

## Multifractal (MF) analysis of extinction coefficient ( $\sigma_e$ ) and its consequences in characterizing atmospheric visibility

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NP3.3

Scaling, Multifractals from Urban to Climate scales, from Theories to Big Data Analysis and Simulations

## Visibility

A measurement of ‘transparency of atmosphere’, measured in km.

- transportation sectors (aviation)
- Freespace optic communications
- understanding regional variations in air quality and climate

## Measurement of visibility

$$F = F_0 e^{\sigma_e x}$$

$$MOR = -\frac{\ln(0.05)}{\sigma_e} = -\frac{2.998}{\sigma_e}$$

Bouger-  
Lambert's law



Koschmeider's  
theory



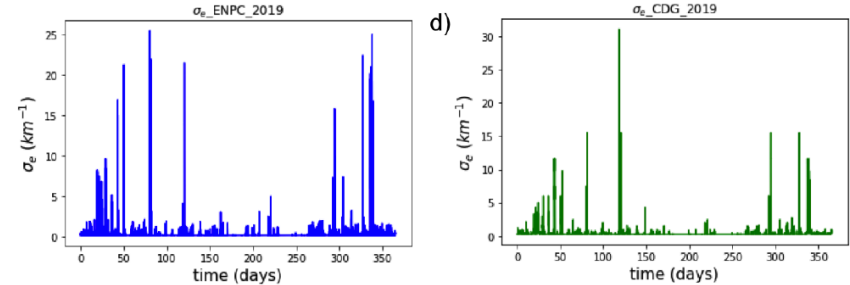
MOR

## Universal Multifractals (UM)

Using the frame work of UM, variability in visibility is studied here from 1 year data at two locations in Paris.

$\sigma_e$  was found to be the underlying field and is influenced by instrument upper threshold

## Data used



| location | data collection time | resolution | data range  |
|----------|----------------------|------------|-------------|
| ENPC     | Jan 2019 – Dec 2019  | 30 s       | 0- 20 km    |
| CDG      | Jan 2019 – Dec 2019  | 30 min     | 0.1 - 10 km |

## UM analysis

- On  $\sigma_e$  as it is the objectively measured quantity by visibility measuring instruments
- Results are biased by instrument upper threshold
- Simulations to reproduce this bias for retrieving original UM parameters

Visibility ( $MOR/\sigma_e$ ) shows a highly non-Gaussian distribution, which makes their characterization difficult

The framework of Universal Multifractals (UM) enables to characterize the extreme variability of geophysical fields across scales, with the help of a limited number of parameters with physical meaning (Schertzer and Lovejoy (1987)).

$$\lambda = \frac{\text{outer scale}}{\text{observation scale}}$$

Statistical moments ( $q$ ) of a scale invariant field ( $\varepsilon$ ) scale with resolution ( $\lambda$ )

$$\langle \varepsilon_\lambda^q \rangle \approx \lambda^{K(q)}$$

Moment scaling function ( $K(q)$ ) characterized using only two parameters,

$$K_c(q) = \frac{C_1}{\alpha - 1} (q^\alpha - q)$$

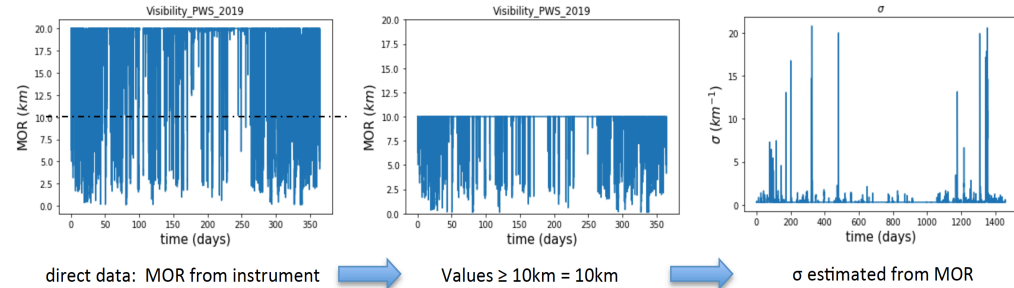
multi-fractality index  $\alpha$  and mean intermittency co-dimension  $C_1$ .

Since the governing non-linear equations of atmosphere such as Navier-Stokes posses scale invariance, it is assumed here that behaviour of light attenuating particles inherits similar scaling properties and hence exhibit multifratcal behavior (Schertzer and Tchiguirinskaia(2020)).

