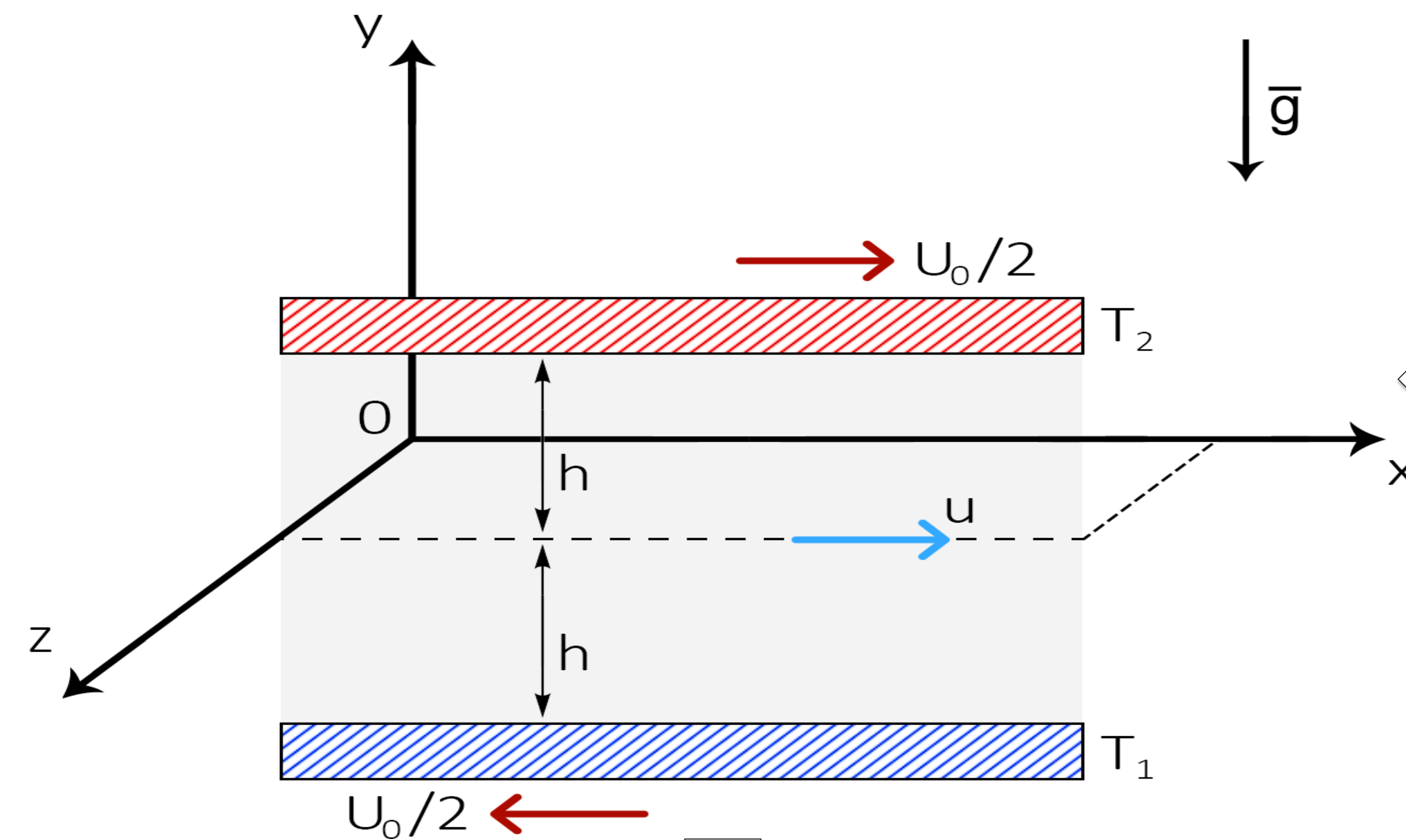


Optimal energy growth in stably stratified turbulent Couette flow

Grigory V. Zasko, Andrey V. Glazunov, Evgeny V. Mortikov, Yuri M. Nechepurenko, Pavel A. Perezhogin

Considered problem: stratified turbulent Couette flow



Motivation: large-scale inclined structures in stably stratified turbulent shear flows

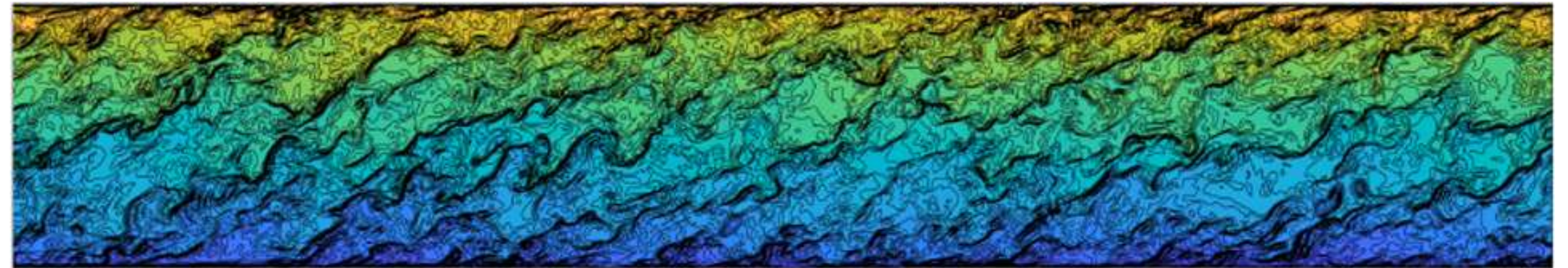


Fig.: Snapshot of temperature isolines in (x, y) section of a stably stratified turbulent Couette flow (DNS). $Re = 4 \times 10^4, Ri = 0.03$.

This phenomenon apparently influences statistical features of stratified turbulence but **has not yet been explained!**

