



# How to build reach-averaged rating curves for remote sensing discharge estimation? The potential of periodic geometry hypotheses

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# Objectives

Estimating discharge for ungauged rivers:

- using only remote sensed data (e.g., SWOT<sup>1</sup> mission);
- without any prior information;
- using a non-uniform steady-state and morphological hypotheses.

# How to estimate discharge?

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## Direct problem

- ✓ Channel and floodplain bathymetry
  - ✓ Bed roughness
  - ✓ Inflow discharge
- 2D stationary Shallow Water Equations (SWE) → Water surface elevations



## Inverse problem

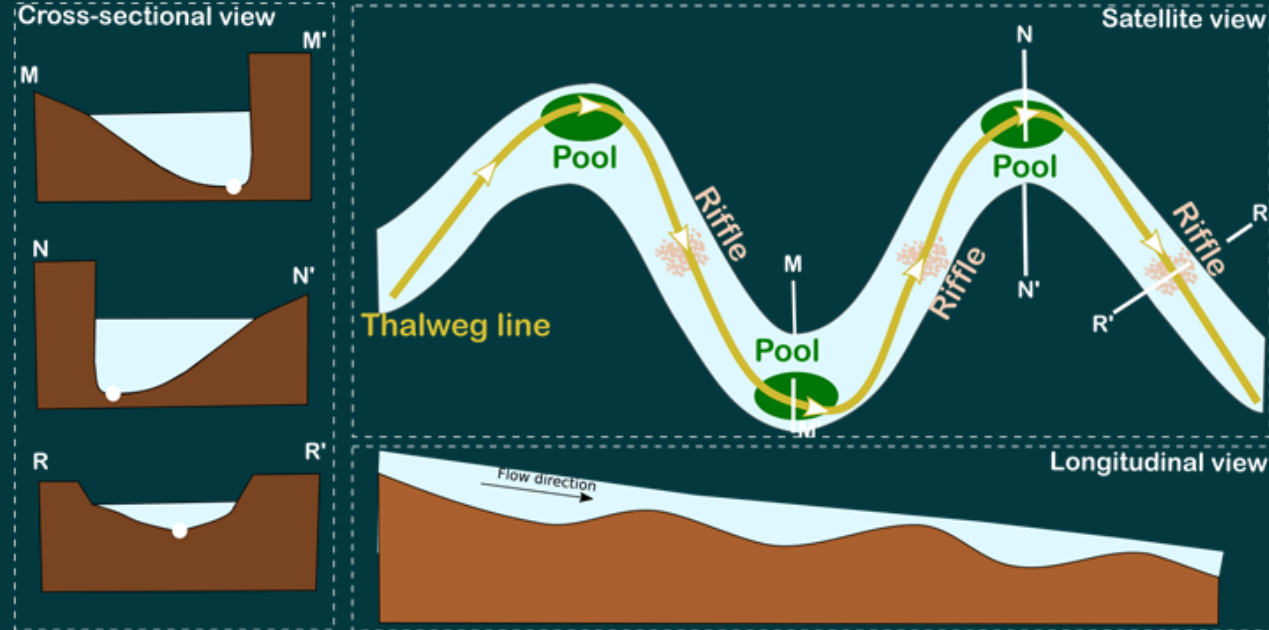
- ✓ Water surface elevations
  - ✓ Floodplain bathymetry
  - ✓ Channel bathymetry
  - × Bed roughness
  - × Downstream Boundary Conditions (rating curve)
- Discharge

# Periodic Equivalent Model

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Morphological variability characterized by repeated alternating units<sup>1</sup> (e.g., pools and riffles)

- periodic alternating pools and riffles ;
- asymmetrical cross-sections according to the position of pools and symmetrical cross-sections for the riffles ;
- a meandering form of the reach.



Direct model

Generator of  
the geometry

S.W.E solver

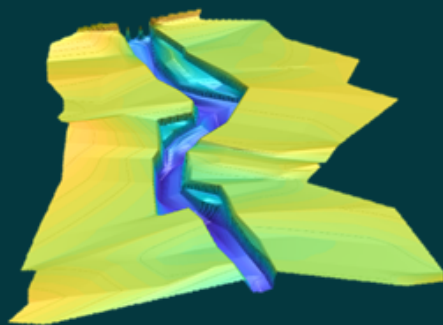
Curvature based asymmetry<sup>3</sup>, Kinoshita curves<sup>2</sup>, and sinusoidal functions

Basilisk<sup>4</sup>: Suitable with periodic boundary conditions

<sup>1</sup>Mahdade et al. (2020), <sup>2</sup>Kinoshita (1961), <sup>3</sup>Deutsch et Wang (1996), <sup>4</sup>Popinet (2013):[www.basilisk.fr](http://www.basilisk.fr)

