

# Finite-frequency effects for imaging underground cavities



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und Geophysik

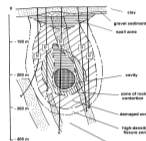
We study finite-frequency effects that arise in cavity detection. The task comes along with the Onsite-Inspection part for the Comprehensive Nuclear Test Ban Treaty (CTBT), where the remnants of a potential nuclear test need to be identified.

In such nuclear tests, there is preexisting knowledge about the depths at which nuclear tests may take place, and also about sizes that such cavities can attain. The task of cavity detection has consistently been a difficult one in the past, which is surprising, since a cavity represents one of the strongest seismic anomalies one can ever have in the subsurface.

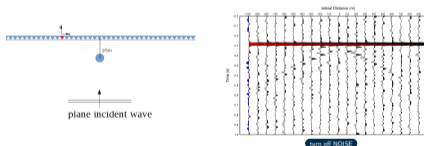
A conclusion of this study is that considering finite-frequency effects are rather promising for cavity detection, and that it is worthwhile to take them into account. We utilize an analytical approach for the forward problem of the a seismic wave interacting with a underground cavity in order to develop an inversion routine that finds and detects an underground cavity utilizing the transmitted wave-field.

## On Site Inspection

- For the realization of the **Comprehensive Nuclear Test Ban Treaty (CTBT)** techniques are needed to detect and verify possible violations of the treaty.
- In case a suspicious event is registered by the IMS, **On-Site Inspections (OSI)** can be conducted to investigate the location area for remnants of an nuclear explosion.
- "**Resonance Seismometry**" is listed in the treaty as technique for detecting cavities.
- Which wave-field interactions could be exploited for cavity detection?



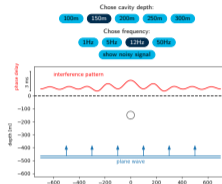
## Setting



- An analytic solution (Korneev and Johnsson, 1993) is used to study the interaction of a plane P-wave with a cavity
- Exact solution allows quick evaluation of finite frequency effects

## Phase Delay Explorer

- The phase delay pattern observed at the surface depends on the depth of the cavity and the single dimensionless parameter  $\frac{R}{\lambda}$  [ $R$ : cavity radius,  $\lambda$ : wavelength]
- In this example we set  $R = 30$  m and  $v_p = 3000$  m/s
- Different scenarios can be explored by choosing cavity depth and frequency



## Detectability

- We tested for different geometries, if the finite frequency pattern is capable for cavity detection.
- A detectability parameter is defined in order to test the sharpness of the misfit function and the right location of its minimum

We define

$$Detectability = \frac{\int L(x, y) K(x, y)}{\int L(x, y)}, \text{ with}$$

$$K = \exp\left(-\frac{(x-x_0)^2 + (y-y_0)^2}{2\sigma^2}\right) \text{ and } L(x, y) = \exp(-10^6 \cdot m(x, y)^2).$$

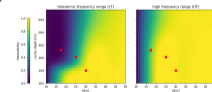
$K$ : Kernel function centered at true location

$L$ : Localization function

$m$ : misfit

Geometries of Real Nuclear Cavities

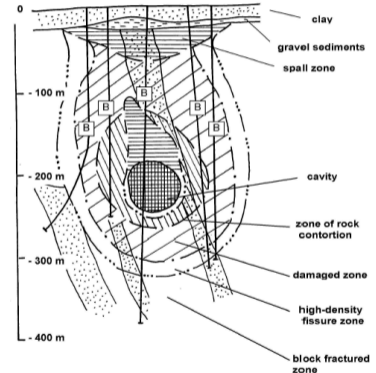
Place	Radius (m)	Depth (m)
a) Salpazar test site, Semipalatinsk	30 m	200 m
b) GNDOME, New Mexico	17.4 m	361 m
c) Maccheri, Nevada	25 m	306 m



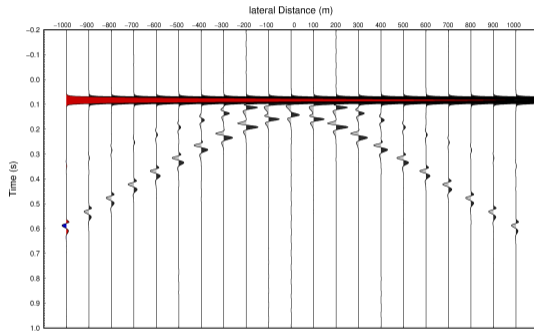
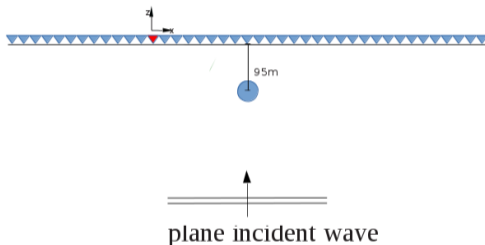
Detectability for teleseismic frequency range (left) and high frequency range (right). Red dots show radius and depth of real nuclear cavities (see table left).

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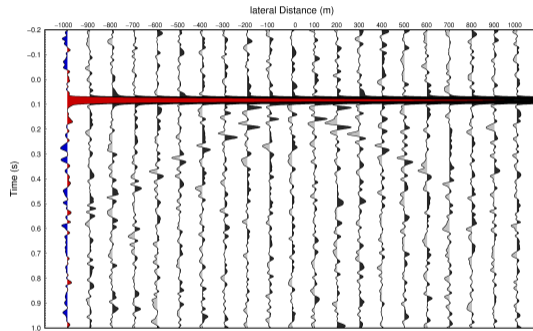
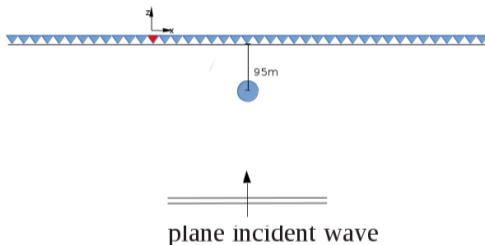
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add NOISE

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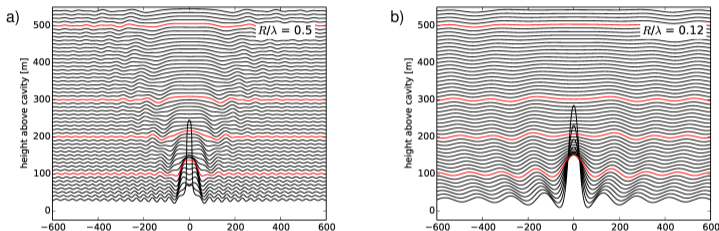
turn off NOISE

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## Finite Frequency Effect of a Cavity

- ▶ When a plane P-wave passes a cavity, the wave has to propagate around the obstacle and gets delayed.
- ▶ Phase delay is strongest and most focussed directly behind the cavity. With distance the magnitude of the phase delay reduces but spreads laterally (wavefront healing).
- ▶ Interference of waves diffracted on either sides of the cavity are causing a phase delay pattern.