Methods

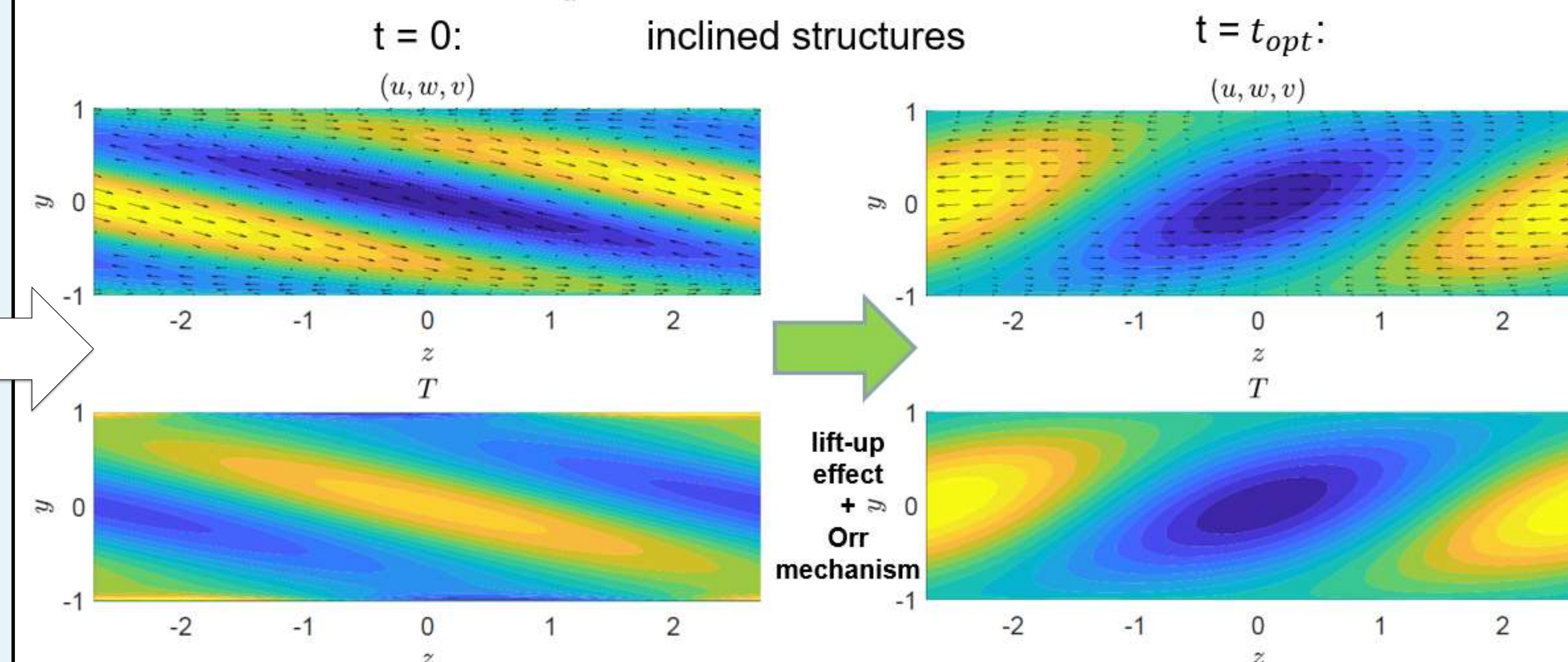
DNS: mean turbulent flow.

Unsteady RANS equations with eddy viscosity and diffusivity: mean turbulent flow is a steady state.

Linearization around the steady state: **mean turbulent flow is linearly stable.**

Linear nonmodal stability analysis
(search for **optimal disturbances** which gain maximum energy growth on finite time intervals).

Results: optimal disturbance at $Re = 4 \times 10^4, Ri = 0.03$.



Inclined structures = inclined streaks.

Obukhov length scale characterizes the emergence of such structures. Kinetic energy dissipation is small during its development.

Conclusion

Emergence of inclined structures in stably stratified turbulent Couette flow is described based on linear model.

Acknowledgments

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- Glazunov, A.V., Mortikov, E.V., Barskov, K.V., Kadantsev, E.V., Zilitinkevich, S.S. 2019. Izv. Atmos. Ocean. Phys., 55:312-323.
- Zasko, G.V., Glazunov, A.V., Mortikov, E.V., Nechepurenko, Yu.M. 2020. Russ. J. Num. Anal. Math. Model., 35: 35-53.



Optimal energy growth in stably stratified turbulent Couette flow

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Supplementary material for live presentation at
EGU General Assembly 2021.

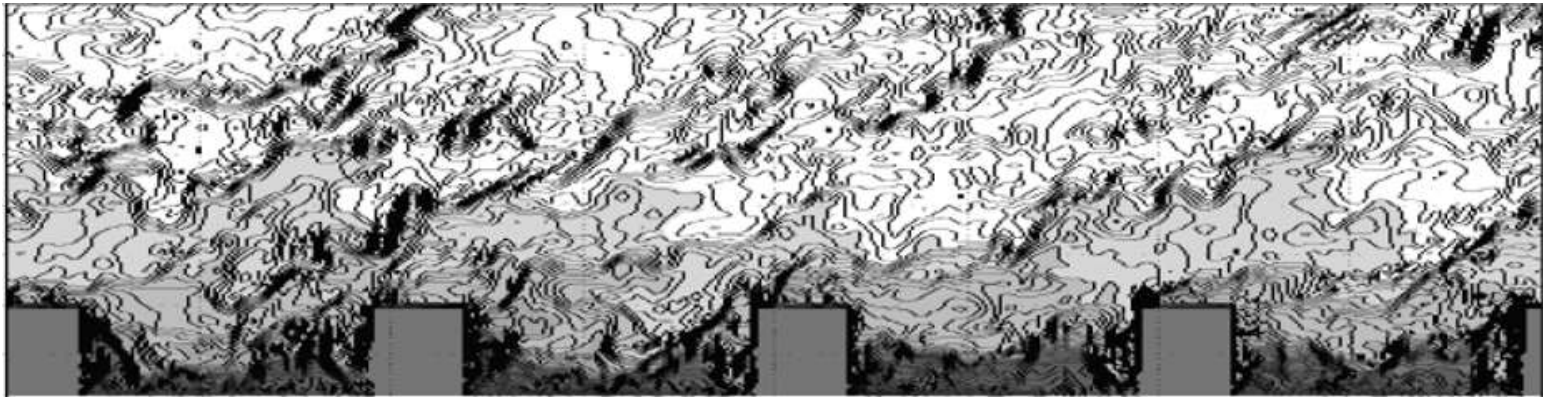
26 April, Online.

Organized structures in stably stratified turbulent flows

Snapshots of temperature isolines in streamwise section are shown.

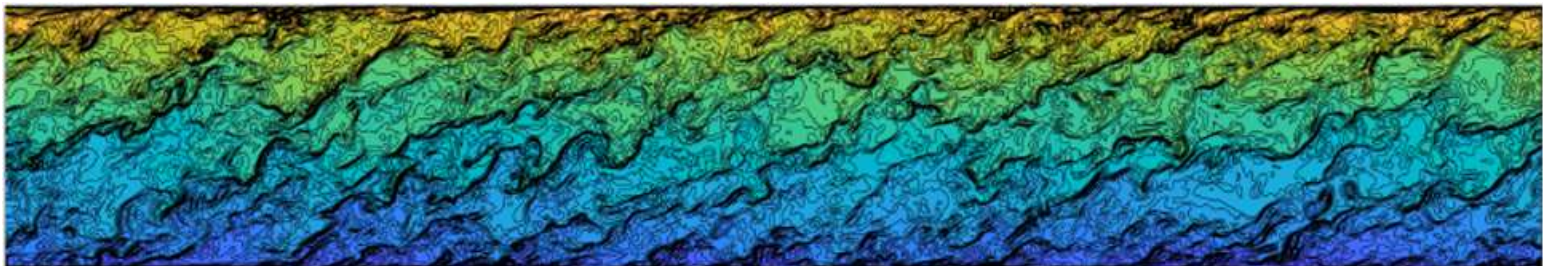
Thin inclined layers with strong stratification are visible.

