

Why rogue waves occur atop depth transitions

(Funded Project: ‘Extreme wind and wave loads on the next generation of offshore wind turbines’)

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Motivation: experiments by Trulsen et al. (2012, PoF)

Random variable: $\tilde{\zeta}_s$

Mean: $m = \langle \tilde{\zeta}_s \rangle$

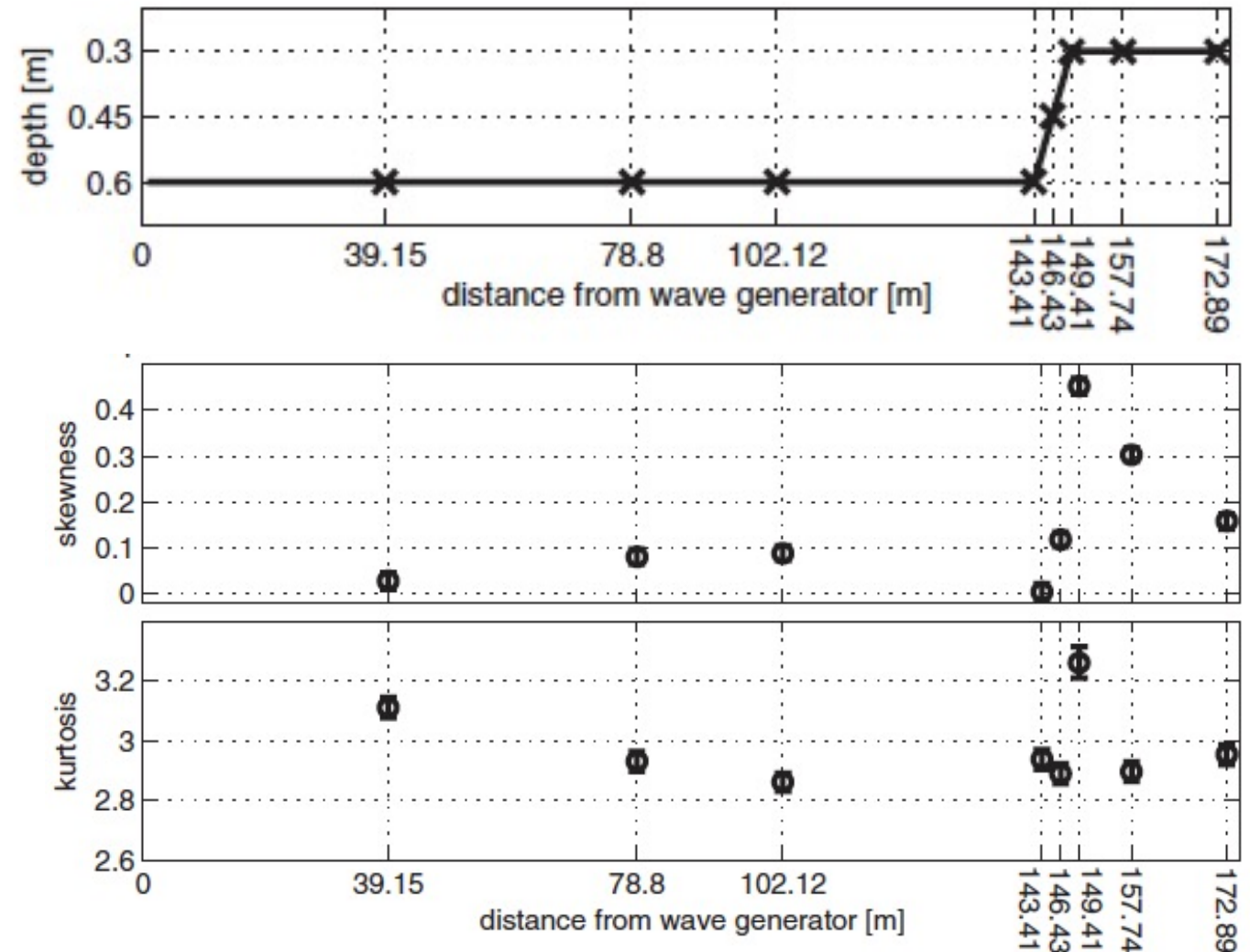
Variance: $v = \langle (\tilde{\zeta}_s - m)^2 \rangle$

