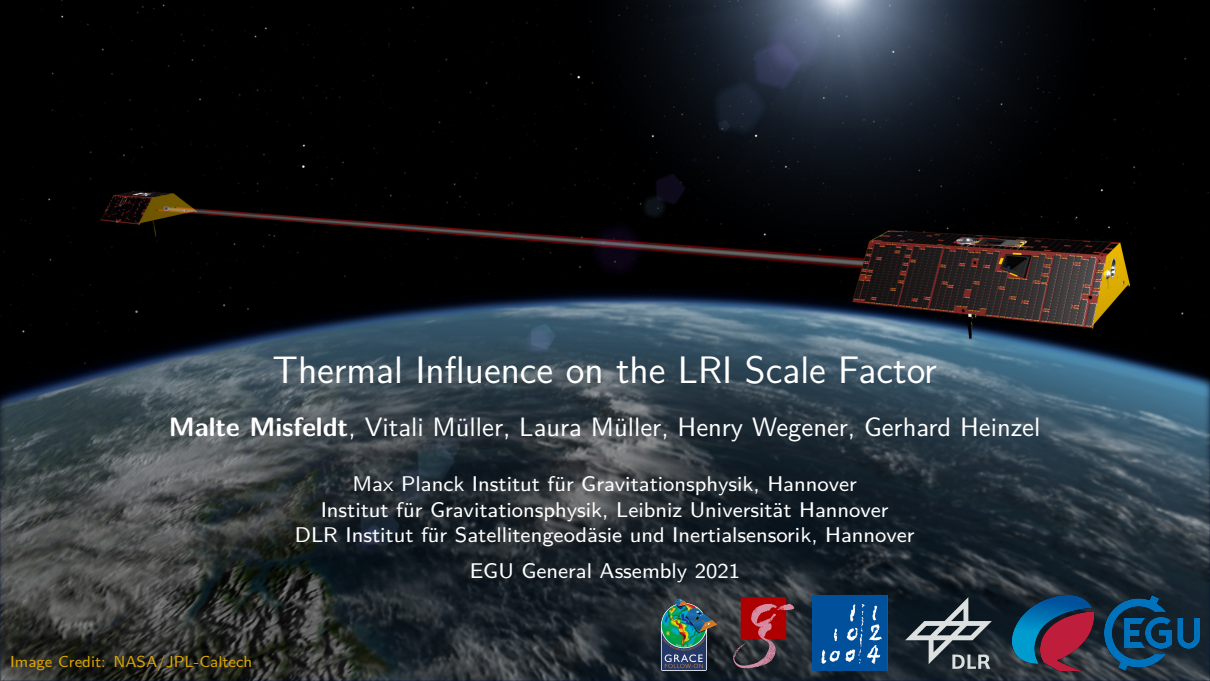

Thermal Influence on the LRI Scale Factor

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Institut für Gravitationsphysik, Leibniz Universität Hannover
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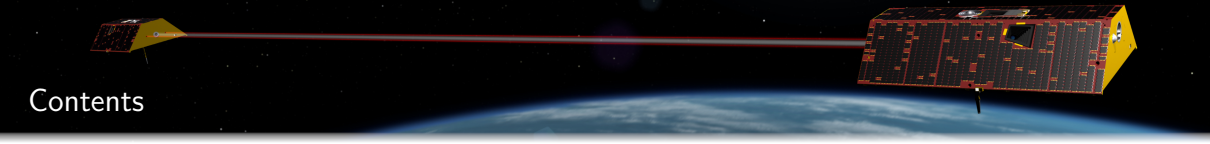
EGU General Assembly 2021

Image Credit: NASA/JPL-Caltech



Contents

- Setup and Status of the Laser Ranging Interferometer
- Scale Factor Estimation
  - Scientific LRI Output
  - Absolute Frequency Computation Methods
  - On-Ground Model
  - In-Flight Application
- Temperature Correlation
- Conclusion & Outlook

Setup and Status of the Laser Ranging Interferometer

Setup

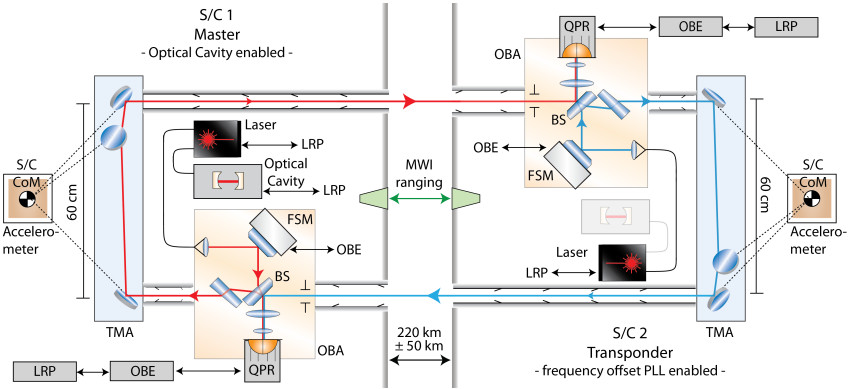


Image Credit: Abich et al. [1]

Setup and Status of the Laser Ranging Interferometer

Status

The Laser Ranging Interferometer (LRI) onboard the GRACE-FO mission successfully demonstrated the **feasibility of inter-satellite laser ranging** [1].

