

# Rupture speed of supershear slip instabilities

David S. Kammer – ETH Zurich

Ilya Svetlizky – Harvard University

Jay Fineberg – The Hebrew University of Jerusalem

# Introduction

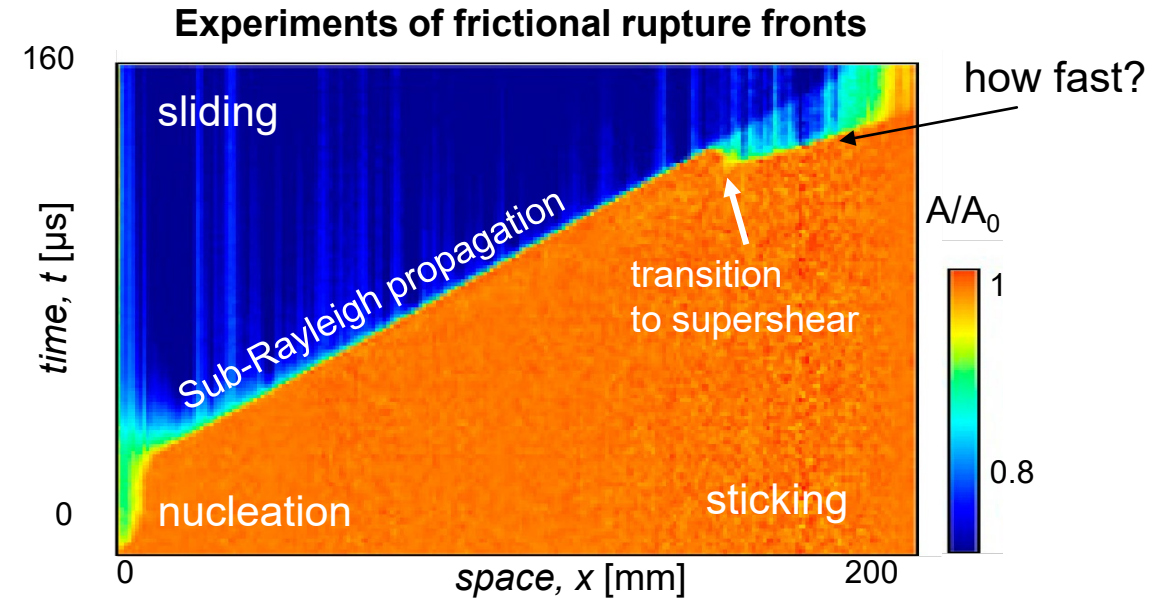
Frictional shear ruptures may propagate faster than the shear wave speed of the adjacent medium.

These supershear ruptures have been

1. predicted from numerical simulations (Andrews, GJR, 1976)
2. observed in experiments (Xia *et al.*, Science, 2004) (Svetlizky *et al.*, PNAS, 2016)
3. observed in nature as supershear earthquakes (Bouchon & Vallée, Science, 2003)

While the transition mechanism to supershear propagation is well understood (see figures on the right), the question of **what determines the evolution of supershear rupture speed** remains open.

Here, we show that supershear rupture speed may be described by a theoretical fracture-mechanics model.



(Svetlizky *et al.*, PNAS, 2016)