Skewness: wave asymmetry

$$s = \langle (\tilde{\zeta}_s - m)^3 \rangle / v^{3/2}$$

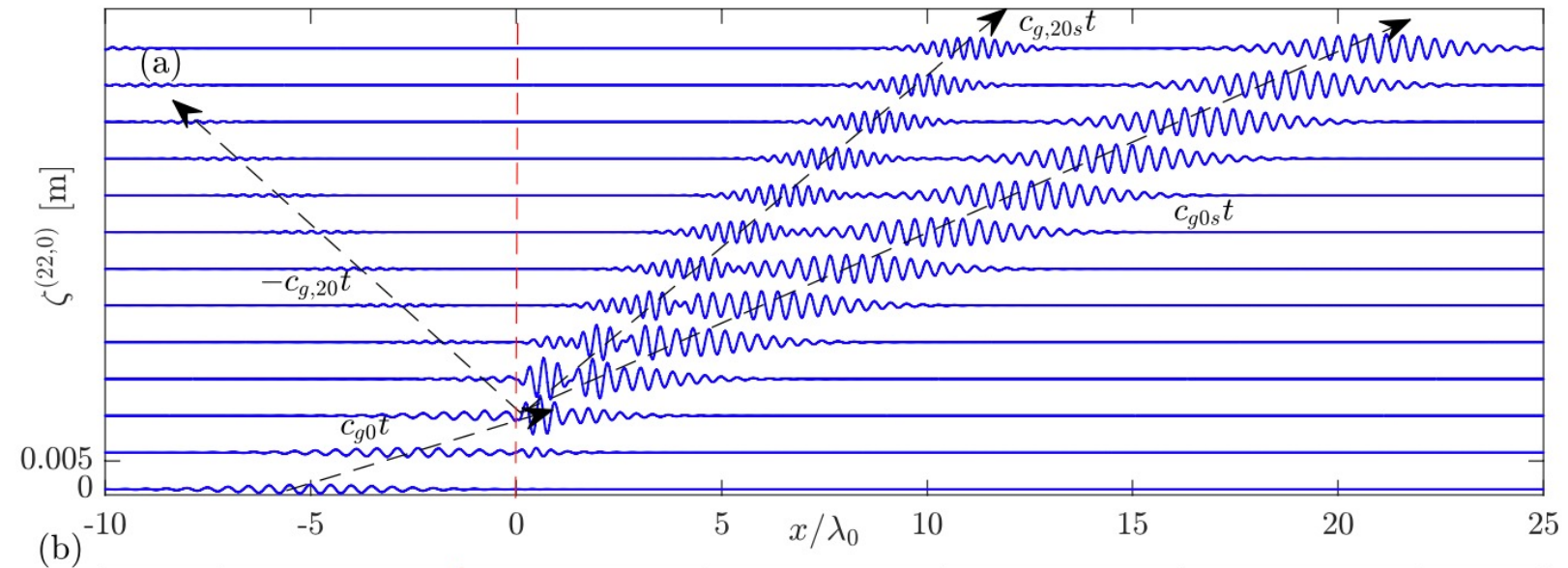
Kurtosis: freak or abnormal waves > 3

$$\kappa = \langle (\tilde{\zeta}_s - m)^4 \rangle / v^2$$

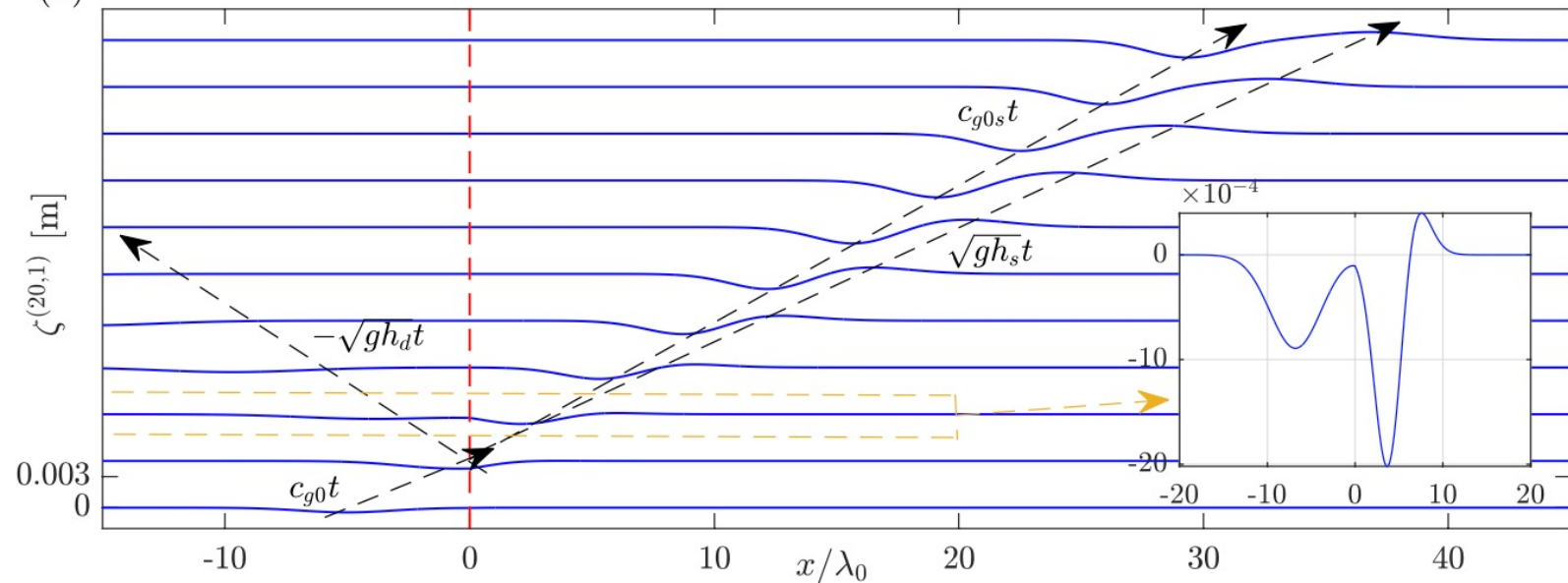


Second-order theory for wavepackets: bound wave release

Super-harmonic
(space,time)-plot

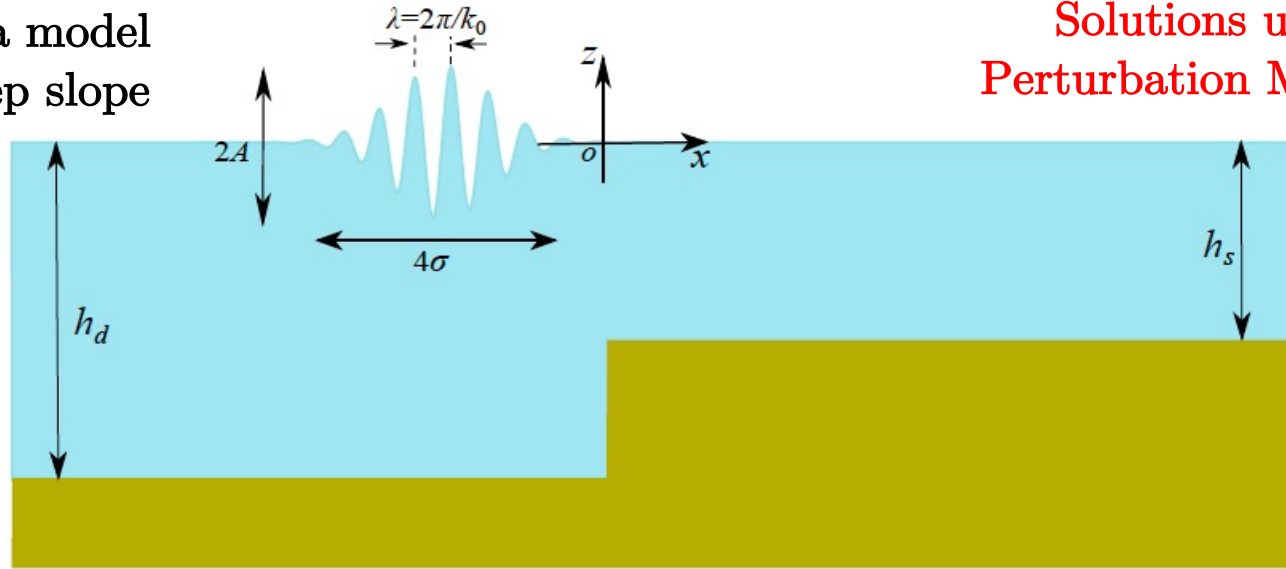


Sub-harmonic
(space,time)-plot



Second-order theory for waves over a step: deterministic model

Step as a model
for a steep slope



Solutions using
Perturbation Methods

Incident wave field

$$\zeta(x, t) = \zeta^{(1)}(x, t) + \zeta^{(2)}(x, t) \text{ for } x < 0, \text{ with} \quad (1a)$$

$$\zeta^{(1)}(x, t) = A \cos(k_0 x - \omega_0 t + \theta_0) \text{ and} \quad (1b)$$

$$\zeta^{(2)}(x, t) = k_0 A^2 [C_{20,b} + C_{22,b} \cos(2(k_0 x - \omega_0 t + \theta_0))], \quad (1c)$$