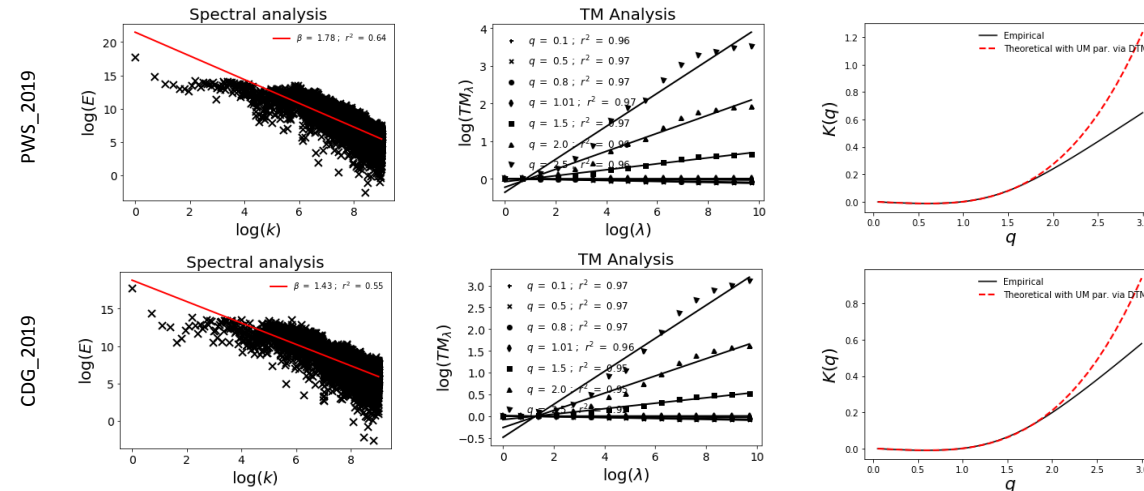
## Effect of threshold on data

As instrument upper limit decreases, number of values equal to that upper threshold increases

- PWS has 20km; CDG has 10km upper threshold
- Cause biases in overall statistical properties



## UM analysis on $\sigma_e$ (with same threshold on PWS and CDG data)



| data                     | PWS   | CDG   |
|--------------------------|-------|-------|
| % of values at threshold | 83.2  | 89.07 |
| D1                       | 1     | 1     |
| R2_TM                    | 0,971 | 0,957 |
| alpha_TM                 | 3,594 | 3,502 |
| C1_TM                    | 0,077 | 0,061 |
| alpha_DTM                | 3,402 | 3,368 |
| C1_DTM                   | 0,076 | 0,059 |
| bet1                     | 1,782 | 1,430 |
| H_1                      | 0,527 | 0,320 |

values of UM parameters

To understand the bias from upper threshold of instrument, MF fields were simulated and were imposed with thresholds of corresponding fields

Data to mimic :

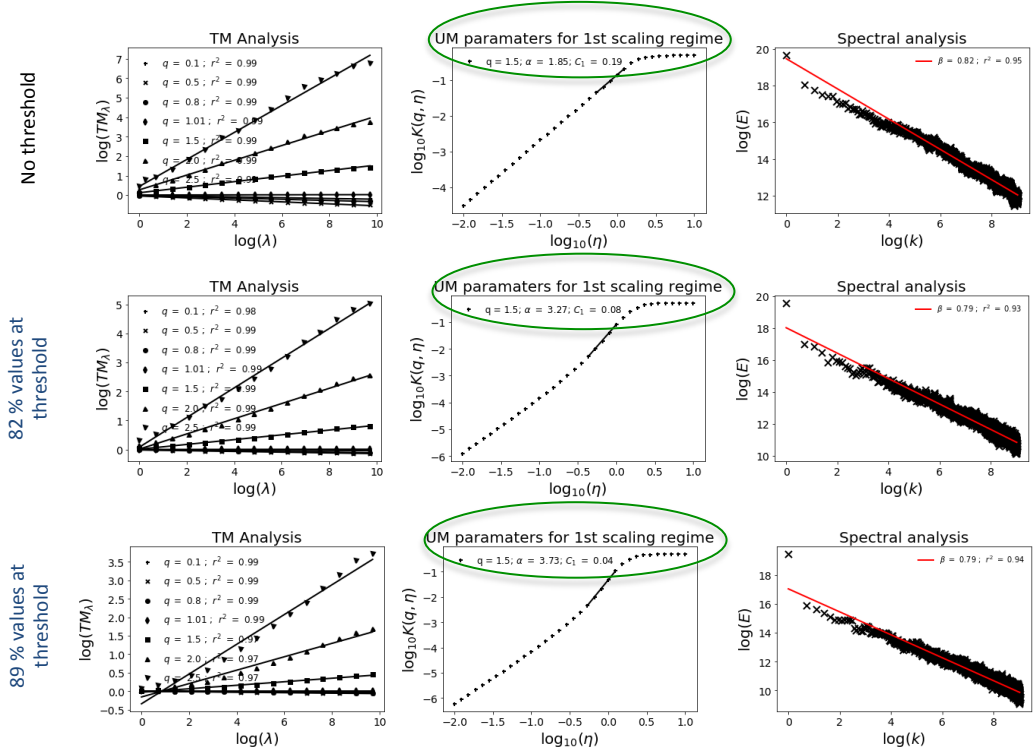
- PWS, 30m, 10km upper threshold
- CDG, 30m, 10km upper threshold

Simulation (Ensemble analysis)

- sample length:  $2^{14}$  (closest length to 2019 data)
- number of samples: 100

Values of UM parameters changes with threshold

- Increase in value of  $\alpha$
- decrease in value of  $C_1$
- Values of spectral slope unchanged



- Using simulated conservative fields having  $\alpha$  1.8 and  $C_1$  0.2, it was possible to retrieve UM parameter values close to that of  $\sigma_e$  at similar thresholds
- The underlying MF field to study visibility is  $\sigma_e$  itself
- UM parameters of the simulated fields were comparable to that of  $PM_{10}$  concentration in nearby station

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