

Wave development and transformation under strong offshore winds: modelling by DNS and kinetic equations and comparison with airborne measurements

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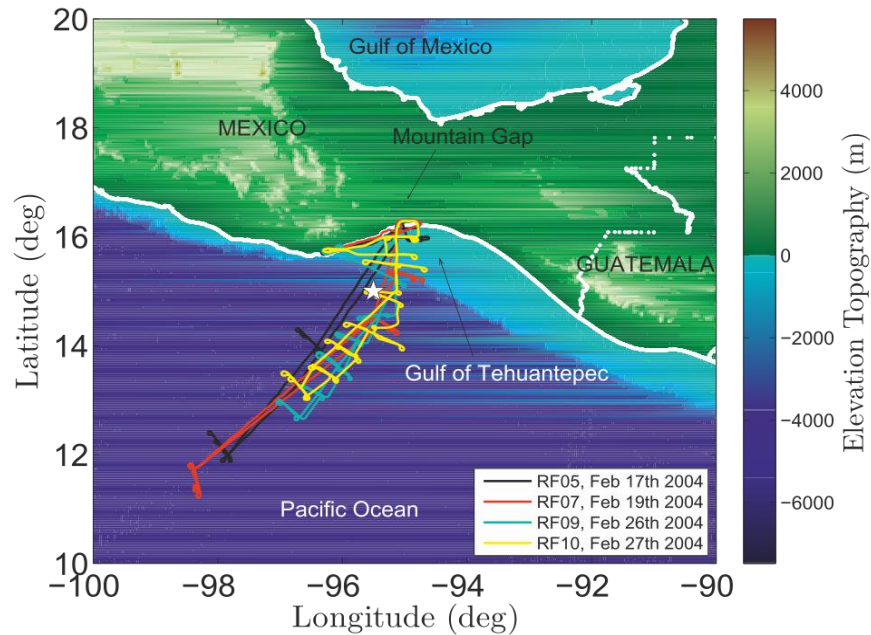
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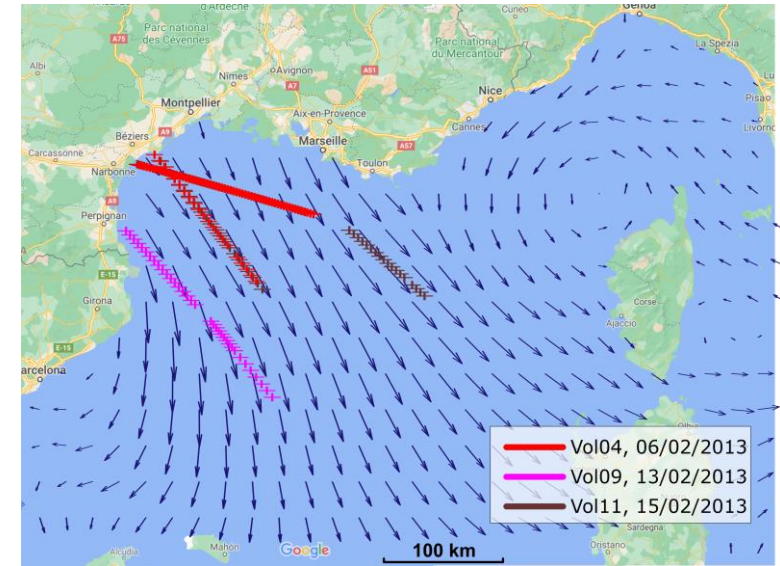
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Airborne measurements of fetch-limited waves during strong offshore winds



- GOTEX experiment, February 2004
- "Tehuano" wind jet, up to 25 m/s
- Measurements of 2D spectra up to 500 km fetch
- Persistent opposing swell
- Modelling starts at 100 km fetch
- Wind measurements available
- Wind slowly decreasing and turning right
- Wind jet quasi-stationary in time



- HyMeX experiment, February-March 2013
- "Tramontane" wind jet, up to 25 m/s
- Measurements of 2D spectra up to 150 km fetch
- No swell
- Modelling starts at zero fetch
- Wind measurements not directly available, but
- ...wind has nearly constant speed and direction
- Wind jet quasi-stationary in time

modelling

We use three different modelling approaches

- **Direct numerical simulation based on the Zakharov equation (DNS-ZE)**

- Not based on any statistical assumptions
- Initial phases are random, averaging of 100 realisations

