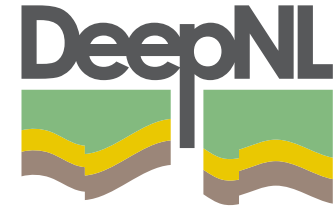




Utrecht University



# Dynamic and quasi-dynamic modelling of earthquake sequences from zero to three dimensions: choose model complexity as needed

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# Objectives

- Simulate and compare seismic cycles on a strike-slip fault with rate-and-state friction law in 0D, 1D, 2D (and 3D), with and without inertia,
- Validate numerical code in different dimensions within the community by solving benchmark problems,
- Clarify the advantages and limitations of these models, identify the appropriate model complexity to solve a specific problem,
- Provide reliable and efficient forward models within a well-structured code library to contribute to the preparatory work for data assimilation to understand seismicity.



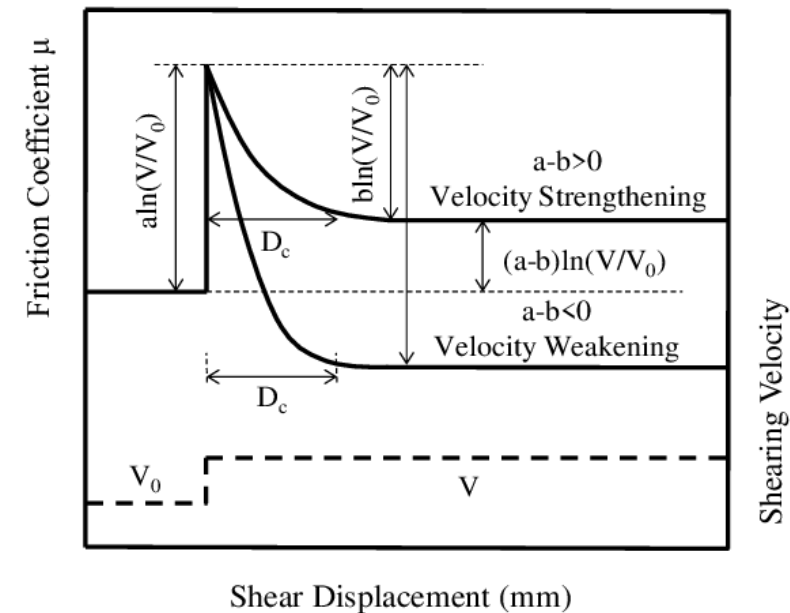
# Methodology

- Rate-and-state friction

$$\tau_s = \mu \sigma_n = \left[ \mu_0 + a \ln\left(\frac{V}{V_0}\right) + b \ln\left(\frac{\theta V_0}{L}\right) \right] \sigma_n$$

$$\theta_{ss} = \frac{L}{V}, \quad \mu_{ss} = \mu_0 + (a - b) \ln\left(\frac{V}{V_0}\right)$$

- Aging law  $\frac{d\theta}{dt} = 1 - \frac{V\theta}{L}$



**Figure:** Rate and state friction law.

# Methodology

- Medium behavior
  - Elastic rheology
  - Mass conservation
  - Momentum conservation
- Boundary condition
  - Rate-and-state friction
  - Aging law
- Adaptive time stepping

e.g.: in 2D, fully dynamic

$$\begin{aligned}\dot{\tau}_{xy} &= G \frac{\partial v_y}{\partial x}, \\ \dot{\tau}_{yz} &= G \frac{\partial v_y}{\partial z}, \\ \dot{v}_y &= \frac{1}{\rho} \left( \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yz}}{\partial z} \right).\end{aligned}$$

or Quasi-dynamic model

$$\begin{aligned}\dot{\tau}_{xy} &= G \frac{\partial v_y}{\partial x}, \\ \dot{\tau}_{yz} &= G \frac{\partial v_y}{\partial z}, \\ 0 &= \frac{1}{\rho} \left( \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yz}}{\partial z} \right).\end{aligned}$$

$$\tau_s = \mu \sigma_n = \left[ \mu_0 + a \ln \left( \frac{V}{V_0} \right) + b \ln \left( \frac{\theta V_0}{L} \right) \right] \sigma_n$$

$$\frac{d\theta}{dt} = 1 - \frac{V\theta}{L}$$

$$dt = \max \left( \min \left( dt_{pl}, dt_{por}, dt_{dx}, dt_{dy}, dt_{vxchange}, dt_{vyschange} \right), dt_{\min} \right)$$



# Garnet

Pranger, 2020

- General Root-finding Algorithm for Time-dependent, Tightly-coupled, Nonlinear problems arising in Earth sciences:
  - a public-domain **code library** with
  - a central finite difference discretization in
  - a **staggered grid** for
  - solving coupled **nonlinear** multi-physics systems in
  - any number of spatial **dimensions** [1-3]
- **Kokkos** and multi-threading on data structure level parallel computing (GPU)
- **PETSc** and MPI point-to-point communication on solver level **parallel computing**