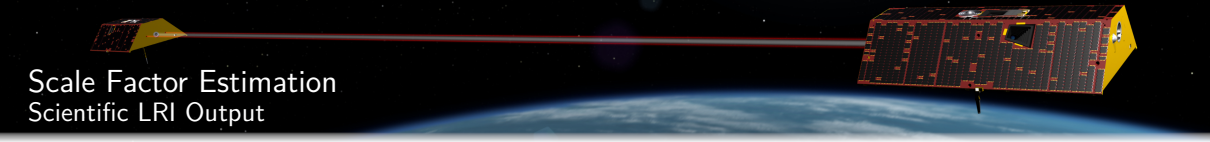
The LRI provides **precise ranging data** with a noise level smaller than $200\,\mathrm{pm}/\sqrt{\mathrm{Hz}}$ at frequencies of 5 Hz.

Long uninterrupted measurements, e.g. for more than 106 days, which is approximately 1650 orbital revolutions.

The LRI team at the AEI Hannover is **analyzing the LRI data** in detail and **developing processing algorithms**.

More on the current LRI status

See EGU21-9870 by Vitali Müller!

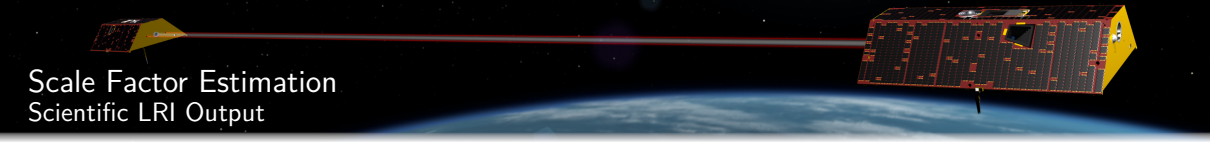


Scale Factor Estimation
Scientific LRI Output

- LRI Ranging Phase: Difference of Transponder and Master phase measurement at corresponding instance of time (note the light travel time τ). For both, a Phase Jump (PJ) removal is applied

$$\varphi_{\mathrm{LRI}}(t) = (\varphi_{\mathrm{T}}(t-\tau) - \mathrm{PJ}_{\mathrm{T}}(t-\tau)) - (\varphi_{\mathrm{M}}(t) - \mathrm{PJ}_{\mathrm{M}}(t))$$

- Simple Conversion: $\rho(t) = \lambda \cdot \varphi_{\mathrm{LRI}}(t)/2$ only holds, if $\lambda = \frac{c}{\nu_M} \equiv \mathrm{const.}$



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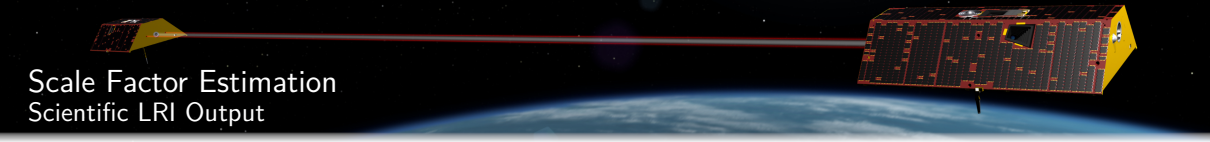
Convert roundtrip
to one-way range



Scale Factor Estimation
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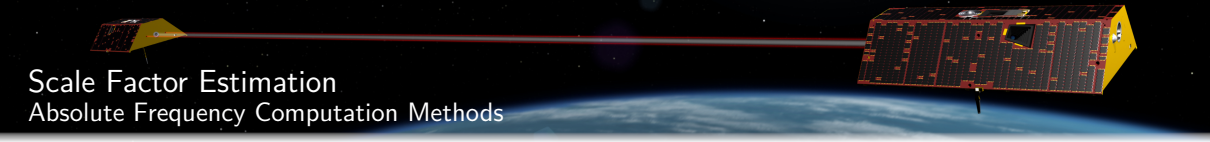
- Simple Conversion: $\rho(t) = \lambda \cdot \varphi_{\mathrm{LRI}}(t)/2$ only holds, if $\lambda = \frac{c}{\nu_M} \equiv \mathrm{const.}$

- Thus, apply scale in range-rate domain:

$$\Rightarrow \quad \dot{\rho}(t) = \lambda(t) \cdot \dot{\varphi}_{\mathrm{LRI}}(t)/2$$

$$\text{and integrate} \quad \rho(t) = \int \dot{\rho}(t)\,\mathrm{d}t$$

- How to determine the absolute frequency $\nu_{\mathrm{M}}(t)$ of the LRI master unit?



