Upscaling sediment-flux-dependent fluvial bedrock incision to long timescales

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Overview

  - Turowski, J. M. (2021). Upscaling sediment-flux-dependent fluvial bedrock incision to long timescales. *Journal of Geophysical Research: Earth Surface*, *126*, e2020JF005880. https://doi.org/10.1029/2020JF005880

Abstract

Fluvial bedrock incision is driven by the impact of moving bedload particles. Mechanistic, sediment-flux-dependent incision models have been proposed, but the stream power incision model (SPIM) is frequently used to model landscape evolution over large spatial and temporal scales. This disconnect between the mechanistic understanding of fluvial bedrock incision on the process scale, and the way it is modelled on long time scales presents one of the current challenges in quantitative geomorphology. Here, a mechanistic model of fluvial bedrock incision that is rooted in current process understanding is explicitly upscaled to long time scales by integrating over the distribution of discharge. The model predicts a channel long profile form equivalent to the one yielded by the SPIM, but explicitly resolves the effects of channel width, cross-sectional shape, bedrock erodibility and discharge variability. The channel long profile chiefly depends on the mechanics of bedload transport, rather than bedrock incision. In addition to the imposed boundary conditions specifying the upstream supply of water and sediment, and the incision rate, the model includes four free parameters, describing the at-a-station hydraulic geometry of channel width, the dependence of bedload transport capacity on channel width, the threshold discharge of bedload motion, and reach-scale cover dynamics. For certain parameter combinations, no solutions exist. However, by adjusting the free parameters, one or several solutions can usually be found. The controls on and the feedbacks between the free parameters have so far been little studied, but may exert important controls on bedrock channel morphology and dynamics.

Background & Approach

- Disconnect between fluvial erosion at the process scale and development of bedrock channel morphology over long time scales
- Address this by explicit upscaling (Lague et al., 2005), by integrating over the distribution of water discharge.

- Introduce an intermediate timescale, where fluctuations in bedload transport rate (Qs) and bed cover (C) average out, and the average shows a well-defined relationship with discharge (Q).
- Explicitly include channel width (W) and erodibility (k).

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General equations

- Inverse gamma distribution to describe distribution of normalized discharge *Q**

$$\text{pdf}(Q^*) = \frac{k^{k+1}}{\Gamma(k+1)} exp\left\{-\frac{k}{Q^*}\right\} Q^{*-(2+k)}$$

- The variability parameter *k* scales inversely with discharge variability
- The long-term mean of a generic parameter X is given by the integral

$$\bar{X} = \int_{Q_{min}}^{Q_{max}} X(Q^*)\text{pdf}(Q^*)dQ^*$$

- Overbars denote long-term means

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Channel width

- Steady state width corresponding to the mean discharge given by (Turowski 2018)

$$\bar{W} = \left(k_e d \frac{\overline{Q_s}}{\bar{I}}\right)^{\frac{1}{2}}$$

- Q_s: sediment supply, k_e: erodibility, I: incision rate, d: scaling length scale
- At-a-station hydraulic geometry (ω_a is a dimensionless exponent)