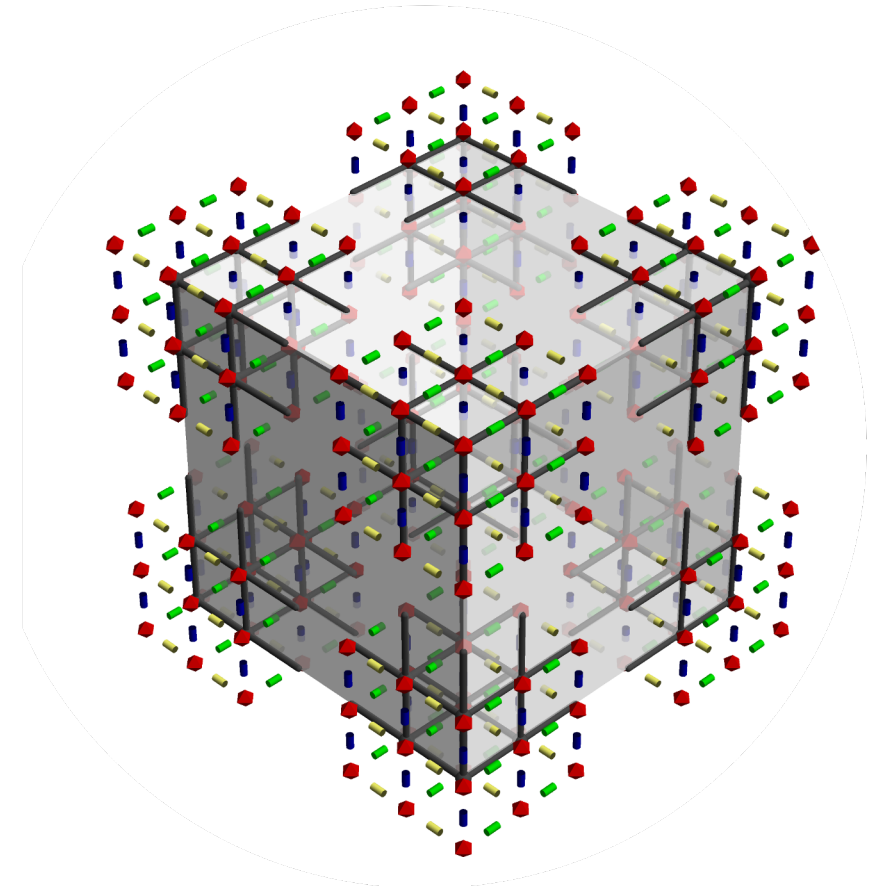


Figure: Staggered spatial discretization in 3D.



# SCEC benchmark