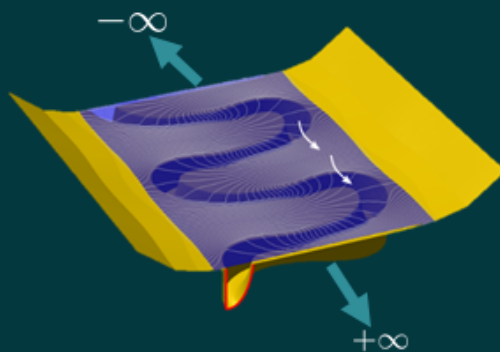
# Periodic geometry vs. Inverse models

## Explicit geometry



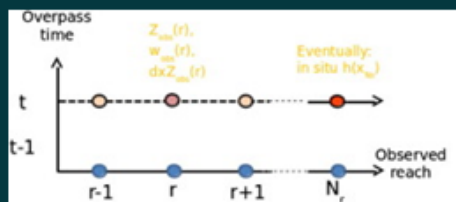
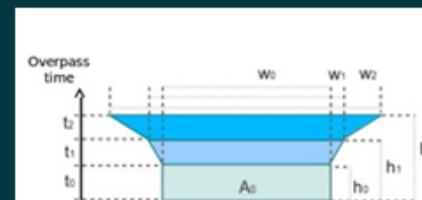
e.g., Oubanas et al. (2018)

## Periodic geometry



- Few parameters to calibrate (covariance of geometrical parameters)
- No in-situ boundary conditions (periodic BC)
- Low computation time

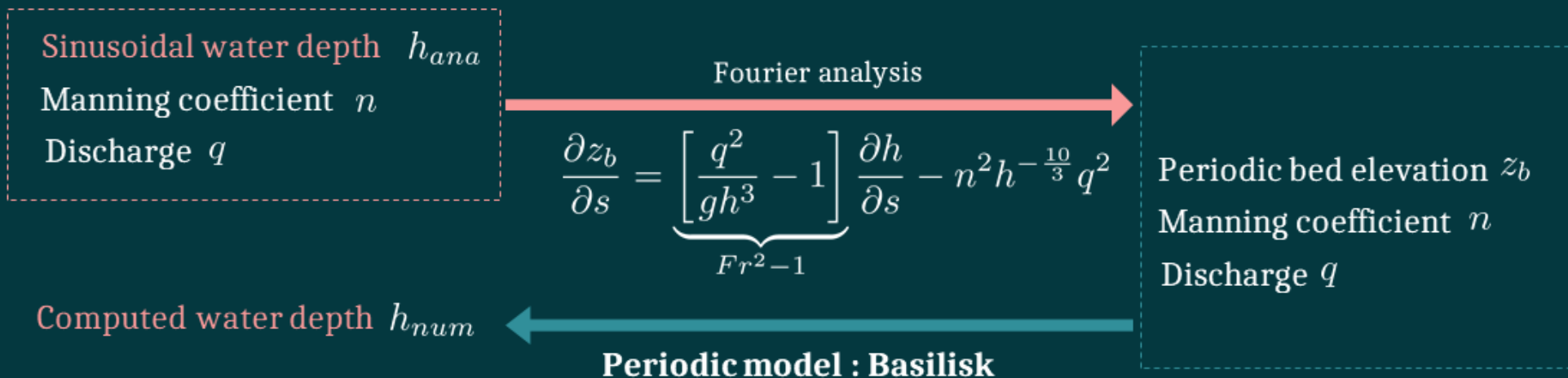
## Uniform geometry



e.g., Garambois and Monnier (2015)

# Test cases with the periodic model

Test of MacDonald et al. (1997) :