# Method

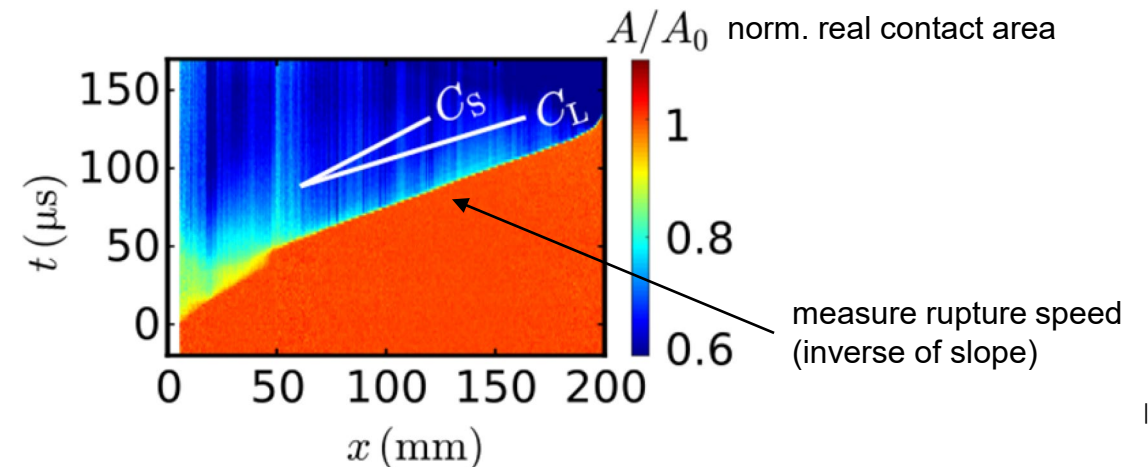
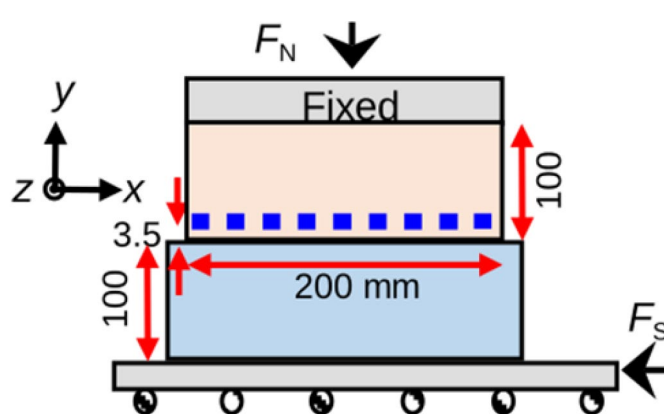
We use numerical simulations and experiments to study the propagation speed of supershear ruptures.

## Simulations:

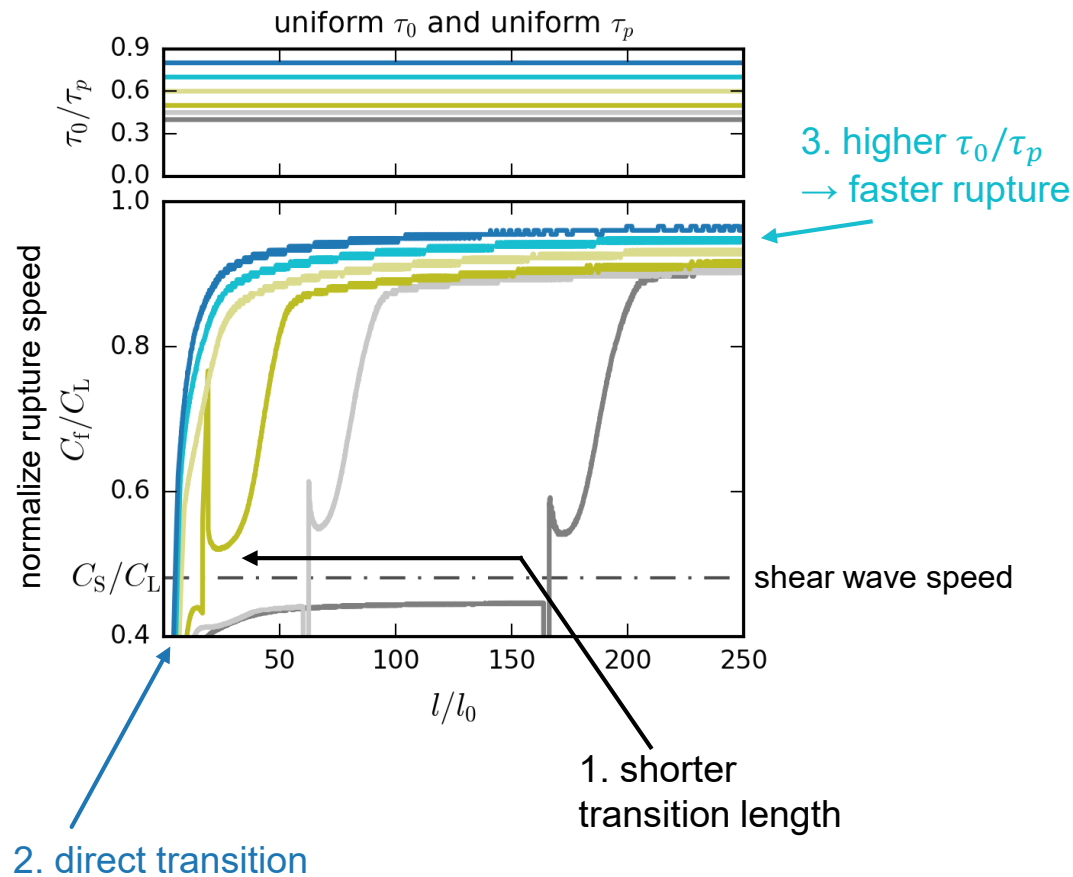
We apply the spectral boundary integral method (Geubelle & Rice, JMPS, 1995) to solve the elasto-dynamic equations for two half-spaces, which are coupled by a cohesive-type interface law describing the evolution of frictional strength.