$$W = \bar{W} {Q^*}^{\omega_a}$$

Bedload sediment supply

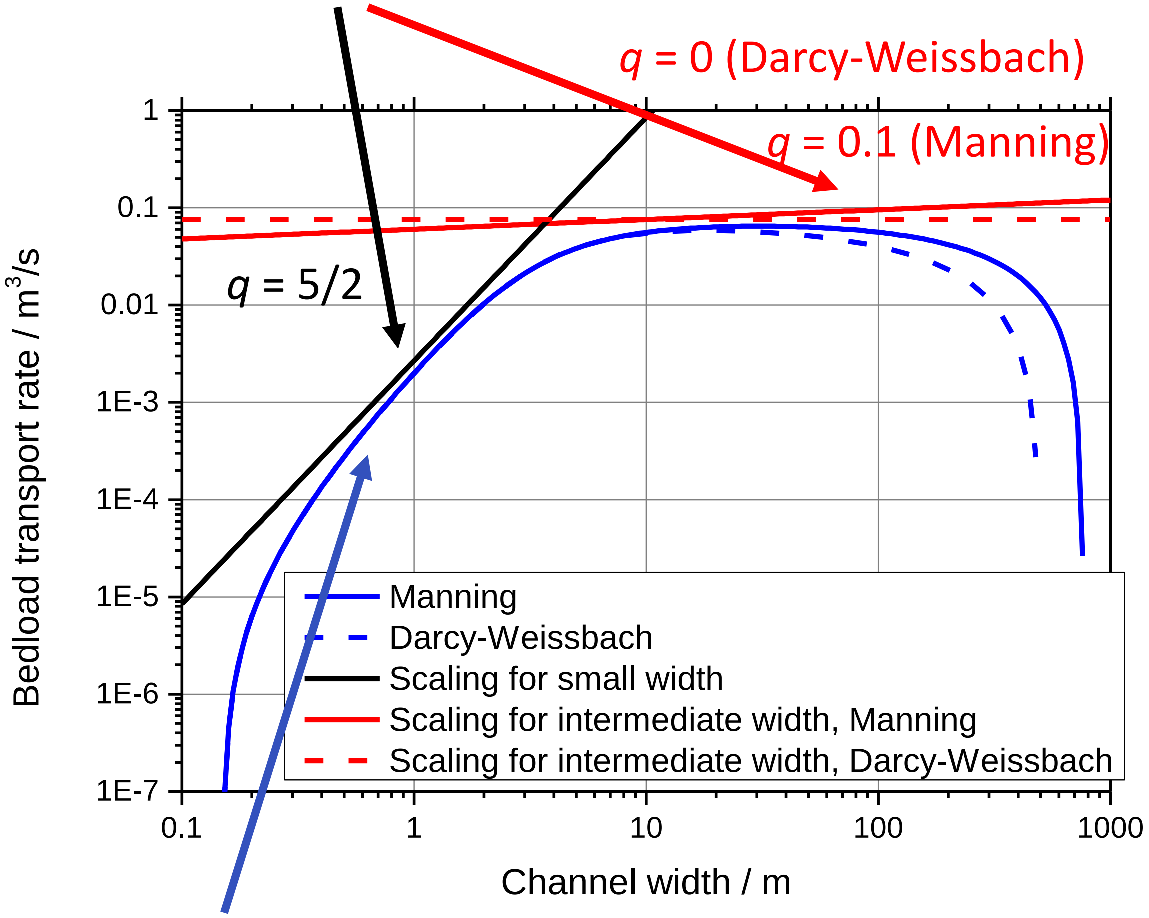
- Discharge-based transport equation

$$Q_s = k_{BL}(Q^m - {Q_{ct}}^m)W^q S^n$$

- Q_{ct}: threshold of motion, S: channel bed slope, m, n, q: exponents
- The exponent q quantifies dependence on width (Figure)
- Upscaled version
  - F_{QS} function of k, q, ω_a, m

