

1. Introduction

Extreme sea levels pose an increasing risk to coastline communities, including nuclear power plants where consequences would be drastic if subject to coastal flooding without suitable defences.

We consider skew surge and maximum tide as components of sea levels that are independent conditional on the time of year [Williams et al., 2016]. These exhibit different seasonal patterns which are ignored in existing methodology.

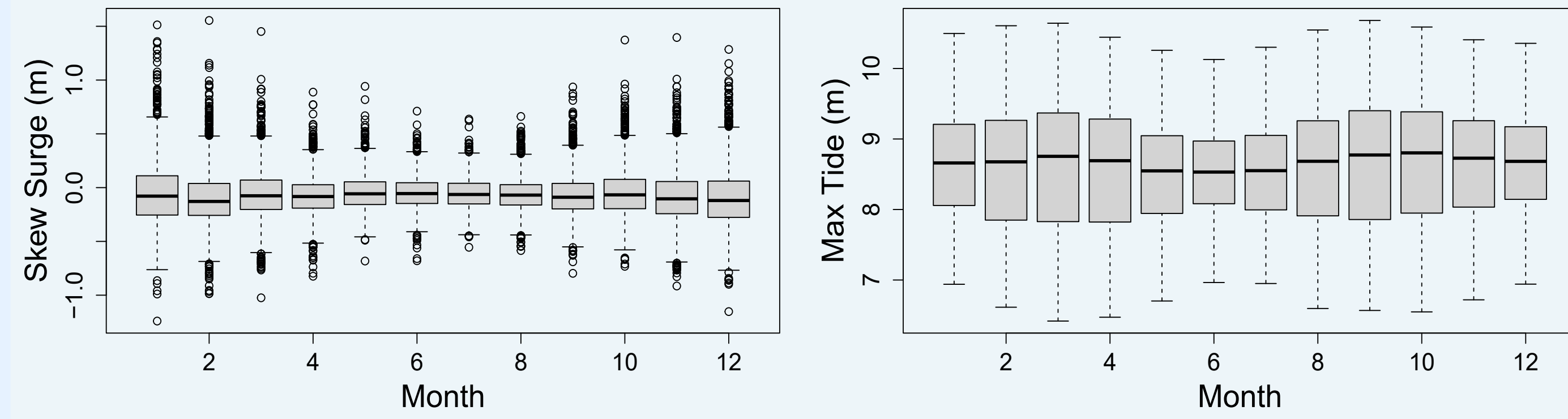


Figure 1: Skew surge and maximum tide observations at Heysham in 1964-2012.

2. Return Levels

Return levels are the levels we expect the annual maxima sea level to exceed every $1/p$ years on average, for p the annual exceedance probability.

We derive estimates from the monthly and annual maxima distributions for the following cases of seasonality:

- Seasonality in both skew surges and tides (M1),
- Seasonality in the skew surges, but not in the tides (M2),
- Seasonality in the tides, but not in the skew surges (M3),
- Ignore seasonality in both skew surges and tides (M4).

3. Tidal Seasonality

We find the probability an observation of max tide is from a month, given it is higher than some threshold u , for $u = 50\%$, **90%**, **95%** and **99.9%** quantile.

- Departures from a common probability over months suggest these are not identically distributed,
- Months where the most extreme high tides are observed have greater values for this probability as u increases.