problem

Erikson et al, 2019

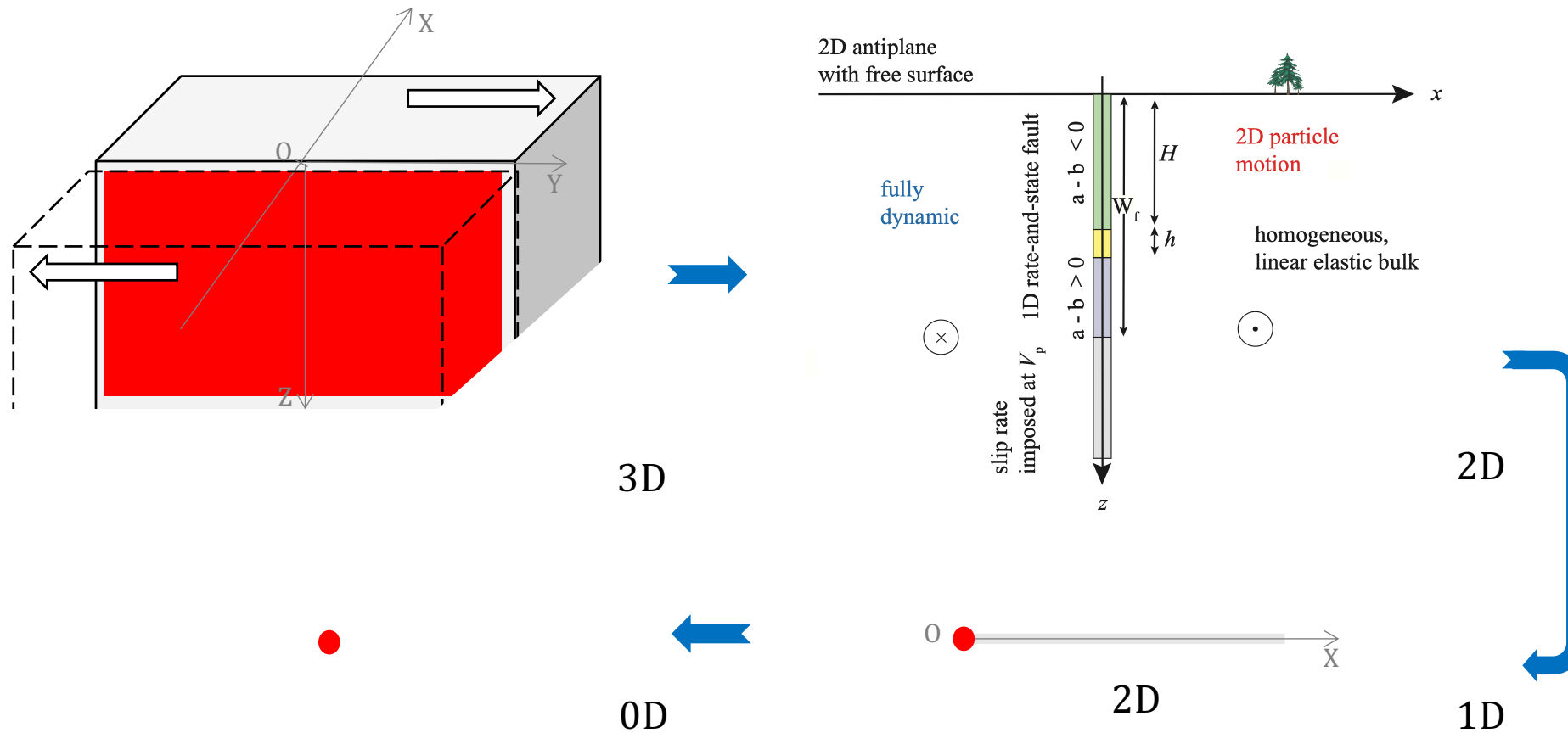
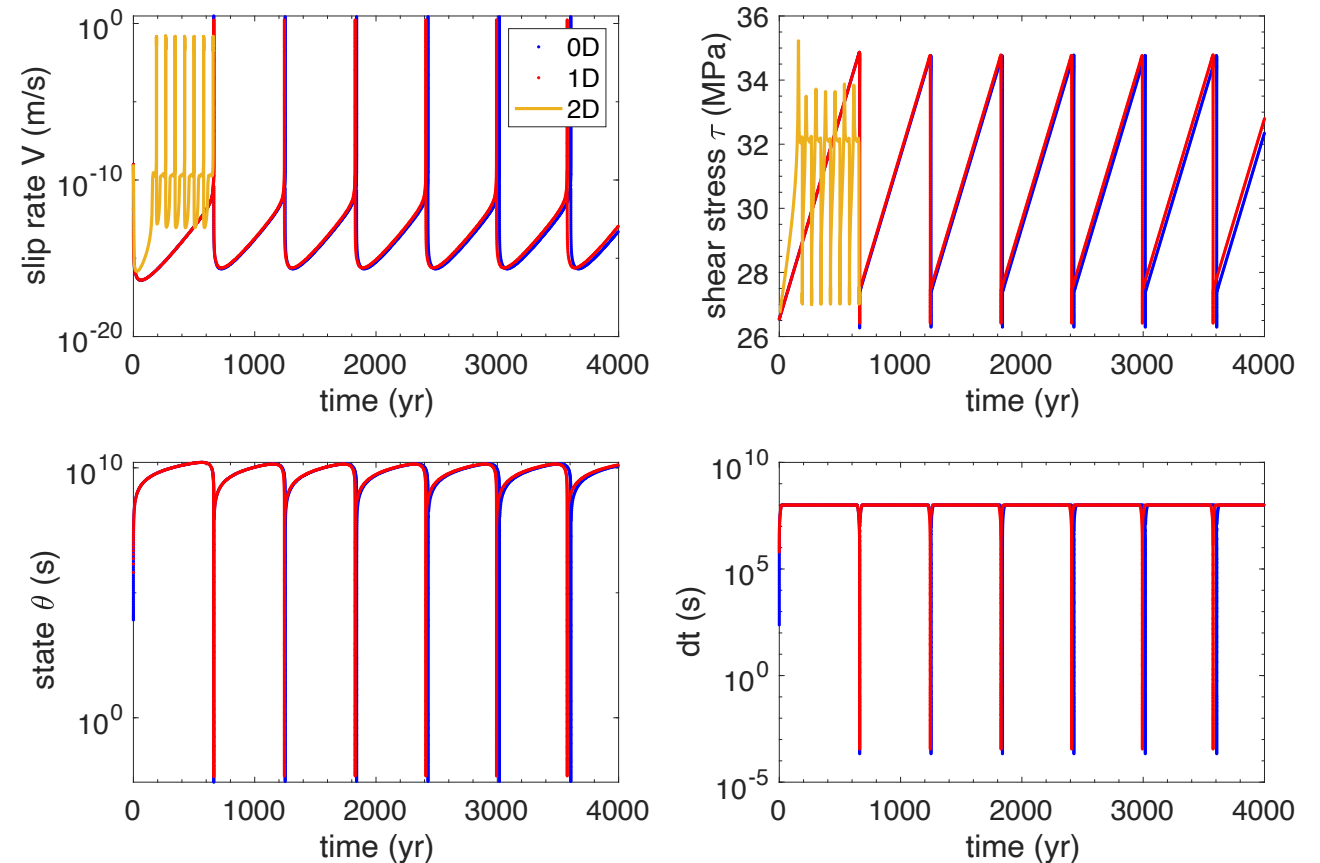