$$\overline{Q_s} = F_{Qs} k_{BL} \bar{W}^q \bar{Q}^m S^n$$

Local power law approximation with exponent q

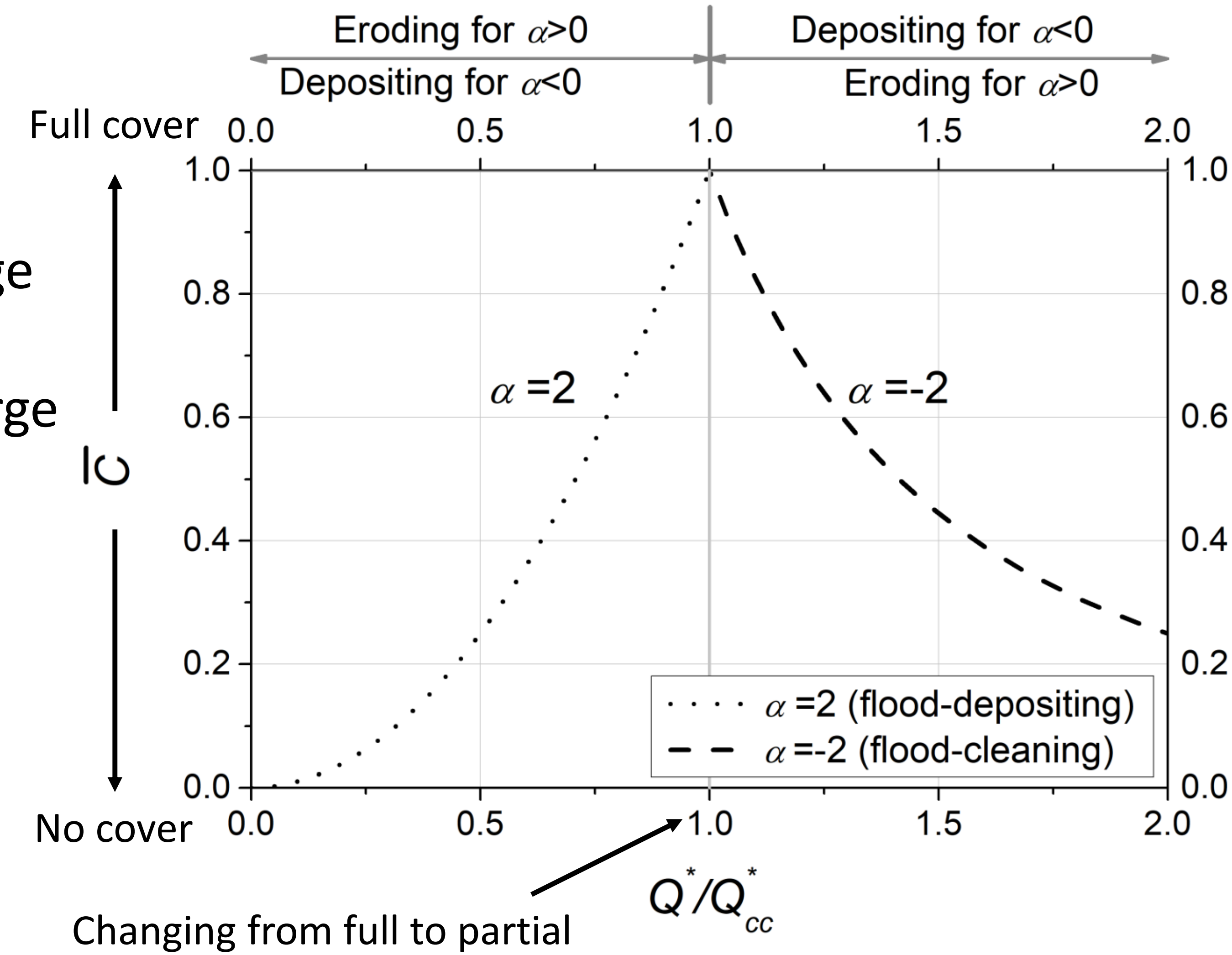


Full curve (blue) based on Meyer-Peter & Müller 1948 transport law

Bed cover

- Cover C can be (Figure)
  - Positively related to discharge (flood-depositing)
  - Negatively related to discharge (flood-cleaning)
- Quantified by exponent α
- Cover threshold Q^*_{cc}: cover changes from full to partial
- Long-term cover