Phase delay:



Characteristics of the wavefield above a cavity. The vertical axis indicates the height above the cavity center, the horizontal axis the distance along the surface. Two wavelengths are considered:  $R/\lambda=0.5$  (a) and  $R/\lambda=0.12$  (b) corresponding for a cavity radius  $R=30$  m to a frequencies of 50Hz and 12Hz, respectively.

- ▶ The "Phase Delay Explorer" demonstrates the influence of wavelength, cavity depth on the interference pattern.

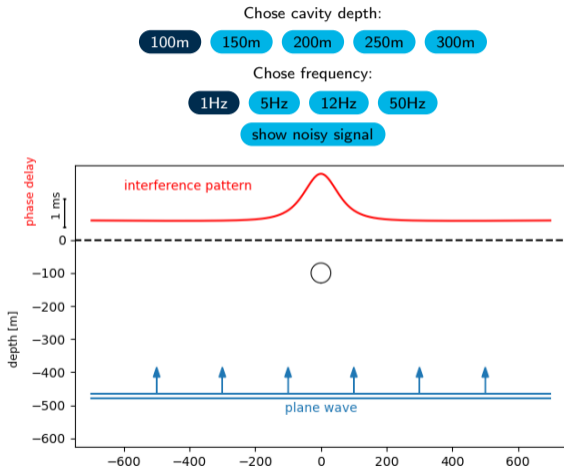
Run Phase Delay Explorer

## Phase Delay Explorer

- ▶ The phase delay pattern observed at the surface depends on the depth of the cavity and the single dimensionless parameter  $\frac{R}{\lambda}$

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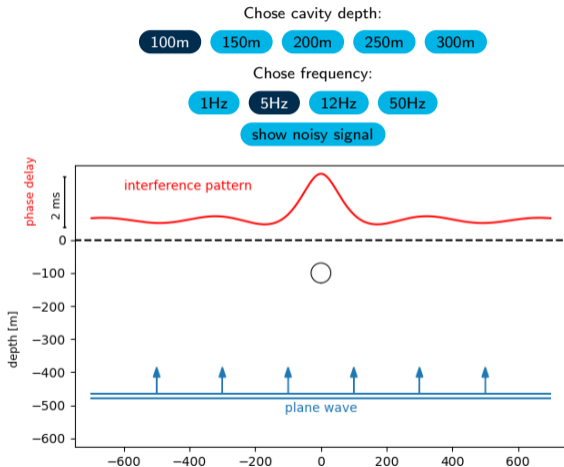


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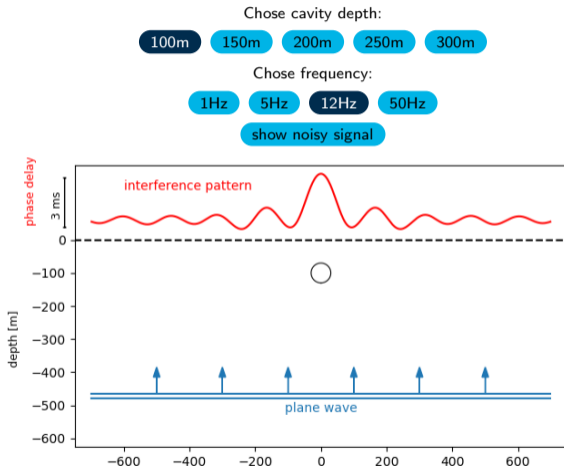


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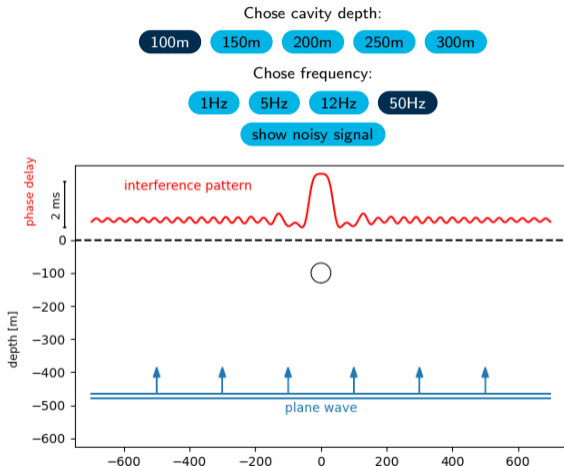


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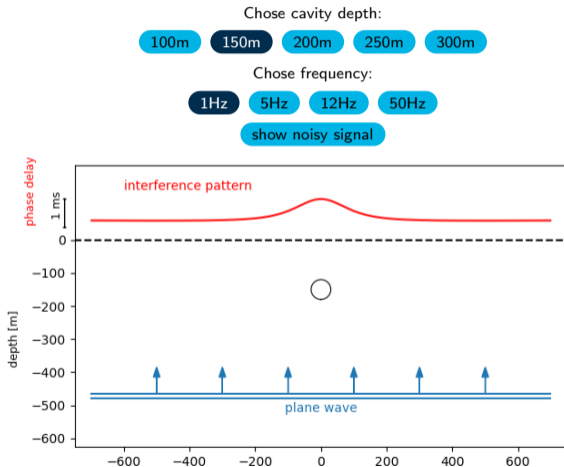