## Experiments:

The experimental setup consists of two PMMA blocks that are brought into contact and subject to normal and quasi-static shear load. A high-speed camera records the dynamic changes of the real contact area  $A(x, t)$ . Further, we determine the local stresses along the interface from strain-gauge measurements (blue squares).



# Results



First, we study a uniform system. Both the applied stress  $\tau_0$  and interface strength  $\tau_p$  are uniform.

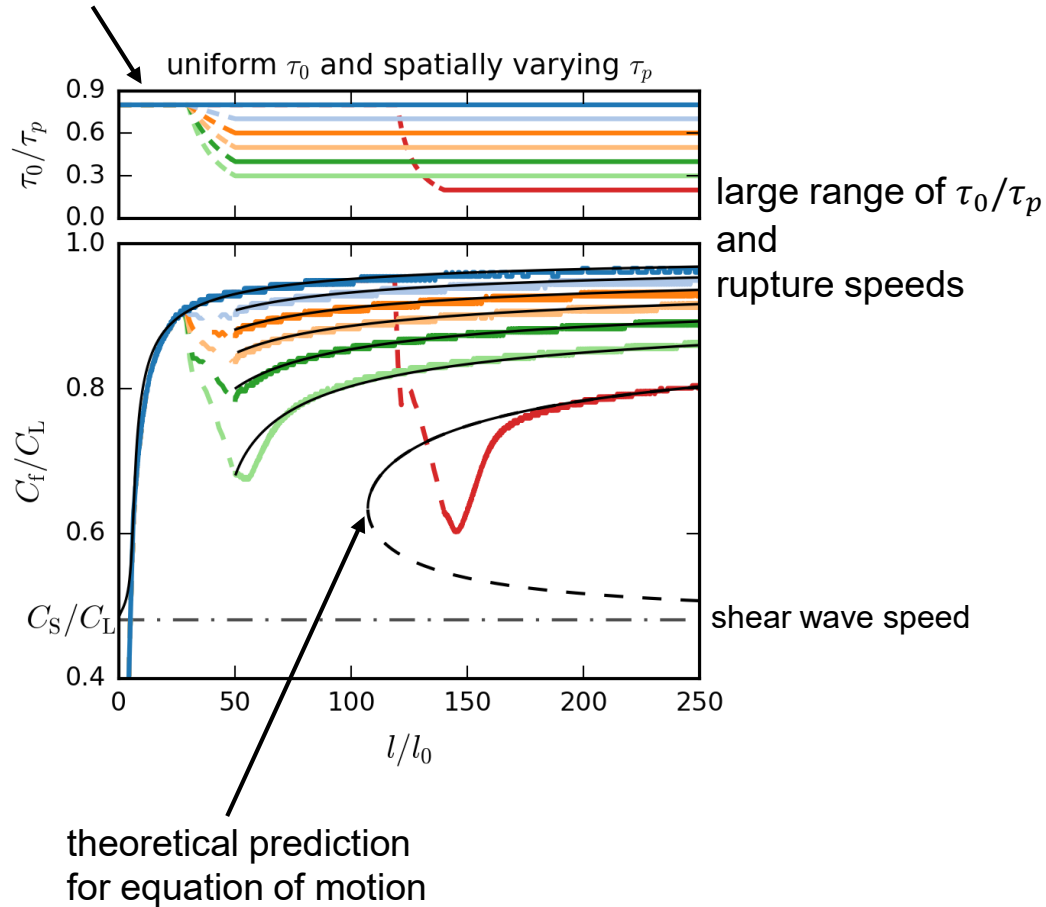
**Simulations** show that

1. Higher  $\tau_0/\tau_p$  result in shorter supershear transition length
2. Very high  $\tau_0/\tau_p$  lead to direct supershear rupture (no sub-Rayleigh propagation)
3. Generally, higher  $\tau_0/\tau_p$  causes faster supershear speed

However, the range of observed supershear speeds is limited because the transition length becomes very long for low  $\tau_0/\tau_p$ .