$$\bar{C} = F_C\left(k, Q^*{}_{ct}, Q^*{}_{cc}, m, \alpha\right)$$



Changing from full to partial cover at Q^*/Q^*_{cc}= 1

See also Turowski et al., 2013

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Incision rate

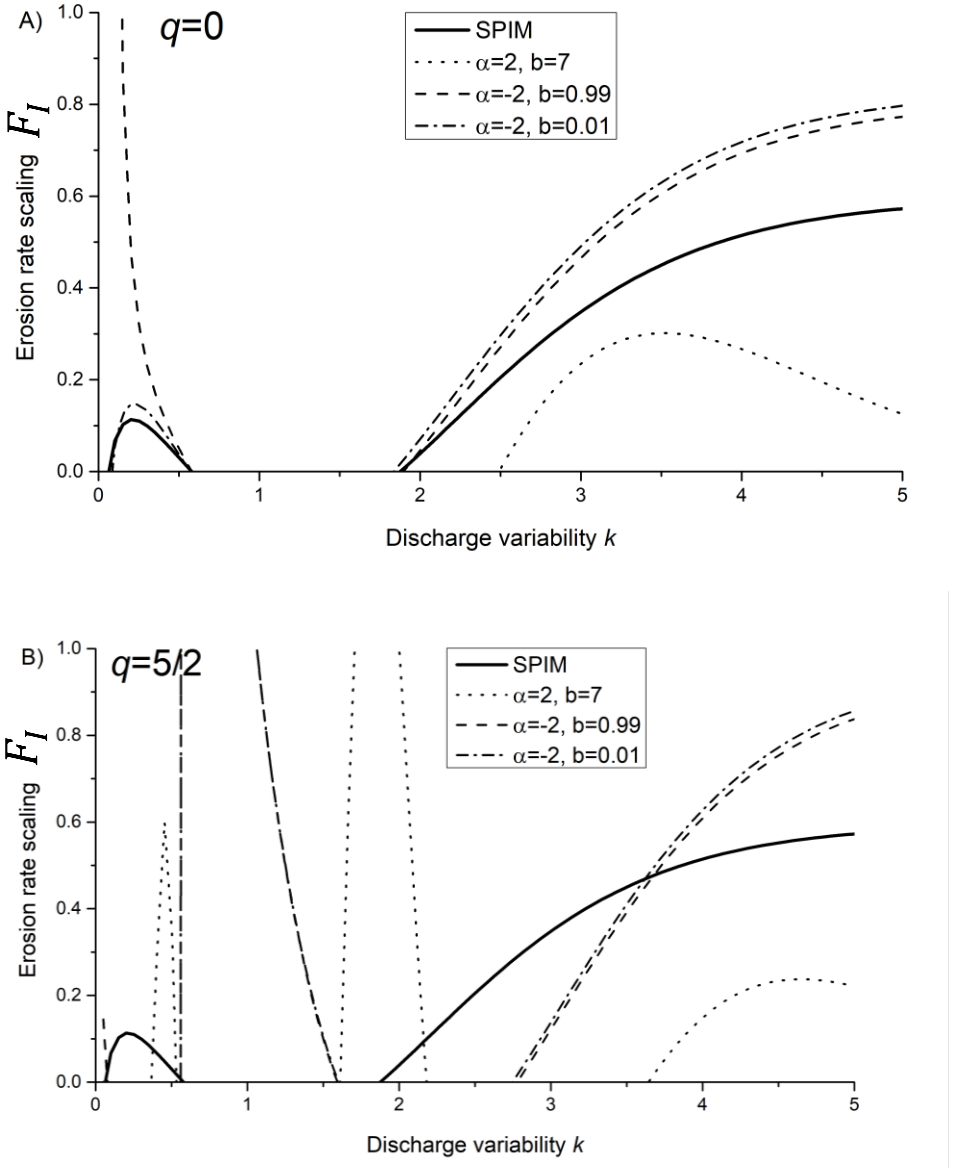
- Sediment-flux dependent incision model
  - Including tools and cover effects

$$I = k_e \frac{Q_s}{W}(1 - C)$$

- Long-term mean incision rate given by

$$\bar{I} = k_e \frac{\overline{Q_s}}{\overline{W}} F_I(k, Q^*{}_{ct}, Q^*{}_{cc}, m, q, \omega_a, \alpha)$$

- Figure: incision rate vs. discharge variability k, in comparison with the stream power incision model (SPIM)



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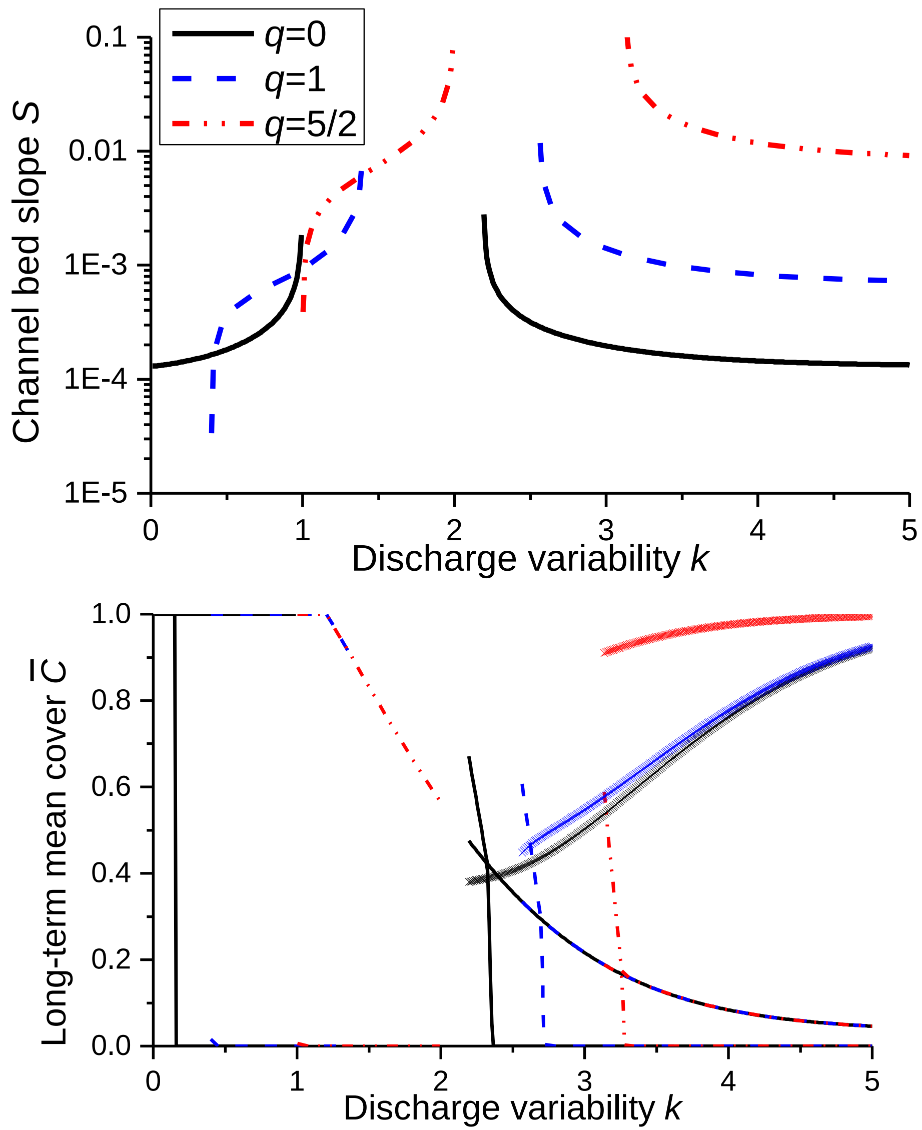
Steady state channel morphology

- Figure: solutions for steady state channel bed slope *S* and long-term cover *C* as function of discharge variability
  - Multiple cases
- Steepness index given by

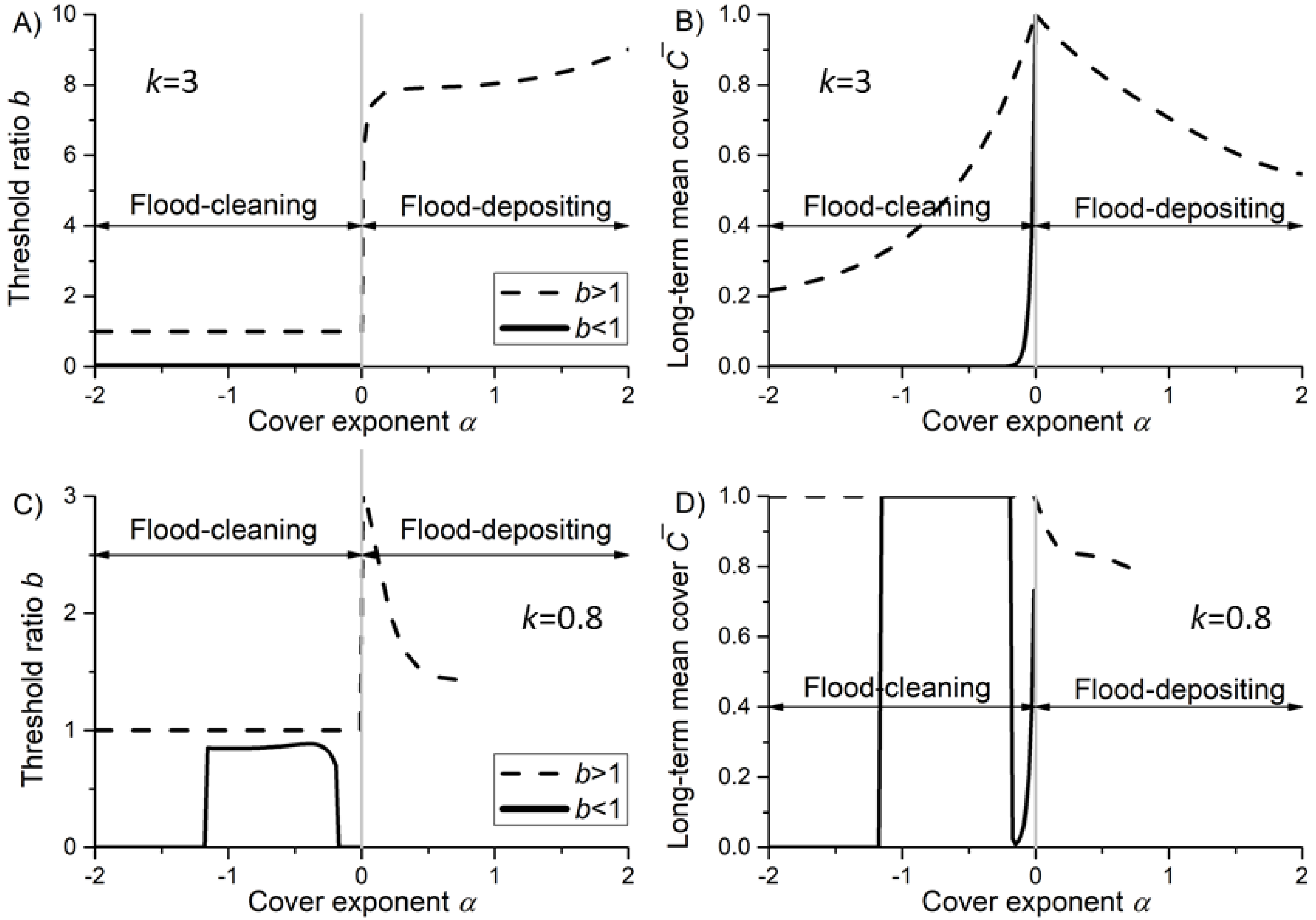
$$k_s = F_{Qs}^{-\frac{1}{n}} \left(\frac{\bar{I}}{k_{BL}}\right)^{\frac{1}{n}} (k_e d)^{-\frac{q}{2n}} k_\beta^{\frac{2-q}{2n}} \bar{R}^{-\frac{m}{n}}$$

- *R*: runoff rate



Thresholds and cover

- Figure: Cover exponent controls
  - Threshold ratio: cover / bedload thresholds
  - b = [Q_{cc}](#) / [Q_{ct}](#)
  - *Note different y-axis scaling!*
- [Cover exponent α](#)



Points to take home

- The model yields solutions similar to those obtained in the stream power paradigm (for q = 0), in addition to other possible solutions.
- The model explicitly resolves forcing behavior and highlights potential dynamic feedbacks that have so far not been considered.
  - Parameters controlling at-a-station hydraulic geometry of channel width, bedload-suspended load partitioning, and bed cover are largely unconstrained in the field.
- For the common case of $q = 0$, steepness is independent of erodibility.
- Look at the full paper in JGR-Earth Surface!