Transmitted wave field

$$\zeta_s^{(1)}(x, t) = A_s (x/c_{g0s} - t) \cos \psi_{0s} + \zeta_{Es}^{(1)}, \text{ with} \quad (3a)$$

$$A_s = |T_0| A (x/c_{g0s} - t), \text{ and} \quad (3b)$$

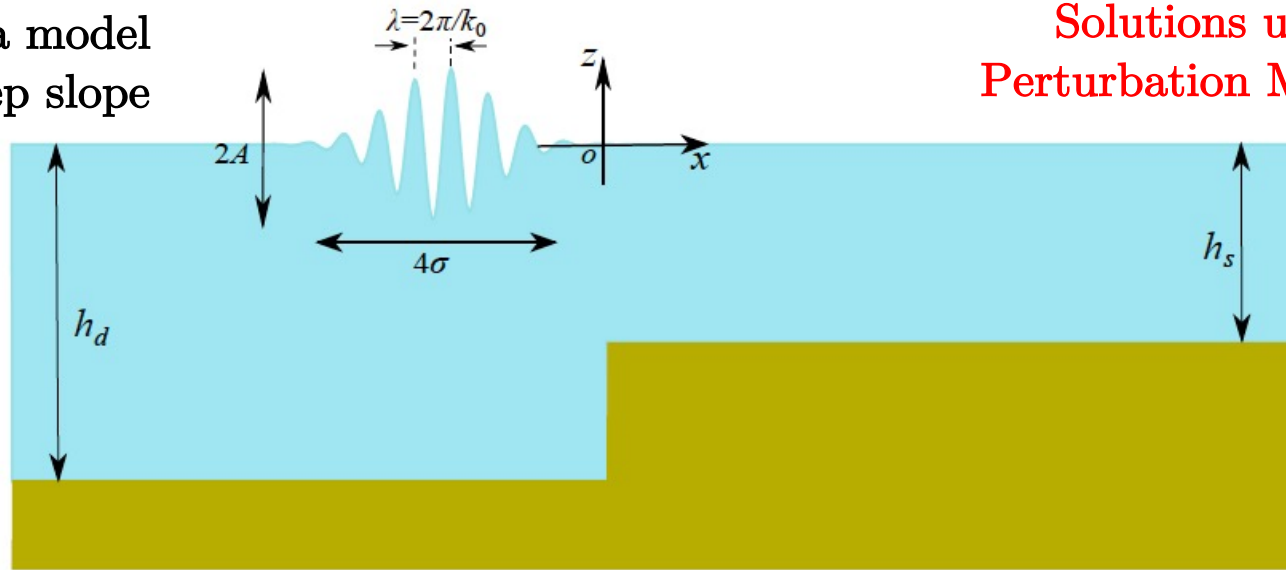
$$\zeta_s^{(2)}(x, t) = \zeta_s^{(20)} + \zeta_s^{(22)}, \text{ with} \quad (3c)$$

$$\begin{aligned} \zeta_s^{(20)}(x, t) = & k_{0s} A_s^2 (x/c_{g0s} - t) C_{20,b} \\ & + k_{0s} A_s^2 (x/\sqrt{gh_s} - t) C_{20,f} + \zeta_{Es}^{(20)}, \end{aligned} \quad (3d)$$

$$\begin{aligned} \zeta_s^{(22)}(x, t) = & k_{0s} A_s^2 (x/c_{g0s} - t) C_{22,b} \cos(2\psi_{0s}) \\ & + k_{0s} A_s^2 (x/c_{g20s} - t) C_{22,f} \cos(2\psi_{0s} + \psi_{22,f}) \\ & + \zeta_{Es}^{(22)}, \end{aligned} \quad (3e)$$

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$$\zeta^{(1)}(x, t) = A \cos(k_0 x - \omega_0 t + \theta_0) \text{ and} \quad (1b)$$

$$\zeta^{(2)}(x, t) = k_0 A^2 [C_{20,b} + C_{22,b} \cos(2(k_0 x - \omega_0 t + \theta_0))], \quad (1c)$$

Sub- and super-harmonic bound waves

Transmitted wave field

$$\zeta_s^{(1)}(x, t) = A_s (x/c_{g0s} - t) \cos \psi_{0s} + \zeta_{Es}^{(1)}, \text{ with} \quad (3a)$$

$$A_s = |T_0| A (x/c_{g0s} - t), \text{ and} \quad (3b)$$

$$\zeta_s^{(2)}(x, t) = \zeta_s^{(20)} + \zeta_s^{(22)}, \text{ with} \quad (3c)$$

$$\zeta_s^{(20)}(x, t) = k_{0s} A_s^2 (x/c_{g0s} - t) C_{20,b} + k_{0s} A_s^2 (x/\sqrt{gh_s} - t) C_{20,f} + \zeta_{Es}^{(20)}, \quad (3d)$$

$$\zeta_s^{(22)}(x, t) = k_{0s} A_s^2 (x/c_{g0s} - t) C_{22,b} \cos(2\psi_{0s}) + k_{0s} A_s^2 (x/c_{g20s} - t) C_{22,f} \cos(2\psi_{0s} + \psi_{22,f}) + \zeta_{Es}^{(22)}, \quad (3e)$$

Bound
waves must
adjust and
release free
waves:
modulated
'beating'

Comparison between experiments, numerics and theory.

Kurtosis:

