Guessing the missing half of a geophysical field with blunt extension of discrete Universal Multifractal cascades

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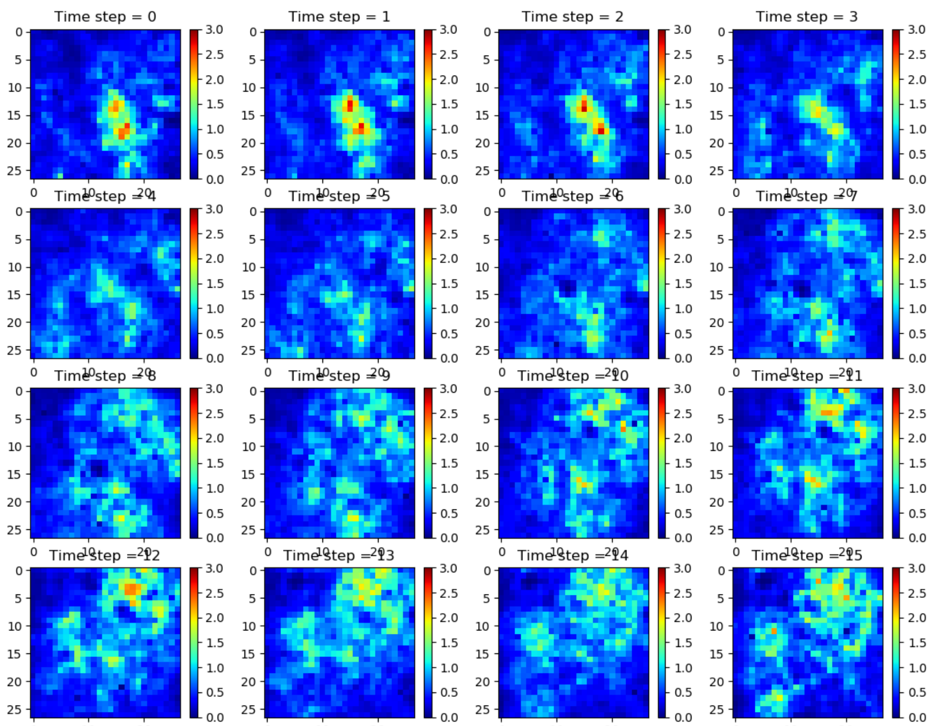
Extension of the blunt discrete UM cascades to space-time processes:



Development of an iterative process to stochastically guess the missing half a multifractal field.

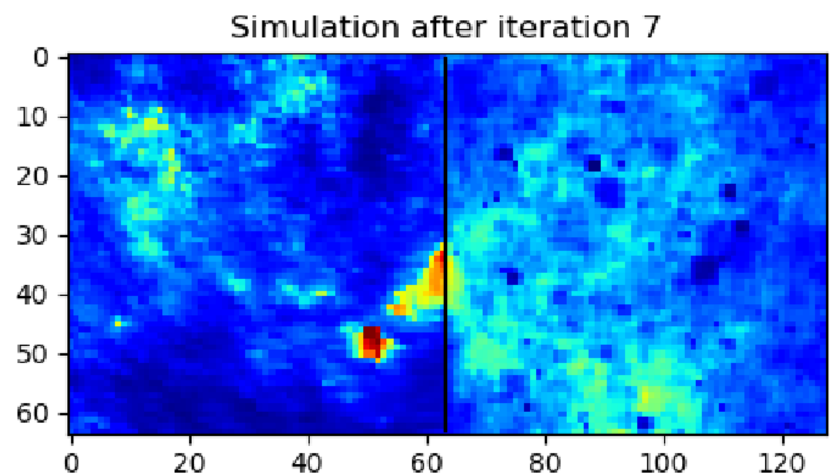


Illustration with a rainfall field (actual data on the left, simulations on the right)

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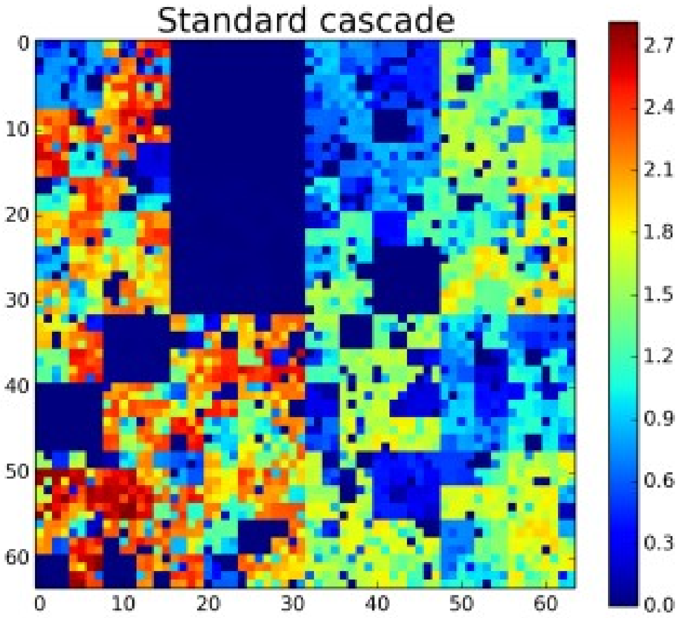
Introduction and position of the problem

Where are the blunt extension of discrete cascades coming from ?

- Ubiquity of scale issues in geosciences
- Often illustrated / handled with discrete cascades
- Such cascades are rather (at best!) pedagogic and suffer from severe, usually neglected, issues notably “translation invariance”
- Continuous in scale cascades were introduced to overcome such difficulty while being directly linked to the underlying process equations.
- A sort of intermediate solution keeping the tree structure is suggested.



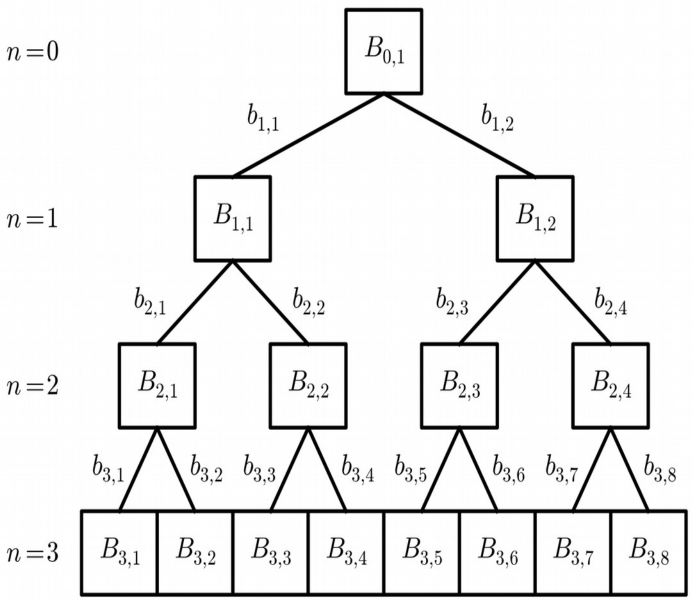
Outline of the presentation:

- Reminder on blunt extension of discrete UM cascade process (Gires et al. 2020)
- Extension to space-time processes
- Application to guessing the missing half of a field, which is a common issue in geosciences

Reminder on blunt extension of discrete UM cascades

Discrete UM cascades

Distributing an "intensity" in space and time



$$B_{n,i} = B_0 \prod_{i=s}^{n} (b_s)$$

Increment

Singularity

$$b_i = \lambda_1{}^{\gamma_{b_i}}$$

$$\begin{aligned} B_{n,i} &= B_0 \lambda_1^{\sum_{s=1}^{n} \gamma_{b_s}} \\ &= B_0 \lambda_n^{(\sum_{s=1}^{n} \gamma_{b_s})/n} \\ &= B_0 \lambda_n^{\gamma_{n,i}} \end{aligned}$$

What is needed :
- How to split a structure into sub-structures ?
- Probability distribution of the random multiplicative increment ?

→ **Both features are scale invariant**

Universal Mulfractals (Schertzer and Lovejoy, 1987):
- Conservative fields fully defined with the help of two parameters:
  - C_1 : mean intermittency
  - α : multifractality
- Moments scale with resolution:

$$\langle B_\lambda^q \rangle \approx \lambda^{K(q)}$$

With : $K(q) = \dfrac{C_1}{\alpha - 1}(q^\alpha - q)$

Resolution

$$\lambda = L/l$$

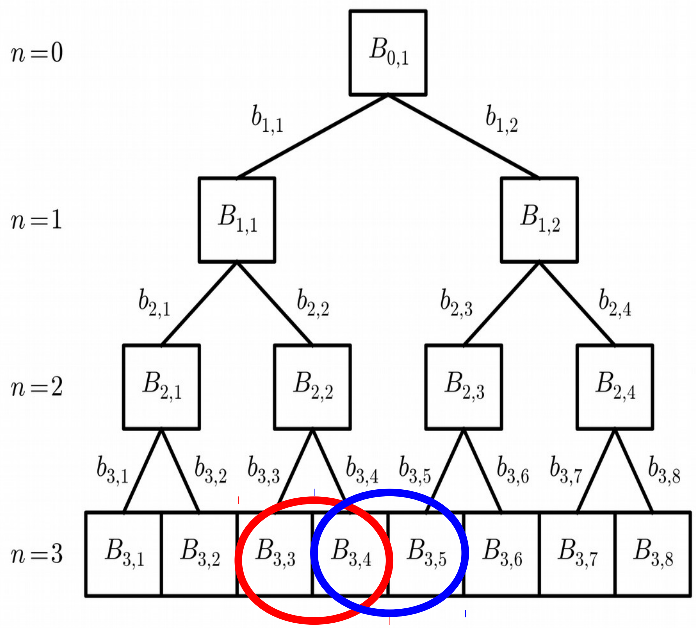
Scaling moment function

Outer scale

Observation scale

Reminder on blunt extension of discrete UM cascades

No translation invariance issue



$$B_{n,i} = B_0 \prod_{i=s}^{n} (b_s)$$

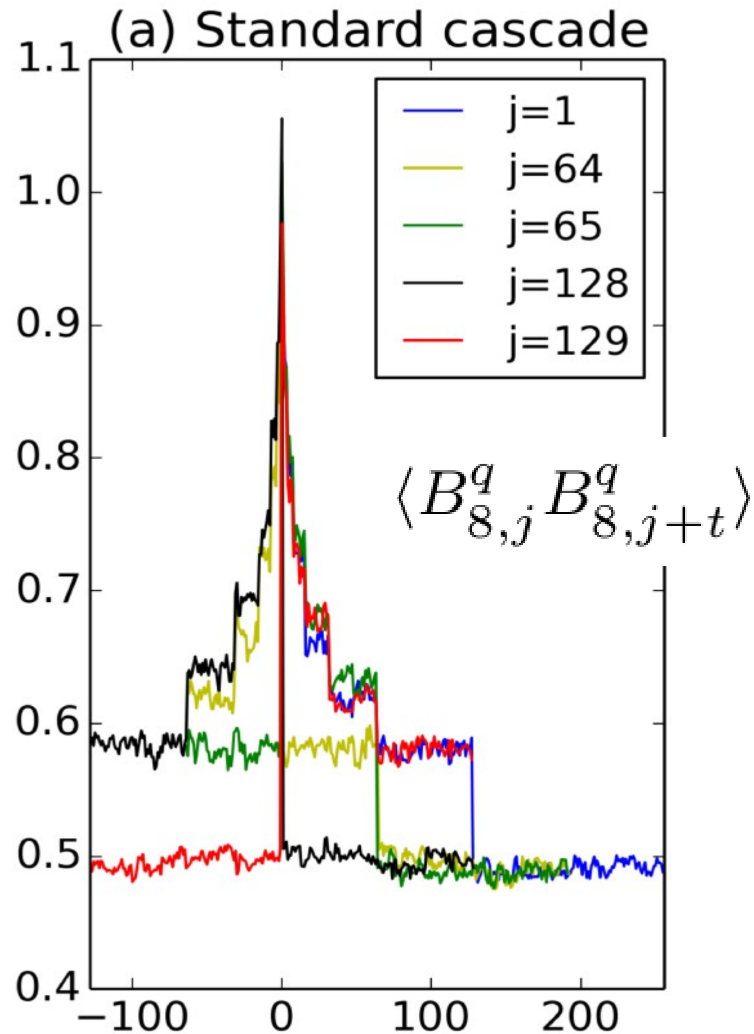


Illustration for 256 = 2^8 time steps series, with α=1.6 and C_1=0.2

Reminder on blunt extension of discrete UM cascades

Description of the process

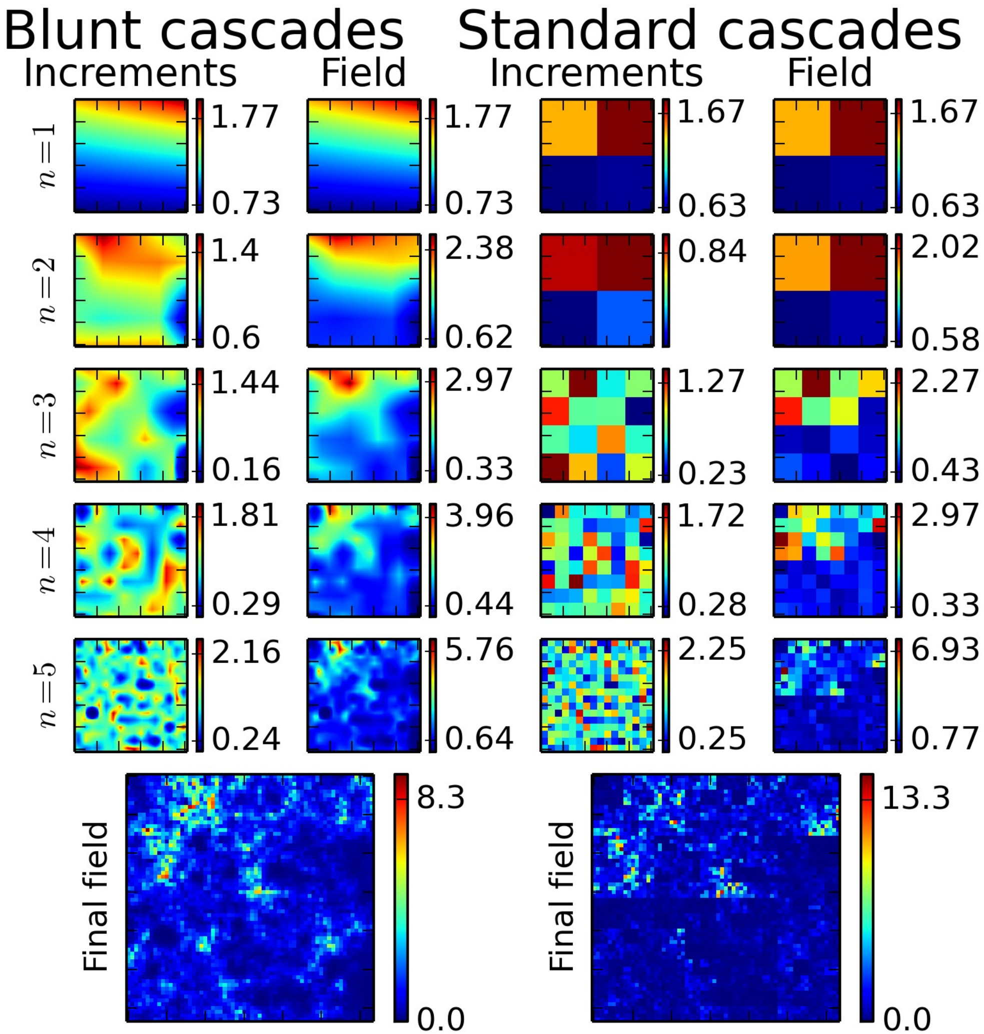


Illustration in 2D, with α=1.6 and C_1=0.2

- Introduction of the final resolution

- Geometric interpolation of the increments ~ linear interpolation of the singularities (moving widow of 2h+1 increments)

- Renormalization to ensure conservation of the mean (on average)

Theoretical expectations (numerically confirmed) :

$$K_{\text{blunt}}(q) = \frac{C_1 S^1(\alpha,h)}{\alpha - 1}(q^\alpha - q) = S^1(\alpha,h)K(q)$$

$$\begin{aligned} \alpha_{\text{blunt}} &= \alpha \\ C_{1,\text{blunt}} &= S^1(\alpha,h)C_1 \end{aligned}$$