Figure: The benchmark problem model setup.

# Comparison

- 0/1/2D quasi-dynamic:
  - In 0D, only quasi-dynamic model exists,
  - In 0/1D, nucleation phase does not exist,
  - in 2D, event repeats faster than in 0/1D,
  - Max/min stress and slip rates, are modeled accurately in lower dimensional models, which are much faster than higher dimensional models.

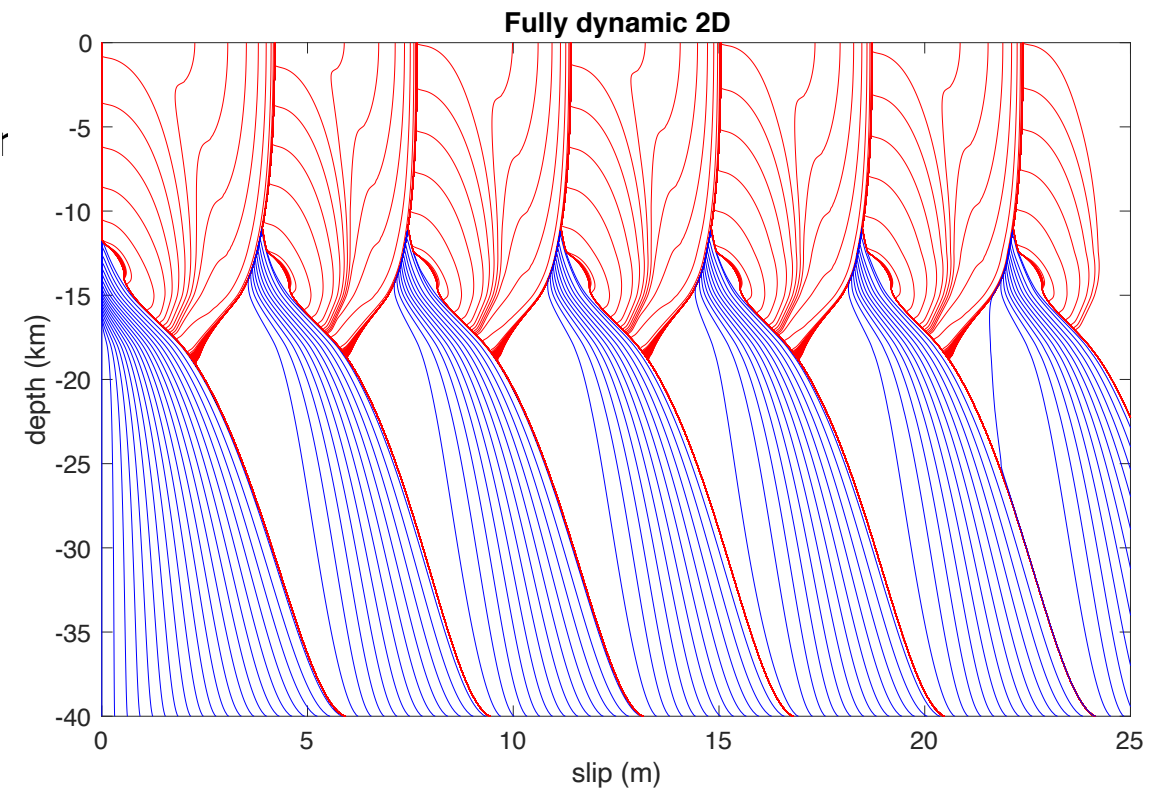
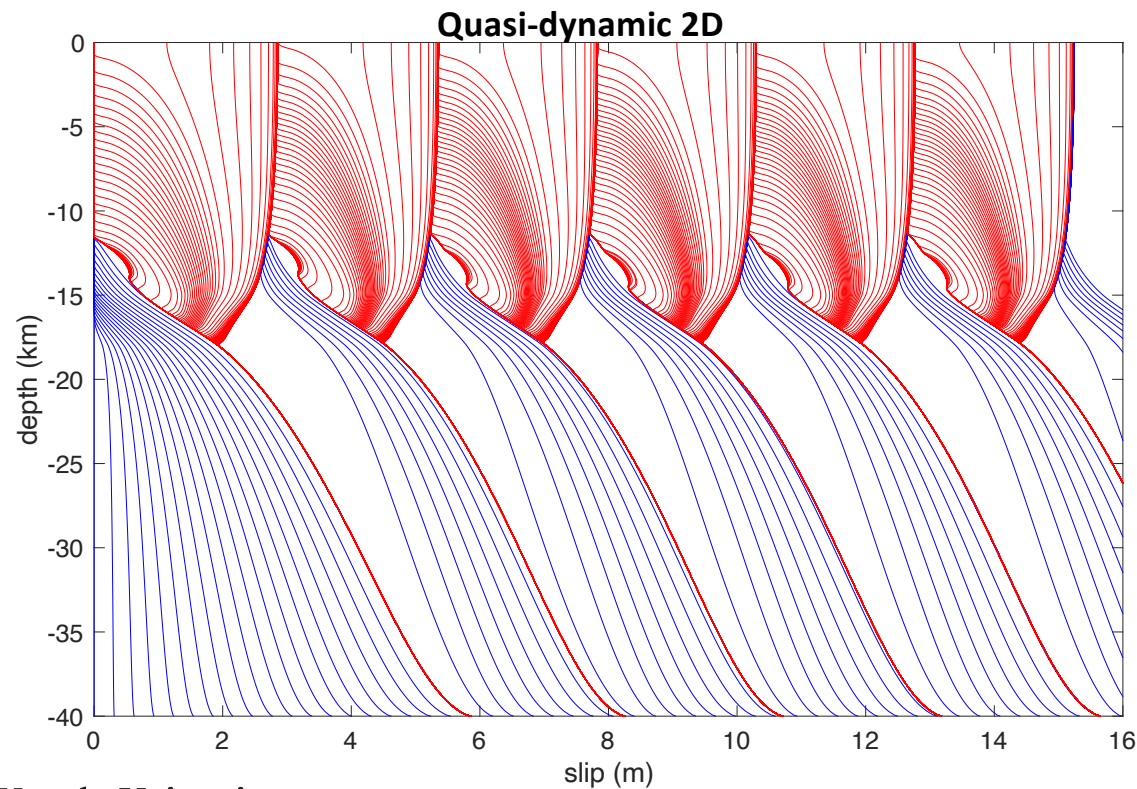


