

Buoyancy-driven flow beneath mid-ocean ridges: the role of chemical heterogeneity

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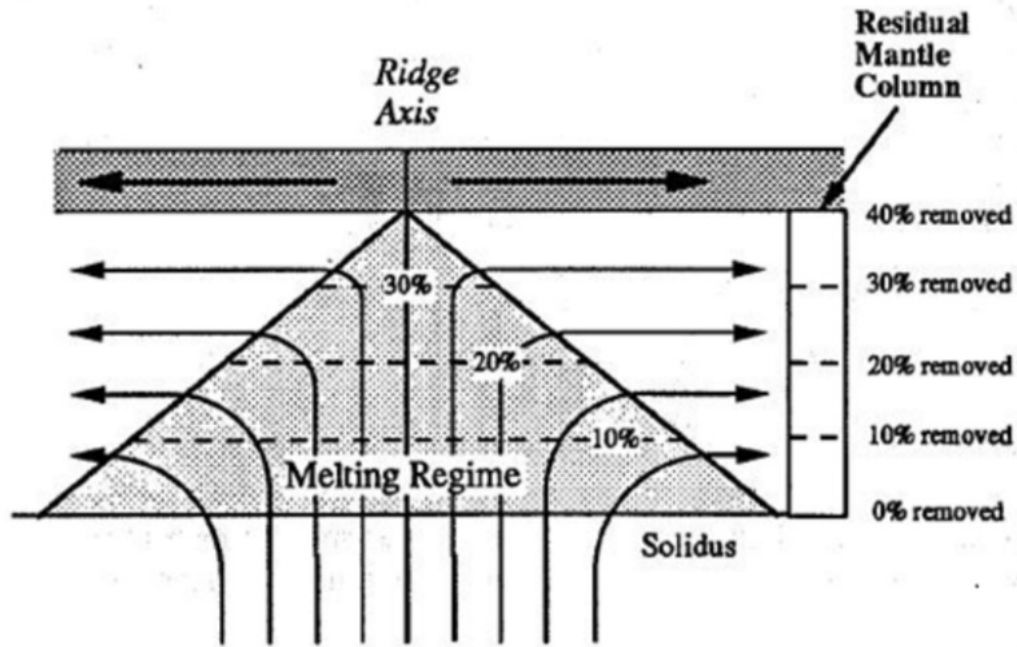
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Motivation

In the **classical model**, mid-ocean ridges (MOR) sit above an asthenospheric corner flow that is **symmetrical** about a vertical plane aligned with the ridge axis.

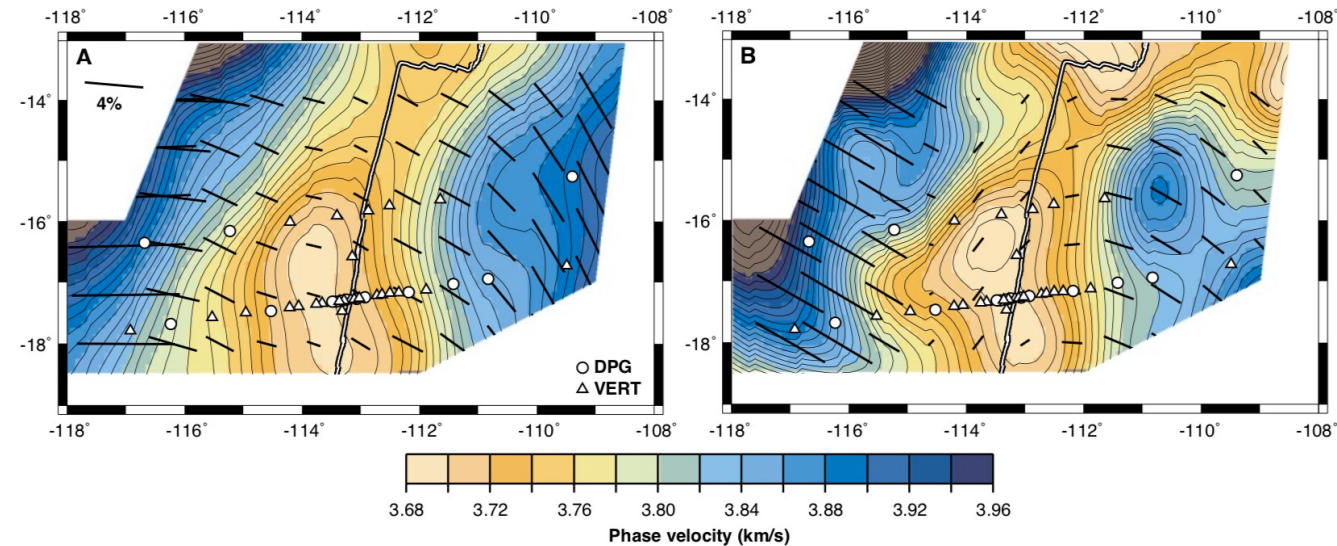
CORNER-FLOW



Langmuir et al. (1992)

However, geophysical observations of MORs indicate **strong asymmetry** in melt production and upwelling across the axis (e.g., Melt Seismic Team, 1998, Forsyth et al., 1998, Toomey et al., 1998, Rychert et al., 2020).