Reminder on blunt extension of discrete UM cascades

Back to the translation invariance issue

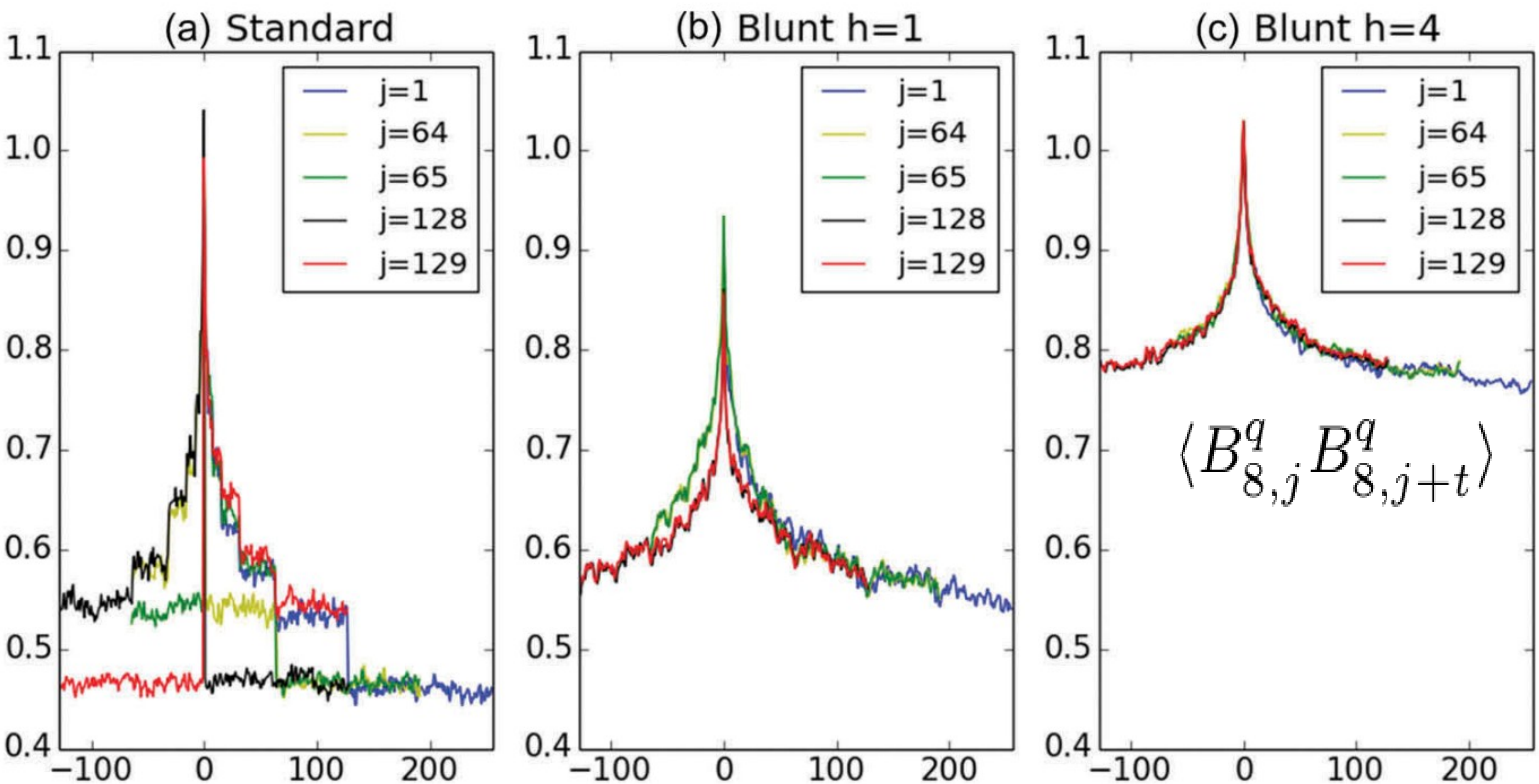


Illustration for 256 = 2^8 time steps series, with α=1.6 and C_1=0.2

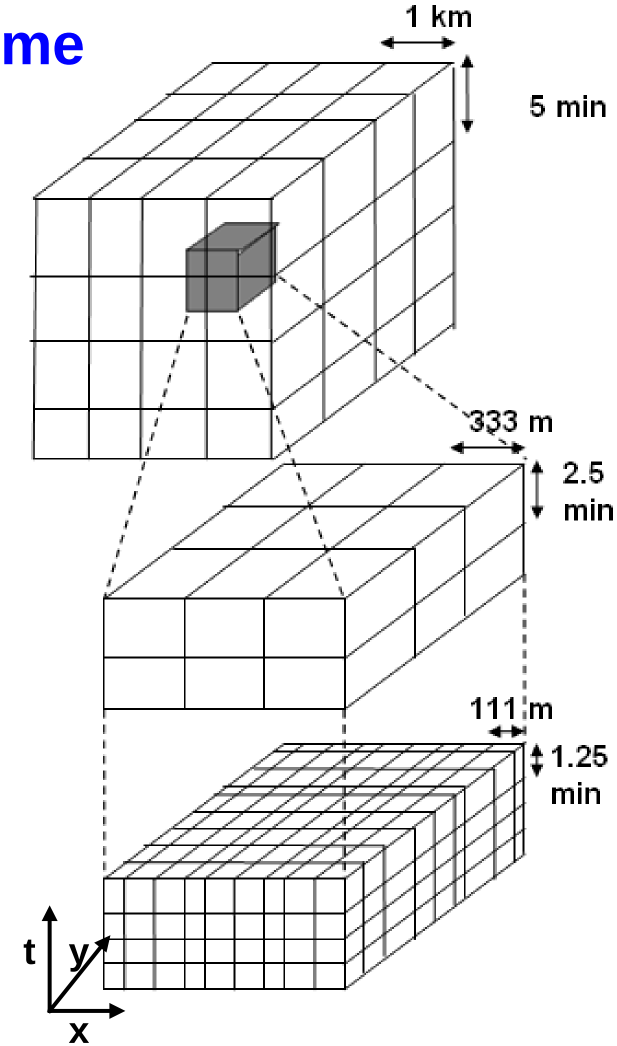
Extension to space-time processes

Introduction of an anisotropy between space and time

***H*$_t$ = scaling anisotropy coefficient**

$\lambda_{xy} = 3$

$\lambda_t = \lambda_{xy}^{1-Ht} \sim 2$



Biaou et al, 2005

Geometric interpolation of sharp increments in 3D boxes

Same equations as before remain valid, with :