1) stably stratified turbulent flow over urban-like surface, LES (A.V. Glazunov, *Izv. Atmos. Ocean. Phys.*, 2014)

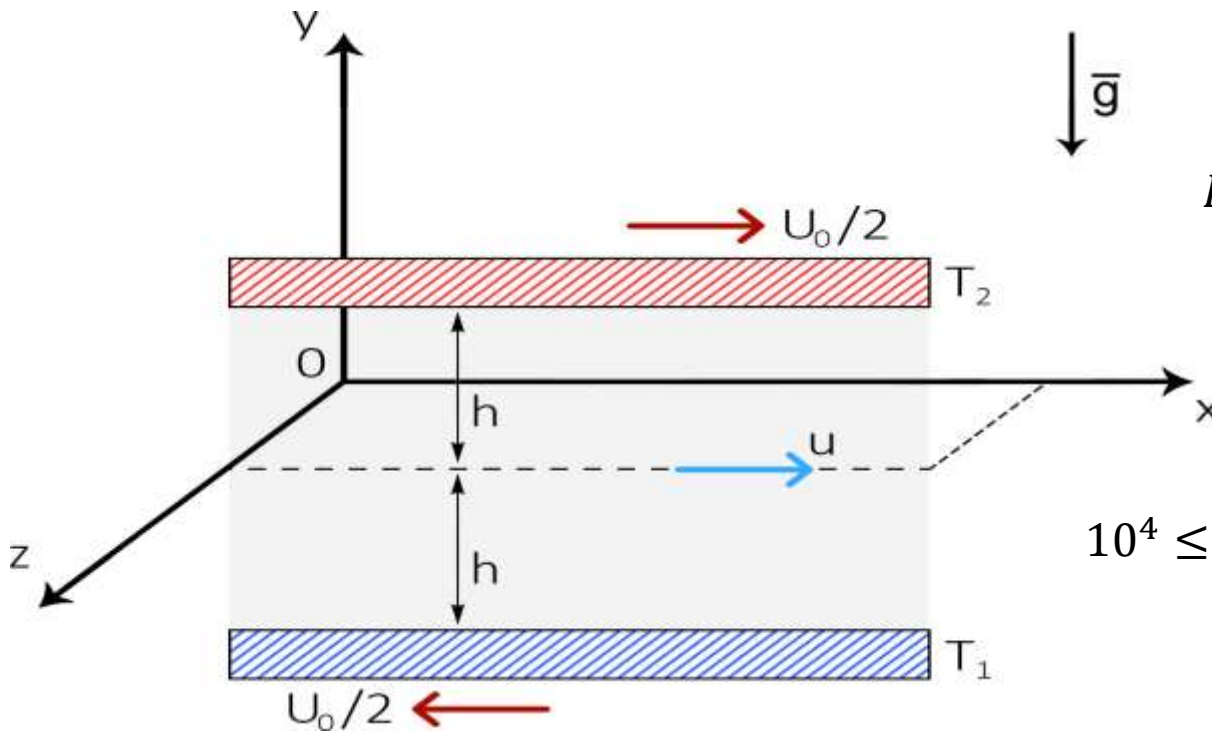


2) stably stratified turbulent Ekman layer, LES (P.P. Sullivan et al, *J. Atmos. Sci.*, 2016)

3) stably stratified turbulent Couette flow, DNS (A.V. Glazunov et al, *Izv. Atmos. Ocean. Phys.*, 2019)



Stably stratified turbulent Couette flow



$$Re = \frac{U_0 h}{\nu}, \quad Ri = g \frac{T_2 - T_1}{T_1} \frac{h}{U_0^2}$$

$$Pr = \frac{\nu}{\mu}$$

$$10^4 \leq Re \leq 6 \cdot 10^4, \quad 0 \leq Ri \leq 0.06$$

$$Pr = 0.7$$

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} + \nabla p - \frac{1}{Re} \Delta \mathbf{u} - (0, RiT, 0)^T = 0$$

$$\frac{\partial T}{\partial t} + (\mathbf{u} \cdot \nabla) T - \frac{1}{Pr Re} \Delta T = 0$$

$$\nabla \cdot \mathbf{u} = 0$$

Evolution equations for large-scale structures (Unsteady RANS):

$$\frac{\partial \tilde{\mathbf{u}}}{\partial t} + (\tilde{\mathbf{u}} \cdot \nabla) \tilde{\mathbf{u}} + \nabla \tilde{p} - \Delta_\nu \tilde{\mathbf{u}} - (0, Ri\tilde{T}, 0)^T = 0$$

$$\frac{\partial \tilde{T}}{\partial t} + (\tilde{\mathbf{u}} \cdot \nabla) \tilde{T} - \Delta_\mu \tilde{T} = 0$$

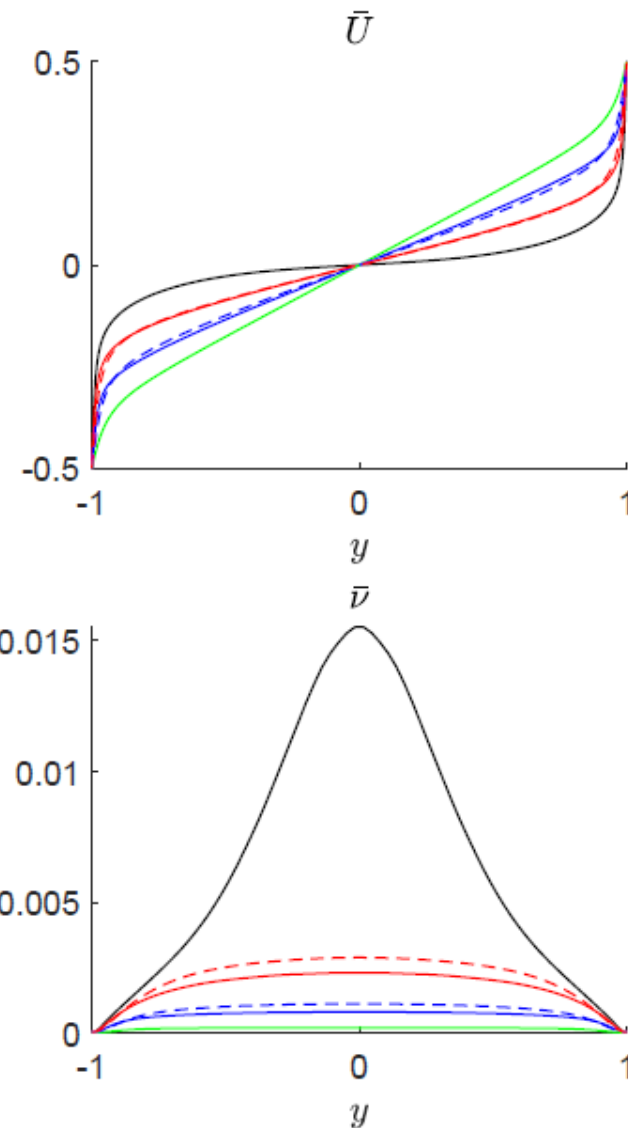
$$\nabla \cdot \tilde{\mathbf{u}} = 0$$

isotropic eddy
viscosity and diffusivity
(based on DNS)

$$\left\{ \begin{array}{l} \Delta_\nu = \bar{\nu} \frac{\partial^2}{\partial x^2} + \frac{\partial}{\partial y} \bar{\nu} \frac{\partial}{\partial y} + \bar{\nu} \frac{\partial^2}{\partial z^2}, \\ \Delta_\mu = \bar{\mu} \frac{\partial^2}{\partial x^2} + \frac{\partial}{\partial y} \bar{\mu} \frac{\partial}{\partial y} + \bar{\mu} \frac{\partial^2}{\partial z^2} \end{array} \right.$$

Steady state (main flow):

$$\tilde{\mathbf{u}} = \bar{\mathbf{u}} \equiv (\bar{U}(y), 0, 0)^T, \quad \tilde{p} = \bar{P}(y), \quad \tilde{T} = \bar{T}(y)$$



Main flow is linearly stable!