# Test cases with the periodic model

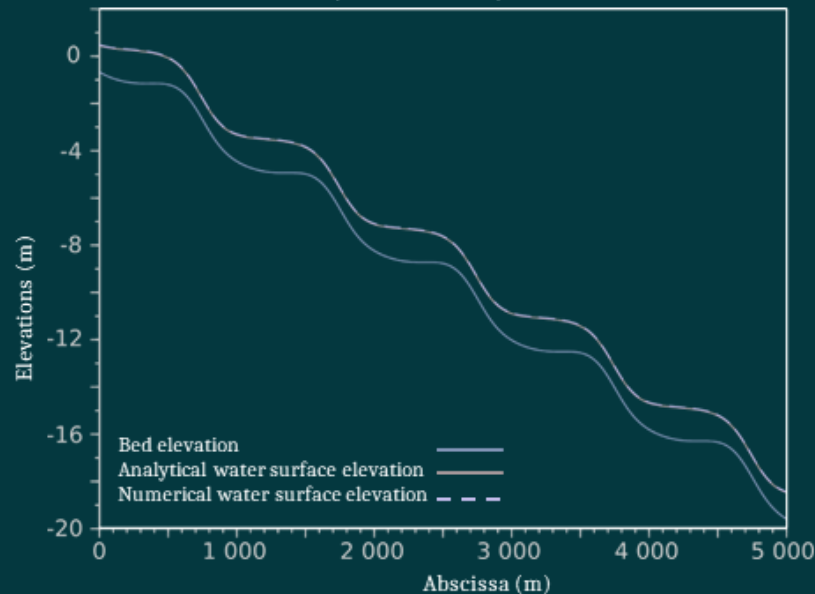
## 1D test-case

Input discharge  $q = 2 \text{ m}^2.\text{s}^{-1}$  1D Wavelength  $\lambda = 1000 \text{ m}$

Periodic Manning Coef  $n(x) = 0.03 - 0.01\sin(k_0x)$

Analytical depth  $h_{ana}(x) = \frac{9}{8} + \frac{1}{4}\sin(k_0x)$

$\text{RMSE}(h_{ana}, h_{num}) = 0.019 \text{ m}$



## 2D test-case

Input discharge  $q = 2 \text{ m}^2.\text{s}^{-1}$

Manning Coef  $n = 0.03$

Analytical depth

$h_{ana}(s) = 1.6 + 0.8(1 + \cos(2k_0s + \phi))$

2D Wavelength  $\lambda_{2D} = 1500 \text{ m}$

Bankfull depth  $h_{bf} = 3 \text{ m}$

Bankfull width  $w_{bf} = 30 \text{ m}$

Sinuosity  $\sigma = 3.0833$

Difference (m)

$h_{num}(x, y) - h_{ana}(s)$

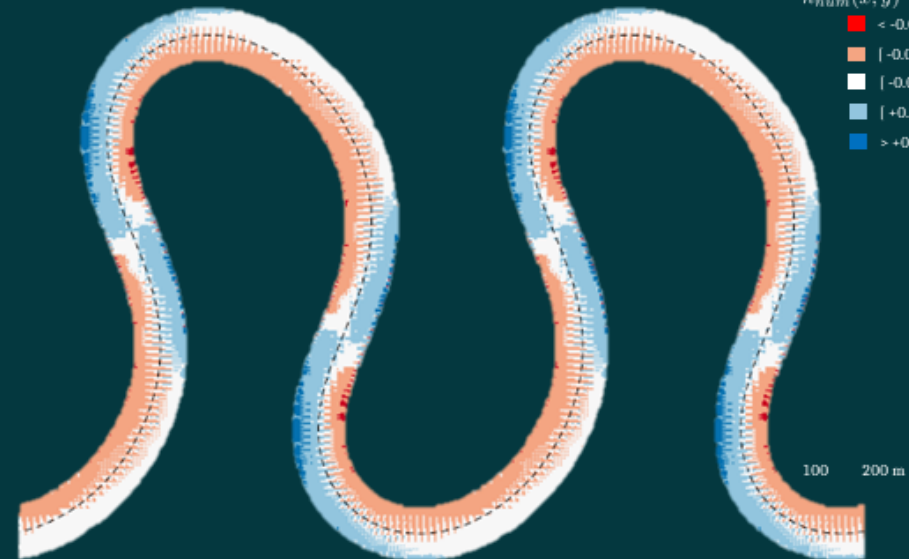
< -0.05

[-0.05 ; -0.01]

[-0.01 ; +0.01]

[+0.01 ; +0.05]

> +0.05



$\text{RMSE}(h_{ana}, h_{num}) = 0.014 \text{ m}$

# Perspectives

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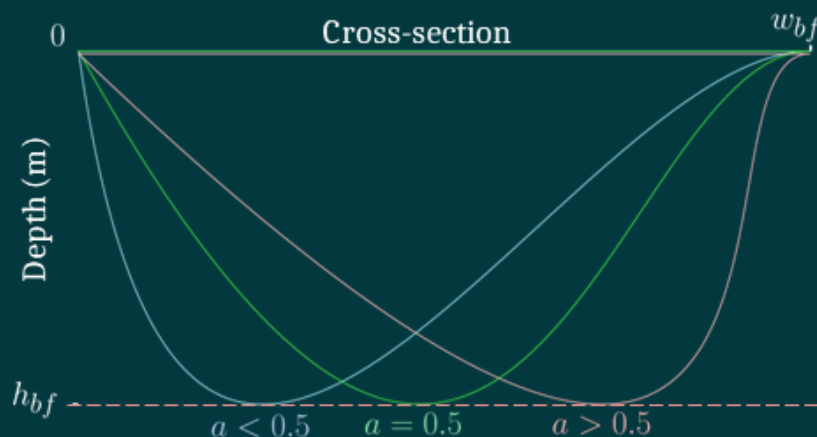
- Develop the inverse algorithm;
- Test the periodic approach on the Garonne river in France.