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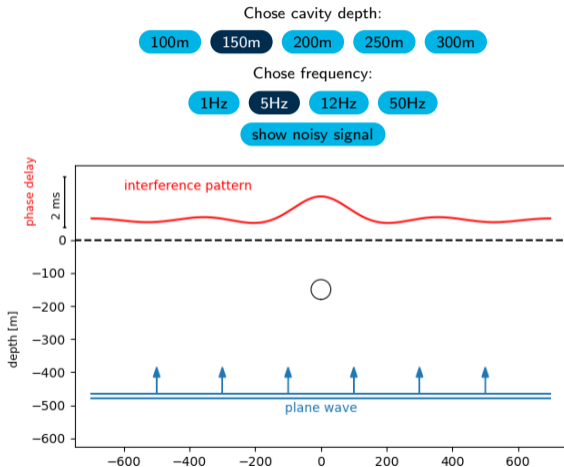


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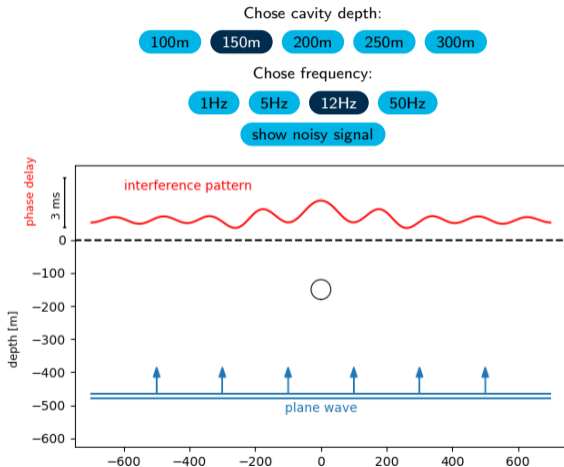


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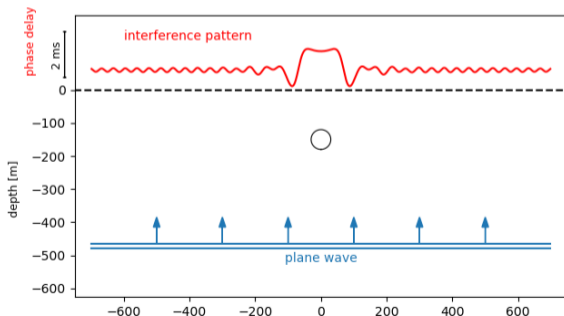
Chose cavity depth:

100m 150m 200m 250m 300m

Chose frequency:

1Hz 5Hz 12Hz 50Hz

show noisy signal

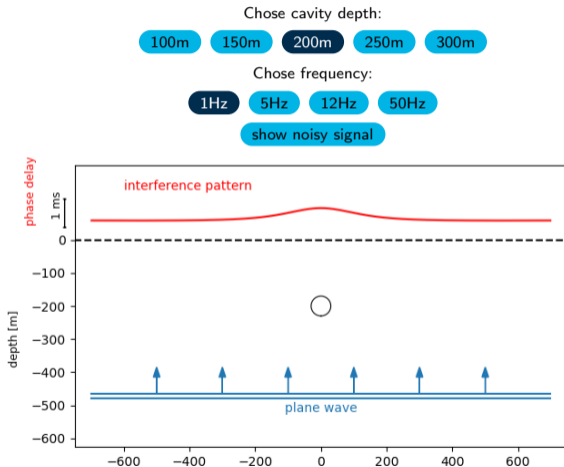


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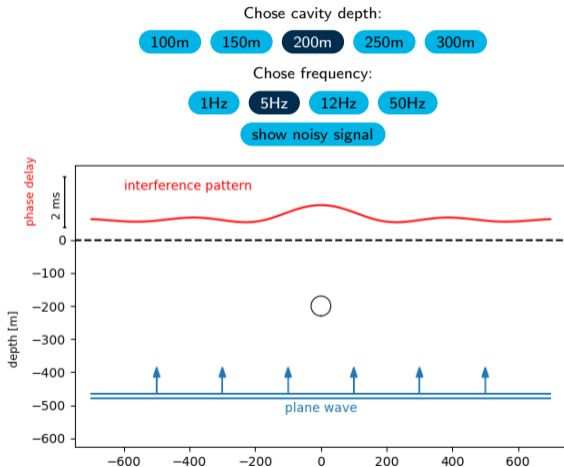


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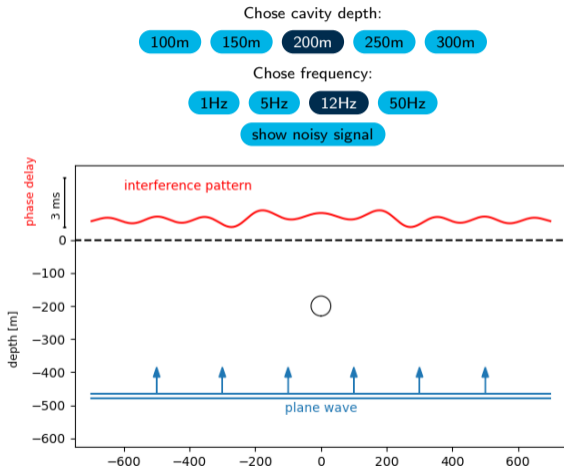


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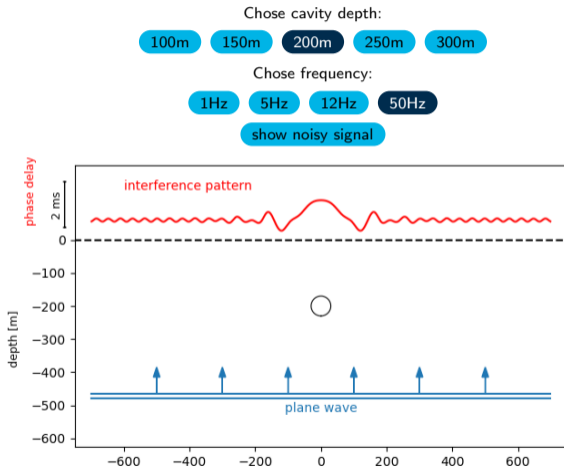


