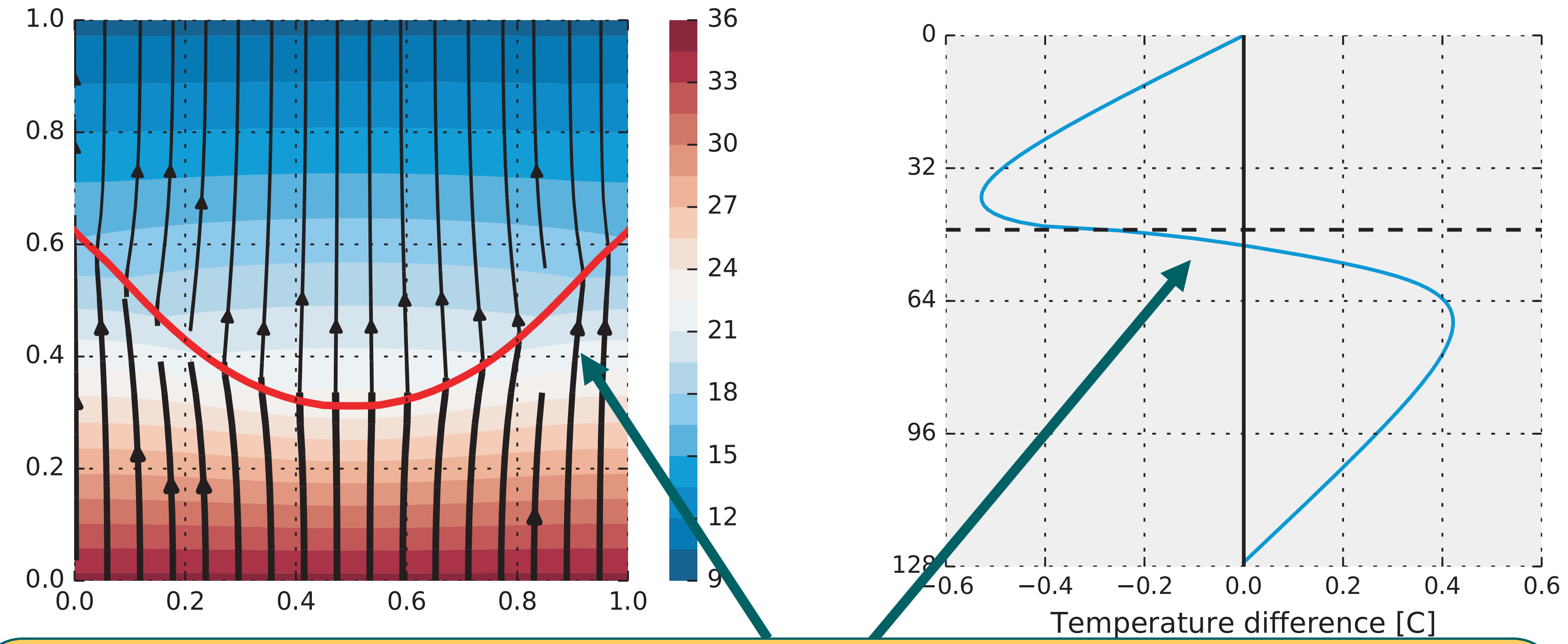
Research Outline:

- Concept: consider a geological domain, **consisting of layers with distinctively different properties** (thermal conductivity λ, density ρ, porosity ϕ, etc.);
- Assume that **layers** are **separated by an interface**;
- In addition, consider a **conductive heat flow field**, affected by layer properties, and temperature observations along vertical drillholes.



Research question: is it possible to detect the interface shape using using an optimization method that directly considers the heat flow field?

Example of deflected temperature field



Deflected temperature field leads to temperature differences at the location of observation points.
Question: can we determine the shape of the interface based on this difference?

Approach

- Describe as **segmentation problem** with **temperature field as additional constraint**
- Can be interpreted as **optimal control problem** with state T and control O:

Given temperature measurements $\tilde{T} : \omega \to \mathbb{R}^+$ on a lower dimensional set $\omega \subset \overline{\Omega} \subset \mathbb{R}^d$, $\mathcal{O}$ should minimize

$$E[\mathcal{O}, T[\mathcal{O}]] = \frac{1}{2} \int_\omega (T[\mathcal{O}] - \tilde{T})^2 \, \mathrm{d}A(x) + \nu \, \mathrm{Per}(\mathcal{O})$$

under the constraint that $T[\mathcal{O}]$ is a weak solution of

$$\begin{aligned} \mathrm{div}((\chi_\mathcal{O} \lambda_1 + (1 - \chi_\mathcal{O}) \, \lambda_2) \, \nabla T) &= 0 \text{ in } \Omega \\ T &= T_D \text{ on } \Gamma_D \\ \nabla T \cdot \nu &= 0 \text{ on } \partial\Omega \setminus \Gamma_D. \end{aligned} \tag{P}$$