# Results

high  $\tau_0/\tau_p$  for direct transition



To increase the range of supershear rupture speeds, we apply a spatially varying interface strength  $\tau_p$ . First, a high  $\tau_0/\tau_p$  causes direct transition, and then at larger rupture length  $l$ , lower  $\tau_0/\tau_p$  enable large range of rupture speeds.

Results from numerical simulations are shown in **color**.

We develop an approximate **theoretical model** for supershear rupture speed by combining a self-similar solution with the cohesive crack solution:

$$\Gamma = G = \frac{\tau_p^2 l}{\mu} \left( \frac{\tau_0}{\tau_p} \right)^{1/g} \tilde{B}(C_f/C_L) \tilde{\Gamma}_D(g)$$

fracture energy

energy release rate

rupture speed

known functions

The theoretical model predicts well the simulation results.

# Results

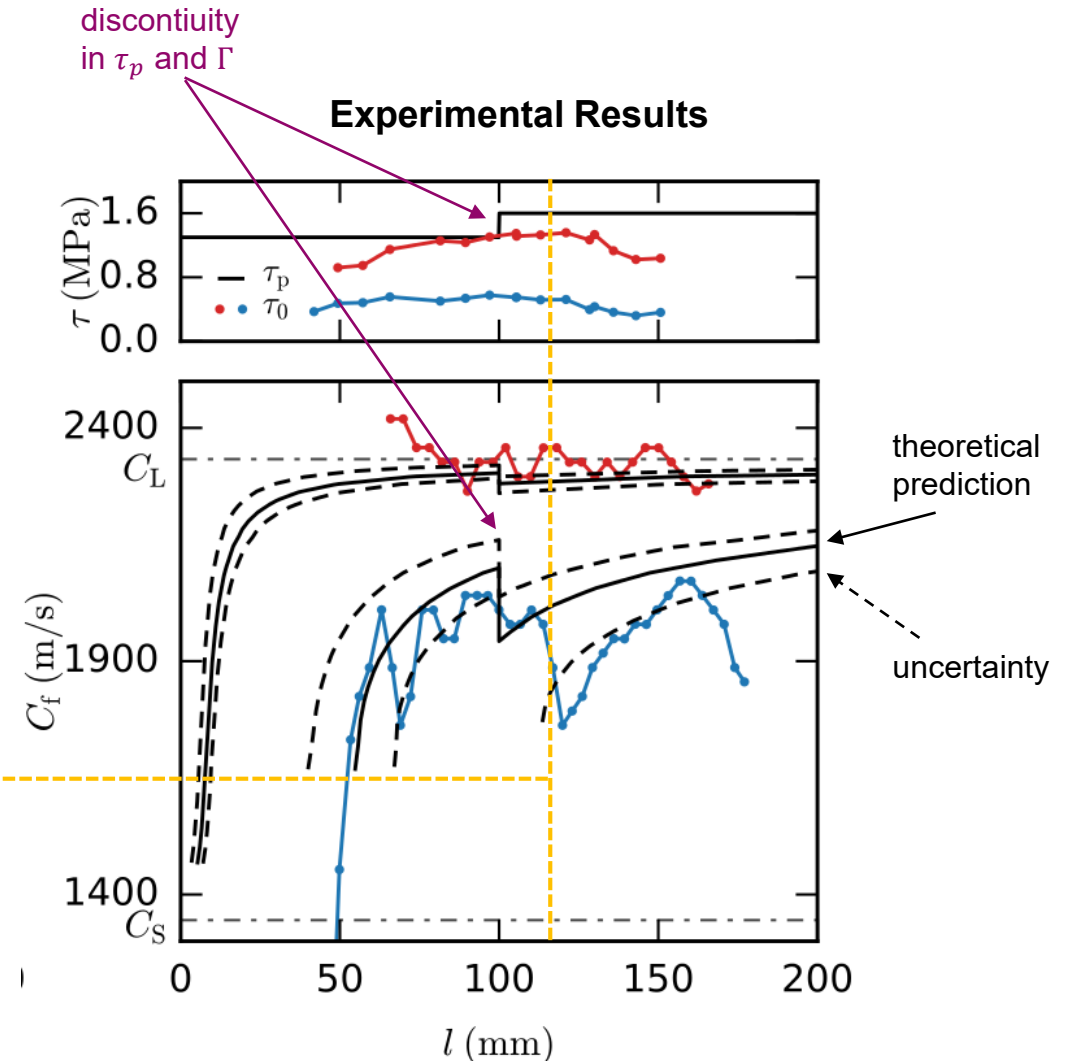
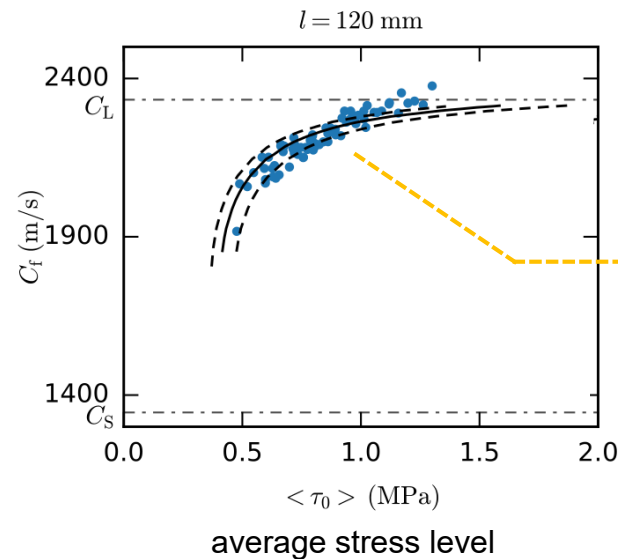
Here, we compare the theoretical model with experimental observations.