Annenkov & Shrira (2013) Large-time evolution of statistical moments of a wind wave field. *J. Fluid Mech.* **726**, 517-546

Annenkov & Shrira (2018) Spectral evolution of weakly nonlinear random waves: kinetic description vs direct numerical simulations. *J. Fluid Mech.* **844**, 766-795

- **Standard (Hasselmann) kinetic equation (KE)**

- The standard WRT algorithm is used, kindly provided by the author, Gerbrant van Vledder

- **Generalised kinetic equation (gKE)**

- Derived without the quasi-stationarity assumption (e.g. for fast-changing winds)
- Tends to the KE in the large-time limit

Annenkov & Shrira (2016) Modelling transient sea states with the generalised kinetic equation. In: *Rogue and Shock Waves in Nonlinear Dispersive Media*, M.Onorato et al (eds), Springer

The primary target of the study is to compare results obtained with different statistical and dynamical models under different sets of assumptions (between themselves, and with measurements)

the challenge

- All numerical models of long-term wind wave evolution are based on the Hasselmann kinetic equation (KE)
- It is well-known from measurements that the spectral shape of wind waves depends on the stage of wave development
- Young waves have relatively large spectral peakedness (JONSWAP peakedness parameter γ around or above 3)
- As waves mature, γ goes through a transition to near 1 at U/c (inverse wave age) near 1
- For even older waves, γ settles to just below 1
- Fully developed waves have Pierson-Moskowitz spectral shape, which is well established statistically (e.g. Babanin & Soloviev 1998)
- But modelling based on the Hasselmann equation shows neither this spectral shape, nor any tendency to it

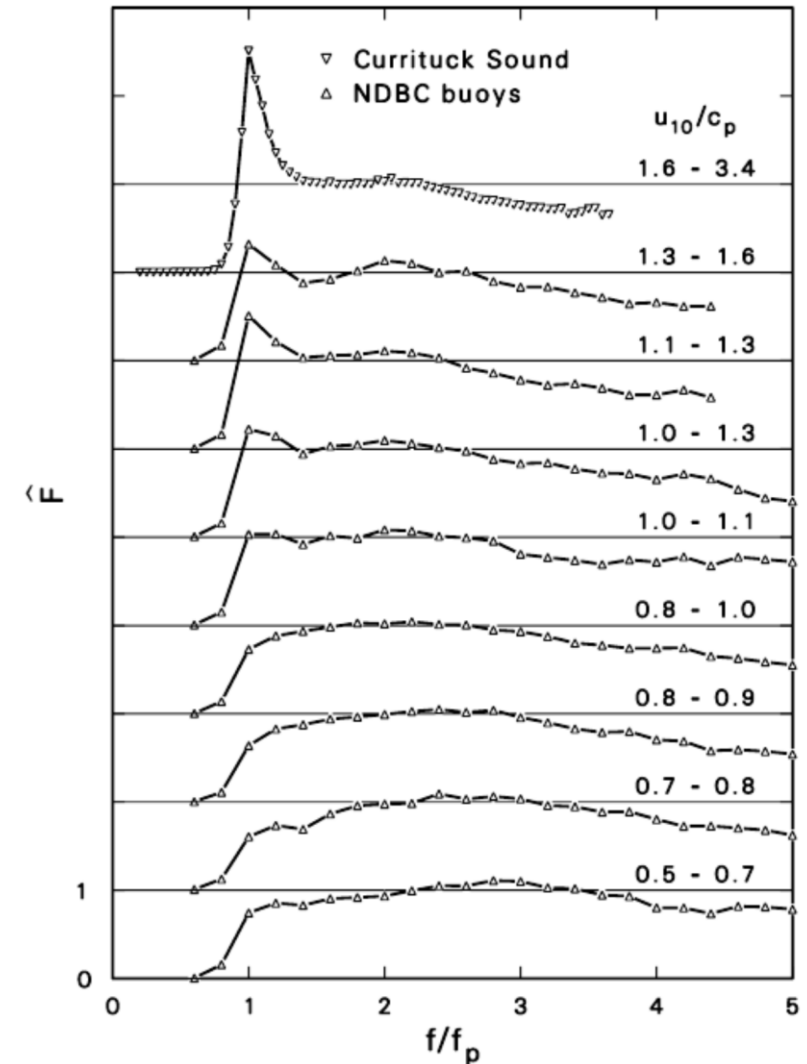


Figure from Long & Resio (2007)