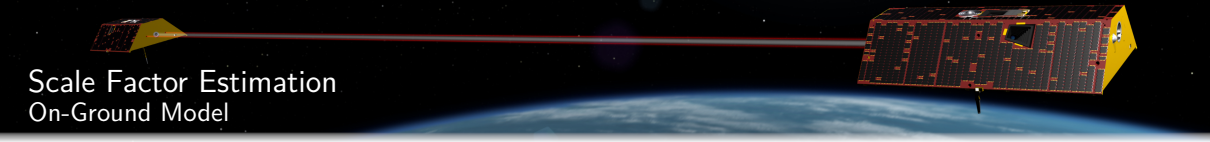
The absolute laser frequency $\nu(t)$ can be derived in two ways:

1. Cross-calibrate the LRI range with the MWI range once per day (used by SDS for LRI1B-v04)
2. Use a physical model based on on-ground measurements and empirical parameters

We split our model in two parts

$$\nu(t) = \underbrace{\nu_{\mathrm{OG}}(t)}_{\text{uses LRI telemetry}} + \underbrace{\nu_{\mathrm{emp.}}(t)}_{\substack{\text{uses empirical parameters from} \\ \text{cross calibration with MWI}}} , \tag{1}$$

where ν_{OG} contains short term variations and $\nu_{\mathrm{emp.}}$ compensates long term drifts, e.g. through aging processes. The drift is in the order of $1\,\mathrm{Hz\,s^{-1}}$, see slide 13.

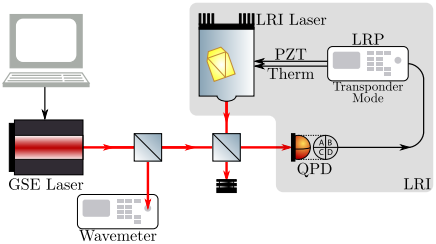


Scale Factor Estimation
On-Ground Model

- For the first term of eq. (1), we use the laser telemetry (i.e. from LHK data product) and the laser temperature (HRT, not yet available in v04) to estimate the absolute laser frequency

$$\nu_{\mathrm{OG}}(t) = \underbrace{\begin{pmatrix} c_{\mathtt{pztIL}} \\ c_{\mathtt{pztOOL}} \\ c_{\mathtt{thermIL}} \\ c_{\mathtt{thermOOL}} \\ c_{\mathtt{lasTRP}} \end{pmatrix}}_{\text{Coupling constants}} \cdot \underbrace{\begin{pmatrix} \mathtt{pztIL}(t) \\ \mathtt{pztOOL}(t) \\ \mathtt{thermIL}(t) \\ \mathtt{thermOOL}(t) \\ \mathtt{lasTRP}(t-\tau) - T_{\mathrm{ref}} \end{pmatrix}}_{\text{Laser telemetry from OG tests}} + \underbrace{\nu_0}_{\text{const.}} \tag{2}$$

- GSE = Ground Support Equipment
- On-Ground Calibration:
  - Lock LRI laser (transponder mode) to GSE laser
  - Monitor GSE lasers frequency with wavemeter
  - Modulate GSE laser frequency
  - Record LRI feedback control loop actuator signals



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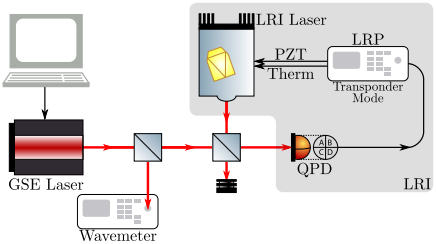
- Fit a linear model (eq. (2))



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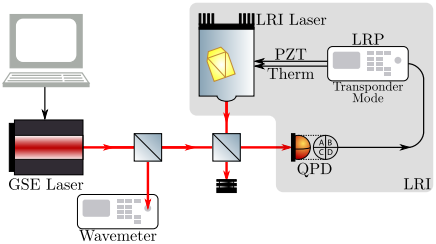
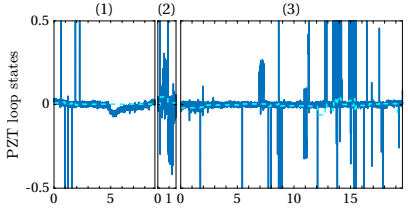
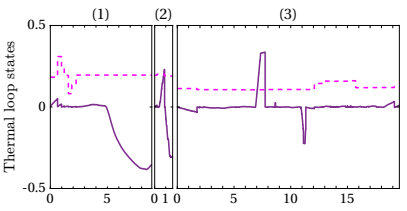
  - Lock LRI laser (transponder mode) to GSE laser
  - Monitor GSE lasers frequency with wavemeter
  - Modulate GSE laser frequency
  - Record LRI feedback control loop actuator signals
- Fit a linear model (eq. (2))
- A-priori model accuracy limited by wavemeter (up to 60 MHz $\approx 2 \times 10^{-7} \cdot \nu$)

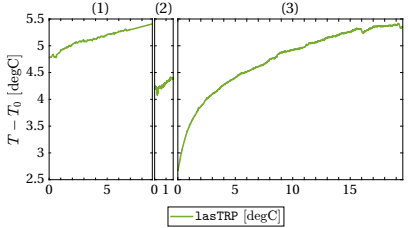


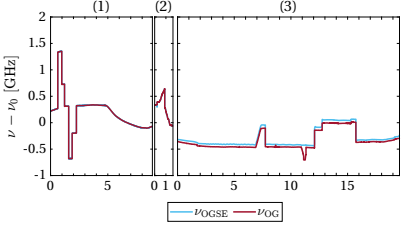
Scale Factor Estimation

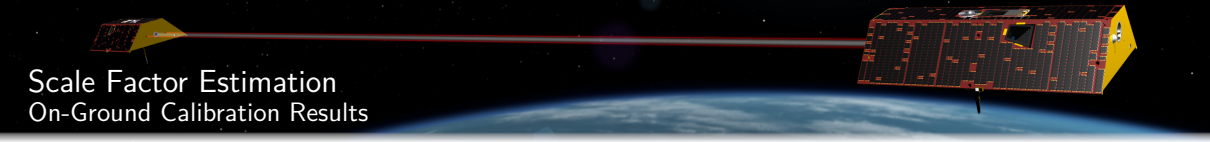
On-Ground Model Calibration Results (GF-1 only)