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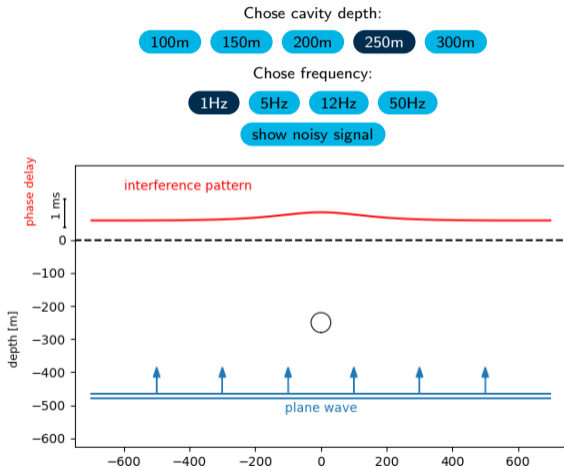


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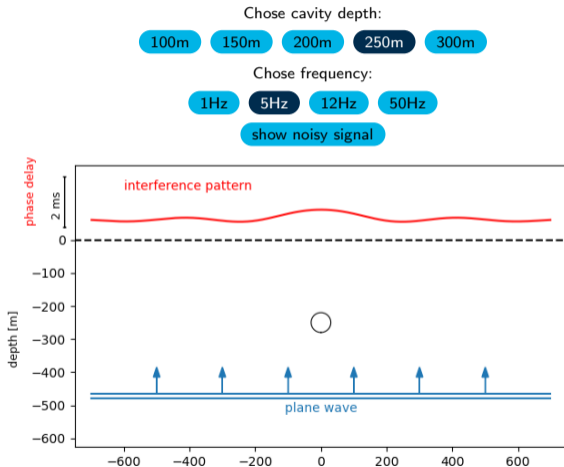


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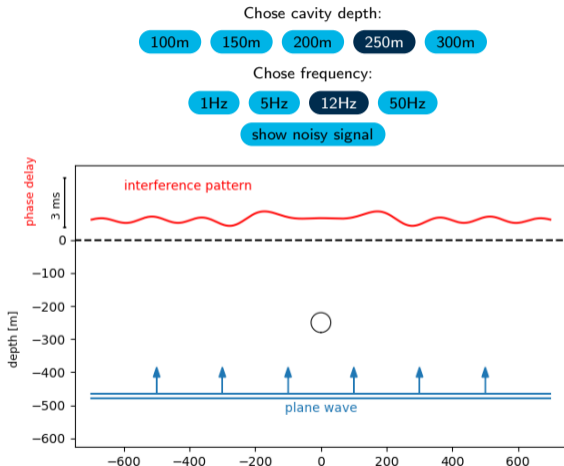


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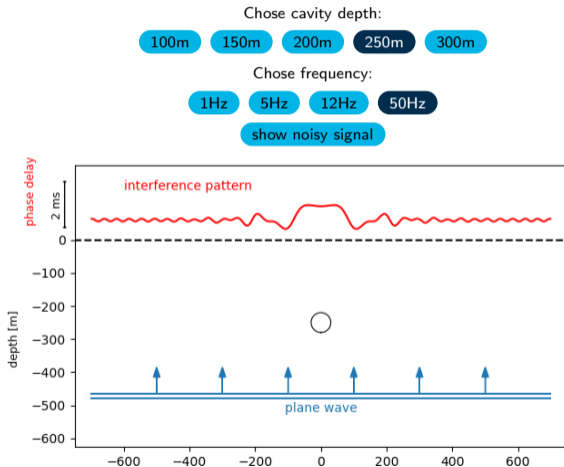


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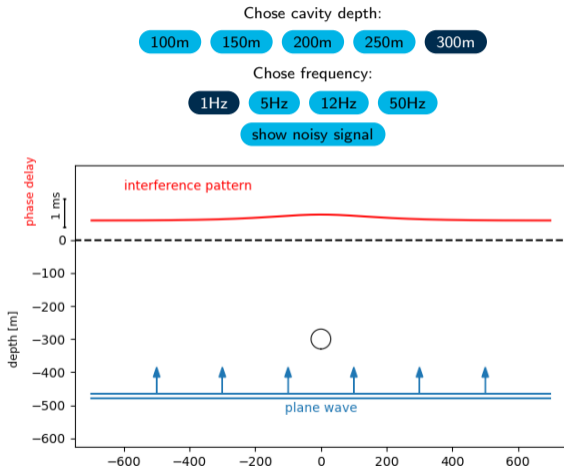


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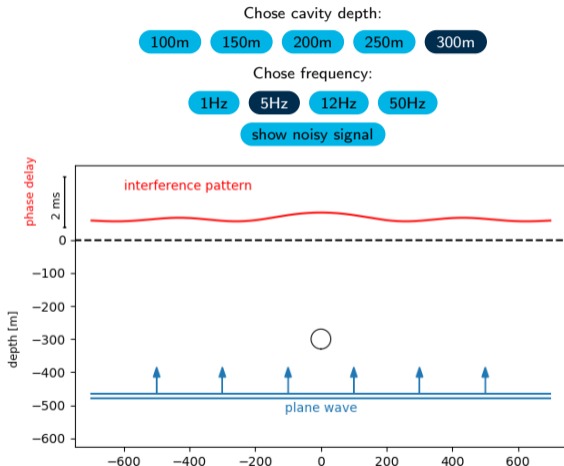


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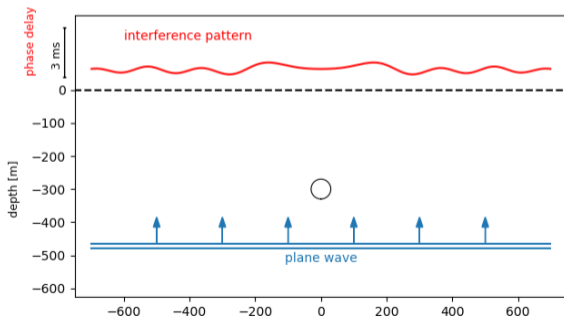
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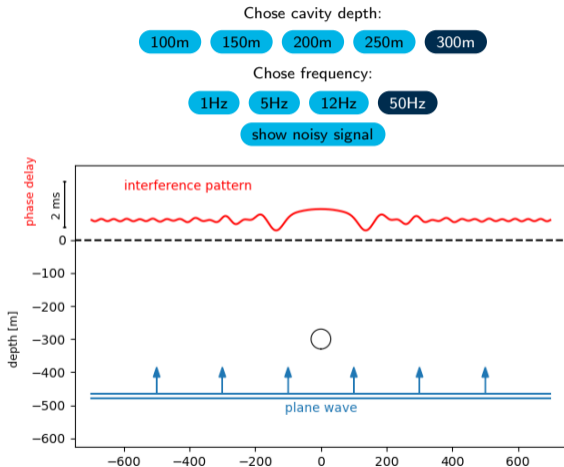