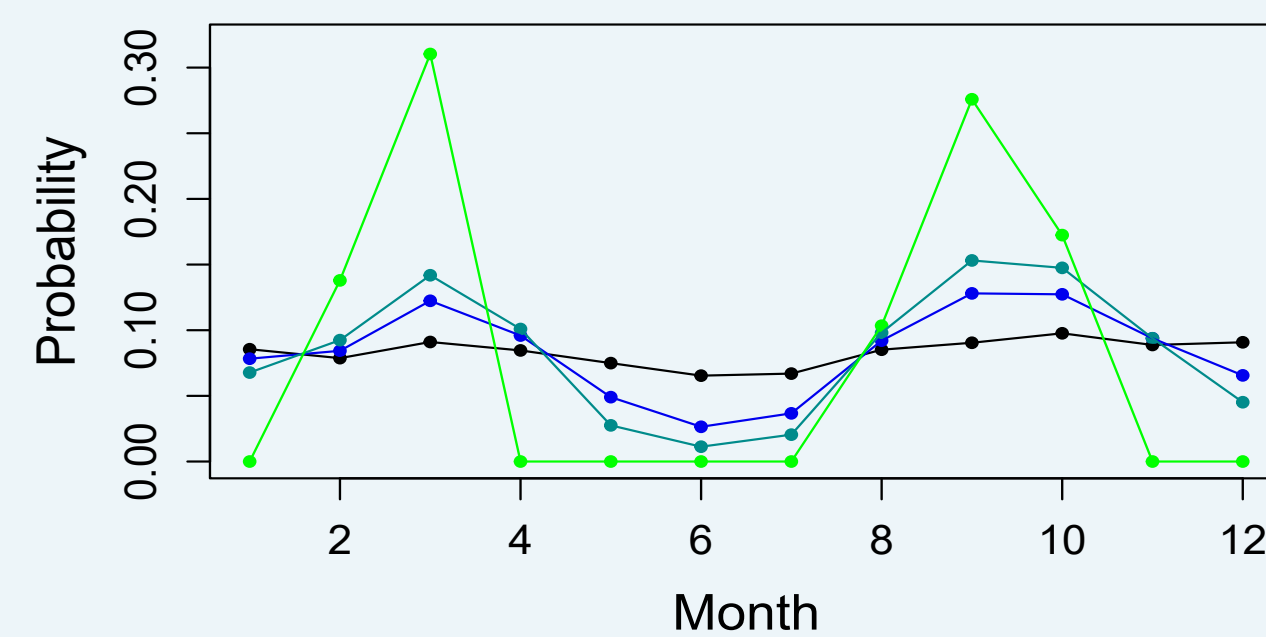


Figure 2: Monthly probabilities of large tides at Heysham.

Let $X_t^{(j,k)}$ denote the maximum tides in month $j = 1, \dots, 12$ and year $k = 1, \dots, K$, for $t = 1, \dots, T_j^{(K)}$ tidal cycles.

- To account for tidal seasonality, we only consider tides that occurred within a given month in our monthly return level estimates,
- Otherwise, we assume annual tidal cycles are stationary and take a tidal sample from any point in the year.

In both cases, we account for annual changes in the tides by averaging over the tidal regimes in all years, assuming equal probability of occurrence.