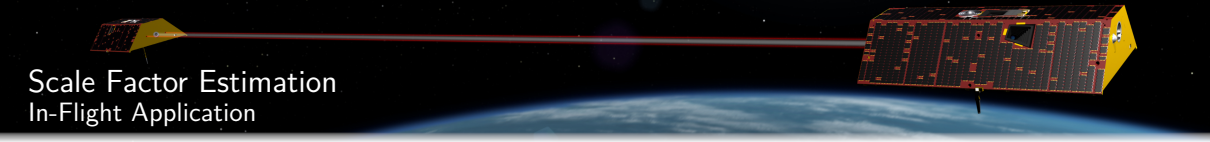




Scale Factor Estimation
On-Ground Calibration Results

Coupling	Unit	Design value	GF-1	GF-2
$c_{\mathtt{pztIL}}$	[MHz]	$2\,\mathrm{V} \cdot 5\,\mathrm{MHz\,V^{-1}} = 10\,\mathrm{MHz}$	-	-
$c_{\mathtt{pztOOL}}$	[MHz]	$18\,\mathrm{V} \cdot 5\,\mathrm{MHz\,V^{-1}} = 90\,\mathrm{MHz}$	-	-
$c_{\mathtt{thermIL}}$	[MHz]	$2\,\mathrm{V} \cdot 500\,\mathrm{MHz\,V^{-1}} = 1000\,\mathrm{MHz}$	1097	1094
$c_{\mathtt{thermOOL}}$	[MHz]	$18\,\mathrm{V} \cdot 500\,\mathrm{MHz\,V^{-1}} = 9000\,\mathrm{MHz}$	9155	8857
$c_{\mathtt{lasTRP}}$	[MHz/K]	-12	-	-
$\nu_{0,\mathrm{air}}$	[MHz]	281 614 840 (GF-1) 281 614 817 (GF-2)	281 614 682.081	281 614 631.999

Measurements taken between 2017 and 2018, now re-analyzed



Scale Factor Estimation
In-Flight Application

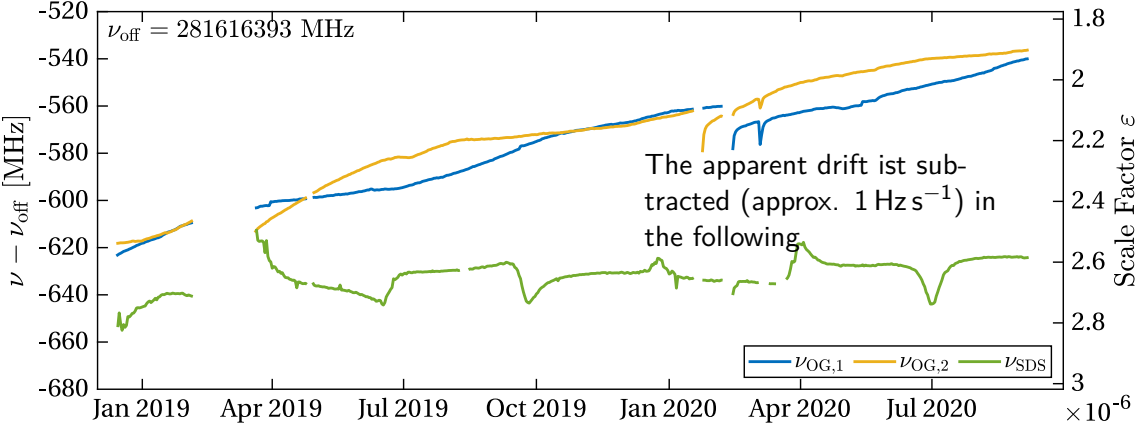
Advantages:

- Telemetry-based modeling may allow to resolve sub-daily variations
- We have a model that is continuous in time and not a daily constant, hence it has no discontinuities at day-bounds
- Our derivation of the scale is independent from the MWI on daily or weekly scales, however we remove long term trends by comparing to MWI measurements.