OBSERVATIONS

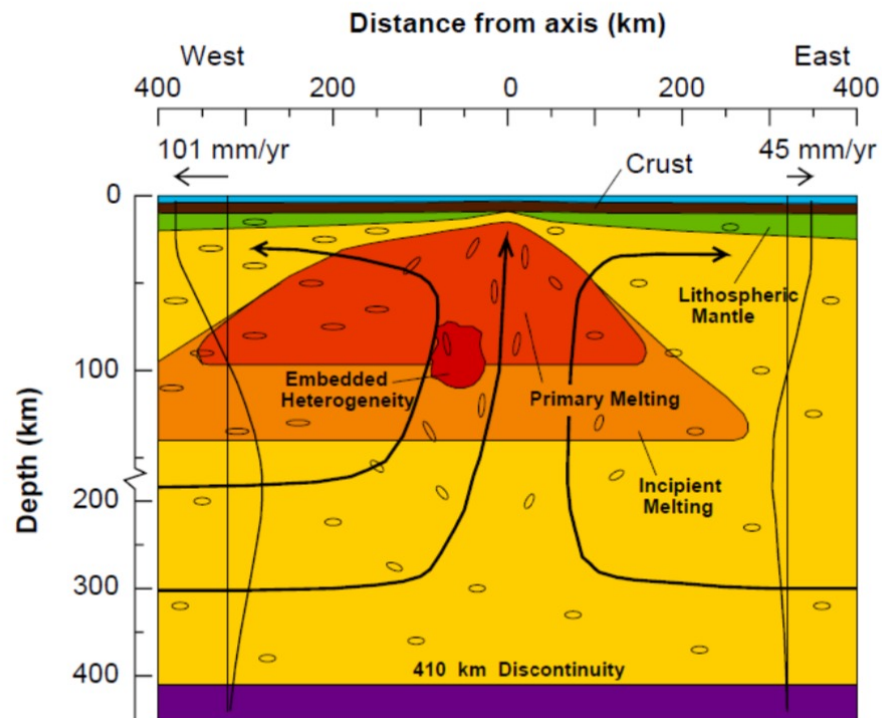


Forsyth et al. (1998)

Motivation

Is upwelling beneath MORs entirely plate-driven (**passive**) OR does it also have a component of buoyancy-driven convection (**active**)?

OBSERVATIONS



MELT Seismic Team (1998)

Models of plate-driven (passive) flow require **unrealistically large forcing**:

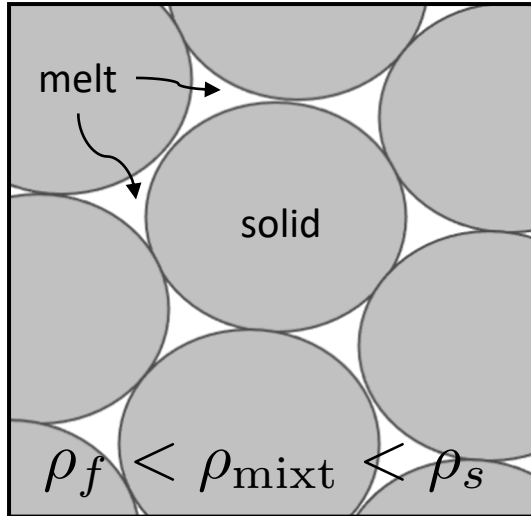
- rapid asthenospheric cross-axis flow (~ 30 cm/yr) at high asthenospheric viscosities ($\sim 10^{21}$ Pa.s),
- temperature anomalies of >100 K beneath the MELT region in the East Pacific Rise (Toomey et al., 2002).

Buoyancy-driven flows are known to produce **symmetry-breaking behaviour** in fluid systems

Sources of buoyancy in the melt-matrix system

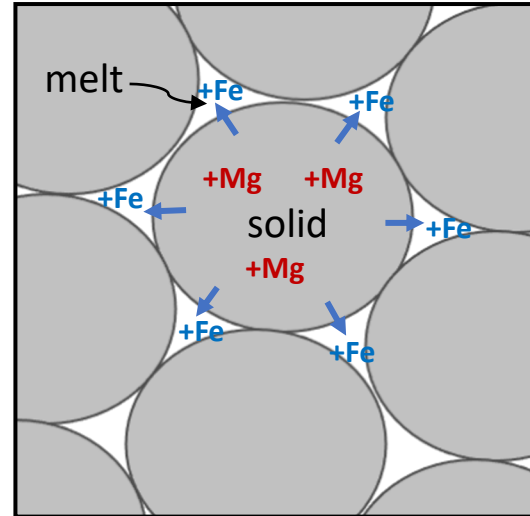
Density variations within the partial-melt zone due to:

Porosity



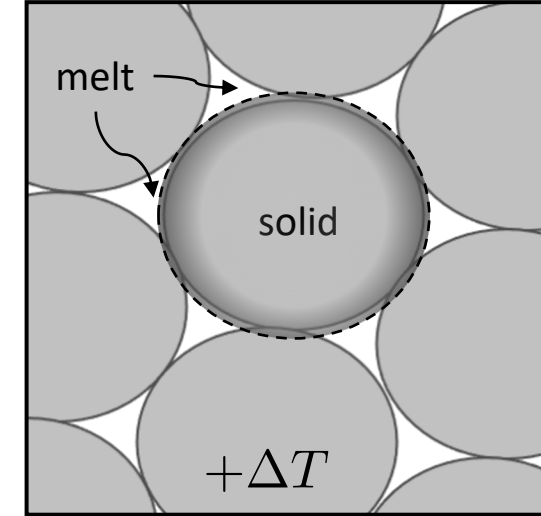
Retention of low-density liquid in the matrix

Composition



Fe/Mg partitioning between melt and residue

Temperature



Thermal expansion

Density equation of state: $\rho_i = \rho_0(1 - \alpha(T - T_0) - \beta(C_i - C_0))$