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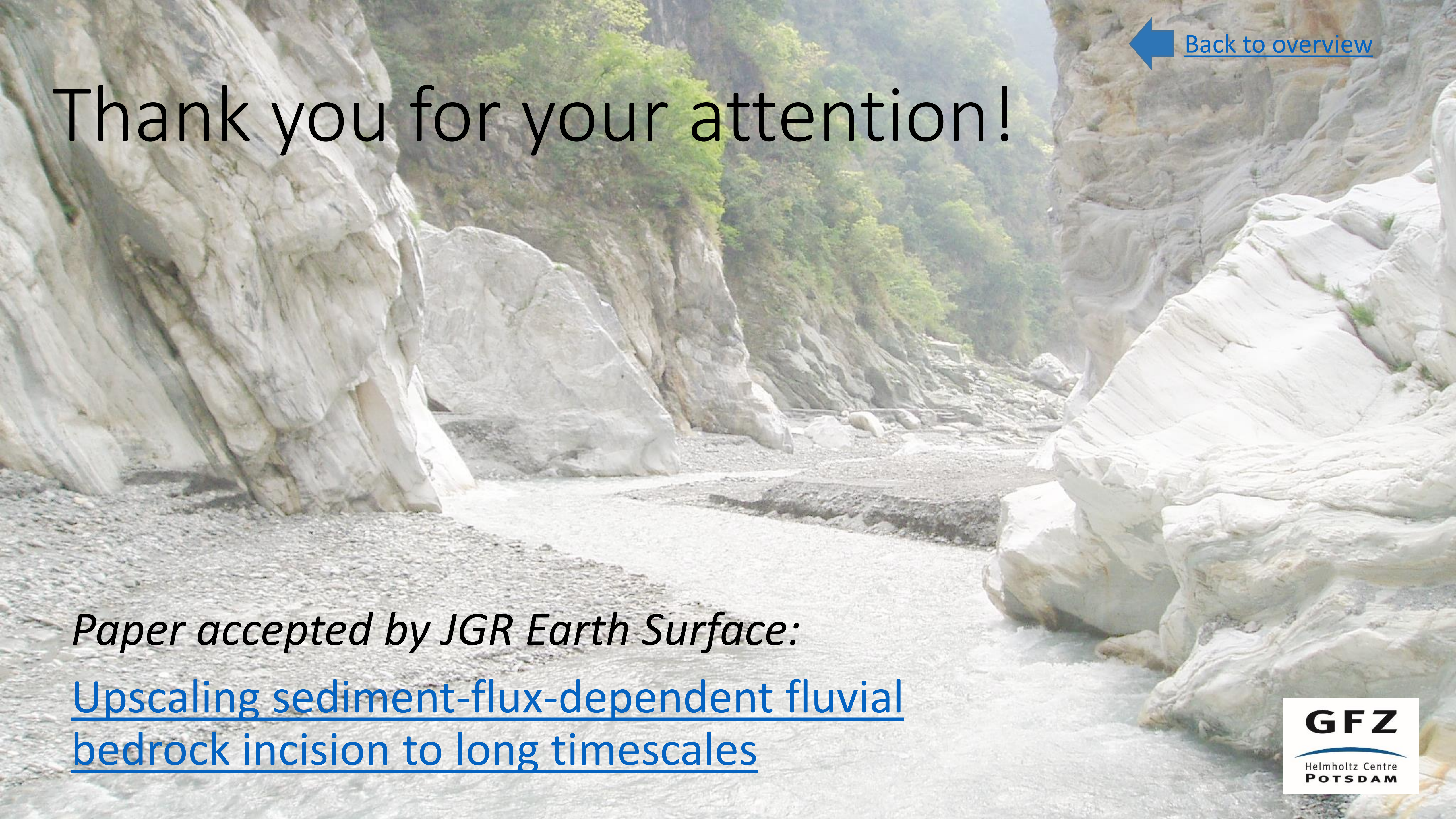
Thank you for your attention!