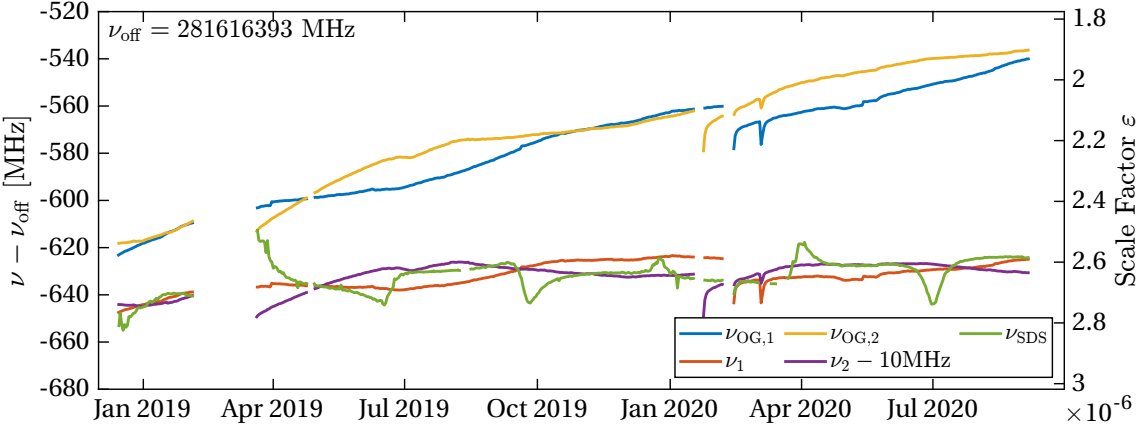
Verfication:

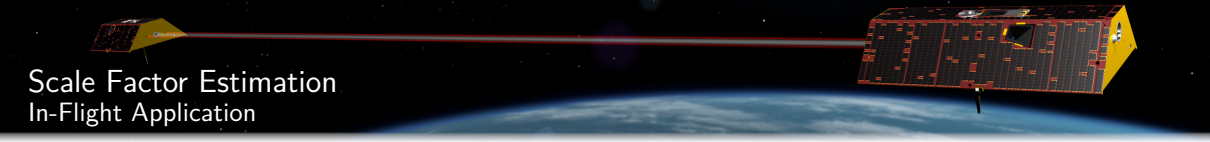
- We compare our resulting LRI range to the MWI, whose scale is well known

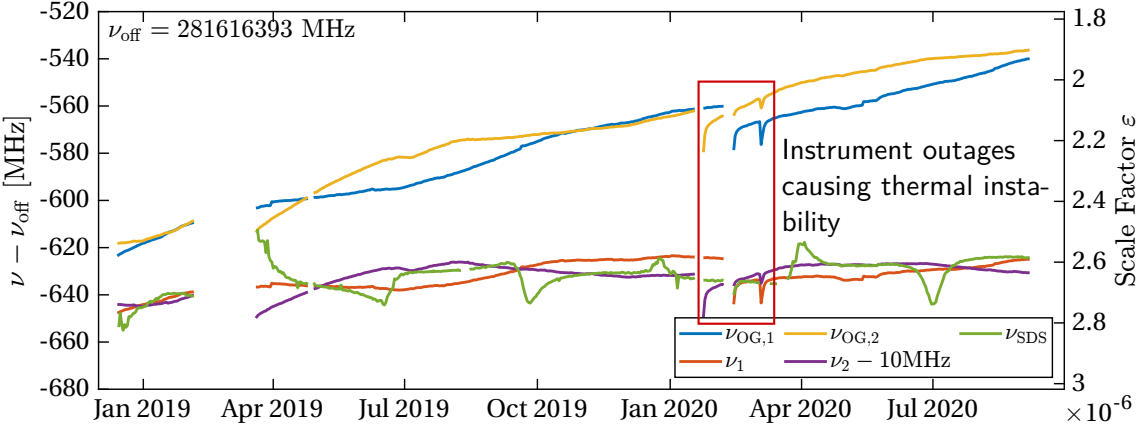
Scale Factor Estimation
In-Flight Application



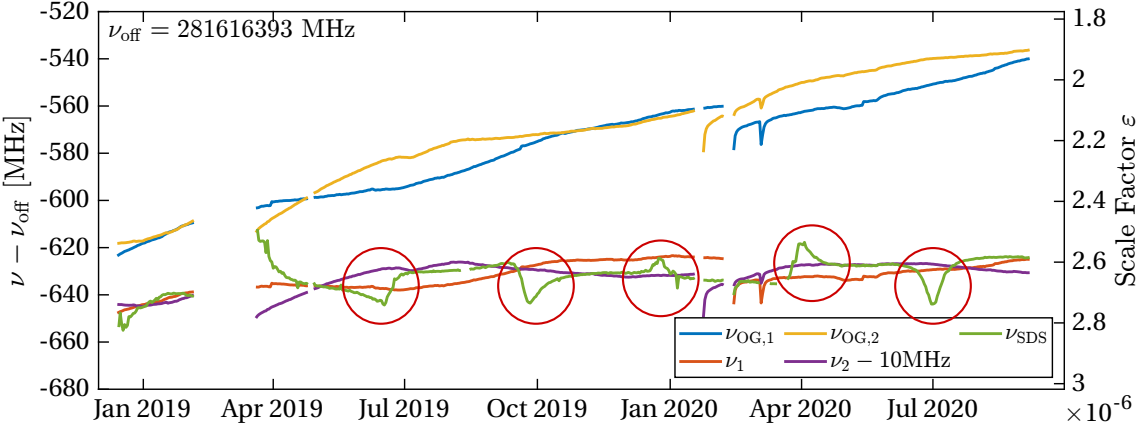
Scale Factor Estimation
In-Flight Application