**Thank you !**  
**Questions ?**

# Generator of periodic geometry

## Cross-section geometry

Curvature based asymmetry



$$z(w) = \begin{cases} h_{bf} \left( 4 \left( \frac{w}{w_{bf}} \right)^b \left( 1 - \left( \frac{w}{w_{bf}} \right)^b \right)^\gamma \right) & \text{if } a \leq 0.5 \quad \text{and } b = -\frac{\ln 2}{\ln(a)} \\ h_{bf} \left( 4 \left( 1 - \frac{w}{w_{bf}} \right)^c \left( 1 - \left( 1 - \frac{w}{w_{bf}} \right)^c \right)^\gamma \right) & \text{if } a \geq 0.5 \quad \text{and } c = -\frac{\ln 2}{\ln(1-a)} \end{cases}$$

Kinoshita (1961), Deutsch et Wang (1996)

## Planform geometry

Kinoshita curve



$$\theta(s) = \theta_0 \sin(k_0 s) + \alpha_s \theta_0^3 \cos(3k_0 s) - \alpha_f \theta_0^3 \sin(3k_0 s)$$

## Thalweg geometry

Sinusoidal functions

$$z_f(s) = z_0 + A_z \cos(2k_0 s + \phi_z)$$



# SWE solver : Basilisk

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Solving the Shallow Water Equations (SWE) in 2D :

$$\frac{\partial}{\partial t} \begin{bmatrix} h \\ hu_x \\ hu_y \end{bmatrix} = -\vec{\nabla} \cdot \begin{bmatrix} hu_x & hu_y \\ hu_x^2 + \frac{1}{2}gh^2 & hu_xu_y \\ hu_xu_y & hu_y^2 + \frac{1}{2}gh^2 \end{bmatrix} - gh \begin{bmatrix} 0 \\ \frac{\partial z_b}{\partial x} \\ \frac{\partial z_b}{\partial y} \end{bmatrix} - \frac{1}{\rho} \begin{bmatrix} 0 \\ \tau_x \\ \tau_y \end{bmatrix}$$

- Finite volume scheme on an adaptive meshes using the software Basilisk<sup>1</sup>;
- Automatically generated mesh with adaptive refinement (by binary tree in 1D or quadratic tree in 2D);
- Flexible handling of periodic boundary conditions.