Paper accepted by JGR Earth Surface:

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General Width Bedload Cover Incision Steady state Thresholds

- A Drainage area [m^2].
- A_c Cross-sectional area of the flow [m^2].
- a Scaling exponent, C-Q^*.
- B Scaling exponent, β-A.
- b Coefficient of proportionality, Q^*_{ct} and Q^*_{cc}.
- C Fraction of covered bed.
- $\langle C \rangle$ Average cover at a given discharge.
- $\overline{C}$ Long-term mean cover.
- D Representative grain size [m].
- d Sideward deflection length scale, reach [m].
- g Acceleration due to gravity [m/s^2].
- H Water depth [m].
- I Instantaneous incision rate [m/s].
- $\overline{I}$ Long-term mean incision rate [m/s].
- k Discharge variability parameter
- k_{bl} Bedload transport efficiency [kg m^{-3m-q}s^{m-1}].
- k_e Bedrock erodibility [m^2/s].
- k_{SPIM} Erodibility in stream power model [m$^{1-3m'}$s$^{1-m'}$].
- k_s Steepness index [m$^{2\vartheta}$].
- K_V Flow velocity coefficient [m$^{1-\delta}$/s].
- k_W Prefactor, downstream hydraulic geometry for width.
- $k\beta$ Prefactor in the relationship of and drainage area [m^{-2B}].
- k_v Rock erodibility coefficient.
- m Discharge exponent in bedload equation.
- n Slope exponent in bedload equation.
- Q Water discharge [m^3/s].
- Q_{max} Maximum water discharge at which erosion occurs [m^3/s].
- Q_{min} Minimum water discharge at which erosion occurs [m^3/s].
- $\overline{Q}$ Long-term mean water discharge [m^3/s].
- Q^* Dimensionless water discharge, normalized by the long term mean discharge
- Q^*_{ce} Critical discharge for the onset of erosion in the SPIM [m^3/s].
- Q^*_{ct} Critical discharge for the onset of bedload motion [m^3/s].
- Q^*_{cc} Critical discharge for the change between a fully and partially covered bed [m^3/s].
- Q_s Upstream sediment mass supply [kg/s].
- $\overline{Q}_s$ Long-term mean bedload supply [kg/s].
- $\langle Q_s \rangle$ Average bedload supply at a given discharge [kg/s].
- Q_t Mass sediment transport capacity [kg/s].
- q Width dependence of transport rate, scaling exponent, Q_s-W.
- R Runoff [m/s].
- S Channel bed slope.
- V Water flow velocity [m/s].
- W Instantaneous channel width [m].
- $\langle W \rangle$ Channel width at the mean discharge [m].
- Y Young's modulus of the bedrock [kg m^{-1}s^{-2}].
- α Scaling exponent, C-Q^*.
- β Fraction of sediment transported as bedload.
- $\bar{\beta}$ Long-term mean of the fraction of sediment transported as bedload.
- γ Dimensionless bedload transport coefficient.
- δ Scaling exponent, flow velocity V-R_h.
- ρ Density of water [kg/m^3].
- ρ_s Density of sediment [kg/m^3].
- σ_T Rock tensile strength [kg m^{-1}s^{-2}].
- ϑ Concavity index, scaling exponent, S-A.
- τ Bed shear stress [N/m^2].
- τ^* Shields stress.
- $\tau_c{}^*$ Critical Shields stress at the onset of bedload motion.
- ω_d Downstream hydraulic geometry exponent for width, scaling exponent $\langle W \rangle$-$\overline{Q}$
- ω_a At-a-station hydraulic geometry exponent for width, scaling exponent W-Q^*