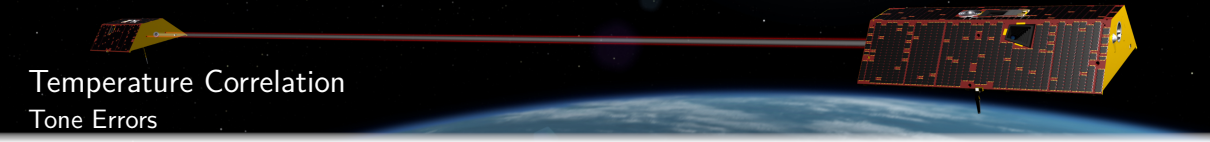





$\nu_{\text{off}} = 281616393$ MHz
$\nu - \nu_{\text{off}}$ [MHz]
Scale Factor ε
$\times 10^{-6}$
-520
-540
-560
-580
-600
-620
-640
-660
-680
1.8
2
2.2
2.4
2.6
2.8
3
Jan 2019
Apr 2019
Jul 2019
Oct 2019
Jan 2020
Apr 2020
Jul 2020
Instrument outages causing thermal instability
$\nu_{\text{OG},1}$
$\nu_{\text{OG},2}$
ν_{SDS}
ν_1
$\nu_2 - 10$MHz

Scale Factor Estimation
In-Flight Application



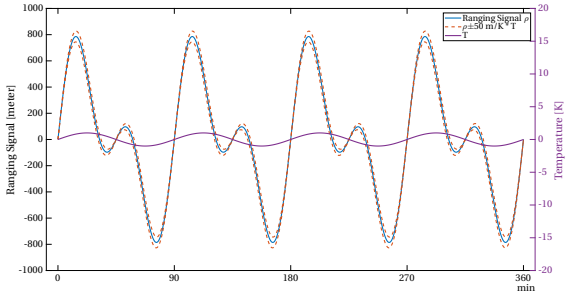


Temperature Correlation

Tone Errors

- Periodic variations of environmental conditions, like temperature, magnetic field, atmosphere, ... may cause so-called **tone errors**.
- Tone errors are **periodic modulations of the instrument output**, occurring mostly at 1/rev and 2/rev.
  - The ranging signal also has dominant 1/rev and 2/rev oscillations
  - If driving tone & ranging signal oscillate in-phase, the scale factor appears to be different!

- Example: 1/rev and 2/rev ranging signal, thermal coupling at 1/rev (highly exaggerated)
- Typical values are in the order of a few µm K^{-1}.



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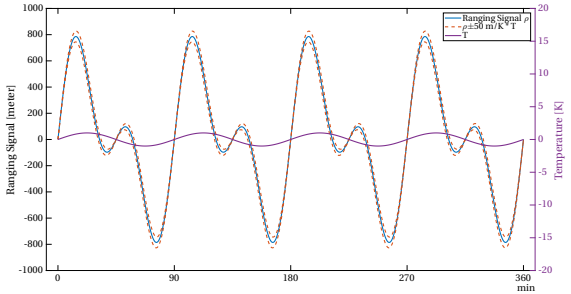
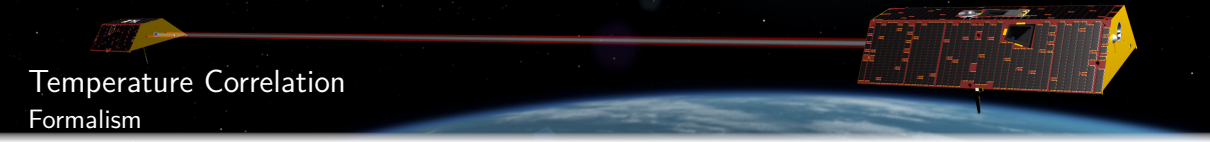


We only investigate tone errors in phase domain! Such errors in frequency domain (such as periodic length changes of the cavity) would be present in the laser actuator signals and hence in the OG-model.



Temperature Correlation

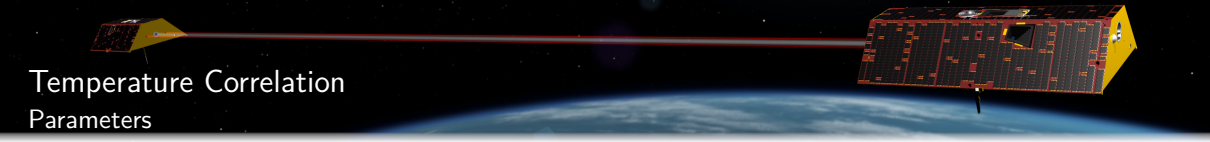
Formalism

- We developed a framework to express the LRI ranging phase

$$\rho_{\mathrm{LRI}} = \rho_{\mathrm{inst.}}(t) - \rho_{\mathrm{LTC}}(t) + \delta\rho_{\mathrm{LPN}} - \underbrace{\int_0^t \varepsilon_{\mathrm{SCF}} \cdot \dot{\rho}(t)\, \mathrm{d}t}_{\text{Scaling Error}} - \underbrace{\Delta t \cdot \dot{\rho}(t)}_{\text{Timing Error}}$$

$$\Rightarrow \rho_{\mathrm{err}} = \varepsilon_{\mathrm{SCF}} \cdot \rho(t) + \Delta t \cdot \dot{\rho}(t) + \mathcal{O}(1\,\mu\mathrm{m}) \approx \rho_{\mathrm{KBR}} - \rho_{\mathrm{LRI}}$$

- Goal: minimize error range by optimizing ν_{OG} and analyze temperature couplings into the apparent laser frequency. After minimization, we can disentangle $\varepsilon_{\mathrm{SCF}}$ and Δt.

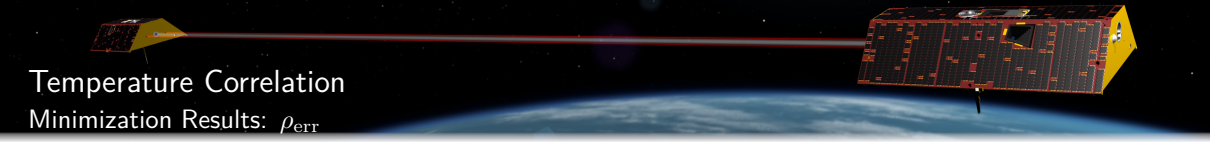


Temperature Correlation

Parameters

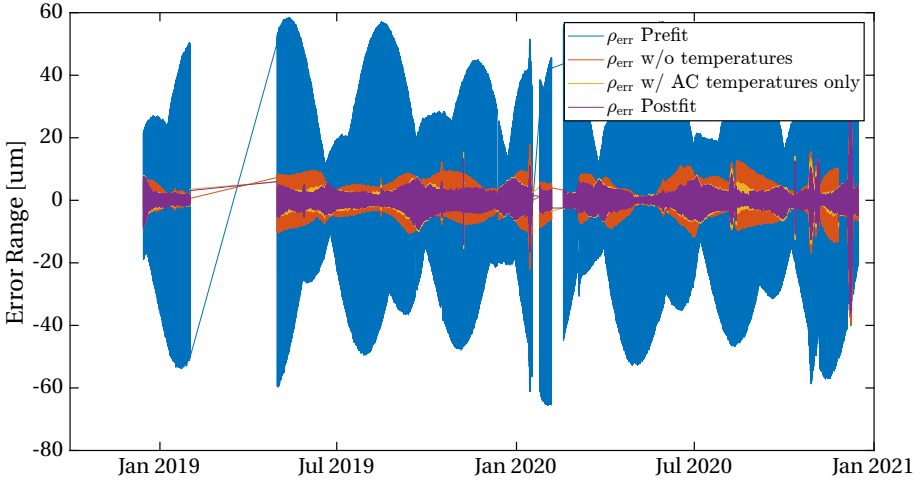
- Refine parameters of the ν_{OG} model with in-flight data, that enters via $\varepsilon_{\mathrm{SCF}} = 1 - \frac{\nu_{\mathrm{true}}}{\nu_{\mathrm{est}}}$
- KBR-LRI time offset Δt
- Frequency Drift a
- Frequency Offsets at specific times
- We apply temperatures two-fold:
  - DC values with MHz / K coupling
  - AC values with m / K coupling
- Furthermore: Time Delay (several 100 sec) to account for temperature propagation time

Algorithm: iteratively pick N sensors that reduce the error range ρ_{err} the most.



Temperature Correlation

Minimization Results: ρ_{err}



RMS residuals:

24.73 µm

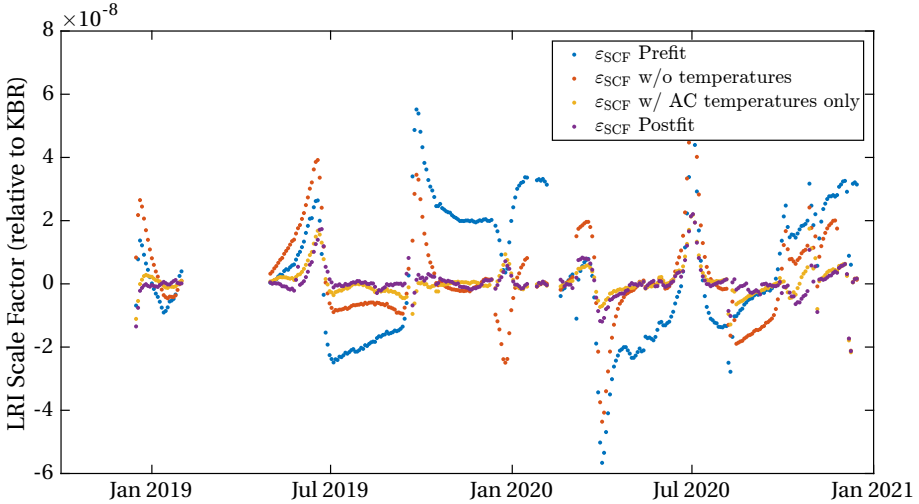
4.48 µm

2.56 µm

2.47 µm

Temperature Correlation

Minimization Results: $\varepsilon_{\mathrm{SCF}}$



RMS residuals:

20.665×10^{-9}

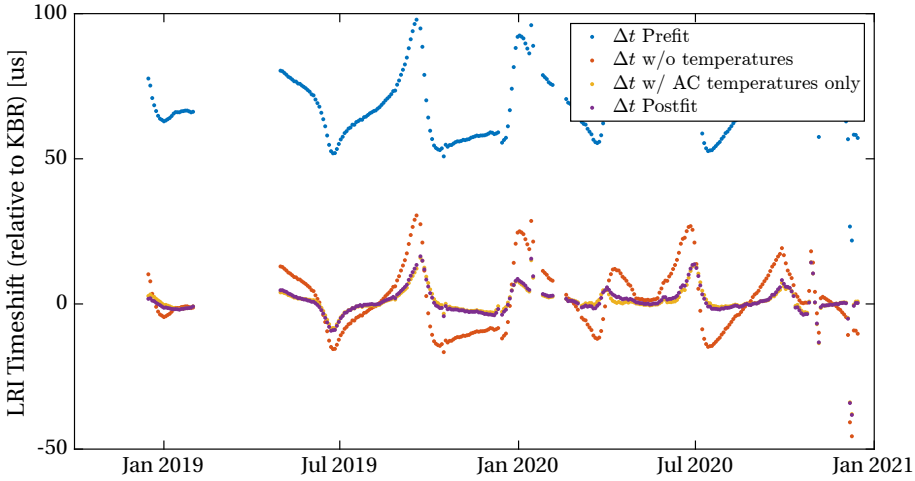
14.629×10^{-9}

4.900×10^{-9}

4.761×10^{-9}

Temperature Correlation

Minimization Results: Δt



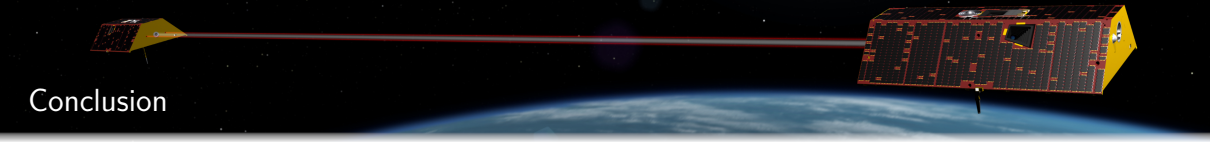
RMS residuals:

69.67 µs

11.25 µs

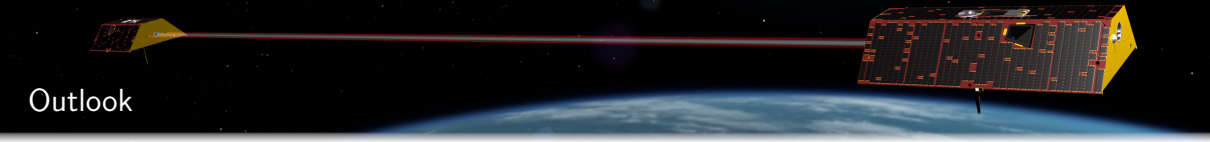
4.96 µs

5.19 µs



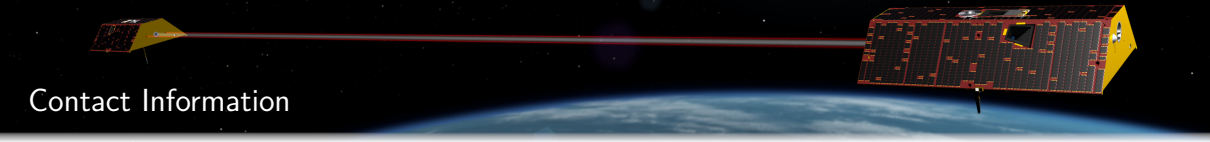
Conclusion

- A telemetry-based model for **estimating the scale factor** was presented, is independent from the MWI data to large extend ($\rightarrow$ NGGM).
- By using the frequency model, the a-priori residual rms scale uncertainties are below 2×10^{-8} with a LRI-KBR time shift of approx. 60 µs.
- We incorporated temperature data into the analysis to minimize the error range ρ_{err}, which further reduced the scale uncertainties to below 5×10^{-9} and the time shift to approx. 5 µs



Outlook

- Verificate model: Show, that 1/rev and 2/rev differences between LRI and KBR are reduced by incroporating temperature data
- Publish results :-)



Contact Information

Feel free to ask questions!

Malte Misfeldt
for the LRI team

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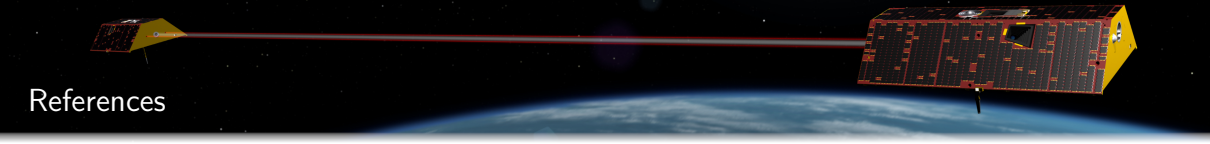

References

[1] Klaus Abich et al. “In-Orbit Performance of the GRACE Follow-on Laser Ranging Interferometer”. In: *Physical Review Letters* 123.3 (July 2019). DOI: 10.1103/physrevlett.123.031101.