Right side: two representative examples are shown.  
Below: a summary of multiple experiments is provided.

Model and experiments agree well.

Note:  
Plane-stress assumption in theoretical model leads to apparent rupture speeds faster than the longitudinal wave speed. Effective wave speed in 3D is higher.

For more details:  
Svetlizky *et al.*, JMPS, 2020.



# Discussion

The theoretical model shows that there is a minimal length for supershear propagation (marked by **orange arrow**). In other words, a rupture shorter than this minimal length cannot propagate at supershear speed.

We tested this by varying the non-uniform  $\tau_p$  profile.

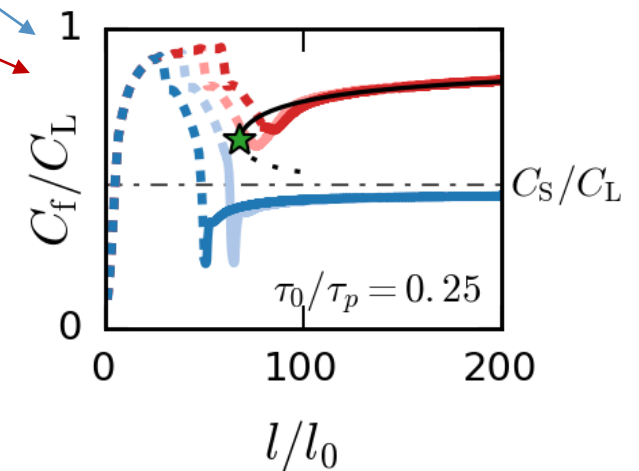
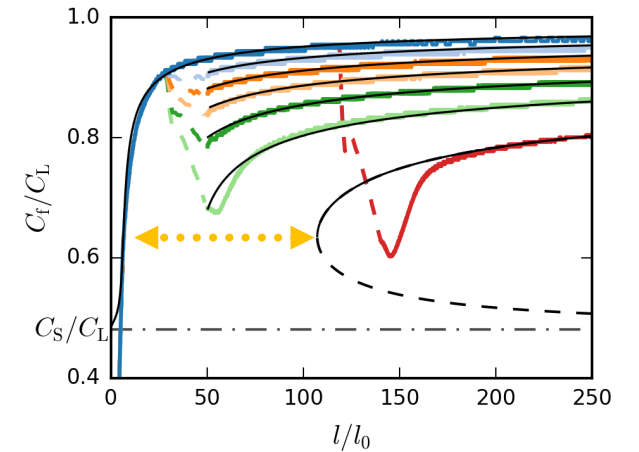
If the region of high  $\tau_0/\tau_p$  stops before this minimal length, the rupture transitions back to the sub-Rayleigh regime.