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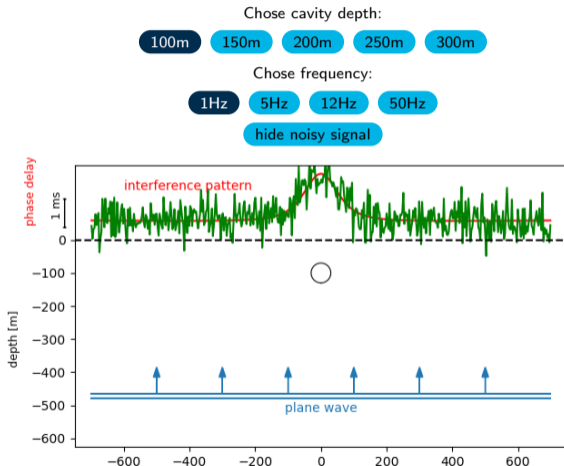


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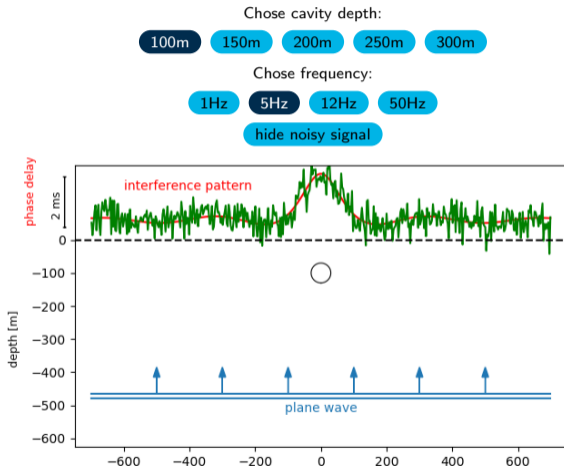


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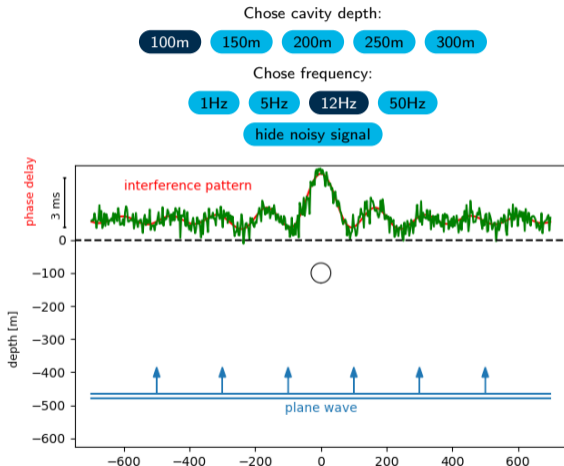


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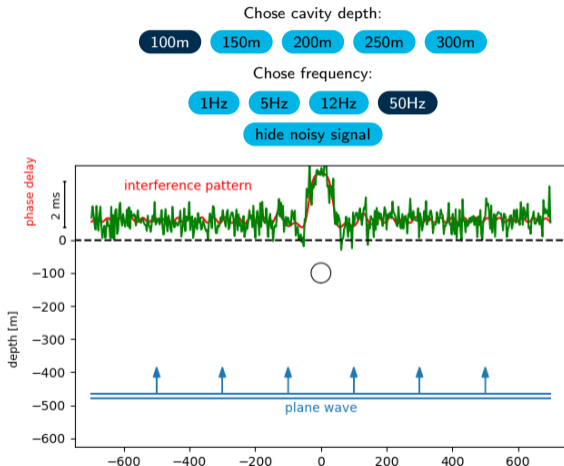


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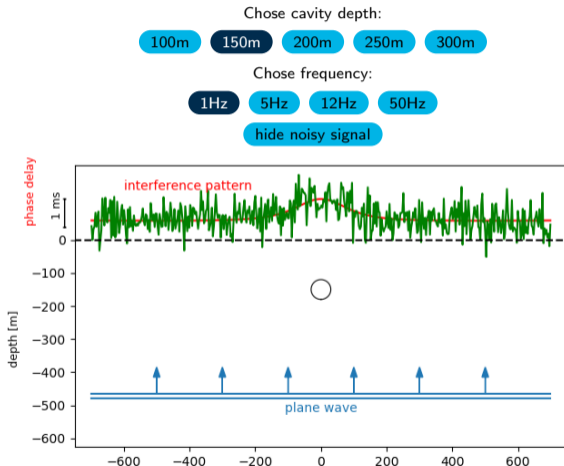


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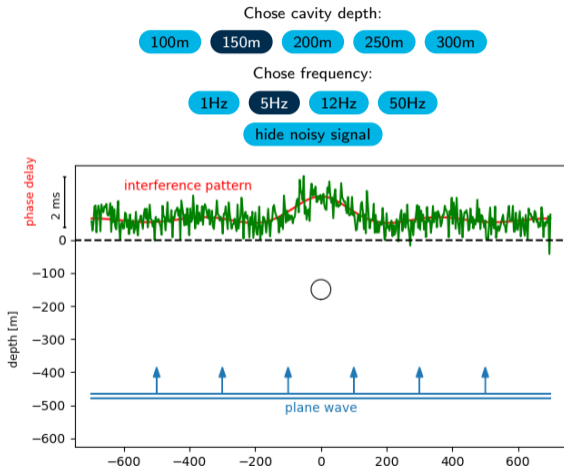