**Figure:** The slip velocity in 0/1/2D quasi-dynamic simulations.



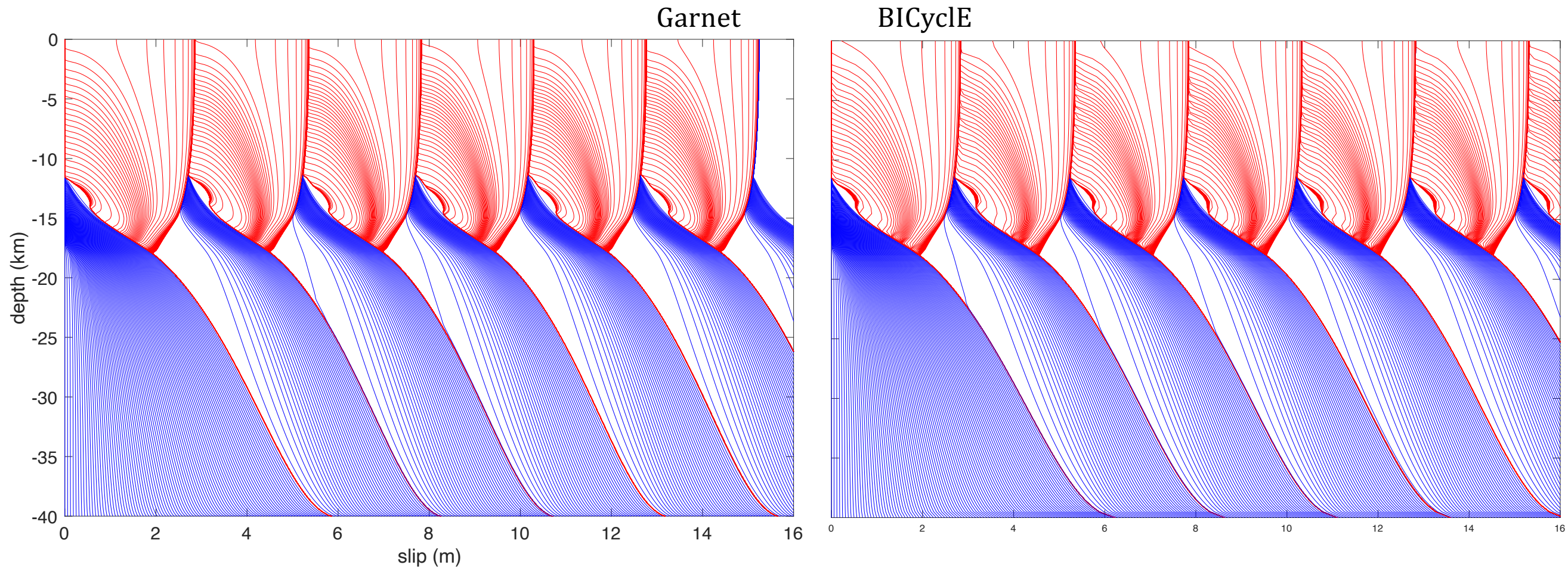
# Comparison

- Quasi- and fully dynamic 2D:
  - Fully dynamic models show larger maximum slip velocity and total slip,
  - Fully dynamic models show sharper wave front and surface reflection phase, as well as larger rupture speed. This makes the coseismic duration in fully dynamic models much shorter.



# Code validation: quasi-dynamic 2D

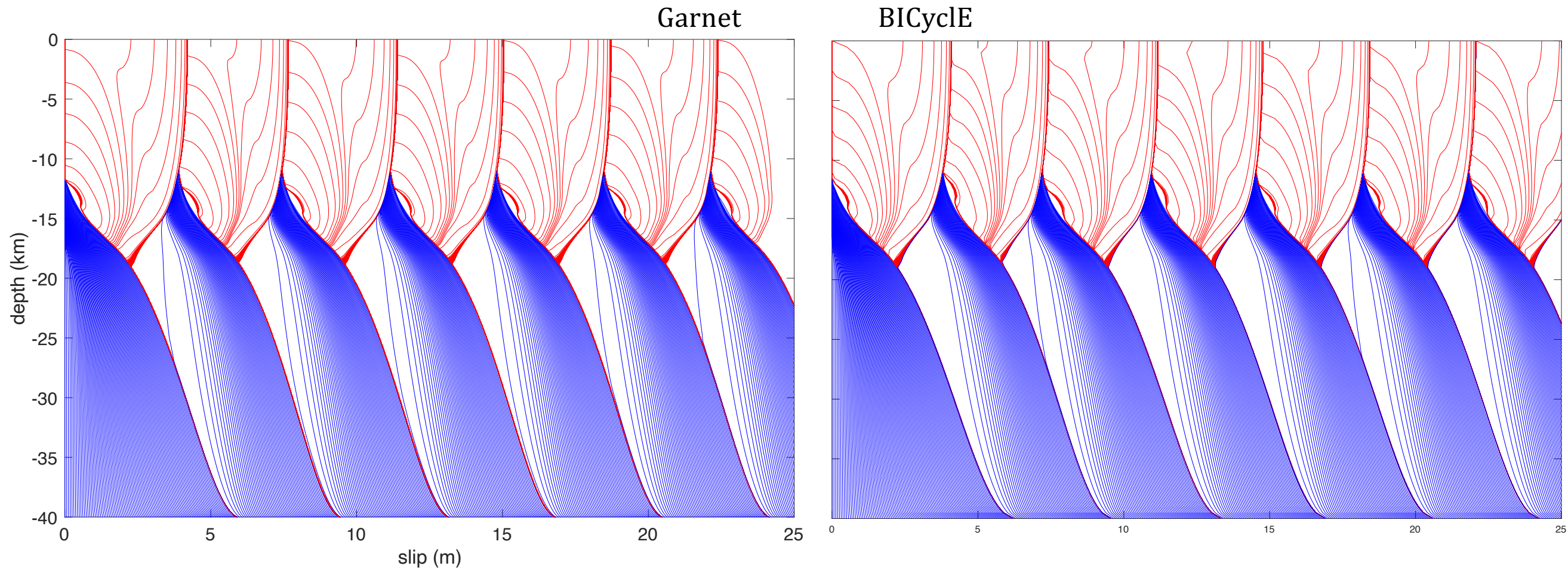
Lapusta et al, 2000, 2009



**Figure:** The comparison of 2D quasi-dynamic seismic cycle modeling between Garnet and BICycle (provided by Valère Lambert, Caltech).