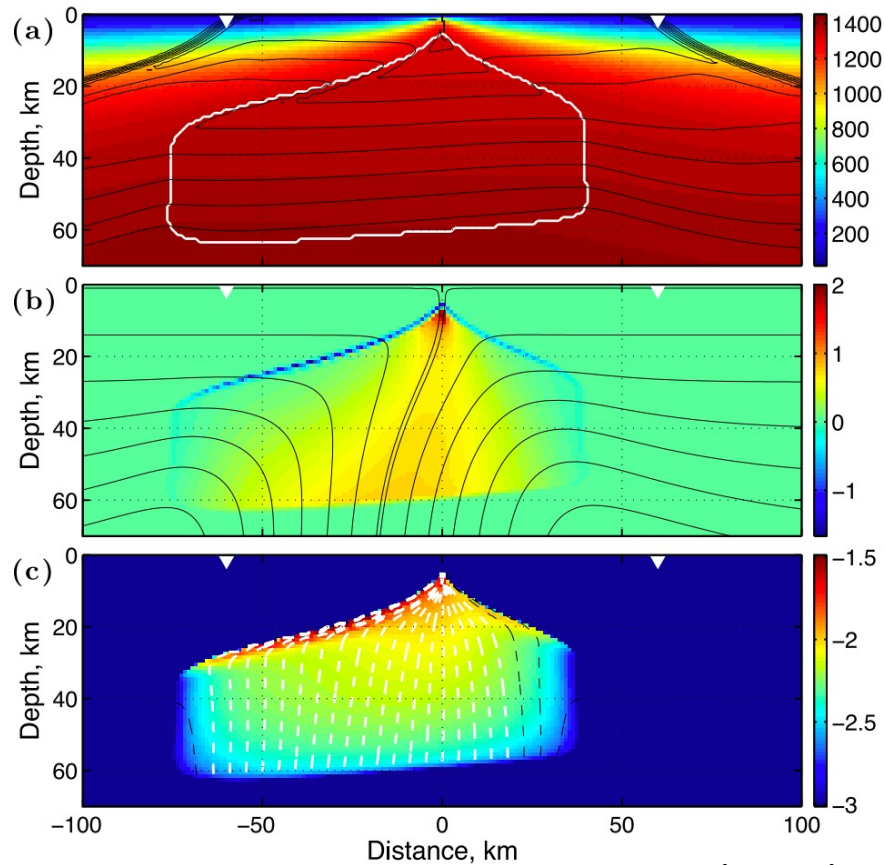
$$\rho_{mixt} = \phi\rho_f + (1 - \phi)\rho_s$$

α - coefficient of thermal expansion,
 β - coefficient of chemical expansion,
 i - phase index (fluid, solid)

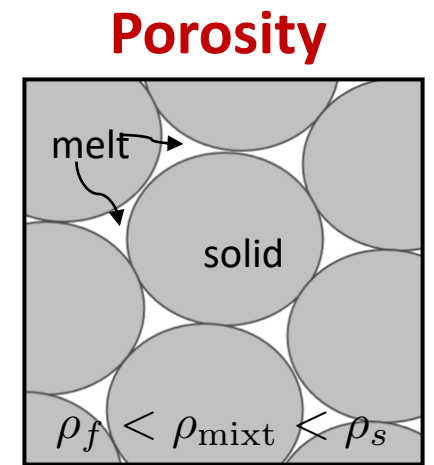
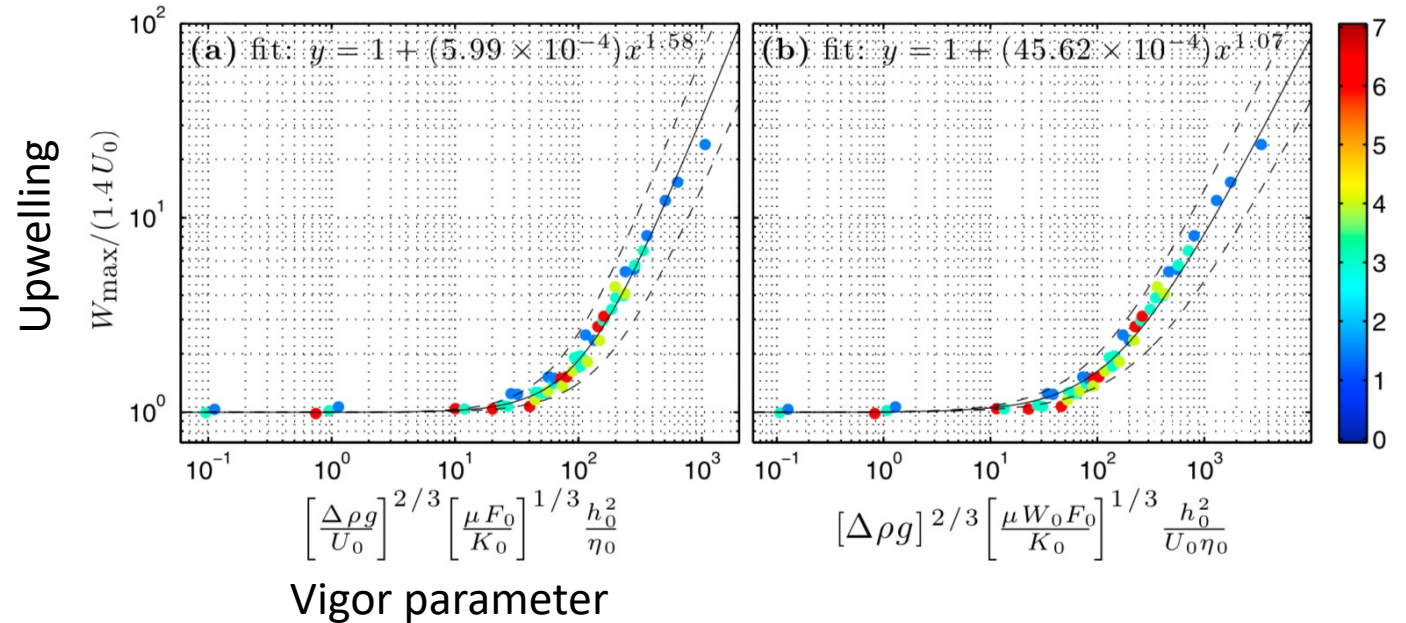
Sources of buoyancy in the melt-matrix system

PREVIOUS WORK

The effect of dynamic retention of low-density liquid in the matrix



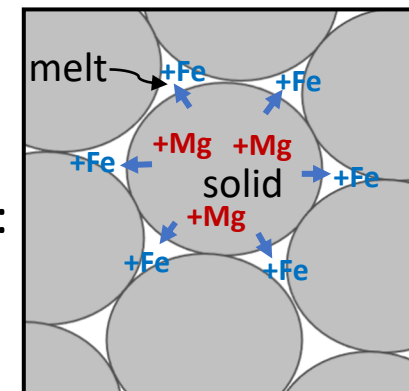
Katz (2010)



Porous buoyancy can induce strong active upwelling and asymmetry at MORs

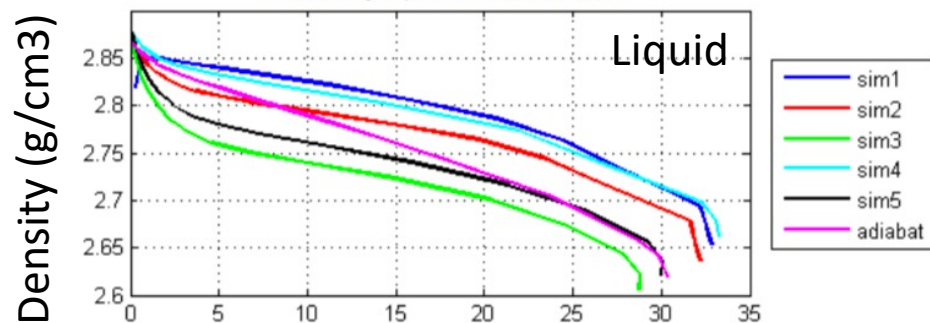
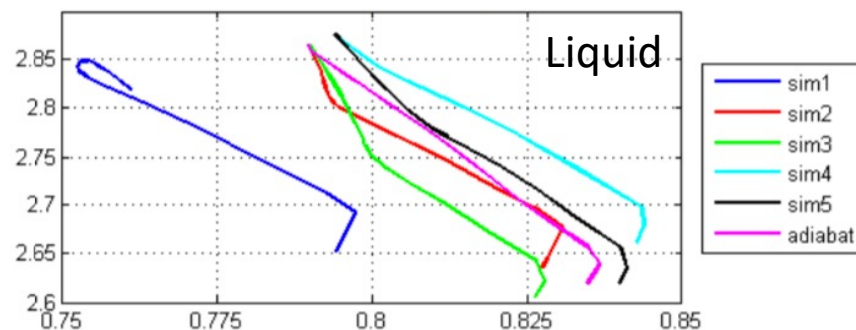
Sources of buoyancy in the melt-matrix system

Composition

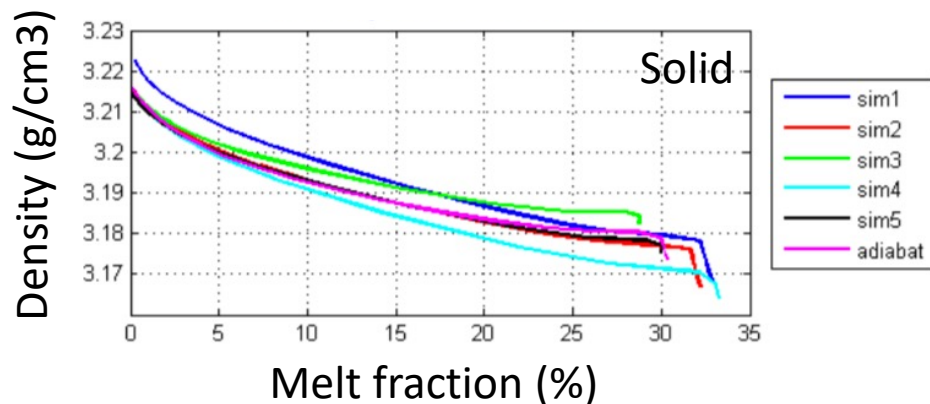
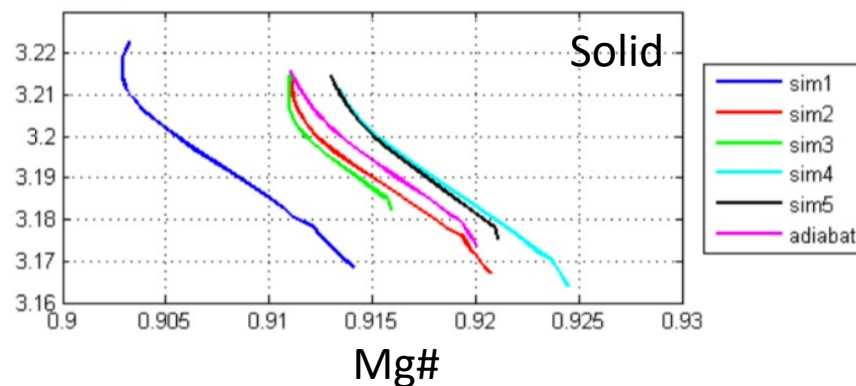


In this study, we also consider the **Fe/Mg partitioning between melt and residue (Mg#)**

Estimate value of beta using pMELTS (Ghiorso et al. 2020) and the density equation of state:



- 2 upper-mantle compositions (Baker and Stolper, 1994),
- different melting paths,
- max degree of melting 30%



Melting simulations
to estimate value of

$$\beta \sim 2$$

Next: Run 2-D simulations of mid-ocean ridges with chemical heterogeneities

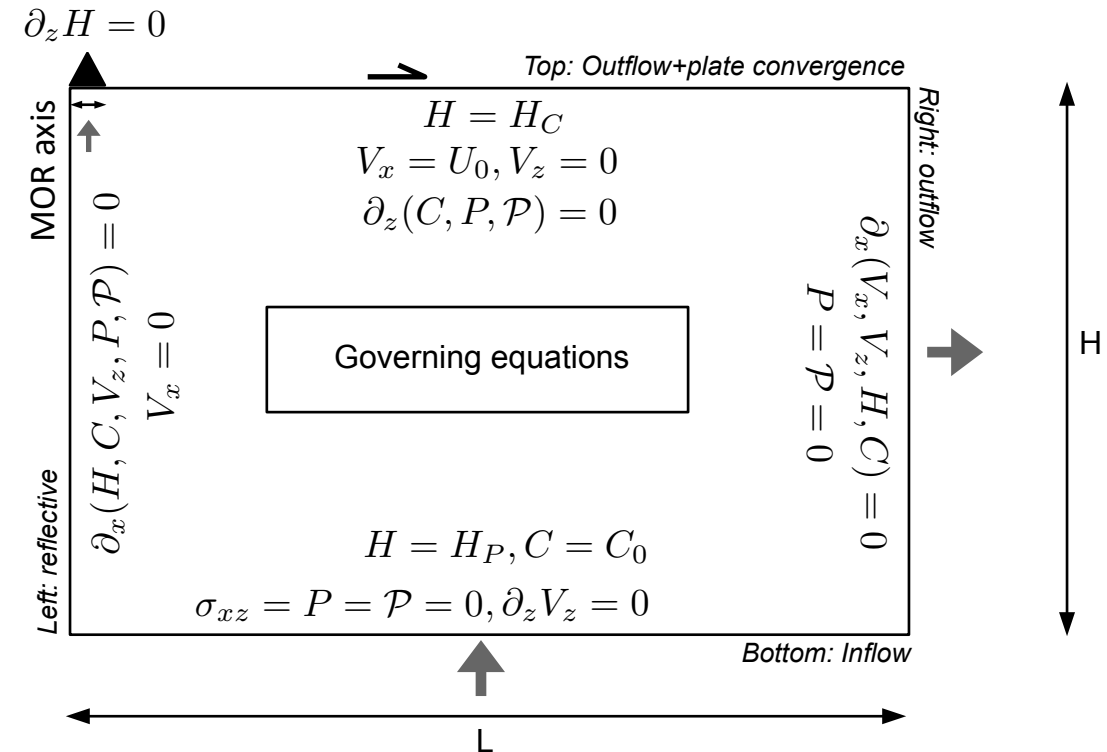
Model setup

Model is built after Katz (2010) using a **finite difference staggered grid framework** for solving partial differential equations (**FD-PDE**) for single-/two-phase magma dynamics (Pusok et al., 2020)

- **Conservation** of mass, momentum, energy and composition for a two-component two-phase flow system,
- **Constitutive equations** for permeability, shear and bulk viscosity (porosity and temperature-dependent), and density,
- Energy and composition are solved using the **Enthalpy method** (Alexiades and Solomon, 1993),
- **Petrology**: a two-component binary phase diagram (a fusible and a less fusible component),

Appendix
(detailed system of equations)

HALF-RIDGE BOUNDARY CONDITIONS

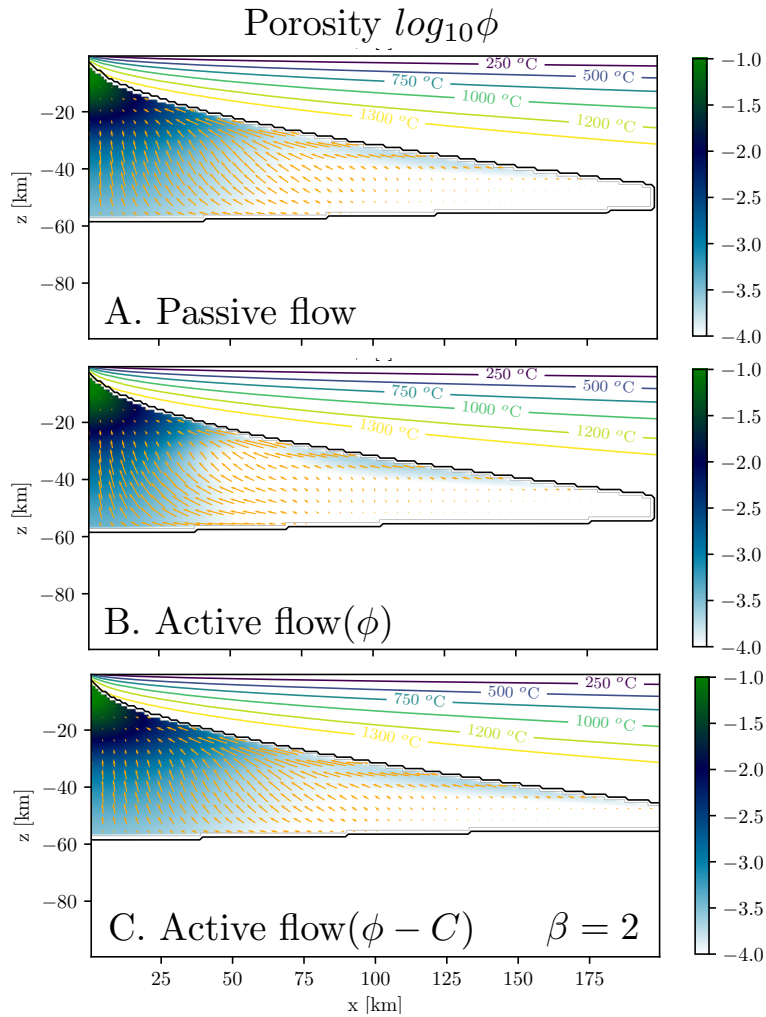
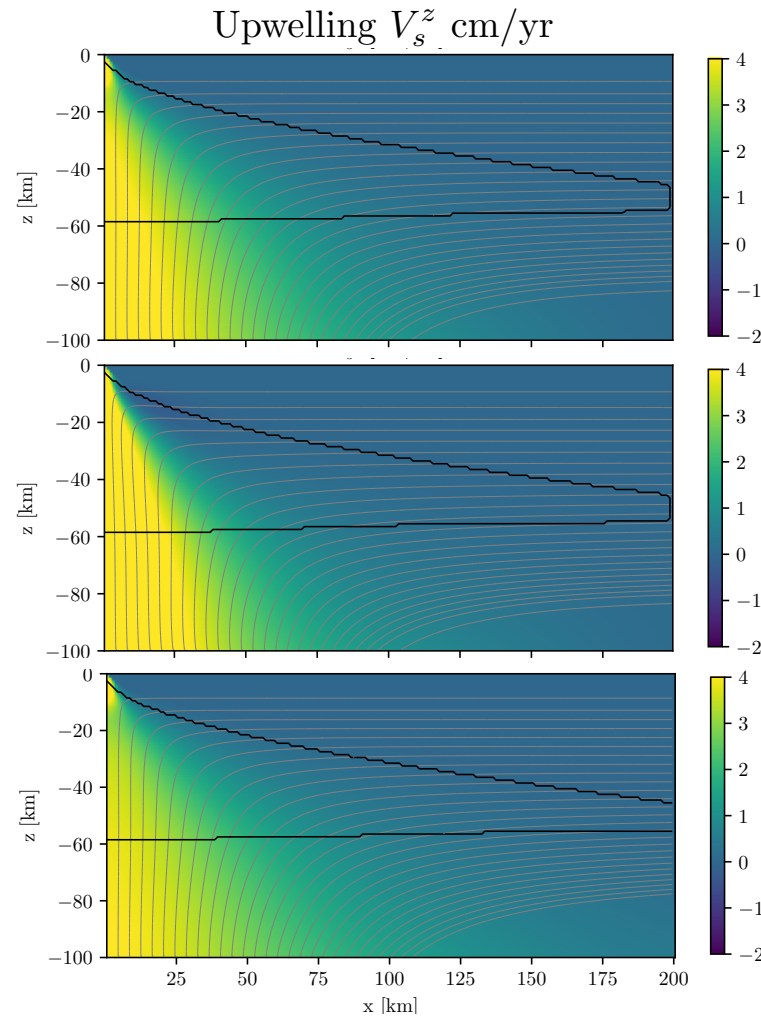