Monthly Sea Level Return Levels at Heysham

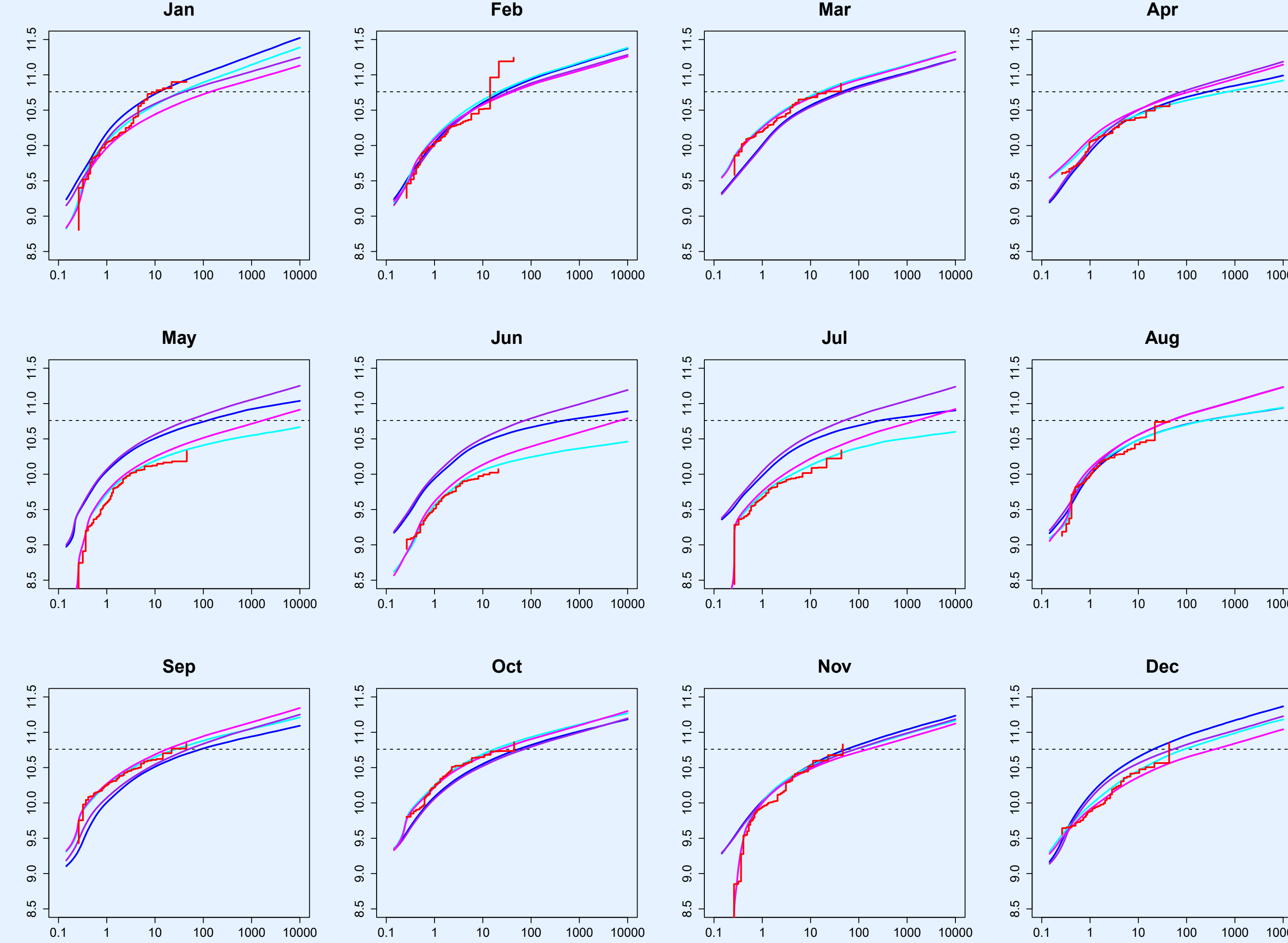


Figure 3: Monthly sea level return level estimates (meters) at various return periods (months), for cases M1, M2, M3, M4 at Heysham. We also show empirical estimates and HAT (black).

4. Skew Surge Seasonality

We define extreme skew surges as exceedances of a monthly threshold. Below this, the monthly empirical distribution is used, whilst excesses (Y) are modelled by a **generalised Pareto distribution** (GPD):

$$\mathbb{P}(Y > y) = (1 + \xi y / \sigma_d)^{-1/\xi}$$

with a fixed shape parameter across months (ξ), but a scale parameter that varies daily (σ_d) to account for seasonality:

$$\sigma_d = a + b \sin\left(\frac{2\pi}{365}(d - \phi)\right) \text{ for } a, b, \phi \in \mathbb{R} \text{ and } d = 1, \dots, 365.$$

When skew surge seasonality is ignored, we fit a GPD with constant shape (ξ) and scale parameters (σ) to exceedances of a common threshold.