## Code validation: fully dynamic 2D

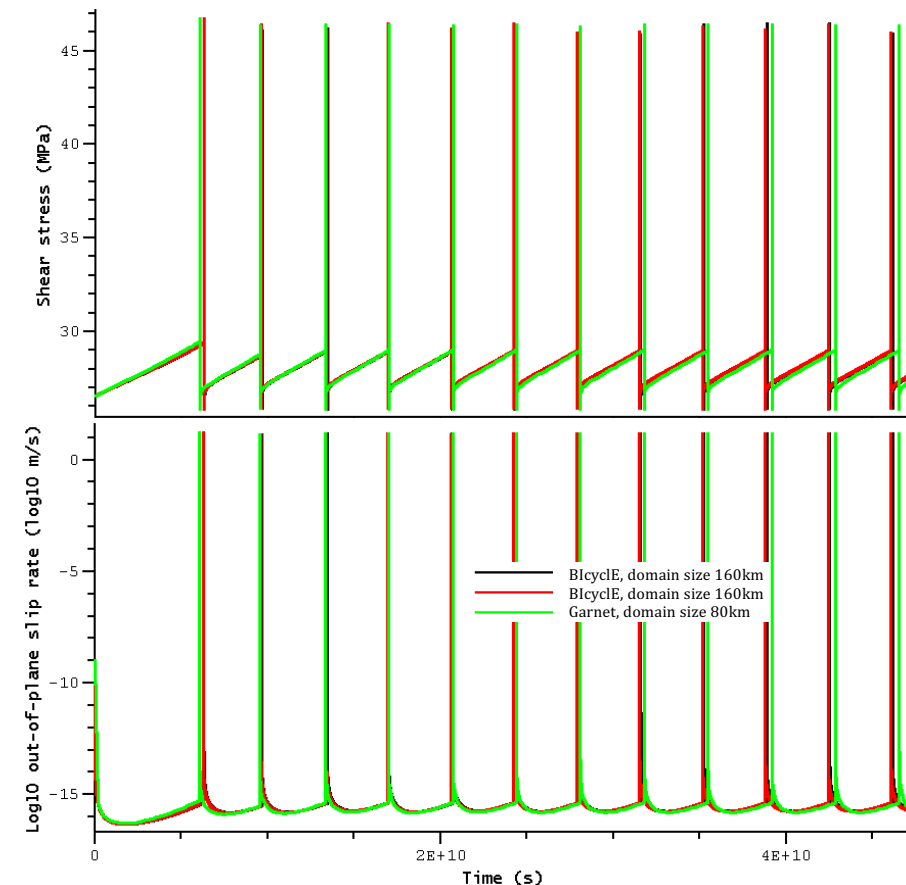
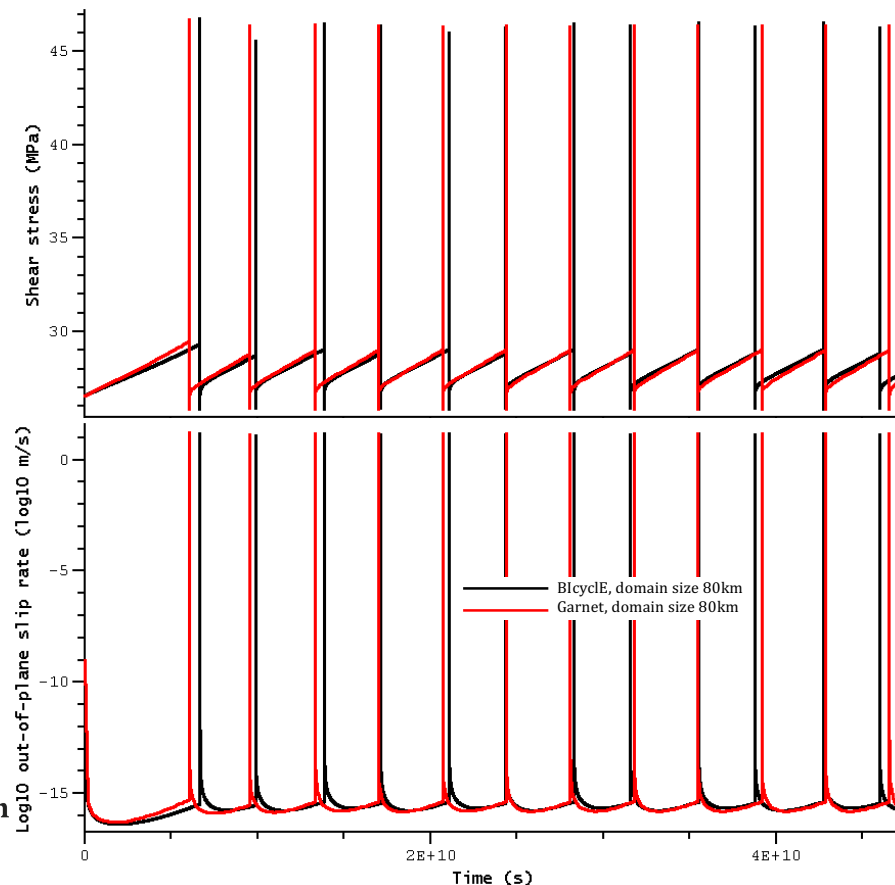


**Figure:** The comparison of 2D fully dynamic seismic cycle modeling between Garnet and BICycle (provided by Valère Lambert, Caltech).