Initial conditions:

- Corner-flow solution
- Half-space cooling solution

Results

Intermediate spreading rate $U_0 = 4$ cm/yr



Reference values:

$$U_0 = 4 \text{ cm/yr},$$

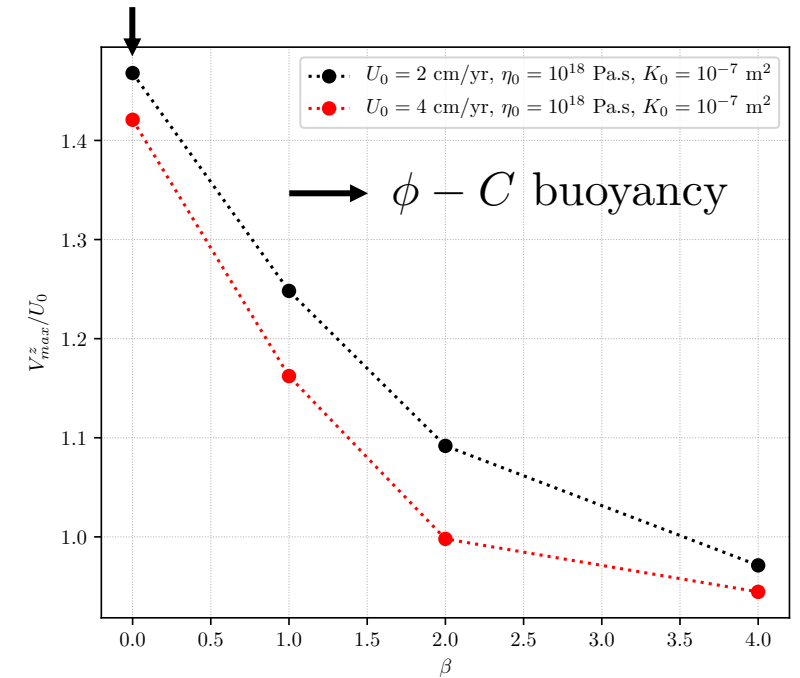
$$T_p = 1350^\circ\text{C},$$

$$\eta_0 = 10^{18} \text{ Pa.s},$$

$$\zeta_0 = 10^{19} \text{ Pa.s},$$

$$K_0 = 5 \times 10^{-9} \text{ m}^2$$

ϕ buoyancy



Porous buoyancy enhances upwelling, compositional buoyancy has the opposite effect

Scaling analysis

Non-dimensional buoyancy numbers (similar to Rayleigh number) can be derived from the momentum equation for solid-phase:

$$\nabla^2 \omega = \nu_\phi \partial_x \phi + \nu_C \partial_x C + \nu_T \partial_x T$$

(lateral gradients in porosity, composition and temperature are an internal source of vorticity)

Porous

Spiegelman (1993),
Katz (2010)