However, if the high  $\tau_0/\tau_p$  region goes beyond this point, supershear propagation is maintained.

The minimal length is approximated by  
For large  $\tau_0$  it does not exist though.

$$l_m \approx \frac{\mu\Gamma}{\tau_0^2} \cdot \frac{1}{\tilde{B}(\sqrt{2}C_S/C_L)}$$

Finally, note that this minimal length determines if a supershear solution exists and should not be confused with the transition length (Andrews, JGR, 1976).



# Conclusion

## Summary of observations:

1. Higher relative pre-stress level leads to faster supershear ruptures (observed in simulations and experiments)
2. A theoretical model predicts quantitatively well the observed supershear rupture speed.
3. At low relative pre-stress level the theoretical model predicts a minimal length required for supershear propagation

## Conclusions:

1. Supershear earthquakes are typically associated with highly stressed faults. Our results show that supershear earthquakes may exist also at lower stress levels, if transition is promoted by local heterogeneities.
2. The proposed theoretical model provides exact conditions for potential of supershear rupture at any given stress level.



# References

- Andrews, D. J. (1976). Rupture velocity of plane strain shear cracks. *Journal of Geophysical Research*, 81(32), 5679-5687. <https://doi.org/10.1029/JB081i032p05679>
- Xia, K., Rosakis, A. J., & Kanamori, H. (2004). Laboratory earthquakes: The sub-Rayleigh-to-supershear rupture transition. *Science*, 303(5665), 1859-1861. [10.1126/science.1094022](https://doi.org/10.1126/science.1094022)
- Svetlizky, I., Muñoz, D. P., Radiguet, M., Kammer, D. S., Molinari, J. F., & Fineberg, J. (2016). Properties of the shear stress peak radiated ahead of rapidly accelerating rupture fronts that mediate frictional slip. *Proceedings of the National Academy of Sciences*, 113(3), 542-547. <https://doi.org/10.1073/pnas.1517545113>
- Kammer, D. S., Svetlizky, I., Cohen, G., & Fineberg, J. (2018). The equation of motion for supershear frictional rupture fronts. *Science advances*, 4(7), eaat5622. DOI: [10.1126/sciadv.aat5622](https://doi.org/10.1126/sciadv.aat5622)
- Bouchon, M., & Vallée, M. (2003). Observation of long supershear rupture during the magnitude 8.1 Kunlunshan earthquake. *Science*, 301(5634), 824-826. [10.1126/science.1086832](https://doi.org/10.1126/science.1086832)
- Svetlizky, I., Albertini, G., Cohen, G., Kammer, D. S., & Fineberg, J. (2020). Dynamic fields at the tip of sub-Rayleigh and supershear frictional rupture fronts. *Journal of the Mechanics and Physics of Solids*, 137, 103826. <https://doi.org/10.1016/j.jmps.2019.103826>
- Geubelle, P. H., & Rice, J. R. (1995). A spectral method for three-dimensional elastodynamic fracture problems. *Journal of the Mechanics and Physics of Solids*, 43(11), 1791-1824. [https://doi.org/10.1016/0022-5096\(95\)00043-I](https://doi.org/10.1016/0022-5096(95)00043-I)