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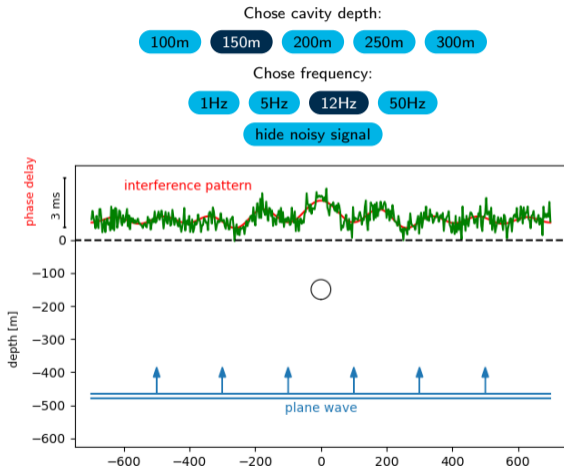


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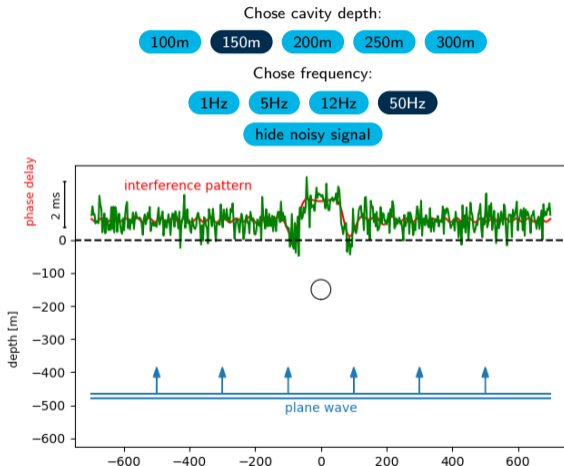


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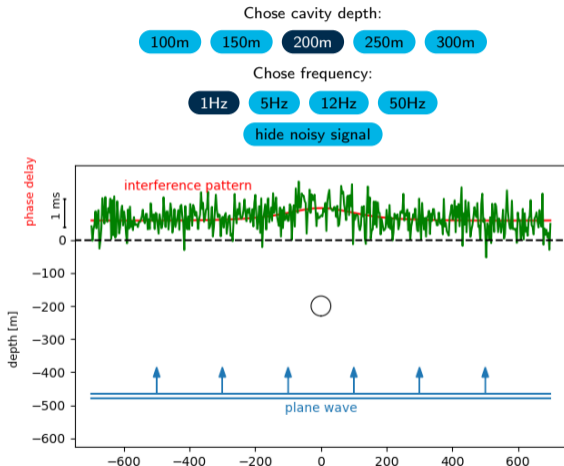


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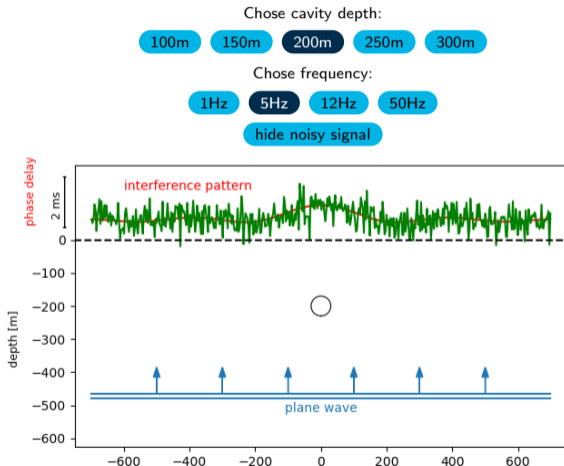


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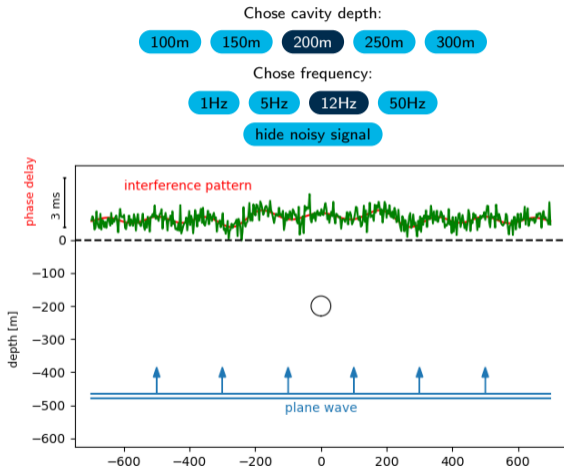


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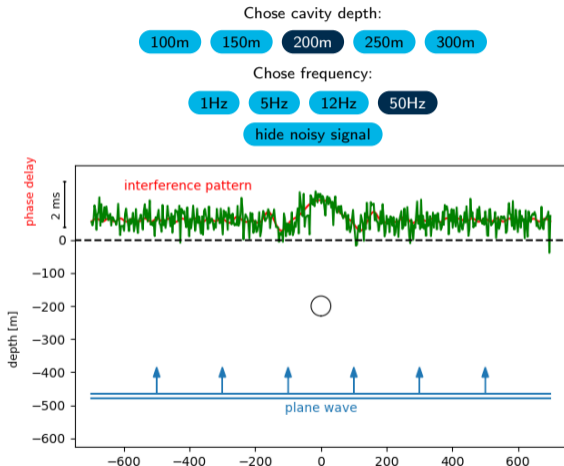


## Phase Delay Explorer

- ▶ The phase delay pattern observed at the surface depends on the depth of the cavity and the single dimensionless parameter  $\frac{R}{\lambda}$

[ $R$ : cavity radius,  $\lambda$ : wavelength]

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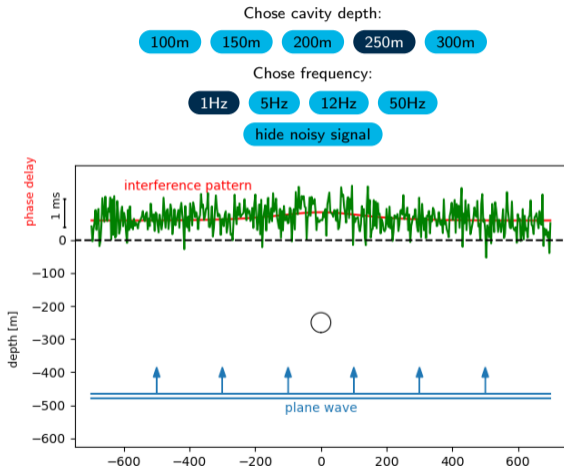