Shape description

- Previously used: Mumford-Shah segmentation, but problem: leads to non-convex optimization problem. Also: optimization with respect to a set is numerically difficult
- **Our approach: Chan-Vese approximation**:

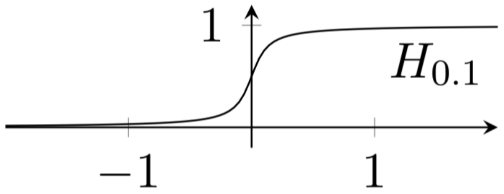
Idea Express $\mathcal{O}$ with a level-set function $\phi : \Omega \to \mathbb{R}$, i.e.

- $[\phi > 0] = \mathcal{O}$
- $[\phi \leq 0] = \Omega \setminus \mathcal{O}$

$$E_{\mathsf{CV}}^{\delta,\epsilon}[\phi] = \int_\Omega H_\delta(\phi) f_1 \,\mathrm{d}x + (1 - H_\delta(\phi)) f_2 + \nu \int_\Omega |\nabla H_\delta(\phi)|_\epsilon \,\mathrm{d}x,$$

where H_δ is the regularized Heaviside function: [Chan, Vese '01]

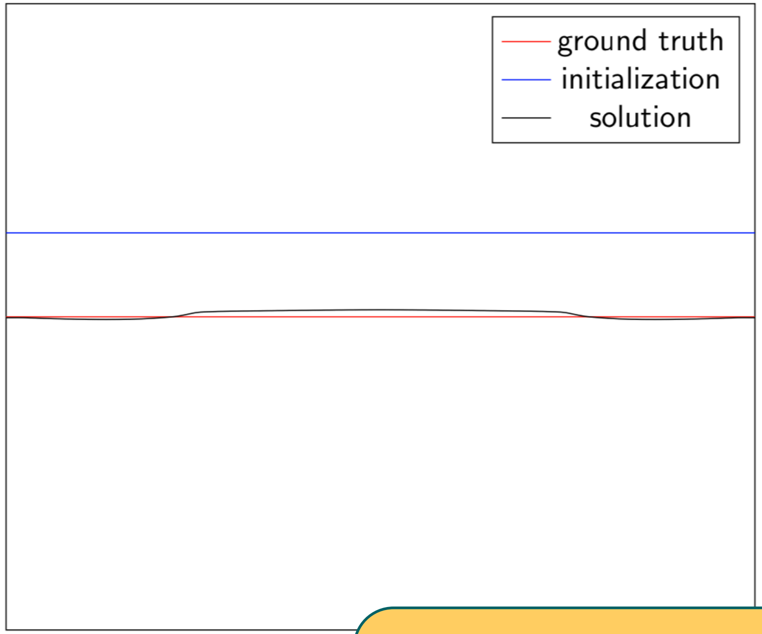
$$H_\delta(s) = \frac{1}{2} + \frac{1}{\pi}\arctan\left(\frac{s}{\delta}\right)$$



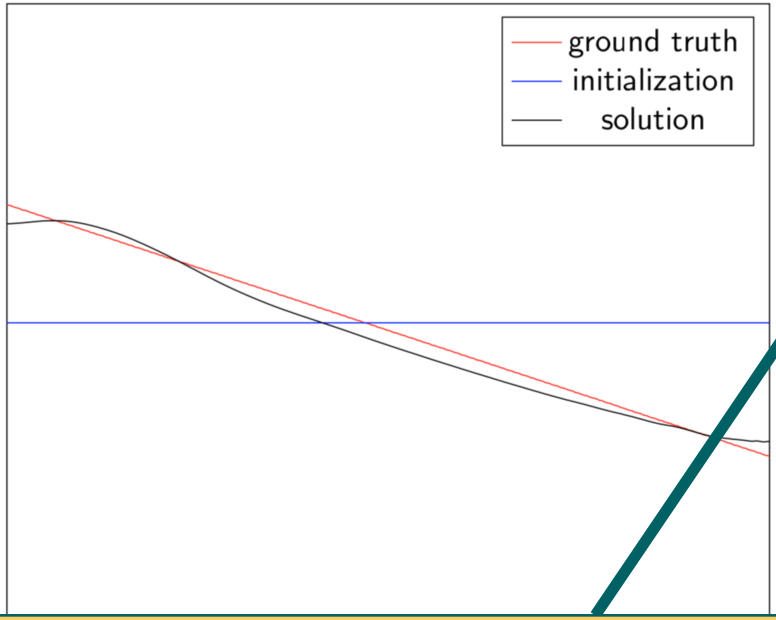
E_{CV} now differentiable, optimization with respect to ϕ is feasible

Results: simple geological scenarios

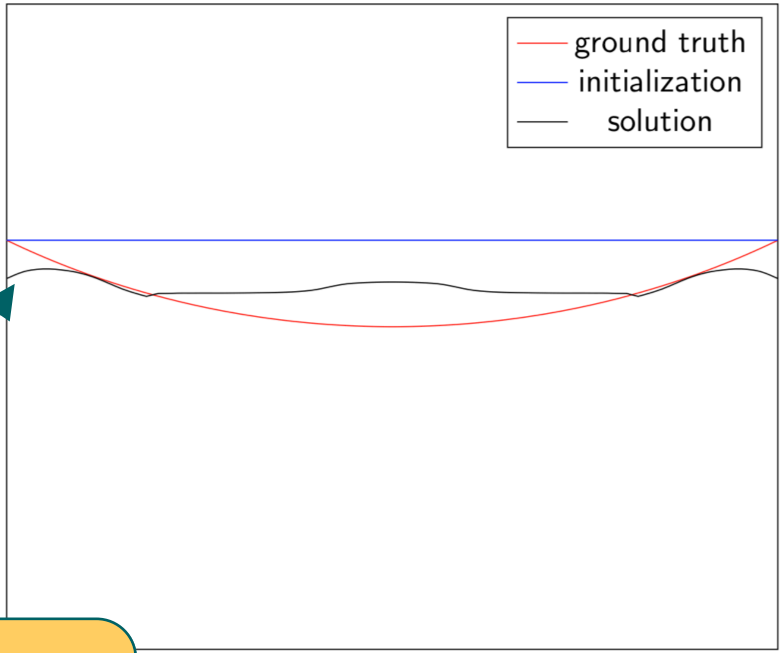
Horizontal layer



Inclined layer



Folded layer

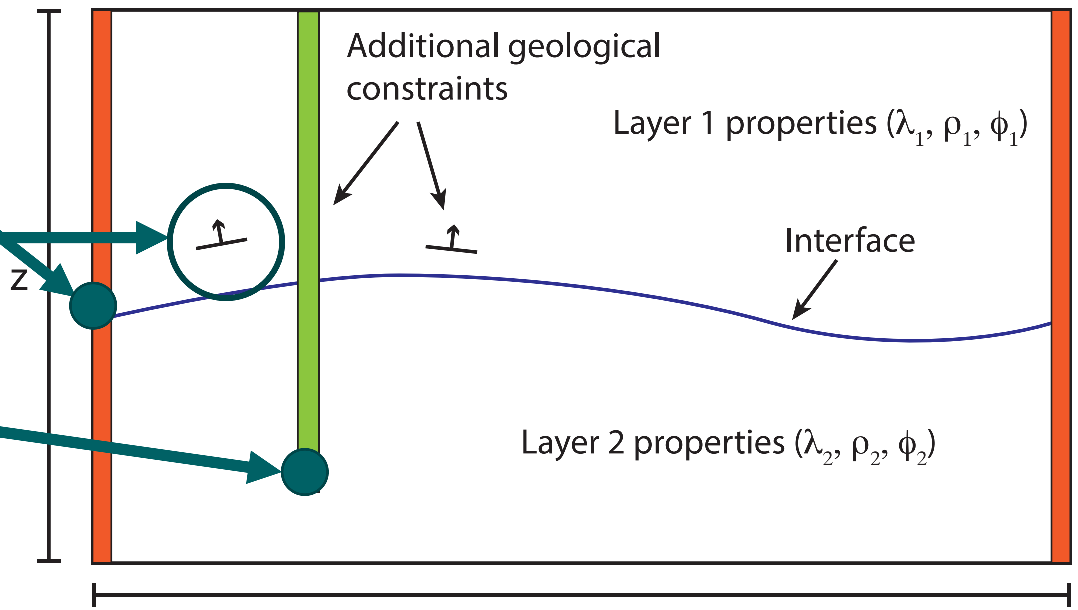


Results promising for simple cases, but **boundary points not respected** (could be expected to be known, in realistic setting) – possible to integrate?