$$S(\alpha,h) = S(\alpha,h)_{xy}^2 S(\alpha,h)_t$$

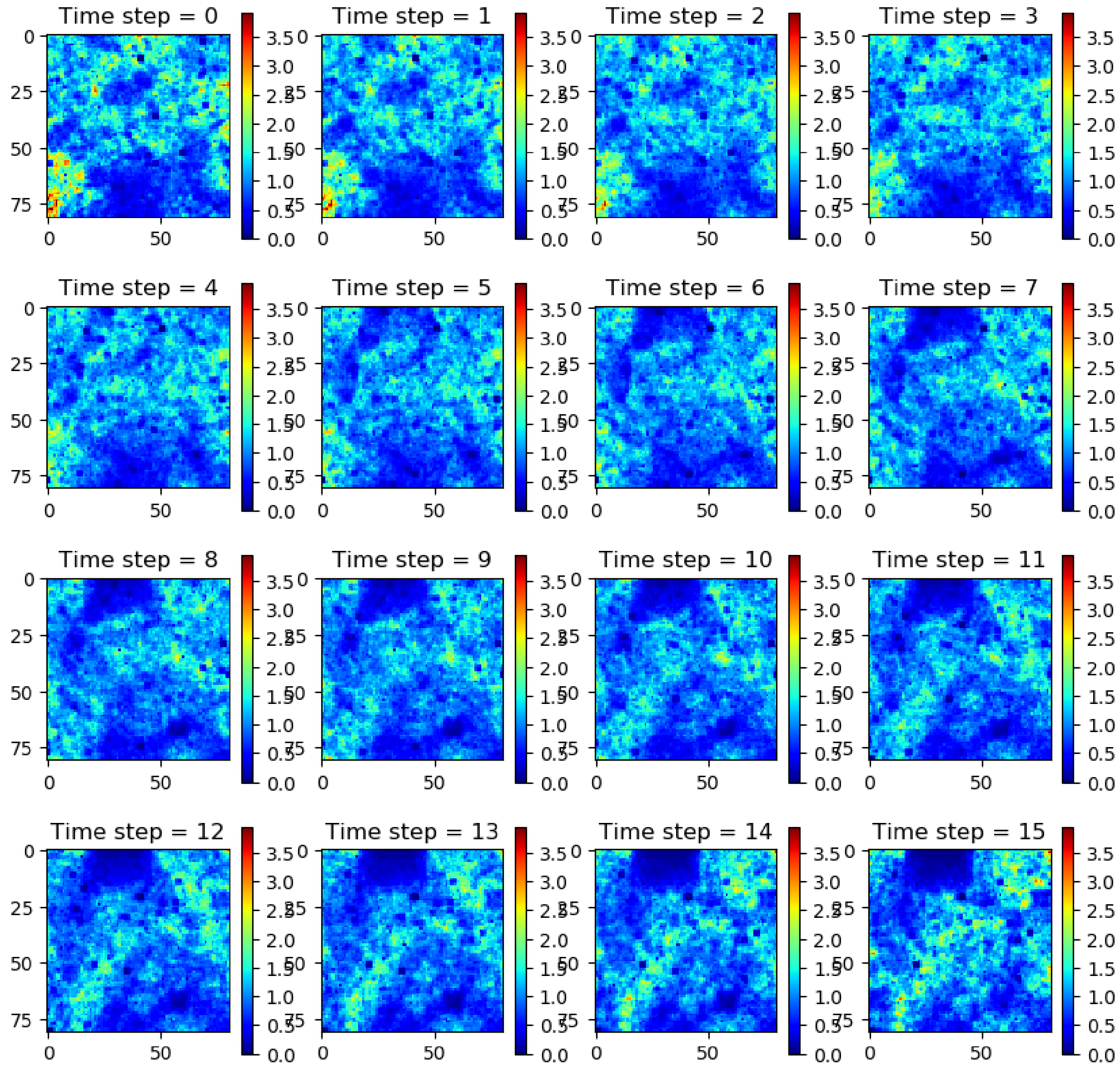
Tuning of renormalization according to voxel to account for side effects

Extension to space-time processes

Illustration

4 cascades steps (final field of size 81 x 81 x 16) with initial input :

α = 1.7
C_1=0.4
h=4

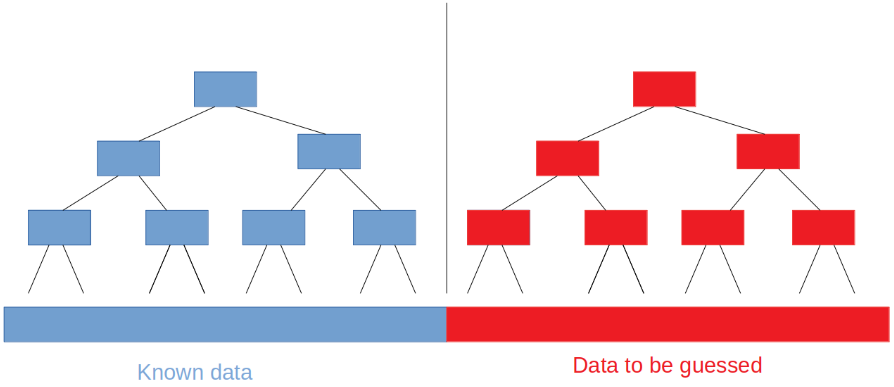


Guessing the missing half

An iterative process (illustrated in 1D)

- Step 1 : computing a first guess of the increments

Stochastically with measured α and C_1 on the available data for the red ones, set to one for blue ones



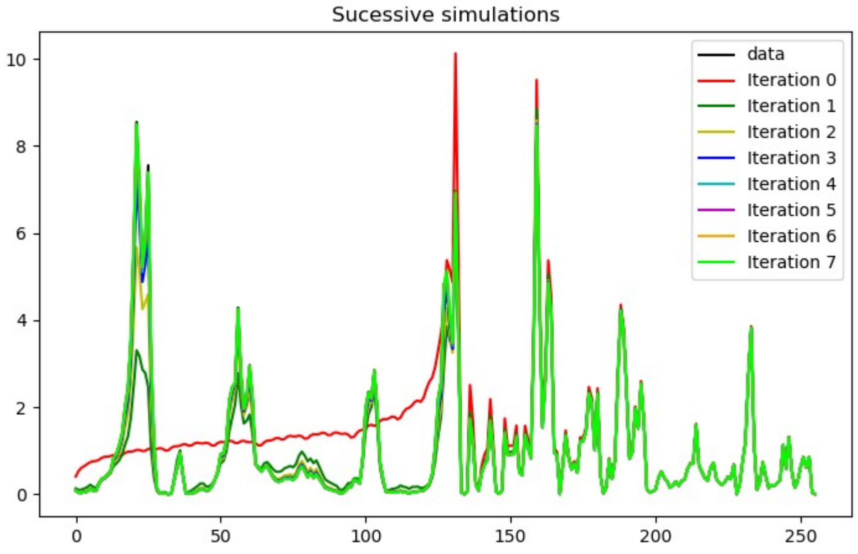
- Step 2 : iterations to tune the "blue" increments so that the implementation of the blunting process of the sharp increments yields the available data on the blue part

(i) Computation of blunt simulation with sharp increments

(ii) Computation of the ratio data/simulation

(iii) Upscaling the "ratio" to assess the corresponding increments and update accordingly the "blue" increments