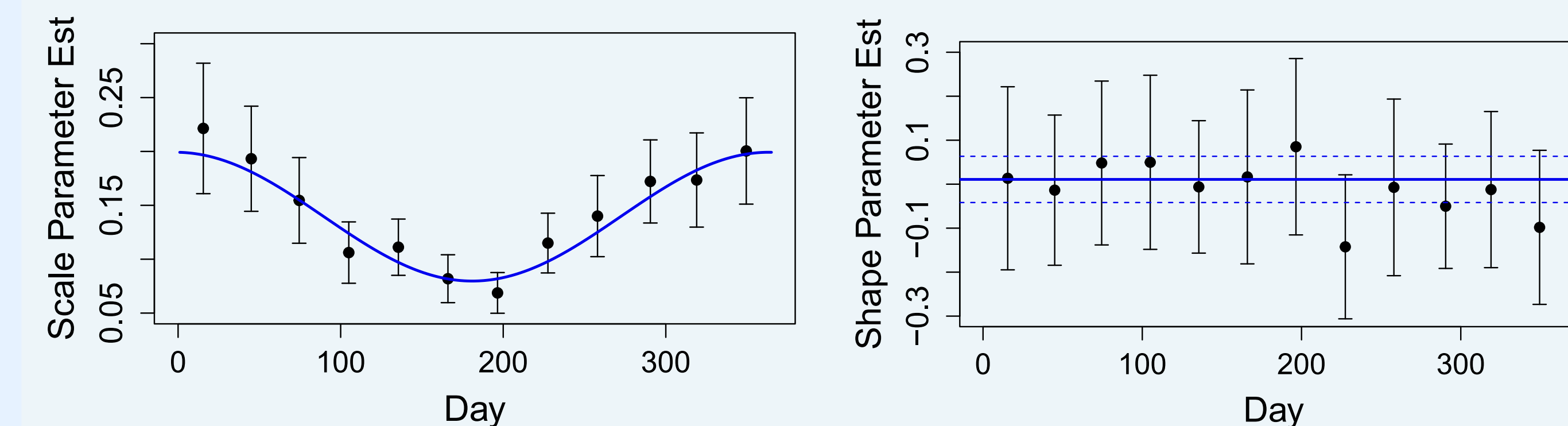


Figure 4: Parameter estimates for the marginal (black) and seasonal GPD at Heysham.

Figure 4 compares our parameter estimates with the marginal model, where each month has a different scale and shape parameter. Our model (F_S) gives a better fit, requires fewer parameters and ensures smooth daily changes.

5. Maxima Distributions of Sea Levels

Monthly Maxima Distribution

We use our skew surge model (F_S) and the known max tide observations ($X_t^{(j,k)}$) to derive the monthly maxima (M_j) distribution of sea levels:

$$P(M_j \leq z) = \frac{1}{K} \sum_{k=1}^K \left(\prod_{t=1}^{T_j^{(k)}} F_S(z - X_t^{(j,k)}) \right).$$

Figure 3 shows monthly return level estimates for all cases of seasonality. The curve for M1 stays close to the empirical estimates, this shows the importance of accounting for seasonality. When seasonality is ignored in both, return levels are overestimated in summer but underestimated at the equinoxes.

Annual Maxima Distribution

- When seasonality is accounted for in either component, we assume independence across months and combine the monthly distributions,
- When seasonality is ignored, we follow the approach of Batstone et al. (2013) and do not consider the monthly maxima distributions.