Hasselmann equation and self-similarity

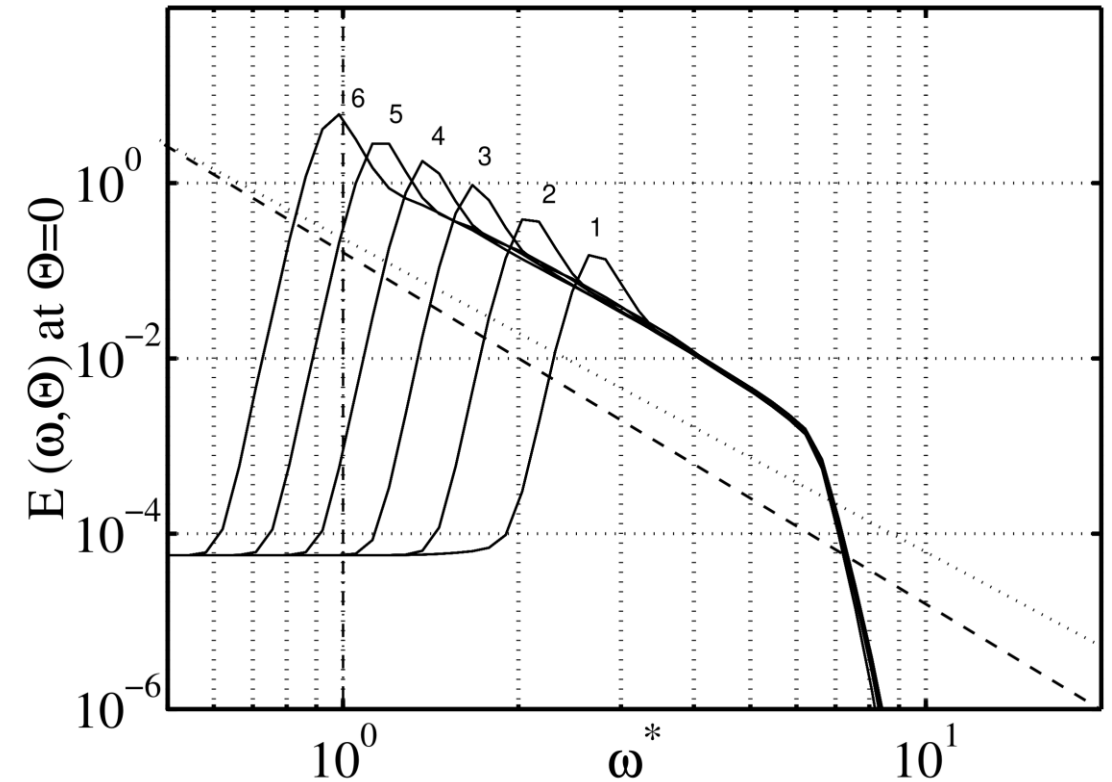
Instead, the Hasselmann equation predicts (under constant wind forcing) perfectly self-similar evolution

$$n(k) = at^{23/11}U(bkt^{6/11})$$

where $U(\xi)$ is the shape function, with an enhanced peak and straight front. Moreover, the Hasselmann equation

$$\frac{\partial n}{\partial t} = 4\pi \int T_{0123}^2 f_{0123} \delta_{0123} \Delta\omega dk_{0123}$$