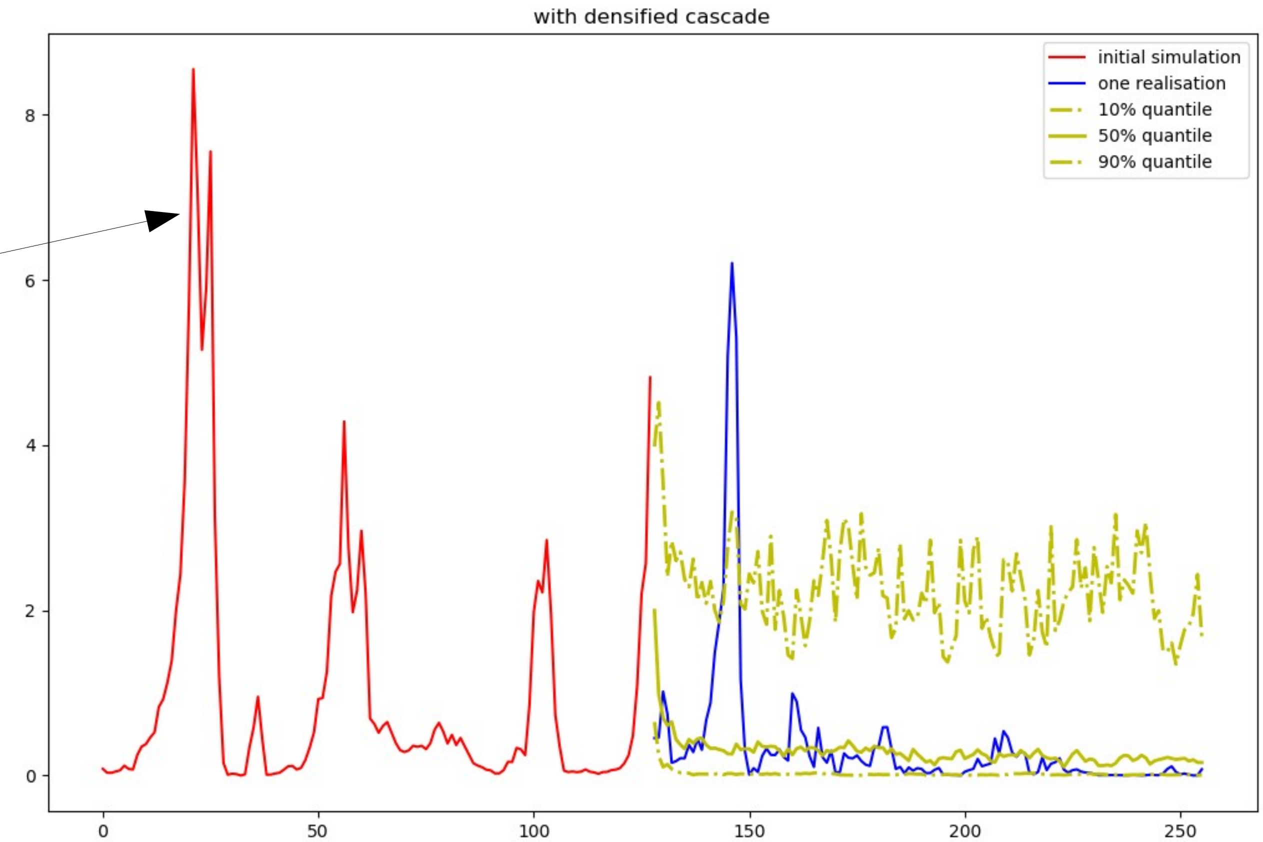
(iv) Repeat the process



Introduction and position of the problem

Implementation in 1D

30 May 2016, disdrometer series (30 s) with α=1.53 and C_1=0.22

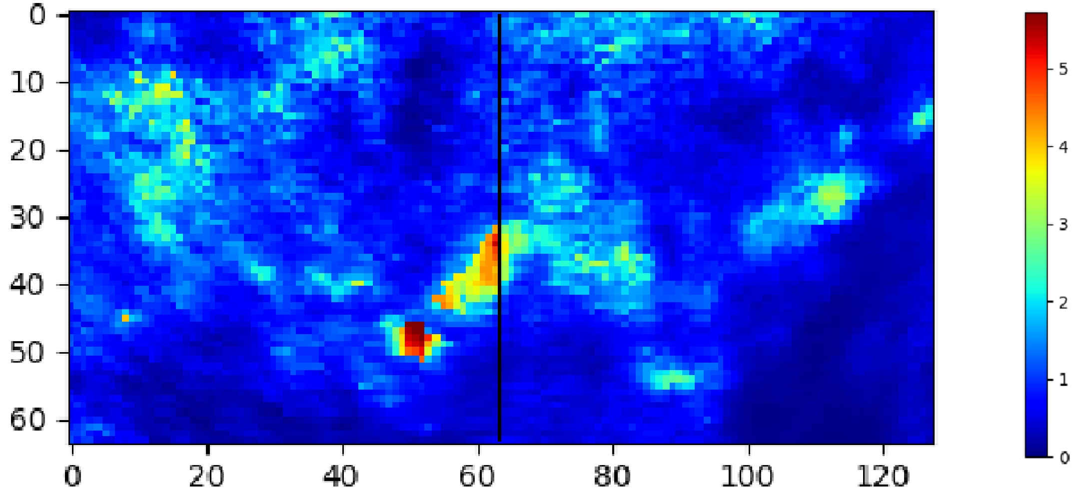


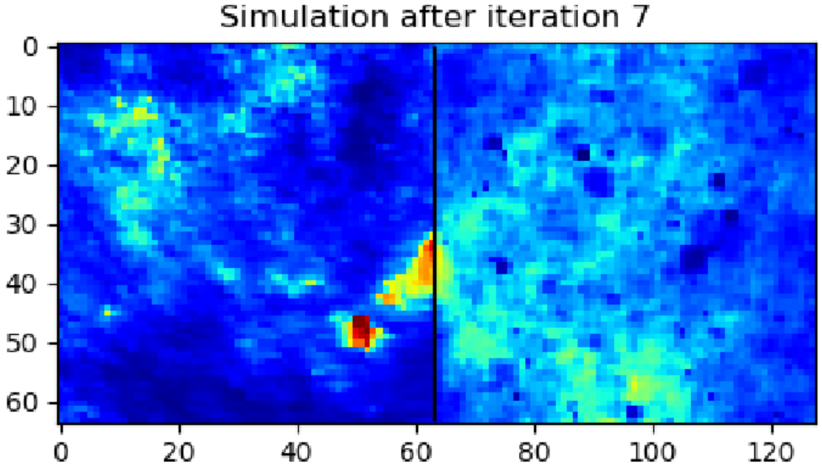
- Stochastic process → an ensemble of potential outcome can be generated
- Display of the quantiles for 100 realisations along with one specific realisation
- Rapid decrease of the expected rainfall after few time steps

Introduction and position of the problem

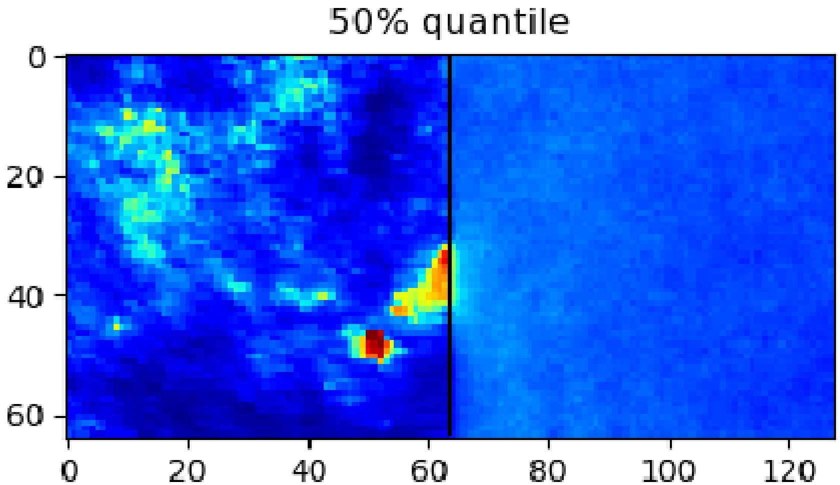
Implementation in 2D

64 x 128 pixels from dual pol X-band radar of Ecole des Ponts ParisTech on 16 September 2015 (pixel size = 250m)





Outcome for one realisation using only the left portion of the field as input.