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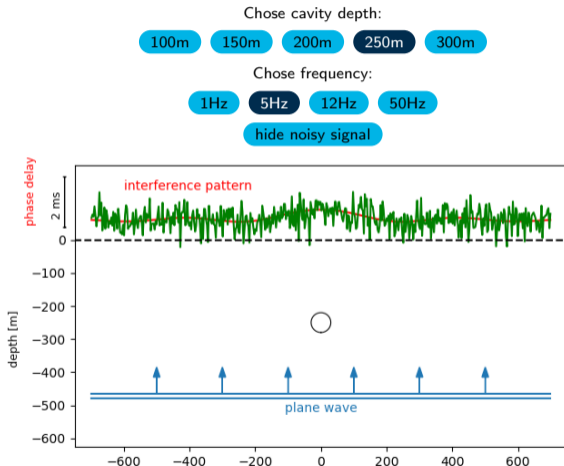


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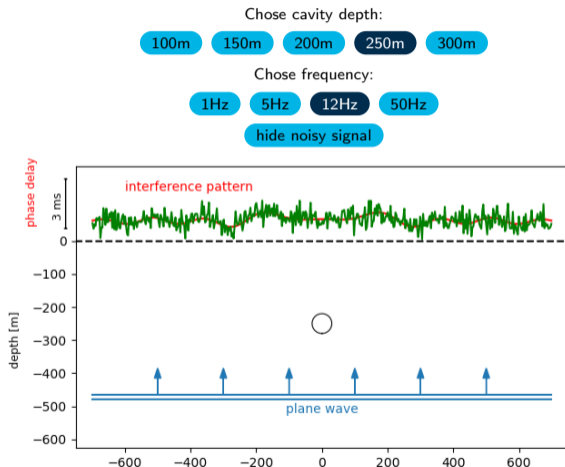


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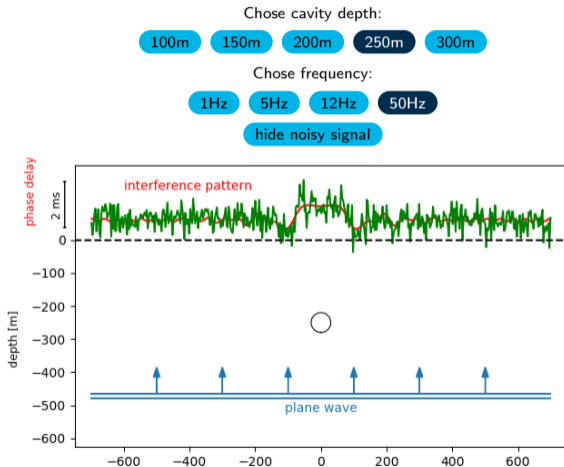


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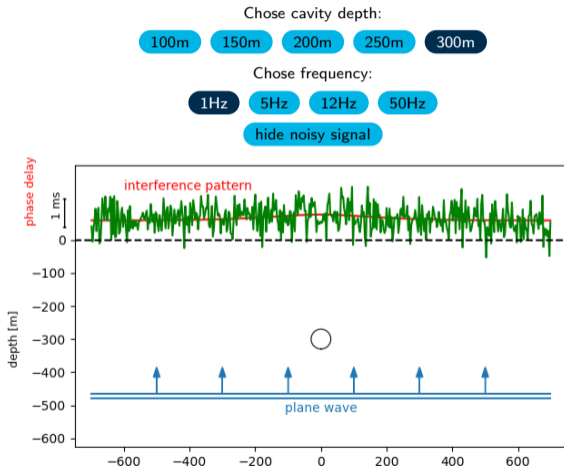


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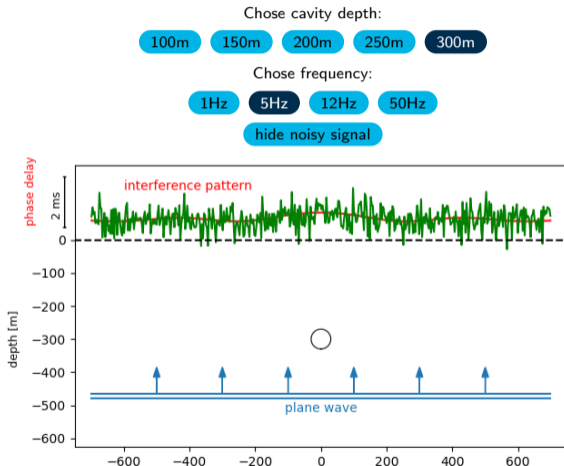


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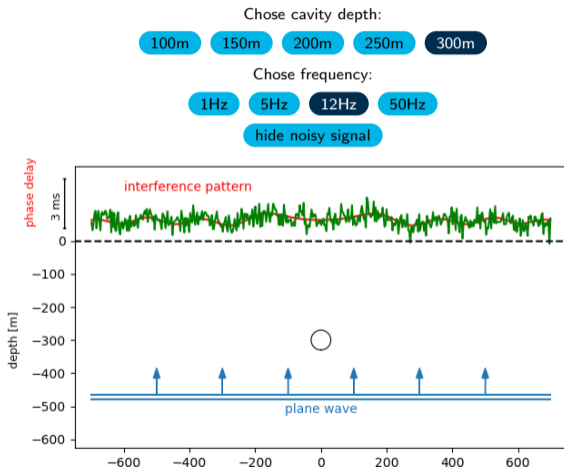


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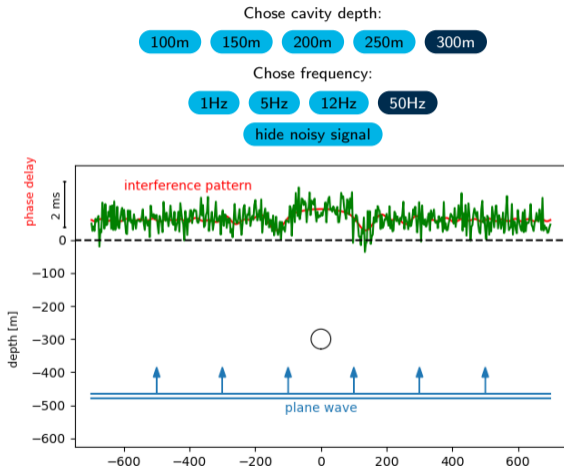