Disturbances of the form:

$$(u', v', w', p', T') = (u_{\alpha\gamma}, v_{\alpha\gamma}, w_{\alpha\gamma}, p_{\alpha\gamma}, T_{\alpha\gamma}) e^{i\alpha x + i\gamma z}$$

Total energy density of disturbance:

$$\mathcal{E}_t = \frac{1}{2} \int_{-1}^1 \left(|u_{\alpha\gamma}|^2 + |v_{\alpha\gamma}|^2 + |w_{\alpha\gamma}|^2 + \frac{Ri}{d\bar{T}/dy} |T_{\alpha\gamma}|^2 \right) dy$$

Maximum energy amplification:

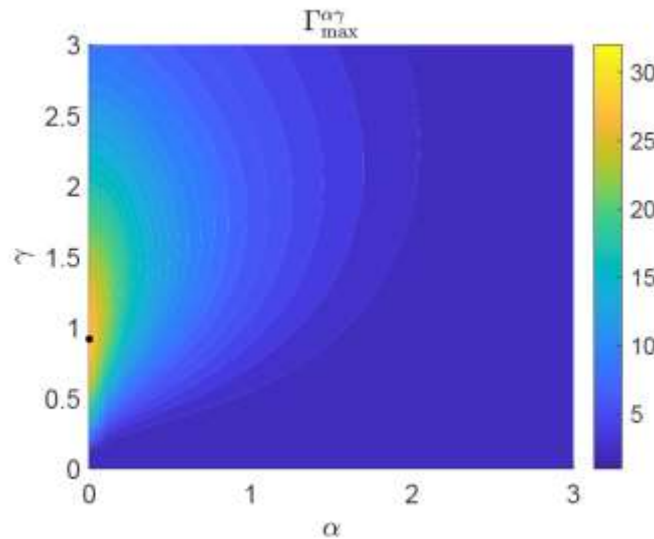
$$\Gamma^{\alpha\gamma}(t) = \max \frac{\mathcal{E}_t}{\mathcal{E}_0}$$

$$\Gamma_{\max}^{\alpha\gamma} = \max_{t \geq 0} \Gamma^{\alpha\gamma}(t), \quad \Gamma_{\max} = \max_{\alpha, \gamma} \Gamma_{\max}^{\alpha\gamma}, \quad (\alpha_{\text{opt}}, \gamma_{\text{opt}}, t_{\text{opt}}) = \arg \max_{\alpha, \gamma, t} \Gamma^{\alpha\gamma}(t)$$

Initial disturbance, at which $\Gamma_{\max}$ is attained,
will be called **optimal disturbance**.

Optimal disturbance at $Re = 4 \times 10^4, Ri = 0.01$

(close to neutral stratification)



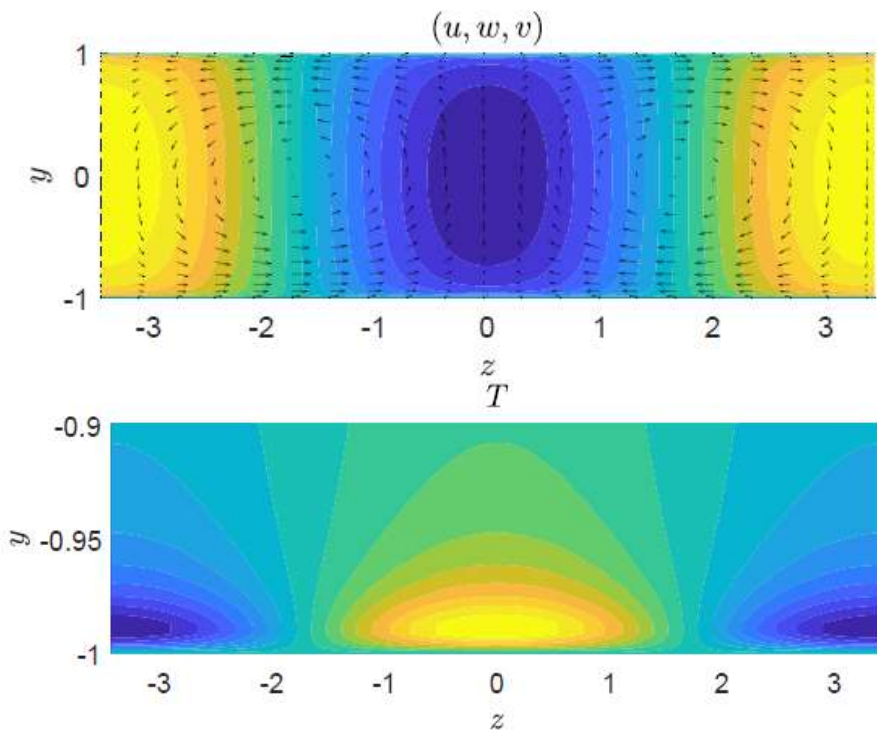
$$\Gamma_{\max} = 28.3$$

$$\alpha_{\text{opt}} = 0$$

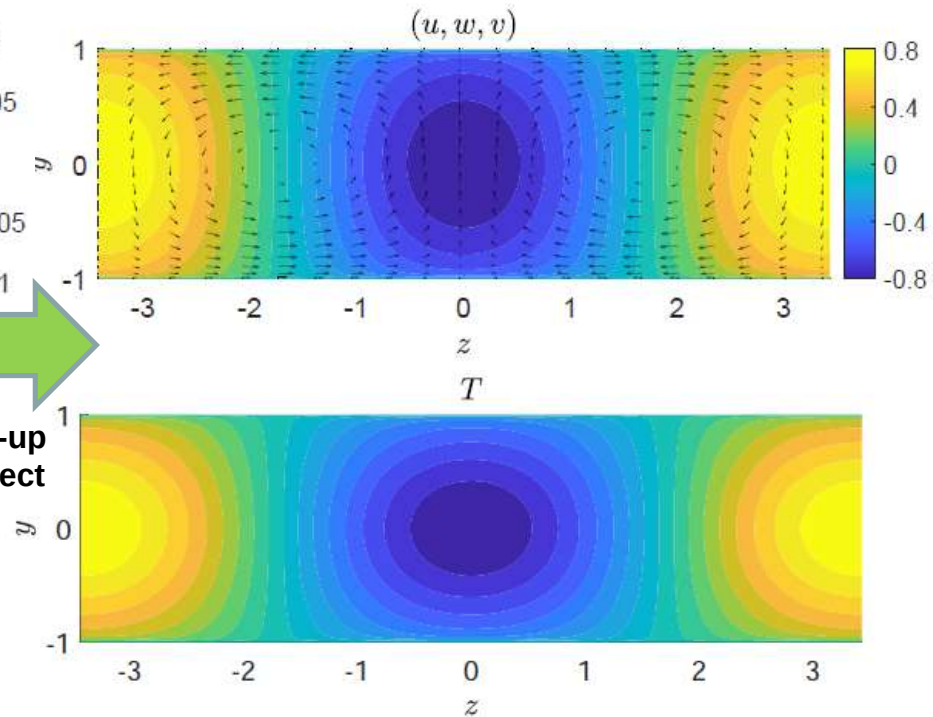
$$\gamma_{\text{opt}} = 0.9$$

$t = 0$, rolls:

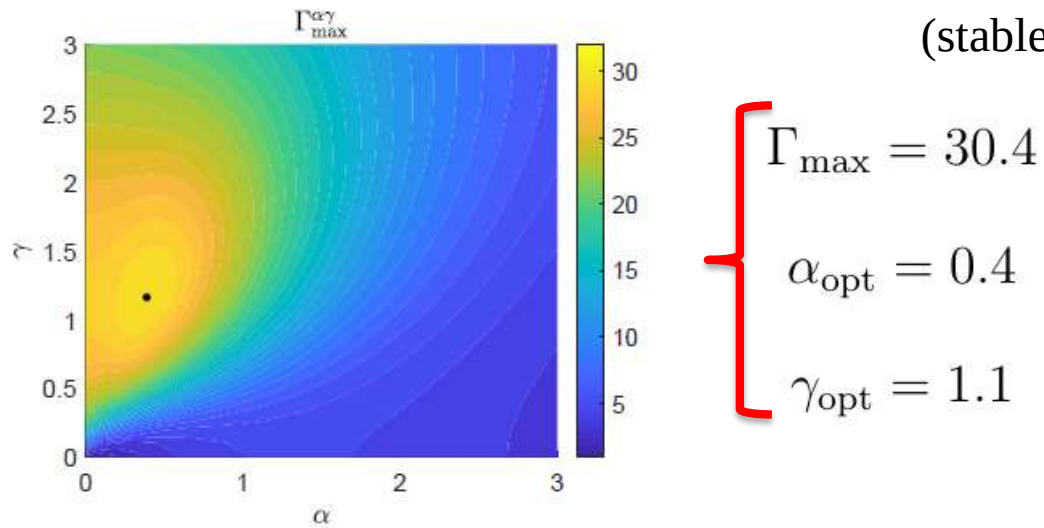
$t = t_{\text{opt}}$, streaks:



lift-up
effect



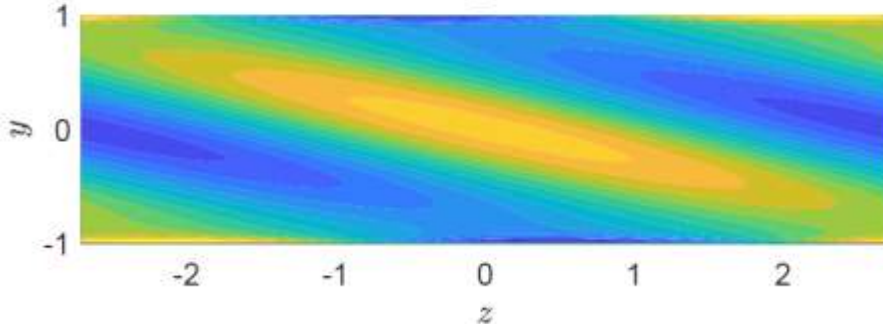
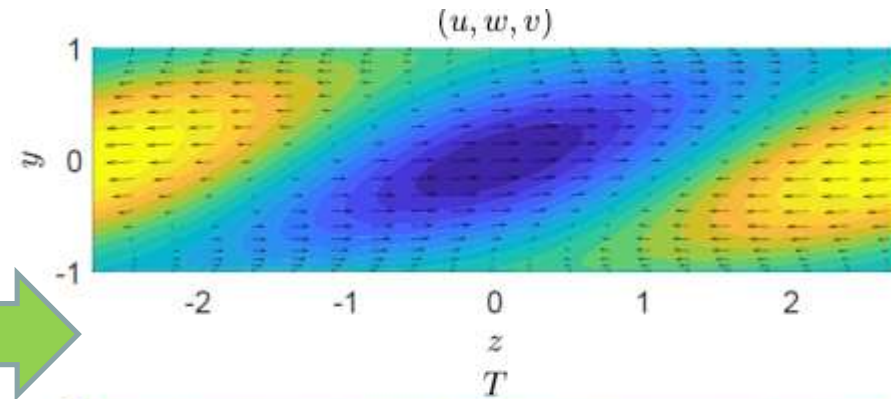
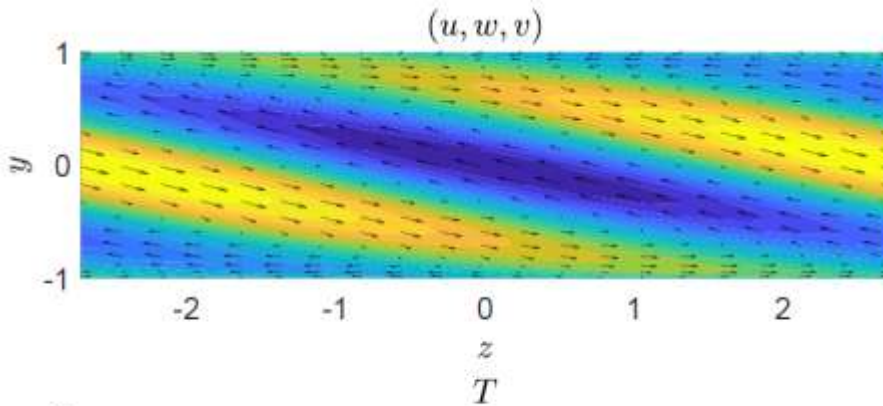
Optimal disturbance at $Re = 4 \times 10^4, Ri = 0.03$ (stable stratification)



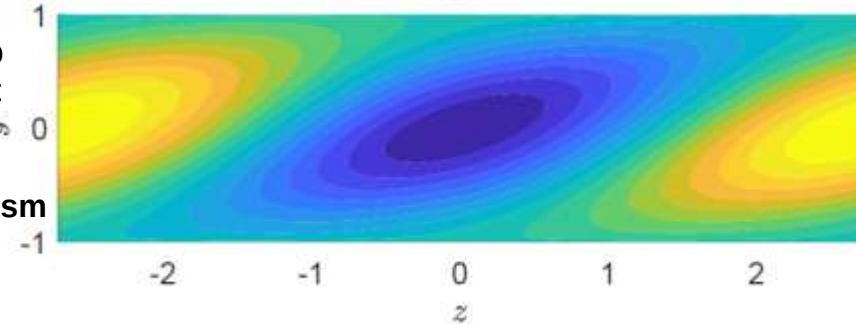
$t = 0$:

inclined structures

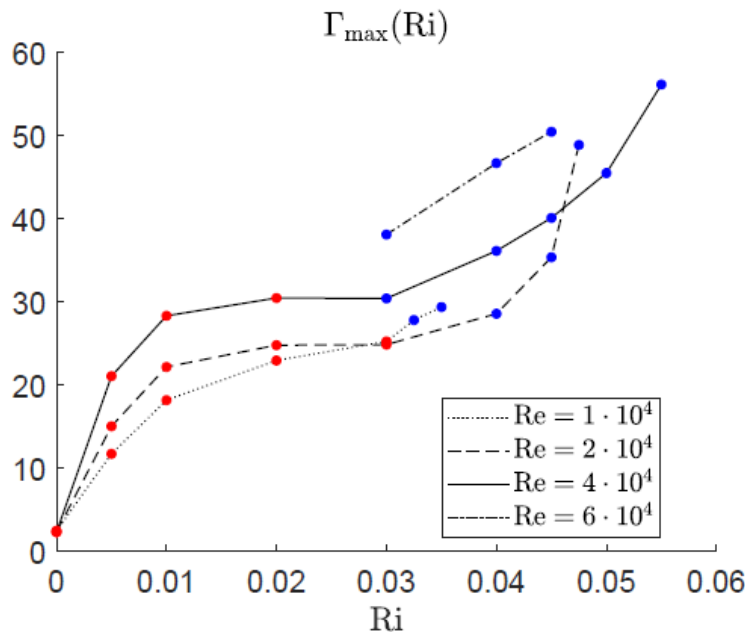
$t = t_{\text{opt}}$:



lift-up
effect
+
Orr
mechanism

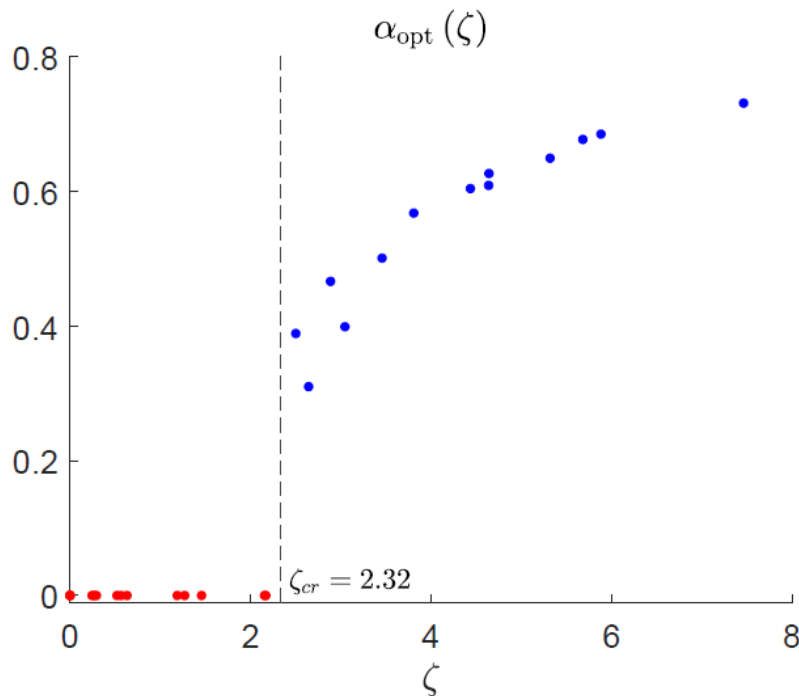


Dependence on Reynolds and Richardson numbers



red dots = streamwise rolls/streaks ($\alpha_{\text{opt}} = 0$)
 blue dots = inclined structures ($\alpha_{\text{opt}} > 0$)

Transition between two types of structures occurs with increasing Ri .



dimensionless Obukhov length scale:

$$L = -u_{\tau}^3 / (\text{Ri} F_T)$$

stability parameter:

$$\zeta = 1/L$$

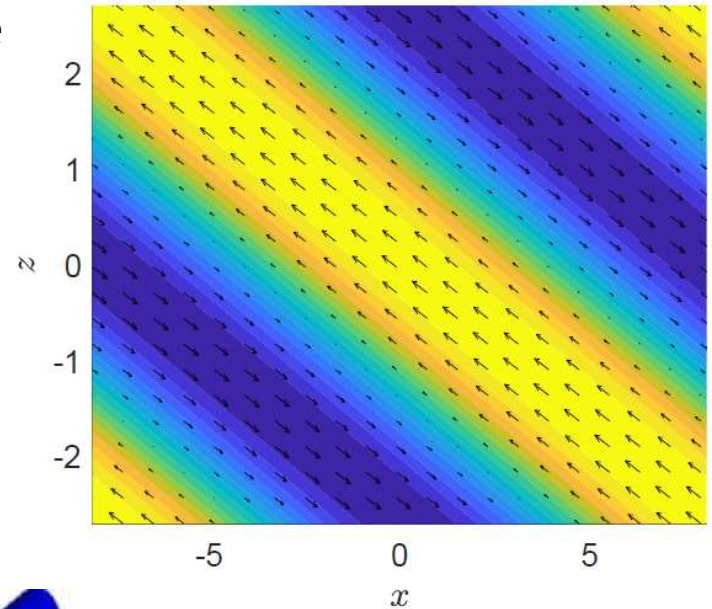
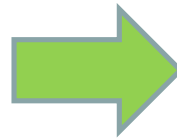
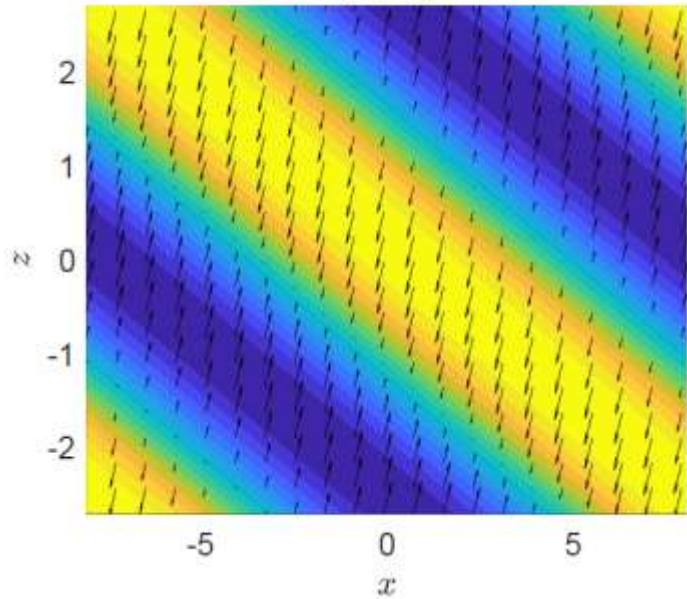
Transition depends solely on Obukhov length scale.

Inclined structures = inclined rolls/streaks!

$t = 0:$
 (u, w)

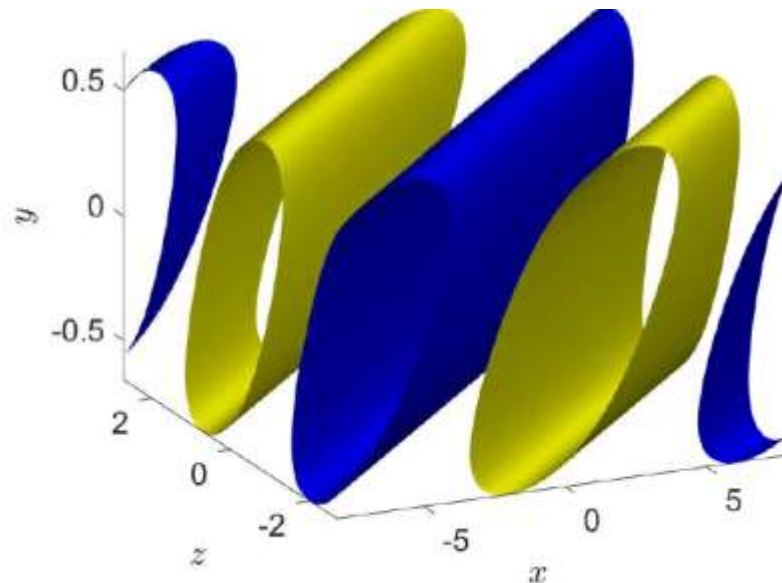
optimal disturbance at
 $Re = 4 \times 10^4, Ri = 0.03$,
view from above

$t = t_{opt}:$
 (u, w)



3D picture.

isosurfaces of
temperature
at $t = t_{opt}$:



Linearized equations of disturbance evolution:

$$\frac{\partial u_{\alpha\gamma}}{\partial t} + i\alpha\bar{U}u_{\alpha\gamma} + \frac{d\bar{U}}{dy}v_{\alpha\gamma} + i\alpha p_{\alpha\gamma} - \Delta_v^{\alpha\gamma}u_{\alpha\gamma} = 0,$$

$$\frac{\partial v_{\alpha\gamma}}{\partial t} + i\alpha\bar{U}v_{\alpha\gamma} + \frac{\partial p_{\alpha\gamma}}{\partial y} - \Delta_v^{\alpha\gamma}v_{\alpha\gamma} - RiT_{\alpha\gamma} = 0,$$

$$\frac{\partial w_{\alpha\gamma}}{\partial t} + i\alpha\bar{U}w_{\alpha\gamma} + i\gamma p_{\alpha\gamma} - \Delta_v^{\alpha\gamma}w_{\alpha\gamma} = 0,$$

$$\frac{\partial T_{\alpha\gamma}}{\partial t} + i\alpha\bar{U}T_{\alpha\gamma} + \frac{d\bar{T}}{dy}v_{\alpha\gamma} - \Delta_\mu^{\alpha\gamma}T_{\alpha\gamma} = 0,$$

$$i\alpha u_{\alpha\gamma} + \frac{\partial v_{\alpha\gamma}}{\partial y} + i\gamma w_{\alpha\gamma} = 0,$$

Evolution equation of disturbance energy:

$$\frac{d\mathcal{E}_t}{dt} = - \underbrace{\int_{-1}^1 \frac{d\bar{U}}{dy} \operatorname{Re}\{v_{\alpha\gamma}u_{\alpha\gamma}^*\} dy}_{\text{growth term}} + \dots (\text{diss. terms})$$

If >0 , then growth of disturbance energy is possible!

where

$$\mathcal{E}_t = \frac{1}{2} \int_{-1}^1 \left(|u_{\alpha\gamma}|^2 + |v_{\alpha\gamma}|^2 + |w_{\alpha\gamma}|^2 + \frac{Ri}{d\bar{T}/dy} |T_{\alpha\gamma}|^2 \right) dy$$

Physical mechanisms of disturbance energy growth

$$\frac{d\mathcal{E}_t}{dt} = - \underbrace{\int_{-1}^1 \frac{d\bar{U}}{dy} \operatorname{Re}\{v_{\alpha\gamma} u_{\alpha\gamma}^*\} dy}_{\text{Lift-up effect}} + \dots (\text{diss. terms})$$

Lift-up effect :

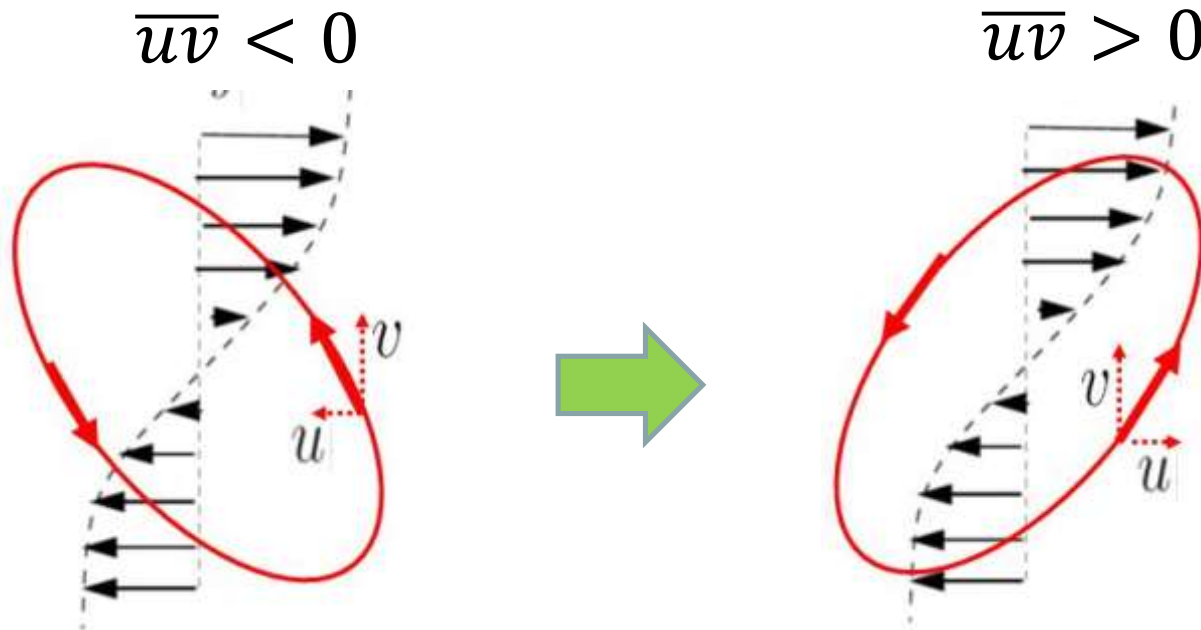
Ellingsen, Palm, 1975
Landahl, 1981

- 3-D streamwise elongated disturbances ($\alpha = 0$)
- Rotation in cross section (w, v) amplifies streamwise velocity of disturbance from the main flow.

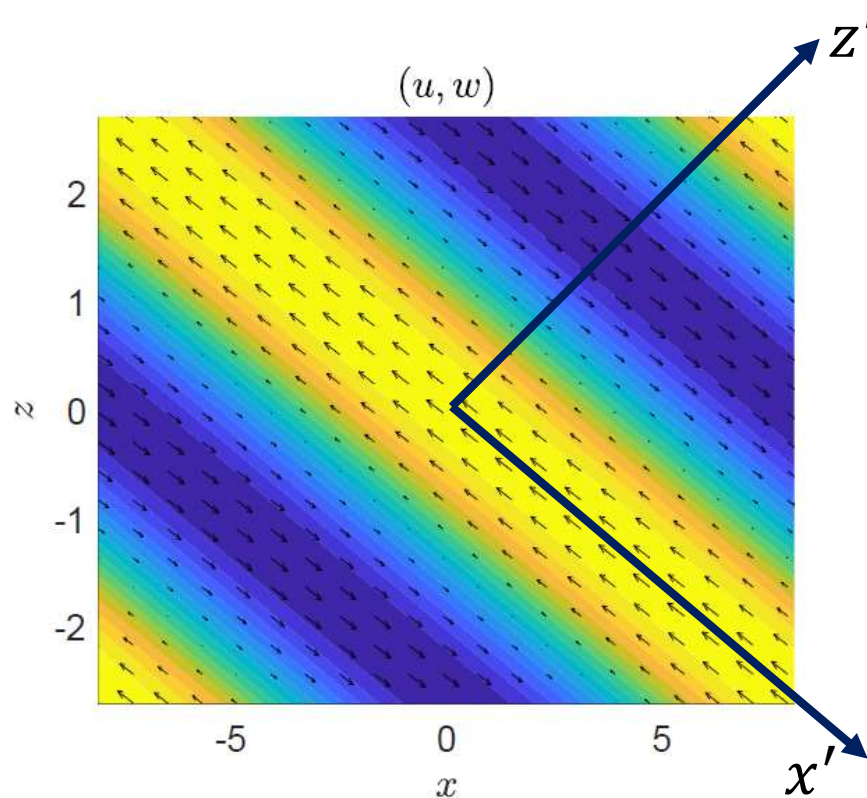
Orr mechanism:

Orr, 1907

- 2-D disturbances ($w = 0$)
- Tilted vortex gain energy from the main shear accompanied by the change of its slope.