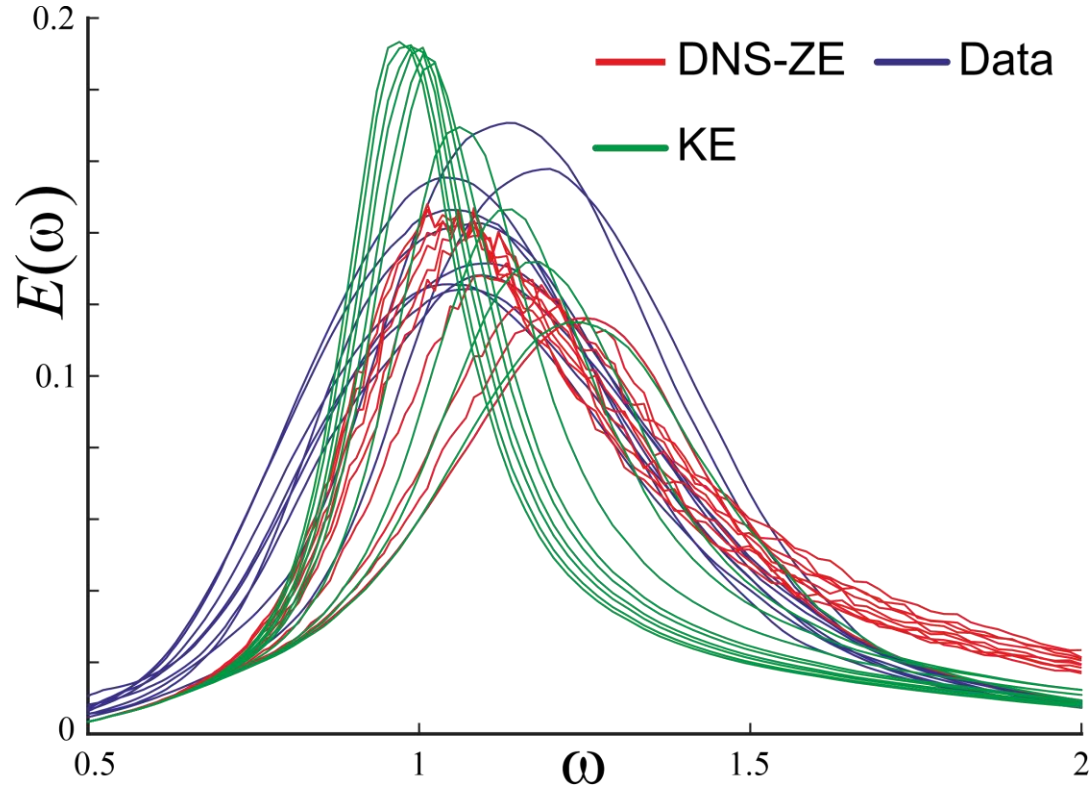
where $f = n_2 n_3 (n_0 + n_1) - n_0 n_1 (n_2 + n_3)$, is homogeneous with respect to spectrum, and thus predicts the same spectral shape for all levels of nonlinearity (even infinitesimal), and the downshift rate has a simple rescaling law (Zakharov *et al* 2015)



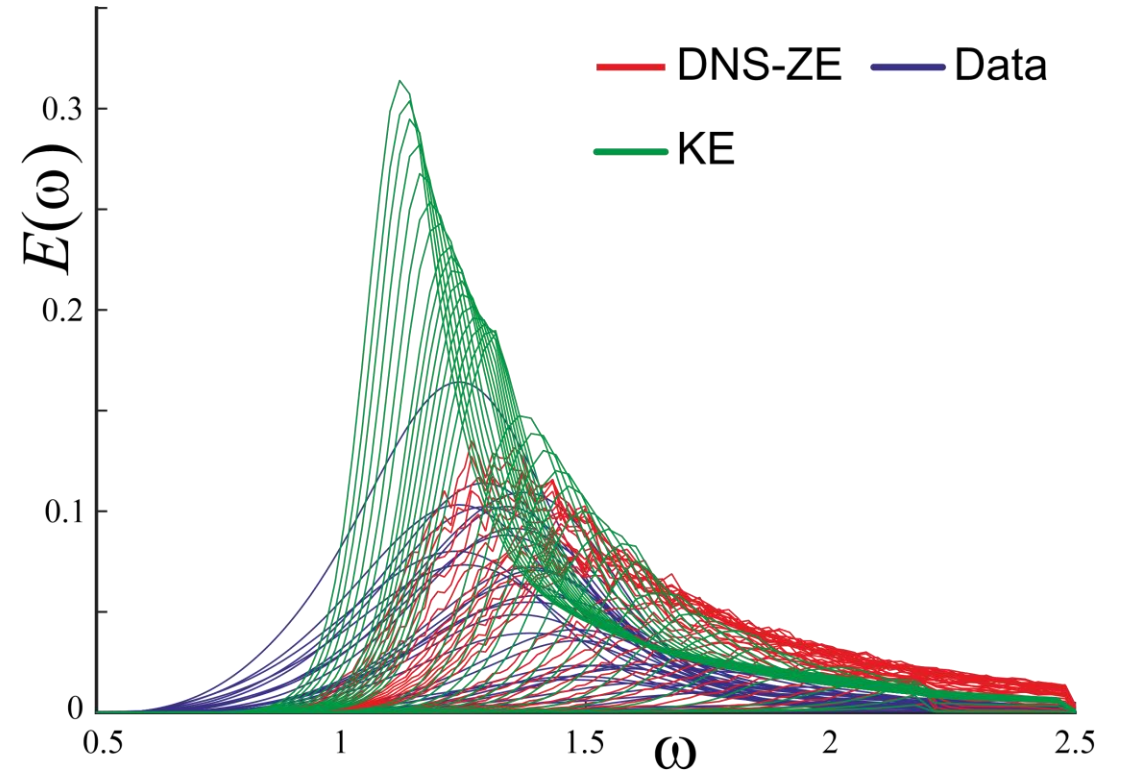
Wind wave spectral development as numerical solution of the Hasselmann equation. Figure from Badulin, Puskarev, Resio & Zakharov (2005)

spectral development

GOTEX



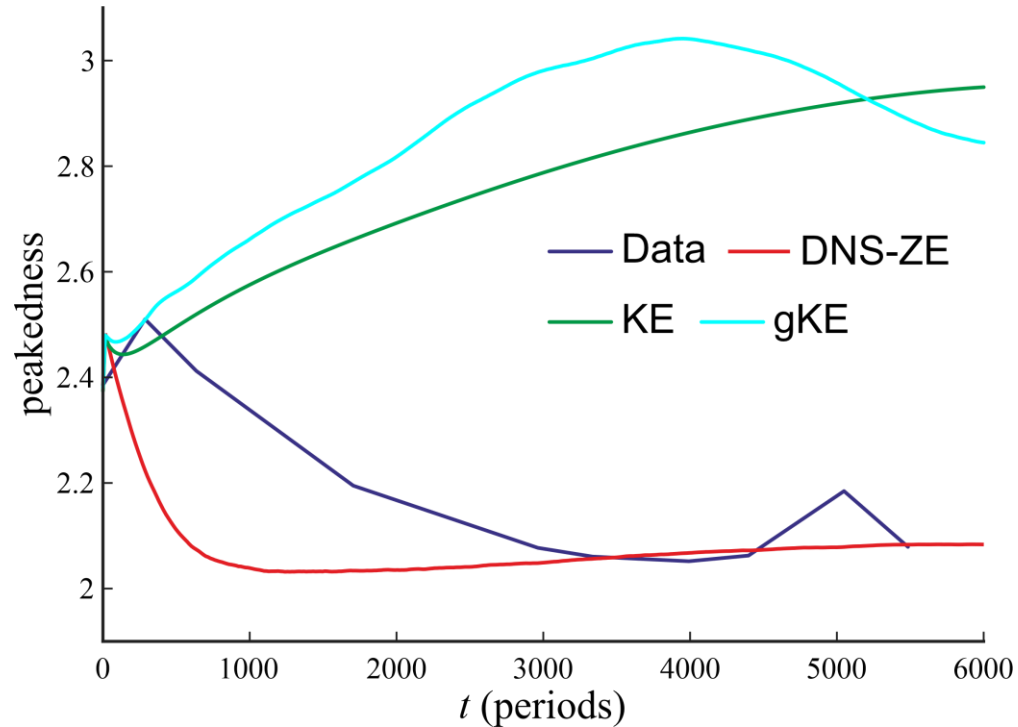
HyMeX



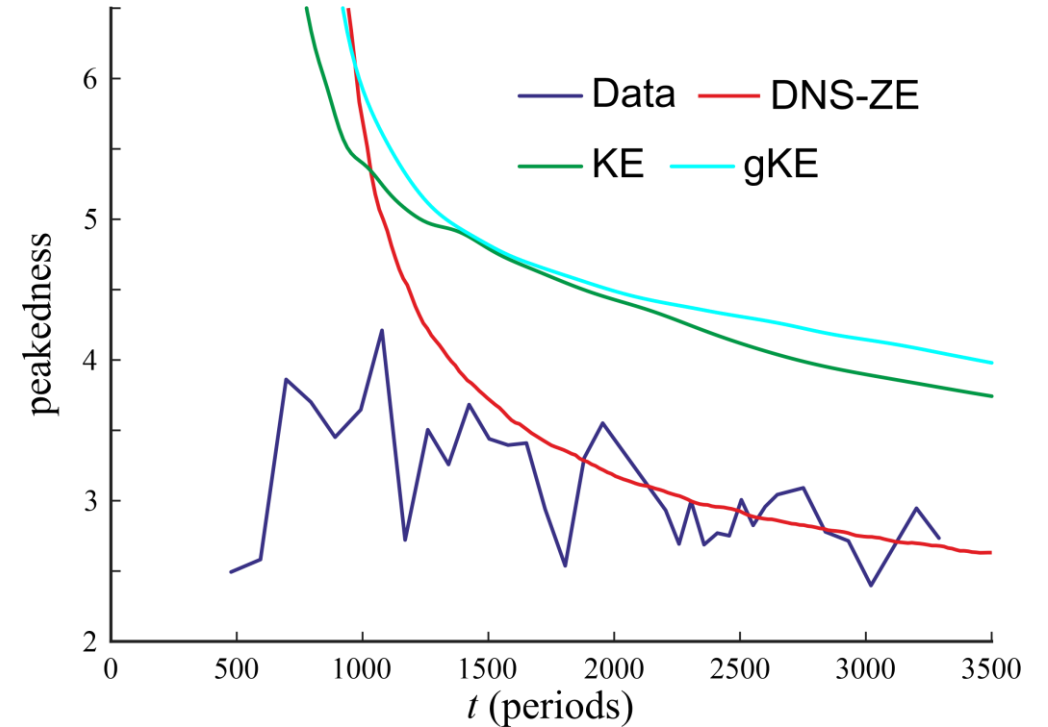
DNS spectra do show self-similar behaviour (under constant wind), and do respect the known asymptotics of wave growth. But they are wider than the KE spectra, have less steep, more rounded spectral front, and wider and lower peak. **Crucially, the DNS spectra appear to be in much better agreement with the measurements.**

peakedness of spectra

GOTEX



HyMeX

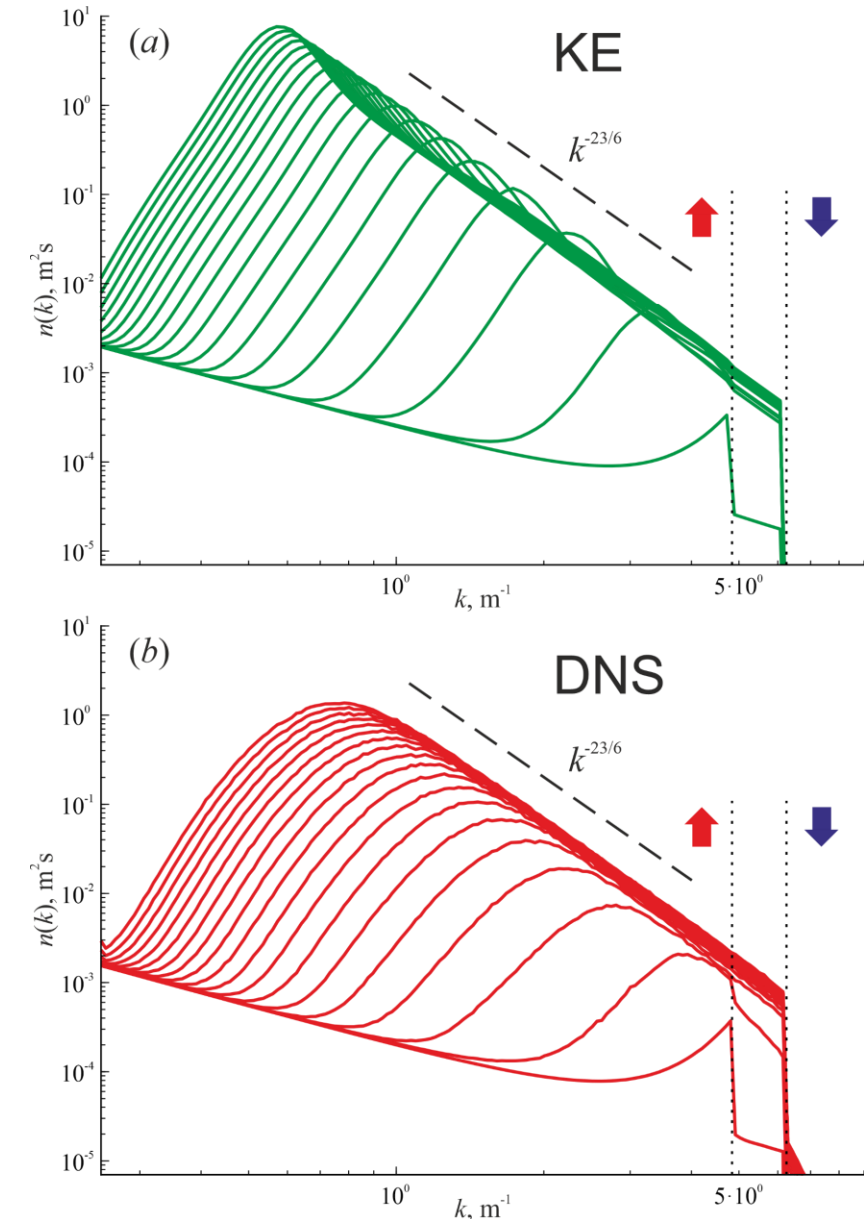


Goda (1970) introduced parameter of spectral peakedness $Q_p = \frac{2 \int \omega S(\omega)^2 d\omega}{[\int S(\omega) d\omega]^2}$

For large times, DNS shows good agreement with measurements, while the kinetic equations considerably overestimate Q_p

so what is the reason for this difference of spectral shapes?

- The KE is derived using a statistical closure: the six-order correlator I_{012345} is reduced to a lengthy expression containing pair correlators and fourth-order cumulants (Zakharov 1999)
- Then, only the leading term is retained
- This is equivalent to assuming infinitesimal non-gaussianity: in fact, the KE can be derived assuming just randomness of initial phases (Onorato & Dematteis 2020)
- Thus, the weak non-gaussianity of a wave field is neglected
- This means that the form of the solution corresponds to the case of infinitesimal amplitude, and the role of finite amplitude effects remains unknown
- The account for non-gaussianity would involve the link between different four-wave cumulants, which would affect the spectral growth rates around the peak

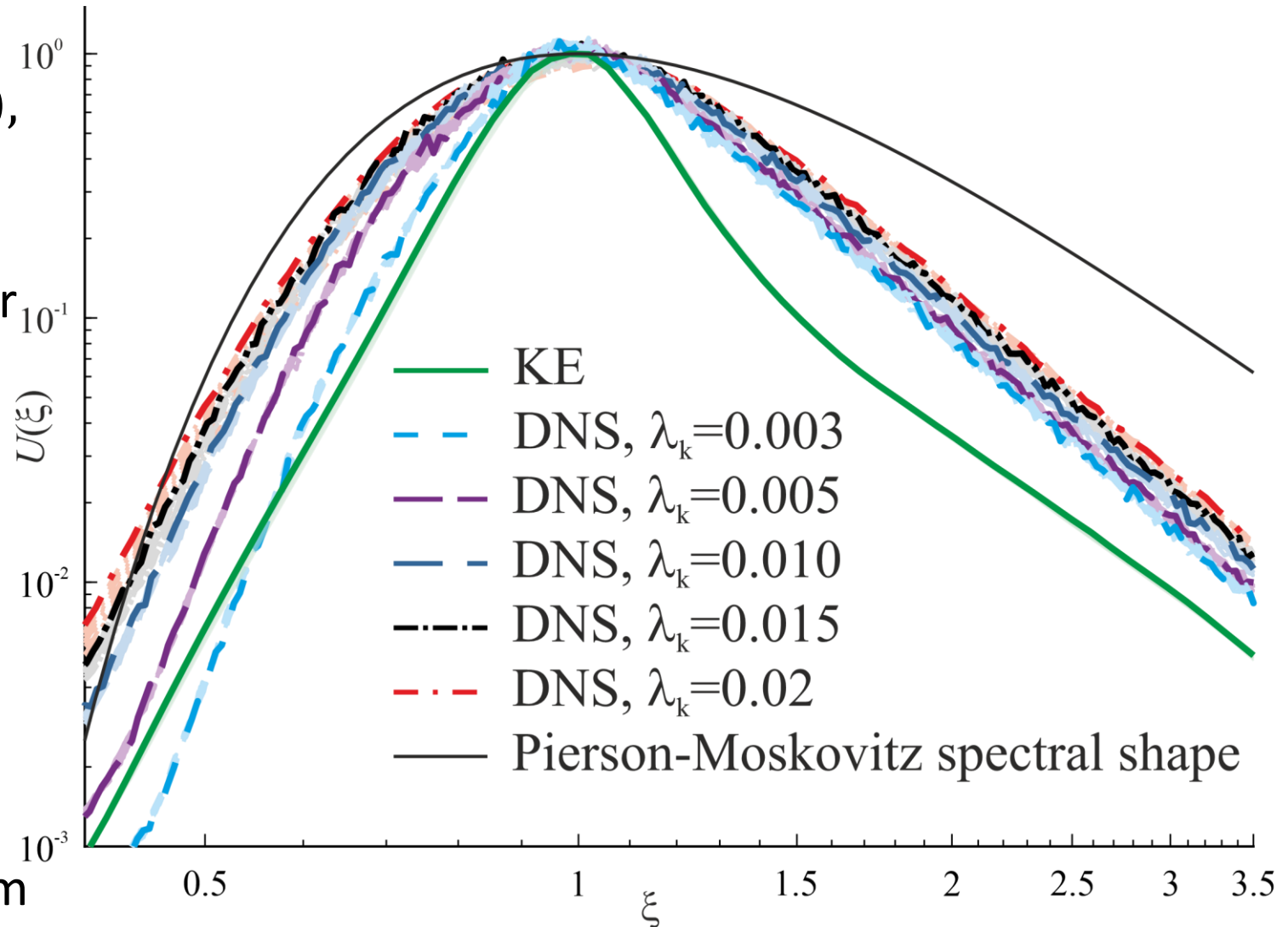


checking the non-gaussianity hypothesis by DNS

- DNS-ZE algorithm is based on the idea of coarse-graining of a wave field (Annenkov & Shrira 2018), which relaxes the resonance condition into $\mathbf{k}_0 + \mathbf{k}_1 - \mathbf{k}_2 - \mathbf{k}_3 = \Delta\mathbf{k}$, in contrast to the standard condition $\Delta\mathbf{k} = 0$.
- Here $|\Delta\mathbf{k}|/k_{min} < \lambda_k \bar{\omega}/\omega_{min}$, and the crucial role is played by parameter λ_k (the coarse-graining parameter)
- If $\lambda_k = 0$, there are no interactions (on a log-spaced grid), and a wave field is gaussian
- When λ_k is increased from zero, the number of wave interactions grows quadratically, the rate of spectral evolution quickly increases and then saturates, on a certain value of λ_k depending on grid resolution
- It is convenient to use λ_k as a way to create wave fields with the same level of nonlinearity, but different levels of non-gaussianity

role of finite non-gaussianity

- We use a very refined 320x70 grid
- When $\lambda_k = 0.003$, the number of interactions is relatively small (10^8), and the wave field is nearly gaussian, with very slow evolution
- With the increase of λ_k , the number of interactions increases to $7 \cdot 10^9$
- We are interested in spectral shape at the advanced stage of the evolution, when the self-similar shape is reached
- For small λ_k the shape function is close to the KE spectral shape
- With the increase of λ_k the shape function approaches a different form resembling Pierson-Moskowitz



conclusions

- Hasselmann equation, on which all wave modelling is based, corresponds to the second-order approximation in nonlinearity, but zeroth-order in non-gaussianity
- Although non-gaussianity is weak, it leads to considerable distortion of the spectral shape
- Integral parameters of a wave field do not appear to be affected, with the error remaining within the uncertainty of wave breaking, which the DNS has to take into account
- Spectral shapes obtained by the DNS are in much closer agreement with measurements

implications

- Huge, for all applications where spectral shape is significant (extreme events, hazard risk assessment, sediment transport models, etc)
- Since wind wave models are carefully optimised against measurements, knowledge of systematic errors can drastically improve the quality of such optimisations
- In the more general context, the insight into the role of non-gaussianity in kinetic models is significant for the whole of the physical kinetics, in particular directly applicable to all wave fields with cubic nonlinearity