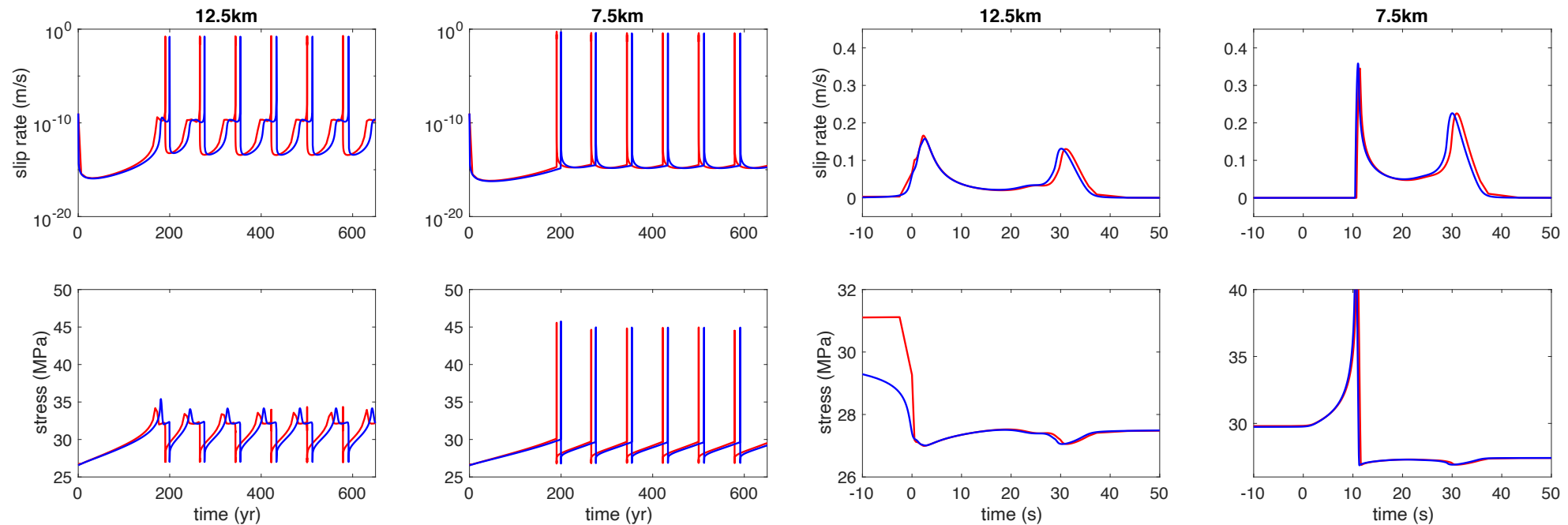


## Code validation: fully dynamic 2D

- Garnet and BIcycleE show similar results, in both quasi- and fully dynamic models.
- Garnet shows more homogeneous events in terms of size and recurrence interval.
- BIcycleE results get closer to Garnet's when the computational domain is enlarged.



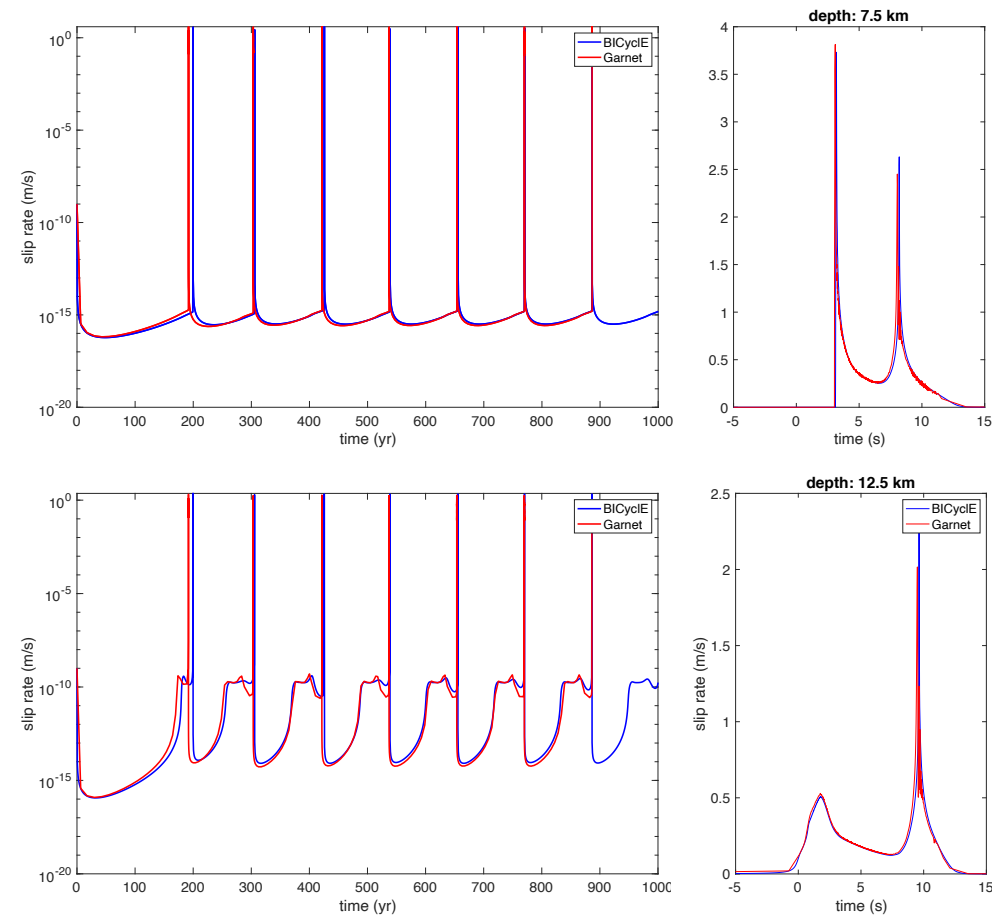
# Code validation: quasi-dynamic 2D



**Figure:** The comparison of 2D quasi-dynamic seismic cycle modeling between (red) Garnet and (blue) BICycle. Left: The overall time series of slip rate and shear stress; Right: the coseismic time series with time origin reset to the rupture initiation time of the third event at the depth of 12.5 km for better comparison.



# Code validation: fully dynamic 2D



**Figure:** The comparison of 2D fully dynamic seismic cycle modeling between (red) Garnet and (blue) BICyclE. Left: The overall time series of slip rate; Right: the coseismic time series with time origin reset to the rupture initiation time of the third event at the depth of 12.5 km for better comparison.