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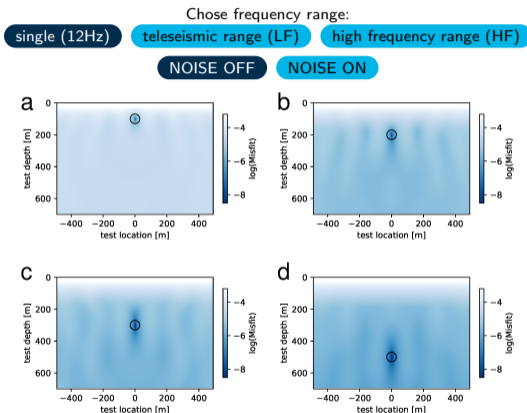
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## Finite Frequency Localization

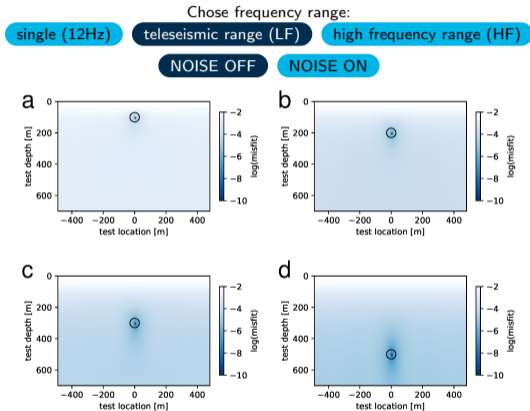
- ▶ Cavity localization is tested by taking into account the phase delay pattern according to the finite frequency nature of the wavefield.
- ▶ For each location on the grid a theoretical phase-delay curve is derived and compared to the phase delay curve corresponding to the cavity location (“observations”).
- ▶ 10% noise is added to the “observations” in order to mimic a realistic scenario.
- ▶ As misfit the root mean square error of theoretical and “observed” phase-delay curves is considered.



Inversion results for different cavity depths a single frequency (12 Hz).

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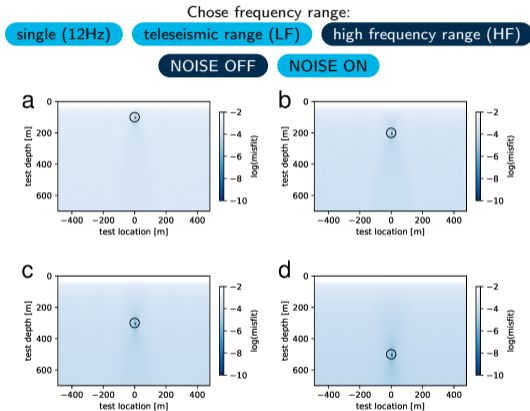
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Inversion results for different cavity depths the teleseismic frequency range (0.2 - 10 Hz).

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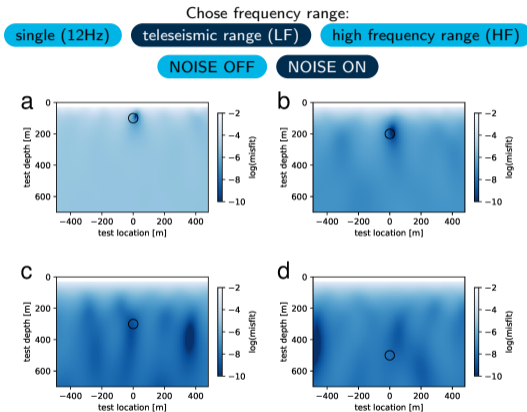
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Inversion results for different cavity depths the high frequency range (10 - 80 Hz).

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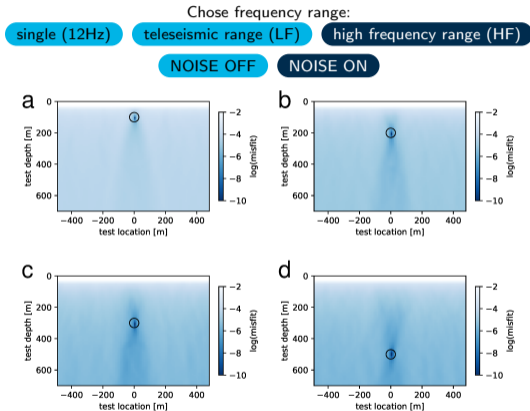
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## Detectability

- ▶ We tested for different geometries, if the finite frequency pattern is capable for cavity detection.
- ▶ A detectability parameter is defined in order to test the sharpness of the misfit function and the right location of its minimum

We define

$$\text{Detectability} = \frac{\int L(x, y) K(x, y)}{\int L(x, y)}, \text{ with}$$

$$K = \exp\left(\frac{(x-x_0)^2 + (z-z_0)^2}{80}\right)^2 \text{ and } L(x, y) = \exp(-10^6 * m(x, y)^2).$$

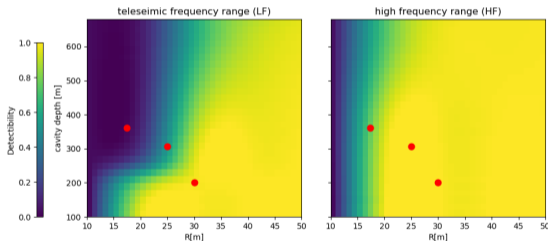
K: Kernel function centered at true location

L: Localization function

m: misfit

Geometries of Real Nuclear Cavities

Place	Radius (m)	Depth (m)
a) Balapan test site, Semipalatinsk	30 m	200 m
b) GNOME, New Mexico	17.4 m	361 m
c) Mackerel, Nevada	25 m	306 m



Detectability for teleseismic frequency range (left) and high frequency range (right). Red dots show radius and depth of real nuclear cavities (see table left).

## Conclusion

- ▶ Finite-frequency effects can be very beneficial for cavity detection and localization.
- ▶ The phase delay pattern of a cavity caused by interference provide a fingerprint for the cavity location and depth, which can be utilized also for relevant geometries of On Site Inspection.
- ▶ Detection of cavities with 30 m radius could be possible for cavities located down to 200 m depth using teleseismic sources.
- ▶ Higher frequency content of the source would be required for deeper cavities.
- ▶ This approach may be utilized by pattern recognition algorithms.