$$\nu_\phi = \frac{g \Delta \rho \phi_0 h_0^2}{\eta_0 u_0}$$

Compositional

$$\nu_C = -\rho_0 \beta F_0 \Delta C g \frac{h_0 \kappa}{\eta_0 u_0^2}$$

where

ω - vorticity,

g - gravitational acceleration,

$\Delta \rho$ - density difference between phases,

ρ_0 - reference density,

ϕ_0 - characteristic porosity,

h_0 - height of melting column,

η_0 - reference shear viscosity,

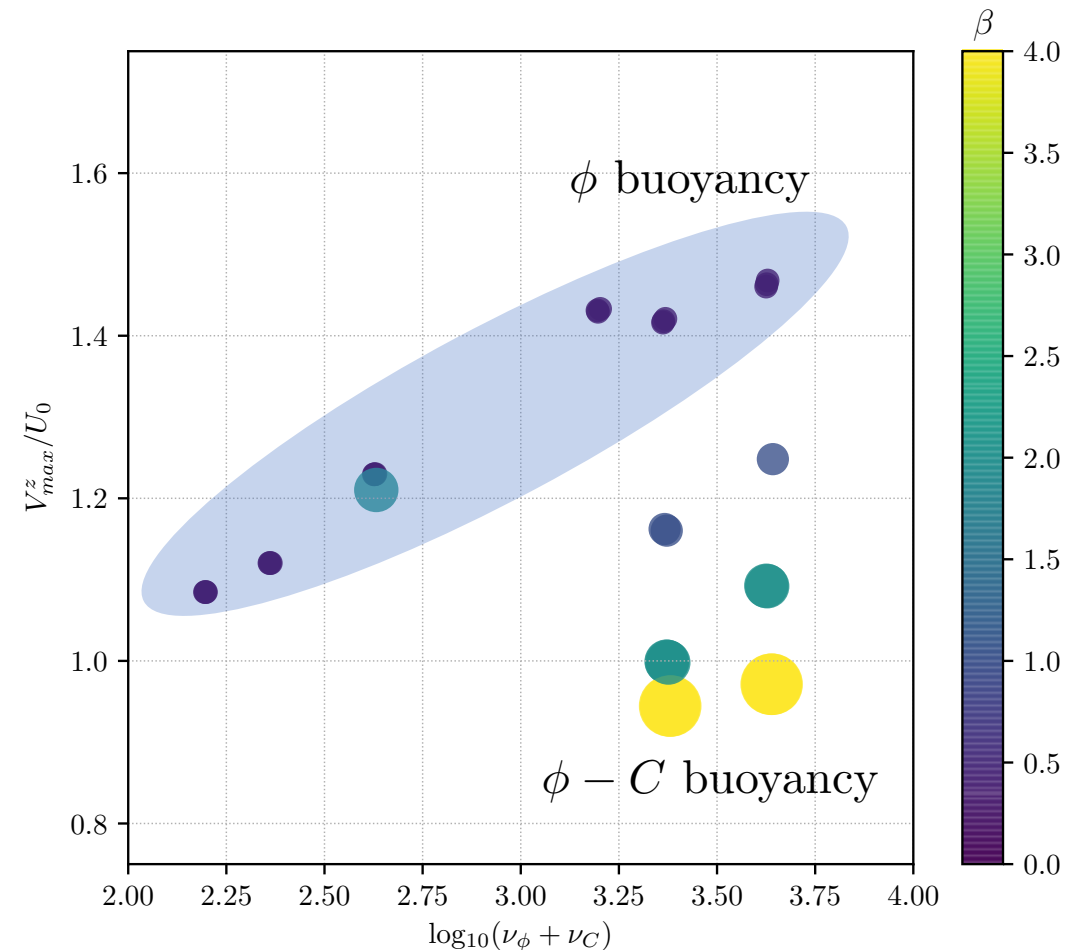
u_0 - half-spreading rate,

β - coeff of chemical expansion,

F_0 - degree of melting at top of column,

ΔC - compositional difference between phases,

κ - thermal diffusivity

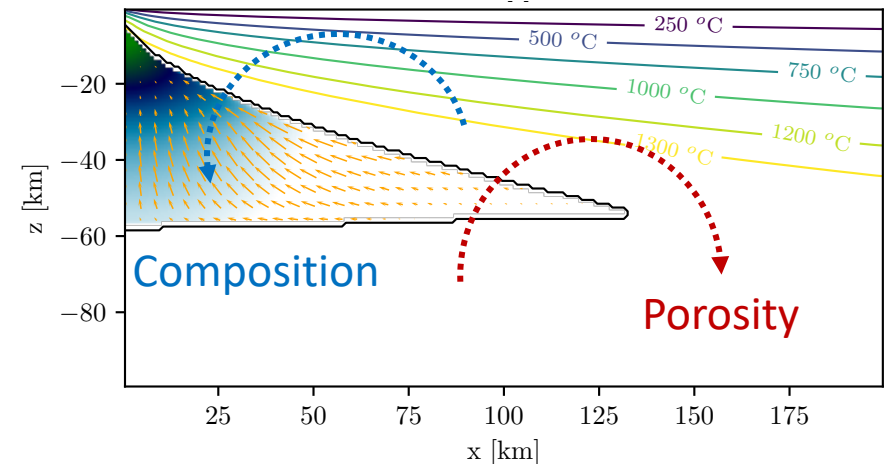


Discussion and Conclusions

- We investigate whether **compositional heterogeneities** (due to Fe/Mg partitioning between melt/solid) can induce **active flow beneath mid-ocean ridges**
- Results show that **compositional buoyancy beneath the ridge is negative and can partially balance porous buoyancy**
- **A scaling analysis** is performed to determine the relative importance of the contribution of compositional and porous buoyancy to upwelling
- Models with both chemical and porous buoyancy are susceptible to asymmetric forcing

References:

- Alexiades and Solomon (1993), *Mathematical Modeling of Melting and Freezing Processes*, 1st Ed.
- Balay et al. (2020), PETSc Users Manual, ANL-95/11-Revision 3.14, <https://www.mcs.anl.gov/petsc/>
- Forsyth et al. (1998), *Science* 280(1235), <https://doi.org/10.1126/science.280.5367.1235>
- Ghiorso et al. (2002), *G-cubed*, 3(5), <https://doi.org/10.1029/2001GC000217>
- Katz (2010), *G-cubed*, 11(Q0AC07), 1-29, <https://doi.org/10.1029/2010GC003282>
- Melt Seismic Team (1998), *Science*, 280(5367), 1215–1218, <https://doi.org/10.1126/science.280.5367.1215>
- Pusok et al. (2020), EGU General Assembly 2020, EGU2020-18690 <https://doi.org/10.5194/egusphere-egu2020-18690>
- Rychert et al. (2020), *JGR Solid Earth*, 125, e2018JB016463. <https://doi.org/10.1029/2018JB016463>
- Spiegelman (1993), *Philos. Trans. R. Soc. London A*, 342, 23–41, <https://doi.org/10.1098/rsta.1993.0002>
- Toomey et al. (2002), *EPSL*, 200(3-4), 287-295, [https://doi.org/10.1016/S0012-821X\(02\)00655-6](https://doi.org/10.1016/S0012-821X(02)00655-6)