Additional geological constraints

Idea: add additional constraints to optimization problem that can be associated to geological observations (or concepts):

- **Interface observations**
  (interface at specified location, e.g. in boreholes)
- **Gradient observation**
  (orientation of geological layer)
- **Off-surface geological layer**
  (observation that specific geological layer exists at location)



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Additional constraints in penalty terms

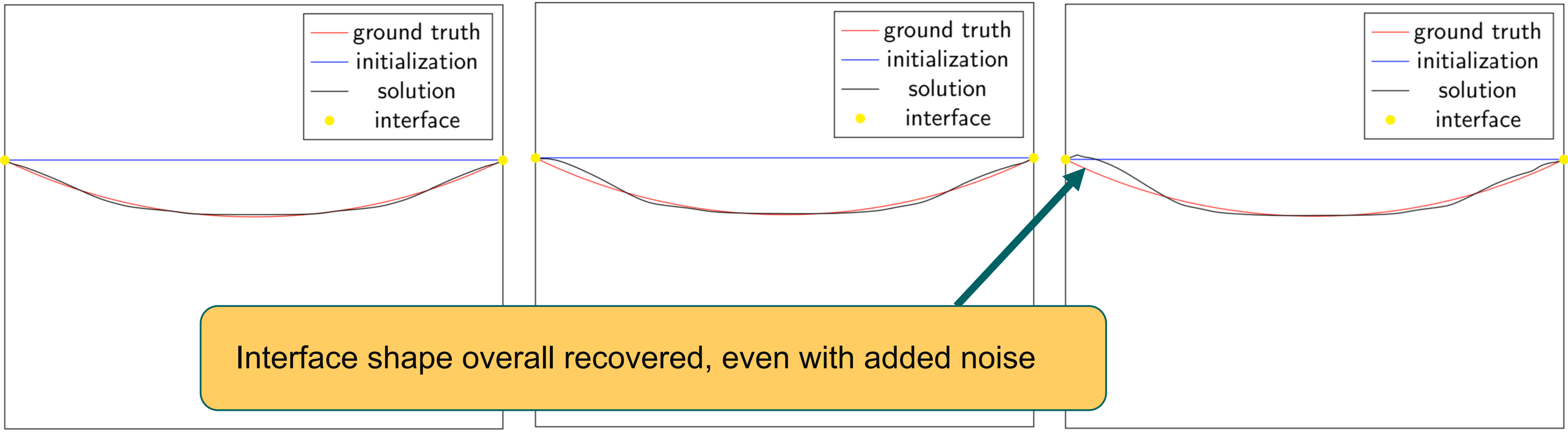
$$G[\phi] = \frac{1}{2}\sum_{i=1}^{N} \|\nabla\phi(p_i) - v_i\|_2^2 + \boxed{\frac{1}{2}\sum_{i=1}^{M} \|\phi(q_i)\|_2^2}$$
$$+ \frac{1}{3}\sum_{i=1}^{K} \max(0, -\phi(p_i^{\mathrm{in}})^3) + \frac{1}{3}\sum_{i=1}^{L} \max(0, \phi(p_i^{\mathrm{out}})^3), \qquad (11)$$

Interface constraint implemented in numerical solution

Constraint	Interpretation	Equation reference
Perimeter term	Length of interface	(5)
Data term	Measurement in boreholes (Squared L2 difference of predicted to calculated state value e.g. temperature)	1^{st} term in (4)
Interface constraint	Set position of interface (Penalizes deviation from zero of Φ at selected points)	2^{nd} term in (11)
Gradient constraint	Define gradient/ geological orientation of interface (penalizes difference of $\nabla\Phi$ to defined values)	1^{st} term in (11)
Observation constraint	Define segmentation outcome at selected points	3^{rd} and 4^{th} term in (11)
Eikonal constraint	Penalizes deviation of size of gradient of Φ from one	(13)

Results: additional interface observation constraint, added noise

No noise | **0.1 °C noise** | **0.2 °C noise**



$\lambda_2/\lambda_1 = 1.5$, Gaussian noise, $\sigma = 0.1°C$

$\lambda_2/\lambda_1 = 1.5$, Gaussian noise, $\sigma = 0.2°C$

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Summary and Outlook

Summary

- Level-set approach along the lines of Chan-Vese
- Derivative of the objective using the bilevel optimization structure
- Geological constraints to include additional information

Outlook

- Use a more realistic PDE to handle real temperature measurements
- Additional geological constraints (hard, projected to $\{\varphi = 0\}$, ...)
- Extension to 3D, but with 1D temperature information only
- Investigate Firedrake as framework for the implementation