# Summary

- Garnet is validated by comparing its results of a benchmark within the community.
  - Garnet shows more homogeneous events in terms of size and recurrence interval, which BIcycleE achieves with larger computational domain.
  - Garnet will be further validated in off-fault plasticity in our future researches.
- Choose model complexity based on problem statement and objectives.
  - To validate data assimilation codes, a 1D quasi- or fully dynamic model is fast.
  - To implement data assimilation on a laboratory setup, a 2D quasi-dynamic model is useful while a 3D model is still required when the third dimension comes into interest.
  - One could use fast quasi-dynamic model instead of full dynamic one for certain purpose, with in mind that fully dynamic models tend to show larger maximum slip velocity and total slip.



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### Introduction

Large numerical models in 3D are still computational time and memory consuming and they may not be optimal if the aspects of lateral or depth variations within the results are not needed to answer a particular objective. This inspired us to investigate the advantages and limitations of various dimensional models by simulating seismic cycles on a strike-slip fault with rate-and-state friction law in 0D, 1D, 2D and ultimately 3D.

### Garnet

It's a public-domain code library with a staggered grid central finite difference discretization (Figure 1) for solving coupled nonlinear multi-physics systems in any number of spatial dimensions from one to three. [1].

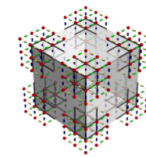


Figure 1 Staggered spatial discretization in 3D.

### Benchmark model setup

This benchmark [2] is a 2D anti-plane problem, with a 1D planar vertical strike-slip fault obeying rate-and-state friction, embedded in a 2D homogeneous, linear elastic half-space with a free surface (Figure 2). The fault has a shallow seismogenic region with velocity-weakening (VW) friction and a deeper velocity-strengthening (VS) region, below which a relative plate motion rate is imposed.

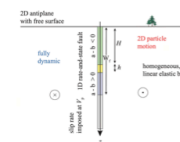


Figure 2 The benchmark problem model setup. (After [2])

### Methodology

- Medium behavior
  - Elastic rheology
  - Mass conservation
  - Momentum conservation
- Boundary condition
  - Rate-and-state friction
  - Aging law
- Adaptive time stepping

$$\Delta t = \min \left\{ \frac{\Delta x}{c}, (1 + \alpha) \Delta t_{\text{min}}, \Delta t_{\text{max}} \right\}$$

### Abstract

We developed a C++ numerical library called GARNET to deal with the various dimensional models in one simulator. By adding dimensions, we simulate a more detailed structure of the seismic cycle. The higher dimensional models present both the validity and the limitations of the lower dimensional ones. However, some important observables, such as the seismic cycle period, maximum/minimum stress and slip rates, are calculated accurately in lower dimensional models, which are much faster than higher dimensional models. We also implemented and compared quasi- and fully dynamic models in the same way. These comparisons clarify the advantages and shortcomings of the models and could provide us with guidance to identify the appropriate model complexity for a specific problem. Finally, we present our results for the SCEC SEAS benchmarks BP1 and 3 and compare them to other participating codes [1].

### Comparison – various dimensional models

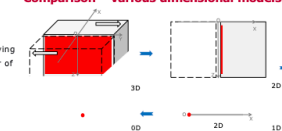


Figure 3 The model setup in 0D, 1D, 2D and 3D.

### Comparison – fully and quasi-dynamic models

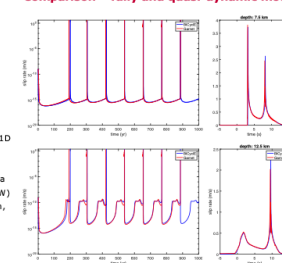


Figure 6 The comparison of 2D fully dynamic seismic cycle modeling between Garnet and BICycle (provided by Valère Lambert, Caltech [3]). Left: The overall time series of slip rate; Right: the coseismic time series with time origin reset to the rupture initiation time of the third event at the depth of 12.5 km for better comparison.

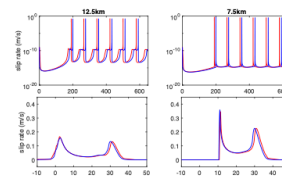


Figure 7 The comparison of 2D quasi dynamic seismic cycle modeling between (red) Garnet and (blue) BICycle (provided by Valère Lambert, Caltech [3]). See Figure 6 for more details.

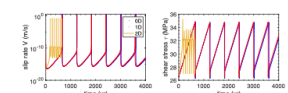


Figure 4 The comparison of 0D, 1D and 2D seismic cycles with radiation damping. Only the results at depth of 13 km in 2D is plotted.

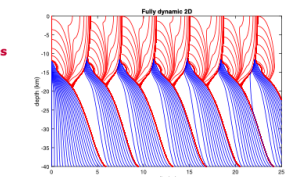


Figure 5 The comparison of 3D seismic cycles in fully (top) and quasi-dynamic (bottom) simulations.

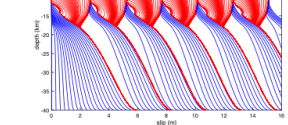


Figure 5 The comparison of 3D seismic cycles in fully (top) and quasi-dynamic (bottom) simulations.

### Summary

- In 0D, only quasi-dynamic model exists since seismic waves cannot be modeled without medium.
- In 0/1D, rate-weakening/strengthening transition is not possible such that nucleation phase is not observable.
- Even given the same material and frictional parameters, the seismic cycle period in 2D models is still much smaller than in 0/1D. However, some important observables, such as the maximum/minimum stress and slip rates, are calculated accurately in lower dimensional models, which are much faster than higher dimensional models.
- Fully dynamic models show larger maximum slip velocity and total slip comparing to quasi-dynamic models.
- Fully dynamic models show sharper wave front and surface reflection phase, as well as larger rupture speed. This makes the coseismic duration in fully dynamic models much shorter.

### References

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