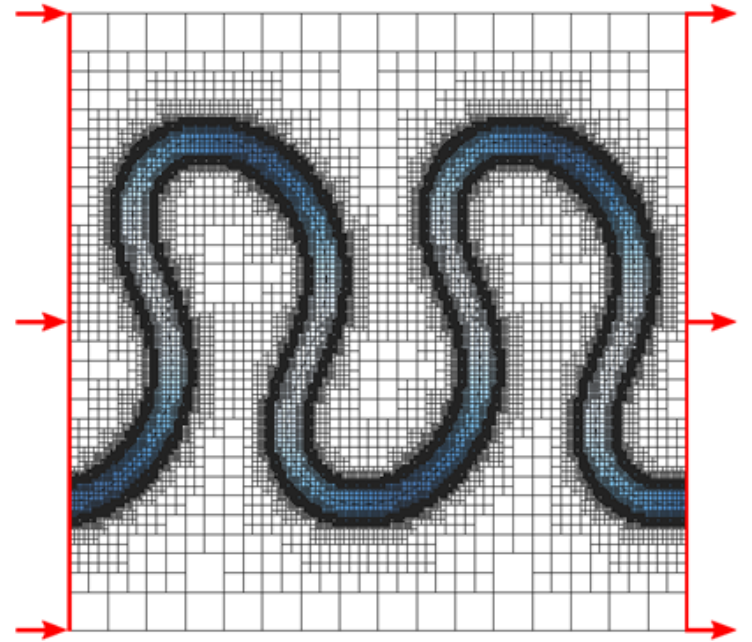
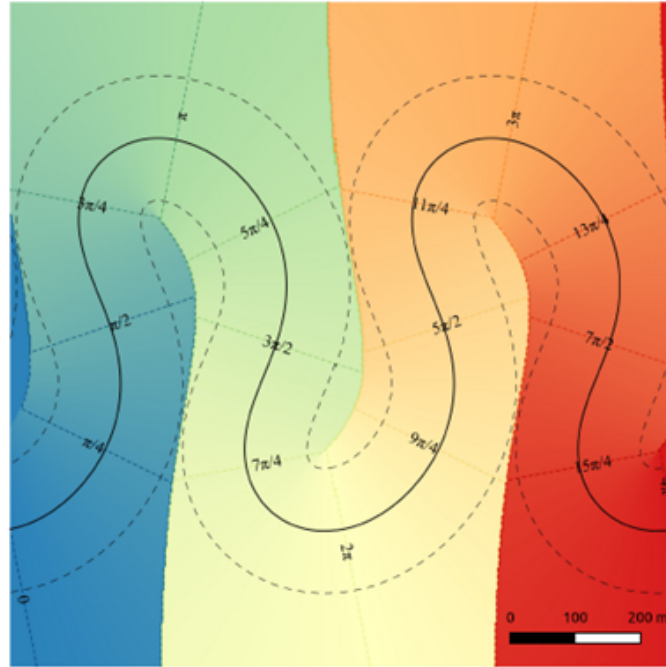
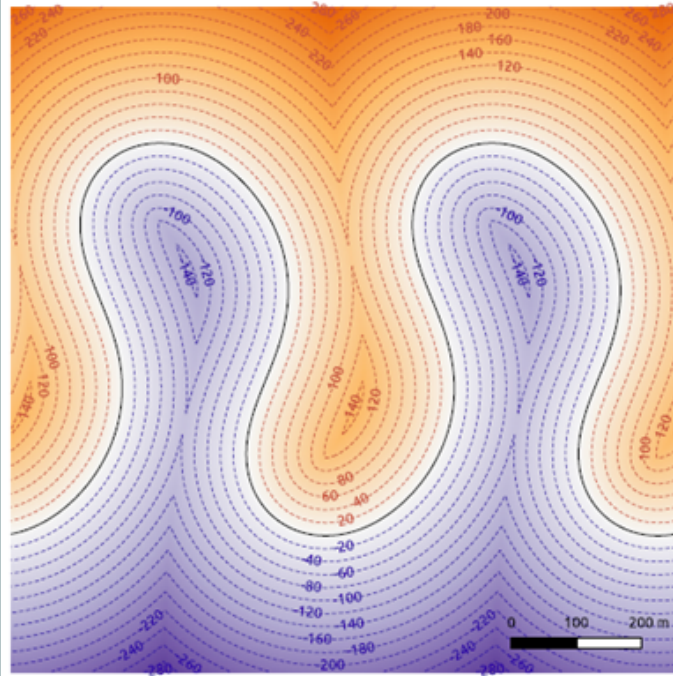
→ Dealing with the technical difficulty of modeling periodic geometric patterns



<sup>1</sup>[www.basilisk.fr](http://www.basilisk.fr), Popinet (2013)

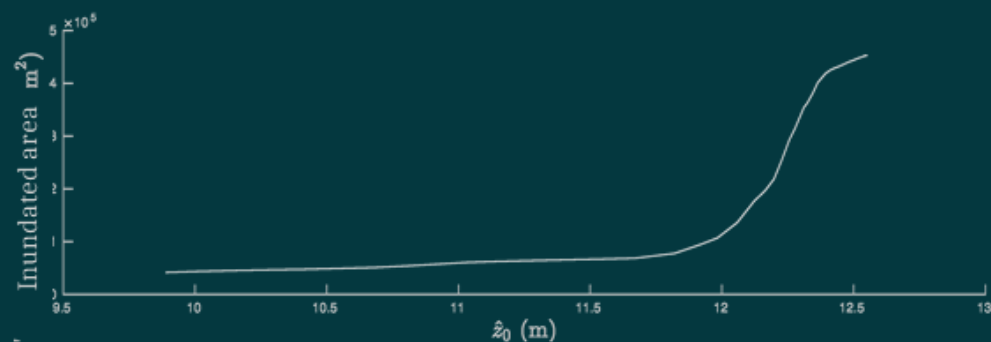
# SWE solver : Basilisk

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# Inverse periodic model

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linear regression  
constant of the water  
surface elevation

Observed signatures  
(ex : SWOT)

First guess of unknown parameters:  
discharge, Manning coefficient, bed  
elevation parameters, bankfull  
depth

Build the  
periodic  
geometry

Compute  
water depths  
with basilisk

Direct model

Parameters'  
optimization

Error computation : simulated  
and observed hydraulic  
signatures