Rotation of the coordinate system is made to separate actions of two mechanisms



Main flow has 2 components
in rotated coordinates:

Disturbance has the form:

$$\theta = \arcsin(\alpha/\gamma')$$

$$\gamma' = \sqrt{\alpha^2 + \gamma^2}$$

$$x' = x \cos \theta - z \sin \theta$$

$$z' = x \sin \theta + z \cos \theta$$

$$u'_{\alpha\gamma} = u_{\alpha\gamma} \cos \theta - w_{\alpha\gamma} \sin \theta$$

$$w'_{\alpha\gamma} = u_{\alpha\gamma} \sin \theta + w_{\alpha\gamma} \cos \theta$$

$$\bar{U}' = \bar{U} \cos \theta$$

$$\bar{W}' = \bar{U} \sin \theta$$

$$(u'_{\alpha\gamma}, v_{\alpha\gamma}, w'_{\alpha\gamma}, p_{\alpha\gamma}, T_{\alpha\gamma}) e^{i\gamma' z'}$$

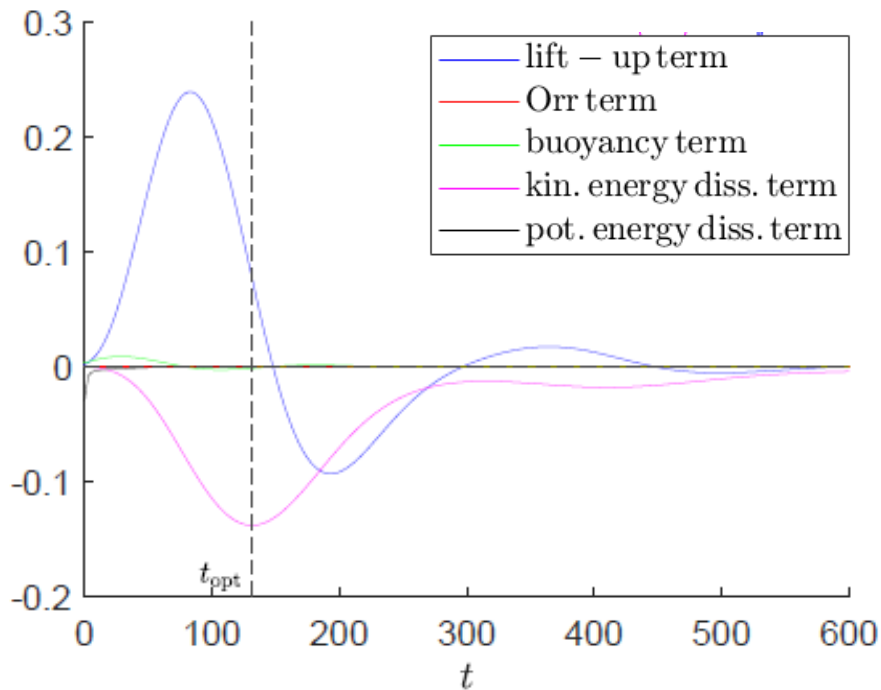
Evolution equations of disturbance kinetic and potential energy in rotated coordinates:

$$\begin{aligned}
 \frac{d\mathcal{E}_t^K}{dt} = & \underbrace{- \int_{-1}^1 \frac{d\bar{U}'}{dy} \operatorname{Re}\{v_{\alpha\gamma} u'_{\alpha\gamma}{}^*\} dy}_{\text{lift-up term}} - \underbrace{\int_{-1}^1 \frac{d\bar{W}'}{dy} \operatorname{Re}\{v_{\alpha\gamma} w'_{\alpha\gamma}{}^*\} dy}_{\text{Orr term}} + \\
 & + Ri \underbrace{\int_{-1}^1 \operatorname{Re}\{T_{\alpha\gamma} v'_{\alpha\gamma}{}^*\} dy}_{\text{buoyancy term}} - \\
 & \underbrace{\int_{-1}^1 \bar{\nu} \left[\gamma'^2 (|u'_{\alpha\gamma}|^2 + |v_{\alpha\gamma}|^2 + |w'_{\alpha\gamma}|^2) + \left| \frac{\partial u'_{\alpha\gamma}}{\partial y} \right|^2 + \left| \frac{\partial v_{\alpha\gamma}}{\partial y} \right|^2 + \left| \frac{\partial w'_{\alpha\gamma}}{\partial y} \right|^2 \right] dy}_{\text{kinetic energy dissipation term}}, \\
 \\
 \frac{d\mathcal{E}_t^P}{dt} = & - Ri \int_{-1}^1 \operatorname{Re}\{v_{\alpha\gamma} T_{\alpha\gamma}{}^*\} dy - Ri \underbrace{\int_{-1}^1 \frac{\bar{\mu}}{d\bar{T}/dy} \left[\gamma'^2 |T_{\alpha\gamma}|^2 + \left| \frac{\partial T_{\alpha\gamma}}{\partial y} \right|^2 \right] dy}_{\text{potential energy dissipation term}}.
 \end{aligned}$$

Energetics of optimal disturbances

$Re = 40000, Ri = 0.01, \alpha = 0$

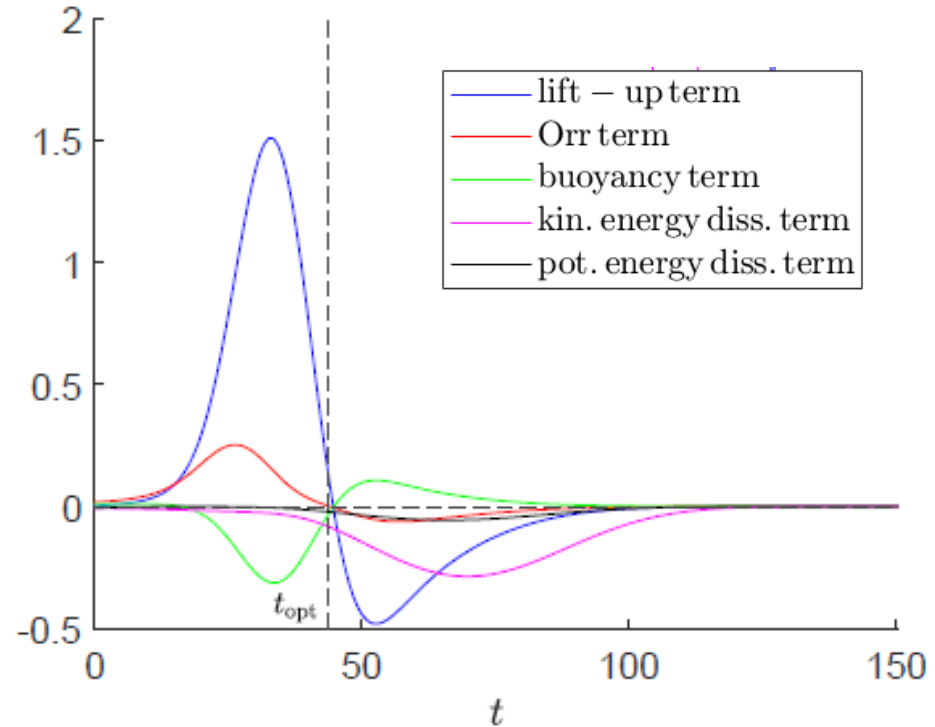
streamwise rolls/streaks



- Dissipation due to lift-up effect occurs after t_{opt} ;
- Moment of maximum kinetic energy dissipation is close to t_{opt} .

$Re = 40000, Ri = 0.055, \alpha \neq 0$

inclined structures



- Moment of maximum kinetic energy dissipation occurs after t_{opt} ;
- Dissipation of kinetic energy is small during development of optimal disturbance;
- Main contribution to energy growth comes from lift-up effect;
- Wave-like behavior of buoyancy term (internal gravity waves!?).