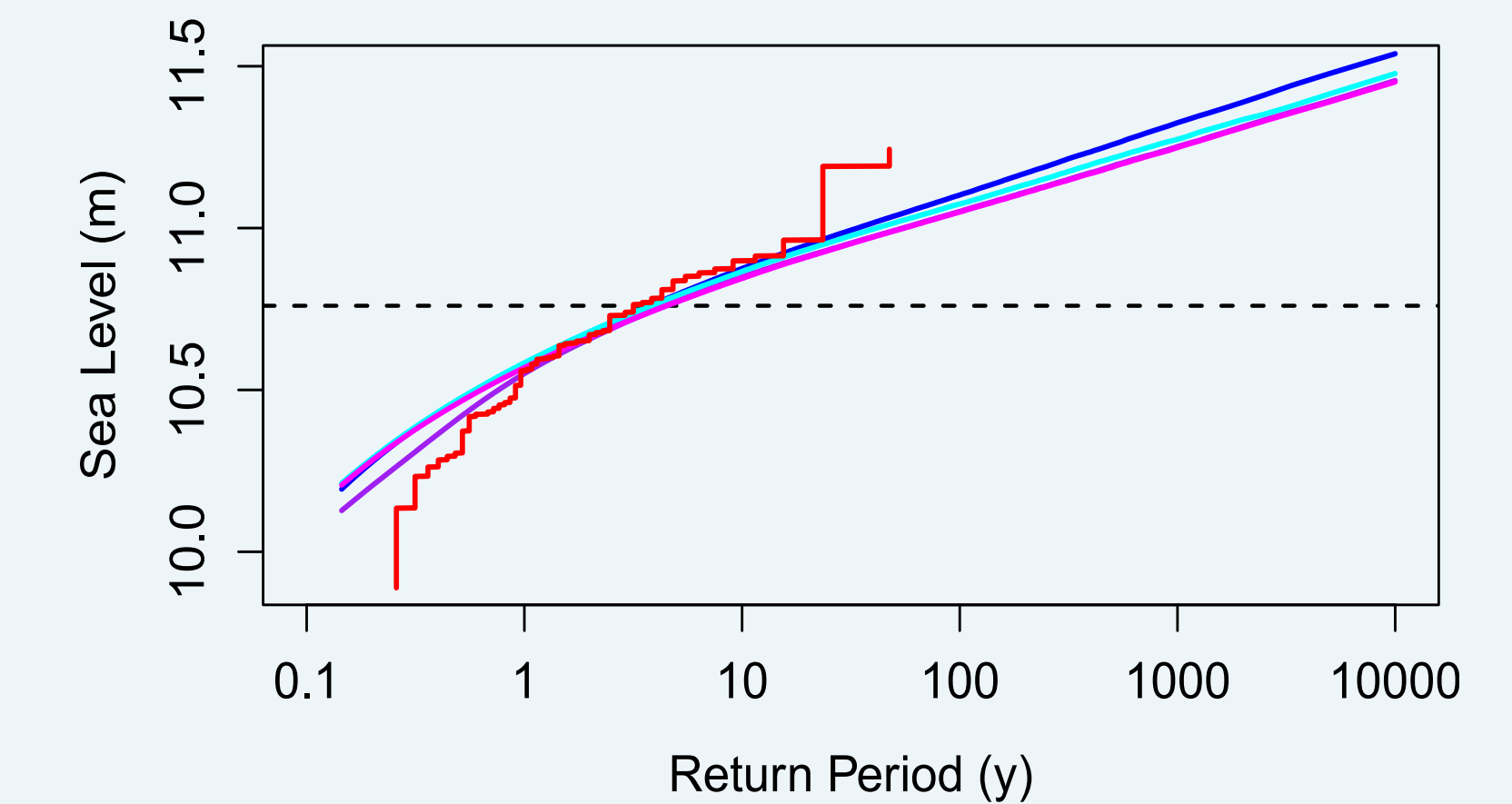


Figure 5: Return level estimates at Heysham for M1, M2, M3, M4, empiricals and HAT (black).

Return levels for the four cases remain relatively close together in Figure 5. The empirical estimates follow the M1 curve where both aspects of seasonality are accounted for, up until the two largest events of the 48 year study period.

6. Further Work

- Adjacent skew surges were assumed to be independent. We will account for their dependence using the extremal index [Batstone et al., 2013],
- Investigation into how extreme skew surges change over season and years,
- Application to more locations.

7. References

- Batstone, C., Lawless, M., Tawn, J., Horsburgh, K., Blackman, D., McMillan, A., Worth, D., Laeger, S., and Hunt, T. (2013). A UK best-practice approach for extreme sea-level analysis along complex topographic coastlines. *Ocean Engineering* 71:28–39.
- Williams, J., Horsburgh, K. J., Williams, J. A., and Proctor, R. N. (2016). Tide and skew surge independence: New insights for flood risk. *Geophysical Research Letters*, 43(12):6410–6417.