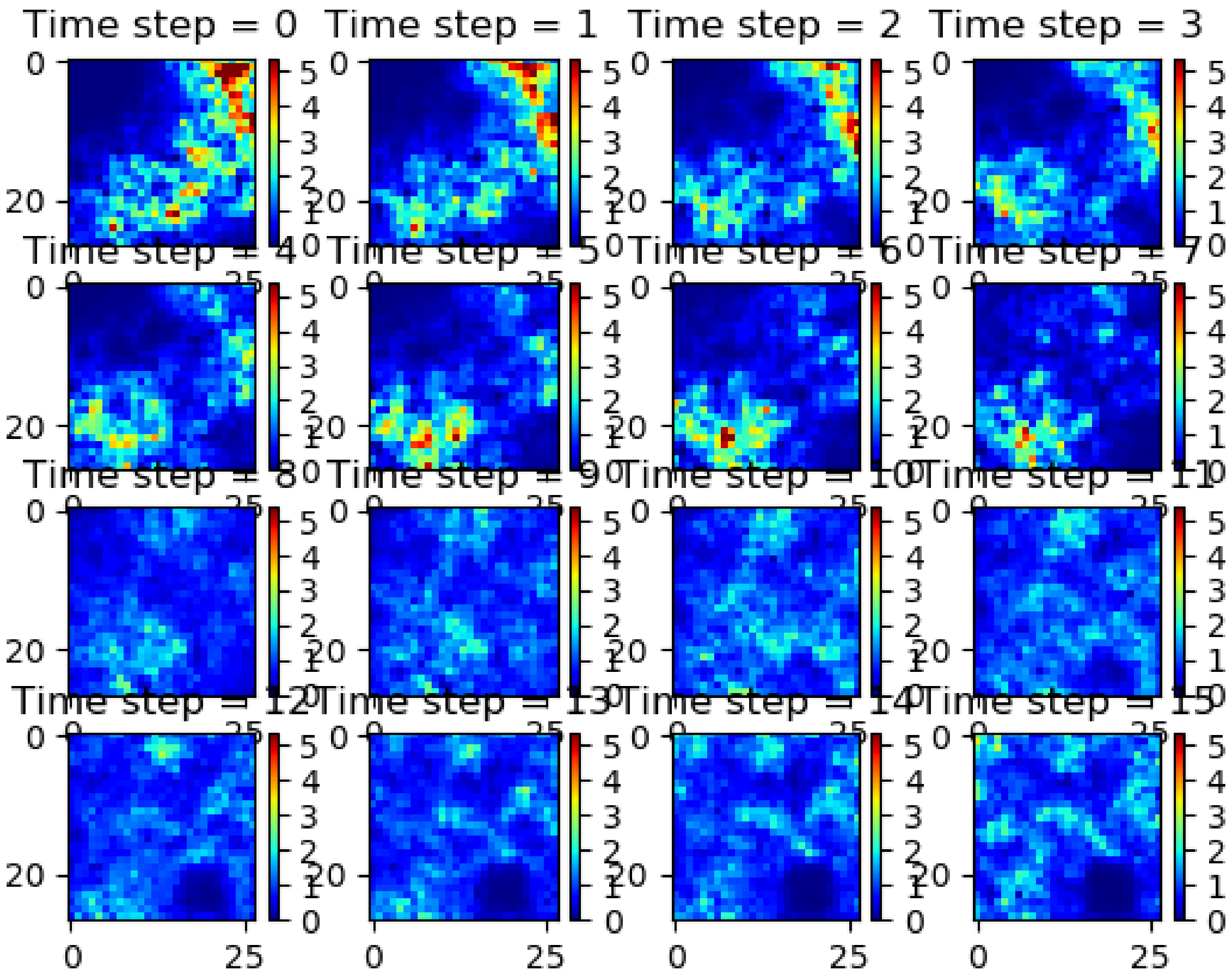
50% quantile over 100 realisations (rapid decrease of rainfall after the transition)

Introduction and position of the problem

Implementation in 3D (with simulations)

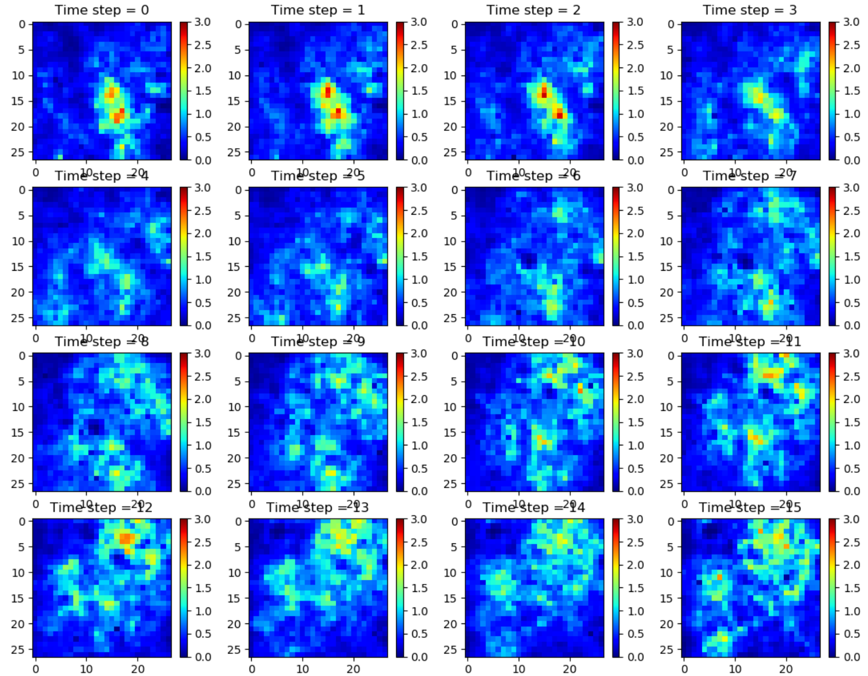
Simulation

Realisation of the next steps with developed process



Conclusion

Summary and perspectives :



- Blunt extension of discrete UM cascades is implemented in space-time :

- Geometric interpolation of the increments ~ linear interpolation of the singularities at each cascade step
- Simulated field exhibit multifractal behaviour, at least with a very good approximation level, in agreement with expectations

- An iterative process is suggested to address a common geophysical issue which is guessing the missing half of a field

- Implementation in 1D, 2D and 3D on illustrative examples
- Further implementation and validation should be carried out
- Advection should be accounted for to switch to actual nowcasting of rainfall fields