Appendix – Theoretical framework

Conservation of mass and momentum

$$-\nabla P_f + \nabla \cdot \eta(\nabla \mathbf{v}_s + \nabla \mathbf{v}_s^T) + \nabla \left[\left(\zeta - \frac{2}{3} \eta \right) \nabla \cdot \mathbf{v}_s \right] + \bar{\rho} \mathbf{g} = 0$$

$$\nabla \cdot \mathbf{v}_s - \nabla \cdot \frac{K}{\mu} (\nabla P_f - \rho_f \mathbf{g}) = 0.$$

Pressure decomposition:

$$P_f = P_L + \mathcal{P} + P,$$

Conservation of energy and composition

$$\frac{\partial \mathcal{H}}{\partial t} + \rho c_p e^{-\frac{\alpha g z}{c_p}} \nabla \cdot \bar{\mathbf{v}} \mathcal{T} = L \rho \nabla \cdot (1 - \phi) \mathbf{v}_s + k e^{-\frac{\alpha g z}{c_p}} \nabla^2 \mathcal{T},$$

$$\frac{\partial C}{\partial t} + \nabla \cdot \phi \mathbf{v}_f C_f + \nabla \cdot (1 - \phi) \mathbf{v}_s C_s = \mathcal{D} \nabla \cdot \phi \nabla C_f,$$

Potential temperature

$$T = \mathcal{T} \exp(-\alpha g z / c_p),$$

for a two-phase, two-component system (Katz, 2008, 2010)

Constitutive equations

Density

Porosity, composition,
temperature-dependent

$$\begin{aligned}\rho_s &= \rho_{s0}(1 - B_s), & B_i &= \alpha(T - T_0) + \beta(C_i - C_0), \\ \rho_f &= \rho_{f0}(1 - B_f),\end{aligned}$$

Permeability

Porosity-dependent

$$K = \frac{d^2}{c} \phi^n = K_0 \phi^n,$$

Kozeny-Carman relationship (Wark and Watson, 1998, Wark, 2003)

Shear viscosity

Porosity, temperature-
dependent

$$\eta = \eta_0 \exp \left[\frac{E_A}{R} \left(\frac{1}{T} - \frac{1}{T_{\eta_0}} \right) - \lambda \phi \right],$$

Karato and Wu (1993), Hirth and Kohlstedt (2003)

Bulk viscosity

Porosity, temperature-
dependent

$$\zeta = \zeta_0 \exp \left[\frac{E_A}{R} \left(\frac{1}{T} - \frac{1}{T_{\eta_0}} \right) \right] \phi^{-1},$$

Cooper (1990), Hewitt and Fowler, (2008), Simpson (2008), Rudge (2018)

Petrology

2-phase, 2-component system

~ the olivine system with 2 end-members (Fayalite and Forsterite)

Solidus
$$C_s = C_0 + \frac{T - (T_0 + P_f/\gamma)}{M_s},$$

Liquidus
$$C_f = C_0 - \Delta C + \frac{T - (T_0 + P_f/\gamma)}{M_f},$$

Enthalpy Method (Alexiades and Solomon, 1993, Katz, 2010)

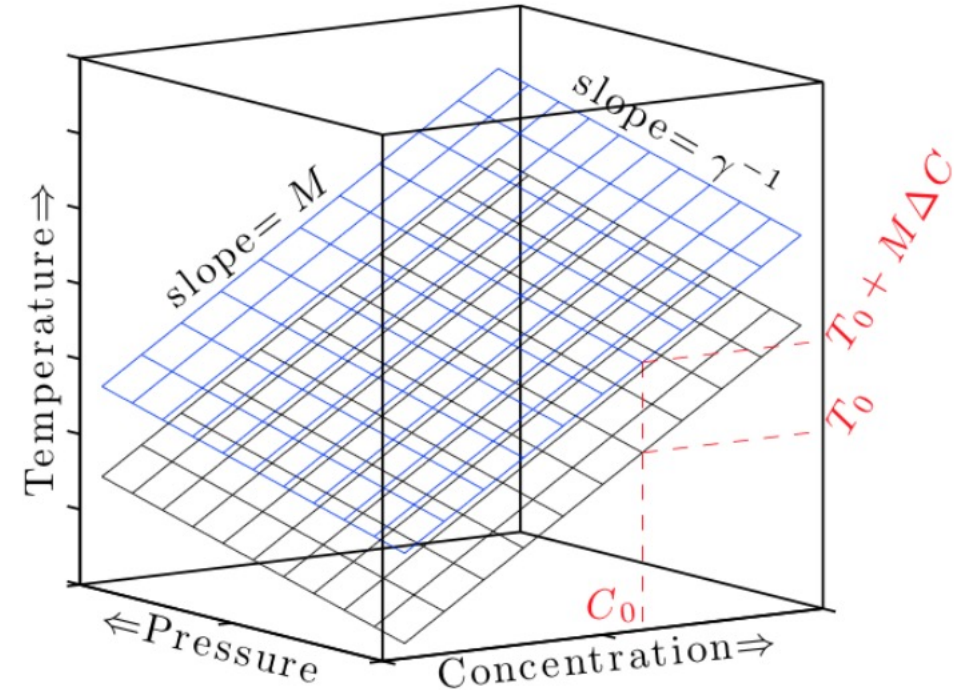
$(H, C, P) > (T, \phi, C_f, C_s)$
$$\mathcal{H} = \rho L \phi + \rho c_p (T - T_0),$$

$$C = \phi C_f + (1 - \phi) C_s,$$

$$C_s = f_s(T, P_f),$$

$$C_f = f_f(T, P_f).$$

$$\phi f_f(T, P_f) + (1 - \phi) f_s(T, P_f) - C = 0.$$



Katz and Weatherley (2012)

Feel free to contact me for details on non-dimensionalisation, model parameters, FD-PDE framework and code.